M1 4/2*

Non-Linear Stochastic Dynamics in the PRA of NPP Structural Components

F. Casciati, L. Faravelli

Università di Pavia, Dipartimento de Meccanica Strutturale, Via Luino 12, I-27100 Pavia, Italy

Abstract

Recent PRA studies made the analyst aware that the contribution of external events to the overall plant risk may not be insignificant. One of the basic elements of the analysis of risk from an external event is the evaluation of the fragility of structural components. Some external actions, as wind and earthquake,provide a dynamic excitation of structural components. In these cases, non-linear stochastic dynamics is individuated as the more appropriate framework for fragility analyses. In particular, the applicability of two quite--general stochastic dynamics procedures to PRA studies is discussed.

1. Introduction

In the design process of a nuclear power plant (NPP), each structural component is generally required to behave elastically during its lifetime. As a consequence the structural response is only investigated by linear analyses.

When a probabilistic risk assessment (PRA) has to be developed, however, the analyst has to investigate the behaviour of the single component also under load levels whose occurrence is very unlikely. Since to design structures which remain elastic under these unlikely action levels would be quite uneconomical, material and/or geometry non linearities have to be allowed for in the analysis.

In a PRA process the complexity of the non-linear mechanical model may become unbearable if associated with the complexity of the probabilistic calculations. Different probabilistic models with different level of sophistication have, therefore, been proposed /1//2//3/. The simplest approaches require the computation of a central value of the response and the availability of expert opinions for the associate uncertainty, while regression schemes (response surface methods) are generally adopted for more accurate analyses /4//5/. These regression schemes are introduced because no analytical expression of the dependence of the response on the basic uncertainty quantities is generally available and numerical investigations become therefore the only way for studying the propagation of uncertainty. This is mainly the case of dynamically loaded structural components.

After a brief review of the special response surface procedure proposed in Refs./5/ and /6/ this paper investigates the possibility of formulating an easier approach to the fragility analysis of dynamically excited components.

The approach is founded on the adoption of a suitable equivalent linearization technique, which provides explicitly the dependence of the response on the input parameters, as all linear stochastic dynamic method does /7/. A numerical example explains the main features of the resulting procedure of fragility analysis.

2. The problem and its governing relations

It is well known in structural reliability theory /7/ that the analyst can conduct his investigation in different spaces: the input space, the output space or a suitable state space. In the last case one separates the response analysis and the strength (or fragility) analysis of any structural component. Fragility, which is defined as the conditional frequency of failure for a given value of the response parameter S, can be therefore evaluated as the frequency for the actual carrying capacity R of the structure being less than S (SSMRP method /2/). Alternatively, fragility can be expressed as the conditional frequency of failure for a given load intensity value W (Zion method/2/): the check is here conducted in the input-space. The frequency of the event $W_R<W$ is evaluated, W_R being the global carrying capacity of the system. In both cases, Ref./2/ suggests for the structural component capacity R or W_R, say r, an expression of the type:

$$r = \check{r}\, V_{rr}\, V_{ru} \qquad (01)$$

where $\check{r}$ is the median capacity, V_{rr} is a random variable reflecting the inherent randomness and V_{ru} is a random variable reflecting the uncertainty in the median value. Under the assumption that both V_{rr} and V_{ru} are lognormally distributed with unit median, then, the fragility can be easily estimated from Eq.(01), once the logarithmic standard deviations of V_{rr} and V_{ru} are known. These standard deviations become therefore the unknowns to be determined by the analyst. Their values, however, were generally selected in previous PRA studies on the basis of analyst judgement rather than by performing accurate uncertainty propagation analyses. This approach can appear to be justified for V_{ru} which arises from the variability due to an insufficient understanding of structural material, errors in the calculated response and the use of engineering judgement. By contrast, the variability of V_{rr} can be obtained in a more rational way by using appropriate methods of propagating input uncertainties. These methods are summarized in Ref./2/, where they are presented "as techniques still being developed" so that the "reader is cautioned that ...there is no generally accepted approach".

In order to provide an improvement in such procedures, the authors and associates have formulated a generalized response surface technique which has been applied to the evaluation of seismic fragility /5//6/; however, this technique can be used whenever the external event causes a mechanical (static or dynamic) action on the system.

2.1 An improved response surface technique

Consider a structural component whose input-output relationship can be represented by

$$z = \mathbb{L}\,(x_1, x_2, \ldots, x_n) \qquad (02)$$

where x_i (i=1,...,n) are the input variables or a function of them (f.i. the logarithm), and z is an output variable. Let z denote the logarithm of the system carrying capacity W_r. $\mathbb{L}$ can either be a functional relationship or represent a long-running computer code. In the latter case, the variable z can be approximated by

$$z \cong g(\underline{x},\underline{\theta}) + \varepsilon = \bar{z} + \sum_{i=1}^{n} b_i \, \Delta x_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \, \Delta x_i \, \Delta x_j + \varepsilon \qquad (03)$$

where $\Delta x_i = x_i - \bar{x}_i$, $\bar{x}_i$ being a reference value for x_i. In Eq.(03) b_i and c_{ij} are sensitivity coefficients which can be determined by using a statistical design and applying a regression fitting procedure to the code outputs resulting from a set of code runs. The term ε is the lack of fit random term.

Critiques of the response-surface technique are that /2/:

i) Eq.(03) may represent a proper approximation of the variable z over a limited range of x-variables; moreover x_i cannot be a stochastic function;

ii) classical first-order formulae for the mean and the variance of z will yield a good approximation only in case where z behaves linearly or exhibits a weak non-linearity as a function of the x_i variables. Denote by $\underline{x}$ the vector of all uncertain quantities, each of which is either a random variable or a random function. In order to reduce the dimension of $\underline{x}$, $\underline{x}$ can be partitioned as

$$\underline{x}^T = \{\underline{x}_1^T \, , \, \underline{x}_2^T\} \qquad (04)$$

where $\underline{x}_1$ is a small vector of influential independent random variables and $\underline{x}_2$ is the remainder vector which contains large subvectors of $\underline{x}$ associated with distributed properties (local resistances, yielding stresses, etc.) as well as random functions as the stochastic external excitation. The random effects on z of its components can be separated by partitioning $\underline{x}_2$ into sub-vectors $\underline{x}_{21}, \underline{x}_{22}, \ldots, \underline{x}_{2n}$; a separate independent random term ε_j is then associated with each subvector $\underline{x}_{2j}$; and Eq.(03) is rewritten as:

$$z \cong g(\underline{x}_1, \underline{\theta}_1) + \sum \varepsilon_j + \varepsilon \qquad (05)$$

Fitting the model of Eq.(05) requires estimation of $\underline{\theta}_1$, ε_j and ε. In Refs./5/ and /6/ the variances of the random effects ε_j and ε (supposed to be normal) were obtained from a lattice-square experiment design followed by an analysis of variance. An augmented factorial design was introduced in order to estimate the fixed-effect coefficients $\underline{\theta}_1$ in Eq.(05): the results have been analysed by standard Bayesian regression with a non-informative flat prior on the parameters θ_{1i}.

Once the model (05) has been fitted, critique i) is by-passed. Point ii) has been removed in Ref./5/ and /6/ by using the so called Level II procedures of propagating uncertainty. For this purpose, the fragility $P(W)=\mathrm{Prob}[W_R<W]$ has been estimated as $\Phi[-\beta(z)]$ (Φ being the normal distribution function) where

$$\begin{cases} \beta(z) = \pm \min \left\{ (\underline{y} - \underline{\mu}_y)^T \, \underline{\Sigma}_y^{-1} \, (\underline{y} - \underline{\mu}_y) \right\}^{\frac{1}{2}} \\ \underline{y} \, : \, \ln W_R(\underline{y}) = z \end{cases} \qquad (06)$$

and $\underline{y}$ is the vector of all the random quantities in the r.h.s. of Eq.(05) ($\underline{x}_1$, $\underline{\theta}_1$, ε_j and ε), assumed to be distributed like a joint normal probability distribution with mean vector $\underline{\mu}_y$ and covariance matrix $\underline{\Sigma}_y$.

2.2 Fragility analysis by equivalent linearization

The response surface procedure summarized in Eq.s (05) and (06) is very fascinating because of its generality: there is no limitation to the number of components of $\underline{x}_1$, and of subvectors or random functions $\underline{x}_{2j}$; moreover any algorithm relating the input variables to the output parameters can be analysed. However, first analyses developed on simple structural components emphasized that two sources of uncertainty are predominant in fragility estimation: the exciting function and the local resistance vector. Since this local resistance vector can be regarded as a limit value to a suitable function of the maximum local inelastic deformation and the associated dissipated energy, the problem becomes to calculate the stochastic output of a deterministic non linear system subject to a stochastic excitation. In this field, an equivalent linearization technique associated with an endochronic model of the material constitutive law has been shown to be very promising. In particular, if one concentrates the inelastic deformations at critical sections, the dynamical problem can be written in the form /10//1/:

$$\begin{cases} \ddot{\underline{u}} + \zeta\, \underline{Q}_3\, \dot{\underline{u}} + \underline{Q}_3\, \underline{u} + \underline{Q}_4\, \underline{\phi} = -\underline{1}\psi(t) & \text{(a)} \\ \dot{\underline{\phi}} = (\underline{Q}_2 - \underline{D})^{-1}(\underline{E}\, \underline{Q}_1\, \underline{u} - \underline{Q}_1\, \dot{\underline{u}} + \underline{E}\, \underline{Q}_2\, \underline{\phi}) & \text{(b)} \\ \underline{Z} = \underline{Q}_1\, \underline{u} + \underline{Q}_2\, \underline{\phi} & \text{(c)} \end{cases} \qquad (07)$$

where $\underline{u}$ is the vector of the displacement associated with inertial forces; $\underline{\phi}$ is the vector of the inelastic deformation; $\underline{Z}$ is the vector associated with $\underline{\phi}$ in the endochronic model; $\underline{Q}_1$, $\underline{Q}_2$, $\underline{Q}_3$ and $\underline{Q}_4$ are suitable structural matrices and $\underline{E}$ and $\underline{D}$ are diagonal matrices whose entries are the linearization coefficients which are known functions of some entries of the covariance matrix $\underline{\Sigma}_{\dot{\phi},Z}$, (with $\dot{\phi}_i$ and Z_i jointly Gaussian). This matrix is related by a linear relation to the covariance matrix $\underline{\Sigma}=\Sigma_{\underline{u},\dot{\underline{u}},\underline{\phi}}$ which can be obtained solving the Liapunov equation associated with Eq.s (07) a) and b) when the constitutive law is non-degrading and the dynamic excitation $\psi(t)$ is a Gaussian white-noise. The r.h.s. of the Liapunov equation contains the constant power spectral density S_o of $\psi(t)$. An iterative solution of the Liapunov equation is required, because the relevant matrix of the coefficients contains the linearization terms which are function of the response variable statistics. Extensions to the non-stationary case are summarized in Ref./9/.

Once the statistics of $\underline{\phi}$ and $\dot{\underline{\phi}}$ have been derived, classical expressions permit one to obtain the distribution of the maximum value $\underline{\phi}_m(t)$ of $\underline{\phi}$ in a period (0,t) /10//1//9//12/:

$$P_{\phi_{m_i}(t)}(\phi,t) \cong \exp\{-\exp[-K(\frac{\phi}{s_\phi} - K)]\} = Q_i(\phi) \qquad (08)$$

with

$$K = (2\ln(t\, 2\, s_{\dot{\phi}}/2\pi s_\phi))^{\frac{1}{2}} \qquad (09)$$

Conversely, while the mean values $\underline{E}$ of the dissipated energy in the inelastic regions is provided directly by the procedure ($\propto \Sigma_{\dot{\phi}Z}$), an estimation of the relevant variance would require cumbersome calculations /11/.

According to Ref./10/, then, at the present time, an estimation of the local probability of failure in (0,t) can only be obtained by using a simplified failure criterion which compares the maximum inelastic rotation with a limit value depending on the mean value of the dissipated energy. Let R_i denote the limit value to the load effect $S_i(\phi_{mi},E_i)$ in the i-th inelastic region. The entries of the vector $\underline{R}$ can be assumed to be either uncorrelated or equicorrelated. In the first case, the global reliability $\mathbb{R}$ of the system can be calculated by classical system reliability formulae (10), once $\phi_i^* = S_i^{-1}(R_i,E_i)$ has been calculated:

$$\mathbb{R} = \prod_{i=1}^{n} \left(\int_{0}^{+\infty} Q_i(\phi^*)\, p_{R_i}(R)\, dR \right) \tag{10}$$

where n is the number of inelastic regions and $p_{R_i}(R)$ is the probability density function of the local strength in the i-th region.

In the equi-correlated case, assume that the R_i are jontly lognormally distributed with the same mean μ_{R_i} and variance $s^2_{R_i}$ and with coefficient of correlation ρ. Then extending the idea of Ref./13/, one can regard each R_i as the product of R° (constant along the structure) by R_i', where R_o and R_i' are two independent lognormally distributed random variables:the first has unit mean value and logarithmic standard deviation $\sqrt{\rho}\, s_{\ln R_i}$; R_i' has mean value equal to the one of $R_i(\mu_{R_i})$ and logarithmic standard deviation $\sqrt{1-\rho}\, s_{\ln R_i}$. In this case the global reliability of the frame is given by

$$\mathbb{R} = \int_{0}^{+\infty} d\,\xi\, p_{\ln R^\circ}(\xi) \prod_{i=1}^{n} \left(\int_{0}^{+\infty} Q_i(\phi_i^*)\, p_{\ln R_i | R^\circ}(\ln R|\xi)\, d(\ln R) \right) \tag{11}$$

where $p_{\ln R_i | R^\circ}$ is the probability density function of $\ln R_i$, given that $\ln R_o = \xi$, i.e. the normal PDF with mean $(\ln \mu_{R_i} + \xi)$ and standard deviation $\sqrt{1-\rho}\, s_{\ln R_i}$.

3. Numerical example

The reinforced concrete frame already analysed by the authors and their associates in Refs. /5//6//10//13/ is considered. The reader is referred for details to Ref./1/. Here Fig.1 shows in solid line the seismic fragility curve obtained in Ref./6/ where six random variables are considered to form the vector $\underline{x}_1$ (mass, stiffness, damping, hardening, yielding stress factor constant along the structure and R°) and three subvector $\underline{x}_{2j}$ (yielding stresses, $\underline{R}'$ and the ground acceleration a(t))are introduced. In the same figure the dashed line has been obtained by means of a more accurate investigation on the effects of the more significant terms of $\underline{x}_1$ (yielding and R°). The small discrepancy points out that the use of canonic experiment designs reducing the error in estimating the model parameters ($\underline{\theta}_1$ in Eq.(05)) is not completely satisfactory in estimating the distribution of z. In particular this numerical example suggests to perform more experiments along the axes of the variables with which more significant values of the parameters θ_1 are associated.

The three points A, B and C of Fig.1 are the results obtained by using Eq.s (07) and (11) for three different values of the maximum peak acceleration. For this purpose one has assumed /9/ (a in inch/sec·sec; S_o in inch·inch/sec·sec·sec)

$$a_{max} = 29\sqrt{S_o}$$

where S_o is the constant power spectral density of the stationary white noise. The earthquake has been therefore modelled as a segment of Kanai filtered white-noise with ω=15.6 rad/sec and ζ=0.69. The duration of the earthquake has been assumed equal to the mean value (9.2 sec.) of the stationary equivalent duration introduced in Ref./6/, where d,ω and ζ were assumed to be independent random variables. The results of the equivalent linearization approach are in good agreement with the ones achieved in Ref./6/, despite that:

1) the Kanai Tajimi parameters and the duration of the ground motion are assumed to be derterministic;

2) the non-stationary character of the motion has been neglected;

3) the system parameters, except the local resistances, are assumed to be deterministic;

4) the static loads have not been considered explicitly;

5) the different reinforcement for positive and negative yielding moment cannot be taken into account (equivalent linearization computations used the lower value for both the yielding limits).

Aknowledgement. This research was supported by grants from the Italian Research Council (CNR) and the Ministry of Public Instruction (MPI).

References

/ 1/ CASCIATI, F., FARAVELLI, L.,"Methods of Non-Linear Stochastic Dynamics for the Assessment of Structural Fragility", submitted for publication, 1985

/ 2/ -, "PRA Procedures Guide", NUREG/CR-2300, 2 Vol., 1983

/ 3/ RAVINDRA, M.K., et.at.,"Sensitivity Studies of Seismic Risk Models", EPRI, NP3562, RP2170-5, 1984

/ 4/ BOHN, M.P., et.al., "Application of the SSMRP Methodology to the Seismic Probabilistic Risk Analysis at the Zion Nuclear Power Plant", Proc. 7th SMiRT, M5/7, Chicago, 1983

/ 5/ VENEZIANO, D., CASCIATI, F., FARAVELLI, L., "Method of Seismic Fragility for Complicated Systems", Proc. 2nd CSNI Spec. Meet. on Prob.Meth. in Seismic Risk Asses. for NPP, Livermore, CSNI Report 76, 1983, 67-88

/ 6/ CASCIATI, F., FARAVELLI, L., VENEZIANO, D.,"Response Surface Method of Seismic Fragility" submitted for publication, 1985

/ 7/ AUGUSTI, G., BARATTA, A., CASCIATI, F., "Probabilistic Methods in Structural Engineering" London: Chapman & Hall, 1984

/ 8/ BABER, T.T., WEN, Y.K., "Stochastic Response of Multistorey Yielding Frames", Earth. Eng. & Struct.Dyn., 10, 1982, 403-416

/ 9/ SUES, R.H. et al., "Stochastic Seismic Performance of Buildings", UILU-ENG-83-2008, SRS 506, 1983

/10/ CASCIATI, F., FARAVELLI, L., "Reliability Assessment for Non-Linear Random Frames", to be published in the Proc. of the Weibull IUTAM Symposium, Stockholm, 1984

/11/ PIRES, J. et al. "Stochastic Analysis of Seismic Safety Against Liquefaction", UILU--ENG-83-2005, SRS504,1983

/12/ YANG, J.N., LIU, S.C., "Distribution of Maximum and Statistical Response Spectra", Journal of Eng.Mech., ASCE, 107, 6, 1981, 1089-1102

/13/ VENEZIANO, D., "Probabilistic Seismic Resistance of R.C. Frames", Proc. 3rd ICOSSAR, Trondheim, 1981, 241-258.

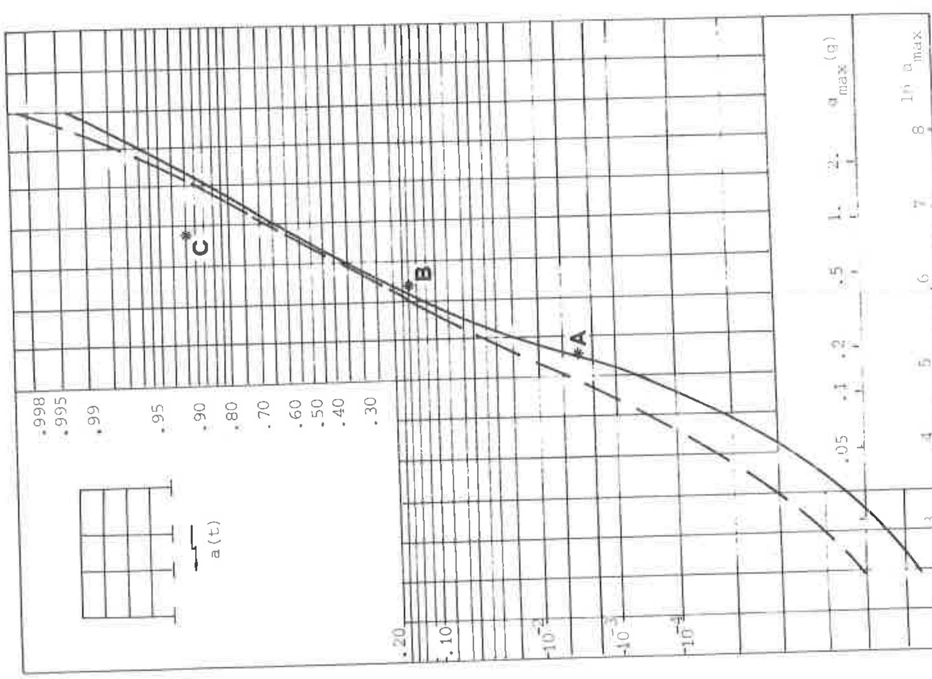

Fig.1 - Seismic fragility plots