ELASTO/VISCO-PLASTIC DEFORMATIONS OF THIN SHELLS OF REVOLUTION

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This paper is concerned with the numerical analysis of large elasto/visco-plastic deformations of thin shells of revolution under axi-symmetrical loading with applications to pressure vessels.

Mechanical behaviour of machinery and structures subjected to loads over an elastic limit has been almost analyzed by the conventional elasto-plastic theory which time does not enter directly into consideration, in spite of the fact that the analytical results from the conventional plastic theory are at variance with experimental ones.

In this paper an analytical method for the problem of shells of revolution widely used under high stress is studied. The method is based on the elasto/visco-plastic theory which viscosity of the material is taken into account in the plastic range.

Many investigations on the analytical method for plastic deformation or creep deformation of shell structures have been carried out for not only simple geometries such as cylindrical shells and spherical shells but also general axi-symmetrical shells, because it is closely related to the analysis and the design of pressure vessels and pressure vessel components. The investigations on elasto-plastic deformations of shell structure, however, are almost all limited to ones based on the conventional plastic theory.

The authors employ as constitutive relation for the shell materials Perzyna's equation which in the elastic range is equal to Hooke's law and in the plastic range considers the viscosity of the material. The method is based on the assumption of plane stress condition and the Kirchhoff-Love hypothesis used in the ordinary thin shell theory, and takes account of geometrical nonlinearity because deformation of thin shells under high stress exceeding elastic limit usually becomes considerably large. The criterion for yielding used in this analysis is the von Mises yield theory. The basic differential equations derived for large elasto/visco-plastic deformation of thin shells of revolution are numerically solved by a finite difference method, and the solutions are obtained by integration of the incremental values.

As a numerical example, the problem of elasto/visco-plastic deformations of pressure vessels composed of a spherical shell, a toroidal shell and a cylindrical shell important in practical use is treated, and the variations of the deformations and the internal forces with time after application of a load are analyzed. The elasto/visco-plastic solutions from the prediction method agree fairly well with experimentally determined values for pressure vessel heads by Findlay and others. The method can be used to generate plasticity solutions in a simple manner when stationary conditions are reached.

1 Introduction

It may be often observed in experiments on pressure vessels, piping etc. that influence of viscosity appears in metals in a plastic range at room temperature. The influence of viscosity in a plastic range becomes remarkable under high stress. However, mechanical behaviour of machinery and structures subjected to loads over an elastic limit has been almost analyzed by the conventional elasto-plastic theory which time does not enter directly into consideration.

In this paper an analytical method for the problem of shells of revolution widely used under high stress is studied. The method is based on the elasto/visco-plastic theory where viscosity of the material is taken into account in a plastic range.

There are many investigations on the elasto-plastic analysis [1] [2], or the creep analysis [3]–[8] of shells of revolution, because it is closely related to the design of pressure vessels and pressure vessel components. The investigations on elasto-plastic deformation of shell structures, however, are almost all limited to ones based on the conventional plastic theory.

The authors employ as constitutive relation of the shell materials Perzyna's equation [9] where in the plastic range the viscosity of the material is considered. The criterion for yielding used in this analysis is the von Mises yield theory. The method is based on the assumption of plane stress condition and the Kirchhoff-Love hypothesis used in the ordinary thin shell theory, and the geometrical nonlinearity is taken into account because deformation of thin shells under high stress exceeding elastic limit usually becomes considerably large.

The basic differential equations derived for elasto/visco-plastic deformation of thin shells of revolution are numerically solved by a finite difference method, and the solutions are obtained by integration of the incremental values.

As a numerical example, the problem of elasto/visco-plastic deformation of pressure vessels important in practical use is treated, and the solutions from the prediction method are compared with the experimentally determined values for pressure vessel heads by Findlay and others [10].

2 Fundamental Equations

The parametric expressions of a middle surface of shells of revolution may be written as:

$$r=r(\xi)\ ,\ z=z(\xi) \qquad \cdots (1)$$

where ξ is the meridional length along the middle surface from a boundary, and r and z are coordinates of the middle surface, as shown in Fig. 1. The relations between the coordinates r, z and the angle φ in Fig. 1 are as follows:

$$r'=\cos(\Phi-\vartheta)\ ,\ z'=\sin(\Phi-\vartheta)\ ,\ (r')^2+(z')^2=1 \qquad \cdots (2)$$

where

$$\Phi=\varphi+\vartheta\ ,\quad '=d/d\xi$$

and ϑ is a change of the angle φ due to deformation.

The relations among internal forces per unit length, N_ξ, Q, H and V are:

$$N_\xi=V\sin\Phi+H\cos\Phi\ ,\quad Q=V\cos\Phi-H\sin\Phi \qquad \cdots (3)$$

The equations of equilibrium are given by:

$$(rV)'+rp_V=0\ ,\ (rH)'-N_\theta+rp_H=0\ ,\ (rM_\xi)'-M_\theta\cos\Phi-rQ=0 \qquad \cdots (4)$$

where M_ξ and M_θ are meridional and circumferential bending moments per unit length, respectively, and p_H and p_V are components of a distributed load per unit area of the middle surface, as shown in Fig. 1.

Denoting a radial component of a displacement by u and a change of the angle φ by ϑ the strains and the changes of curvature of the middle surface are expressed by:

$$\varepsilon_{\xi m}=\frac{u'+\cos\varphi}{\cos\Phi}-1\ ,\ \varepsilon_{\theta m}=\frac{u}{r}\ ,\ \kappa_\xi=\vartheta'\ ,\ \kappa_\theta=\frac{\sin\Phi-\sin\varphi}{r} \qquad \cdots (5)$$

Equations (3)–(5) may be described in rate forms as follows:

$$\left.\begin{aligned} & r\dot{N}_\xi = \dot{\psi}_V \sin\Phi + \dot{\psi}_H \cos\Phi + (\psi_V \cos\Phi - \psi_H \sin\Phi)\dot{\vartheta} \;,\; r\dot{Q} = \dot{\psi}_V \cos\Phi - \dot{\psi}_H \sin\Phi - (\psi_V \sin\Phi + \psi_H \cos\Phi)\dot{\vartheta} \\ & \dot{\psi}_V' + r\dot{p}_V = 0 \;,\; \dot{\psi}_H' - \dot{N}_\theta + r\dot{p}_H = 0 \;,\; (r\dot{M}_\xi)' - \dot{M}_\theta \cos\Phi - r\dot{Q} + \dot{\vartheta} M_\theta \sin\Phi = 0 \\ & \dot{\varepsilon}_{\xi m} = \frac{\dot{u}' + \dot{\vartheta}\sin\Phi + \varepsilon_{\xi m}\dot{\vartheta}\sin\Phi}{\cos\Phi} \;,\; \dot{\varepsilon}_{\theta m} = \frac{\dot{u}}{r} \;,\; \dot{\kappa}_\xi = \dot{\vartheta}' \;,\; \dot{\kappa}_\theta = \frac{\dot{\vartheta}\cos\Phi}{r} \end{aligned}\right\} \quad (6)$$

where

$$\dot{\Phi} = \dot{\vartheta} \;,\; \dot{\psi}_H = r\dot{H} \;,\; \dot{\psi}_V = r\dot{V} \quad \cdots\cdots (7)$$

and the dot denotes a differentiation with respect to time or load.

The strain rate components at a distance η from the middle surface, $\dot{\varepsilon}_\varphi$ and $\dot{\varepsilon}_\theta$ are given as follows by Kirchhoff-Love hypothesis.

$$\dot{\varepsilon}_\xi = \dot{\varepsilon}_{\xi m} - \eta\dot{\kappa}_\varphi \;,\; \dot{\varepsilon}_\theta = \dot{\varepsilon}_{\theta m} - \eta\dot{\kappa}_\theta \quad \cdots\cdots (8)$$

Now, we shall use the elasto/visco-plastic equation by Perzyna [12] for constitutive relations. The Perzyna equation is:

$$\dot{\varepsilon}_{ij} = \frac{1+\nu}{E}\dot{s}_{ij} + \frac{1-2\nu}{E}\dot{S}\delta_{ij} + \gamma\langle\Phi(F)\rangle s_{ij} J_2^{-\frac{1}{2}} \quad \cdots\cdots (9)$$

where the dot denotes the partial differentiation with respect to time, s_{ij}, S and ε_{ij} are a deviatoric stress, a mean stress and a strain, respectively, and E , ν and γ are Young's modulus, Poisson's ratio and a viscosity constant of the material. J_2 is the second invariant of the deviatoric stress tensor. The notation $\langle\Phi(F)\rangle$ is defined as:

$$\langle\Phi(F)\rangle = 0 \quad \text{when } F \leq 0 \;,\; \langle\Phi(F)\rangle = \Phi(F) \quad \text{when } F > 0 \quad \cdots\cdots (10)$$

where function F is:

$$F = \frac{\bar{\sigma} - Y}{Y} \;,\; Y = Y(\bar{\varepsilon}^{vp}) \quad \cdots\cdots (11)$$

In eqs. (11) $\bar{\sigma}$ denotes the von Mises equivalent stress, $\bar{\varepsilon}^{vp}$ is an equivalent visco-plastic strain, and is obtained from the uni-axial tensile test when a strain rate is enough small. Also $F=0$ becomes the von Mises yield surface.

In the plane stress state ordinally assumed in the shell theories the constitutive relation eq. (9) may be written as follows:

$$\begin{Bmatrix}\dot{\varepsilon}_\xi \\ \dot{\varepsilon}_\theta\end{Bmatrix} = \frac{1}{E}\begin{bmatrix}1 & -\nu \\ -\nu & 1\end{bmatrix}\begin{Bmatrix}\dot{\sigma}_\xi \\ \dot{\sigma}_\theta\end{Bmatrix} + \frac{2}{\sqrt{3}}\gamma\left\langle\Phi\left(\frac{\bar{\sigma}-Y}{Y}\right)\right\rangle\frac{1}{\bar{\sigma}}\begin{bmatrix}1 & -\frac{1}{2} \\ -\frac{1}{2} & 1\end{bmatrix}\begin{Bmatrix}\sigma_\xi \\ \sigma_\theta\end{Bmatrix} \quad \cdots\cdots (12)$$

A non-dimensional time $\tau = 2\gamma t/\sqrt{3}$ introduced, eq. (12) can be rewritten as the following equation:

$$\begin{Bmatrix}\dot{\varepsilon}_\xi \\ \dot{\varepsilon}_\theta\end{Bmatrix} = \frac{1}{E}\begin{bmatrix}1 & -\nu \\ -\nu & 1\end{bmatrix}\begin{Bmatrix}\dot{\sigma}_\xi \\ \dot{\sigma}_\theta\end{Bmatrix} + \left\langle\Phi\left(\frac{\bar{\sigma}-Y}{Y}\right)\right\rangle\frac{1}{\bar{\sigma}}\begin{bmatrix}1 & -\frac{1}{2} \\ -\frac{1}{2} & 1\end{bmatrix}\begin{Bmatrix}\sigma_\xi \\ \sigma_\theta\end{Bmatrix} \quad \cdots\cdots (13)$$

where $(\dot{\ }) = d/d\tau$. The non-dimensional time τ will be used in the further equations.

Now, substitution of eq. (13) into eqs. (8) yields

$$\begin{Bmatrix}\dot{\sigma}_\xi \\ \dot{\sigma}_\theta\end{Bmatrix} = \frac{Eh}{1-\nu^2}\begin{bmatrix}1 & \nu \\ \nu & 1\end{bmatrix}\left[\begin{Bmatrix}\dot{\varepsilon}_{\xi m} \\ \dot{\varepsilon}_{\theta m}\end{Bmatrix} - \eta\begin{Bmatrix}\dot{\kappa}_\xi \\ \dot{\kappa}_\theta\end{Bmatrix}\right] - \begin{Bmatrix}\dot{\sigma}_\xi^{vp} \\ \dot{\sigma}_\theta^{vp}\end{Bmatrix} \quad \cdots\cdots (14)$$

where,

$$\begin{Bmatrix}\dot{\sigma}_\xi^{vp} \\ \dot{\sigma}_\theta^{vp}\end{Bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu \\ \nu & 1\end{bmatrix}\begin{Bmatrix}\dot{\varepsilon}_\xi^{vp} \\ \dot{\varepsilon}_\theta^{vp}\end{Bmatrix} \;,\; \begin{Bmatrix}\dot{\varepsilon}_\xi^{vp} \\ \dot{\varepsilon}_\theta^{vp}\end{Bmatrix} = \left\langle\Phi\left(\frac{\bar{\sigma}-Y}{Y}\right)\right\rangle\frac{1}{\bar{\sigma}}\begin{bmatrix}1 & -\frac{1}{2} \\ -\frac{1}{2} & 1\end{bmatrix}\begin{Bmatrix}\sigma_\xi \\ \sigma_\theta\end{Bmatrix} \quad \cdots\cdots (15)$$

The internal forces per unit length of the shell are related to the stresses by the next equations:

$$\{\dot{N}_\xi, \dot{N}_\theta, \dot{M}_\xi, \dot{M}_\theta\} = \int_{-h/2}^{h/2}\{\dot{\sigma}_\xi, \dot{\sigma}_\theta, \dot{\sigma}_\xi\eta, \dot{\sigma}_\theta\eta\}\,d\eta \quad \cdots\cdots (16)$$

From eqs. (6) – (16), the following simultaneous differential equations of second orders with respect to $\dot{\vartheta}$ and ψ_H may be derived:

$$\left.\begin{aligned} & \dot{\vartheta}'' + \frac{(Dr)'}{Dr}\dot{\vartheta}' - \left\{\left(\frac{\cos\Phi}{r}\right)^2 - \nu\frac{(D\cos\Phi)'}{Dr} + \frac{1}{Dr}(\psi_V \sin\Phi + \psi_H \cos\Phi) + \frac{\sin\Phi}{Dr}M_\theta\right\}\dot{\vartheta} - \frac{\sin\Phi}{Dr}\dot{\psi}_H \\ & \qquad = -\frac{\cos\Phi}{Dr}\dot{\psi}_V + \frac{1}{Dr}\left\{\cos\Phi\,\dot{M}_\theta^{vp} - (r\dot{M}_\xi^{vp})'\right\} \end{aligned}\right\} \quad \cdots\cdots (17)$$

$$
\left.
\begin{aligned}
&\dot{\psi}_H'' + \frac{(r/c)'}{r/c}\dot{\psi}_H' - \left\{\left(\frac{\cos\Phi}{r}\right)^2 + \nu\frac{(\cos\Phi/c)'}{r/c}\right\}\dot{\psi}_H - \nu\left\{\frac{\cos\Phi}{r}\psi_V - \frac{\sin\Phi}{r}\psi_H\right\}\dot{\vartheta}' + \left\{-\nu\frac{(\cos\Phi/c)'}{r/c}\psi_V\right.\\
&\left. -\nu\frac{\cos\Phi}{r}\psi_V' + \nu\frac{(\sin\Phi/c)'}{r/c}\psi_H + \nu\frac{\sin\Phi}{r}\psi_H' - \left(\frac{\cos\Phi}{r}\right)^2\psi_V + \frac{\sin\Phi\cos\Phi}{r^2}\psi_H + \frac{\sin\Phi}{r/c}(1+\varepsilon_{\xi m})\right\}\dot{\vartheta}\\
&\qquad = \left\{\frac{\sin\Phi\cos\Phi}{r^2} + \nu\frac{(\sin\Phi/c)'}{r/c}\right\}\dot{\psi}_V + \nu\frac{\sin\Phi}{r}\dot{\psi}_V' - \left\{\frac{(r/c)'}{r/c} + \nu\frac{\cos\Phi}{r}\right\}r\dot{p}_H\\
&\qquad -(r\dot{p}_H)' + \left\{\frac{\cos\Phi}{r} + \nu\frac{(r/c)'}{r/c}\right\}\dot{N}_\xi^{VP} - \left\{\nu\frac{\cos\Phi}{r} + \frac{(r/c)'}{r/c}\right\}\dot{N}_\theta^{VP} + \nu(\dot{N}_\xi^{VP})' - (\dot{N}_\theta^{VP})'
\end{aligned}
\right\} \quad (18)
$$

where

$$C = Eh\ ,\ D = Eh^3/12(1-\nu^2)$$

and

$$\{\dot{N}_\xi^{VP}, \dot{N}_\theta^{VP}, \dot{M}_\xi^{VP}, \dot{M}_\theta^{VP}\} = \int_{-h/2}^{h/2}\{\dot{\sigma}_\xi^{VP}, \dot{\sigma}_\theta^{VP}, \dot{\sigma}_\xi^{VP}\eta, \dot{\sigma}_\theta^{VP}\eta\}d\eta \quad \cdots\cdots (19)$$

Equations (17) and (18) are the fundamental equations for elasto/visco-plastic deformations of axisymmetrical shells. In the equations if the visco-plastic terms are omitted, these reduce to the equations for elastic problem derived by Reissner [11].

3 Numerical Method

A finite difference method is employed for solution of eqs.(17) and (18). The approximations

$$(dx/d\xi)_j = (x_{j+1} - x_{j-1})/2\delta\ ,\ (d^2x/d\xi^2)_j = (x_{j+1} - 2x_j + x_{j-1})/\delta^2 \quad \cdots\cdots (20)$$

are used for every mesh point except boundary points of the shell, where the subscript j refers to the j-th mesh point. The mesh points are taken at equal distance δ. For the boundary points (j =1, N)

$$(dx/d\xi)_1 = (-x_3 + 4x_2 - 3x_1)/2\delta\ ,\ (dx/d\xi)_N = (3x_N - 4x_{N-1} + x_{N-2})/2\delta \quad \cdots\cdots (21)$$

are used. Transforming eqs. (17) and (18) into finite difference equations by the use of eqs. (20), these equations are summarized as:

$$A_j x_{j-1} + B_j x_j + C_j x_{j+1} = d_j \quad (j = 2, 3, \cdots, N-1) \quad \cdots\cdots (22)$$

The boundary conditions at j =1 and N may be prescribed as

$$A_1 x_1 + B_1 x_2 + C_1 x_3 = d_1\ ,\ A_N x_{N-2} + B_N x_{N-1} + C_N x_N = d_N \quad \cdots\cdots (23)$$

where

$$A_j = \begin{bmatrix}(a_{11})_j, (a_{12})_j\\ (a_{21})_j, (a_{22})_j\end{bmatrix},\ B_j = \begin{bmatrix}(b_{11})_j, (b_{12})_j\\ (b_{21})_j, (b_{22})_j\end{bmatrix},\ C_j = \begin{bmatrix}(C_{11})_j, (C_{12})_j\\ (C_{21})_j, (C_{22})_j\end{bmatrix},\ x_j = \begin{Bmatrix}\dot{\vartheta}_j\\ \dot{\psi}_{Hj}\end{Bmatrix},\ d_j = \begin{Bmatrix}d_{1j}\\ d_{2j}\end{Bmatrix},\ j = 1, 2, \cdots, N-1, N$$

and $A_j \sim C_j$ and d_j can be calculated from the solutions at any one stage. Equations (22) and (23) constitute simultaneous equations with N unknown, x_j for j =1,2,. . . N. Once the values of x_j for j =1 ~ N are obtained, the components of internal forces and displacements for j =1 ~ N are calculated. Integrations of eq. (19) are carried out by Sympson's 1/3 rule from the values at K points where are taken for equal distance through shell thickness.

Now, the above numerical method is concerned with a handling of a spatial aspect of the problem. However, in this elasto/visco-plastic problem a temporal aspect also must be considered. The solutions at any stage of the problem are obtained by integration of the increments of internal forces and displacements due to the load increment Δp and the time increment Δt. Appropriateness of values of Δp and Δt is inspected with convergencof the solutions according to variations of Δp and Δt.

4. Pressure Vessels

In this chapter as a numerical example, the elasto/visco-plastic problem of a torispherical pressure vessel head composed of a spherical, a toroidal and a cylindrical shell as shown in Fig. 2 is treated.

The boundary conditions at the point A (j =1) are:

$$\dot{\vartheta}_1 = 0\ ,\ \dot{\psi}_{H1} = 0 \quad \cdots\cdots (24)$$

At the point D remote from the point C the boundary conditions

$$\dot{\vartheta}_N = 0\ ,\ \dot{\psi}_{HN} = 0 \qquad \cdots\cdots (25)$$

are employed [5].

The mechanical property of the material used in calculations are of aluminum (at 35°C):

$$\Phi(F) = \Phi\left(\frac{\bar{\sigma} - Y}{Y}\right) = \frac{\bar{\sigma} - Y}{Y}\ , Y = Y_0 + H'\bar{\varepsilon}^{vp},\ E = 7.45\times10^4 N/mm^2,\ \nu = 0.3\ ,\ Y_0 = 98.1\ N/mm^2,\ H' = E/10 \qquad \cdots (26)$$

The pressure vessel is loaded stepwise as shwon in Fig. 3, and the load is sustained untill the stationary state is reached at each loading stage. The authors regard it as the steady state when $|\dot{\varepsilon}^{vp}_{max}/\varepsilon_{max}| \leq 10^{-4}$ where $\dot{\varepsilon}^{vp}_{max}$ is a maximum viscoplastic strain rate and ε_{max} is a instantaneous maximum strain.

The load increment Δp is determined as $1/\alpha$ of an elastic limit of the shell obtained from a linear elastic theory. The time increment $\Delta\tau$ is decided so that the maximum visco-plastic strain increment $\Delta\varepsilon^{vp}_{max}(=\dot{\varepsilon}^{vp}_{max}\Delta\tau)$ becomes $1/\beta$ of maximum strain ε_{max}. In this example $\alpha = 30$ and $\beta = 60$ are employed.

The meridional mesh number N and the division number K through thickness of the shell are chosen to be $N = 80$ and $K = 19$, respectively. These numerical values are determined from evaluation of convergency of the solutions according to variation of the values and in consideration of capacity of the computer and computing time.

Some of the essential features of the solutions are shown in Figs. 4 – 11.

Finally the authors compare the predicted results with the experimental ones which Findlay and others [13] obtained for pressure vessel head. The dimensions and material constants of the pressure vessel head are as follows:

$$\left.\begin{array}{l} R_1 = 93.51\ mm\ ,\ R_2 = 1455\ mm\ ,\ R_3 = 476.9\ mm\ ,\ h = 14.1\ mm\ ,\ E = 2.00\times10^5 N/mm^2 \\ Y = Y_0 = 262\ N/mm^2\ ,\ \nu = 0.29\ ,\ H' = 0 \end{array}\right\} \qquad \cdots\cdots (27)$$

$\Phi(F)$ is assumed to be the same form as of eqs. (26).

Figure 12 shows the relations between the maximum strain and the internal pressure. In the figure the solid line and the broken line indicate the predicted results by the authors and the experimental results by Findlay, respectively. The chain line is the calculating results by Marcal and Pilgrim [12] from the conventional plastic theory which does not consider the viscosity of the material in the plastic range. It is found from Fig. 12 that the author's results coincide fairly well with the experimental ones by Findlay and others, and also the solutions at the stationary states by the authors almost agree with the calculated results by Marcal and Pilgrim.

5. Conclusions

In this paper the authors analyzed the elasto/visco-plastic deformation of the shells of revolution widely used under high stress by the use of Perzyna's constitutive equations. The authors derived the basic differential equations which took account of the geometrical nonlinearity in the same degree as in the elastic large deflection theory by Reissner. The basic differential equations derived for increments were numerically solved by the finite difference method, and the solutions at any stage were obtained from integration of the increments.

As a numerical example, the problem of elasto/visco-plastic deformations of pressure vessels important in practical use was treated, and the variations of the plastic deformations and the internal forces with time after application of the load were shown. The elasto/visco-plastic solutions from the prediction method agreed fairly well with experimentally determined values for pressure vessel heads by Findlay and others. The method can be used to generate plasticity solutions in a simple manner when stationary conditions are reached.

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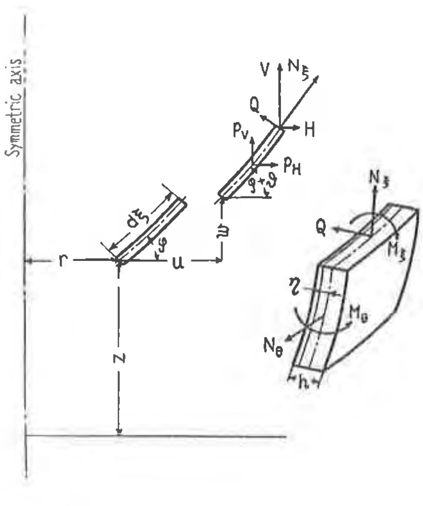

Fig. 1 Notations of shells of revolution.

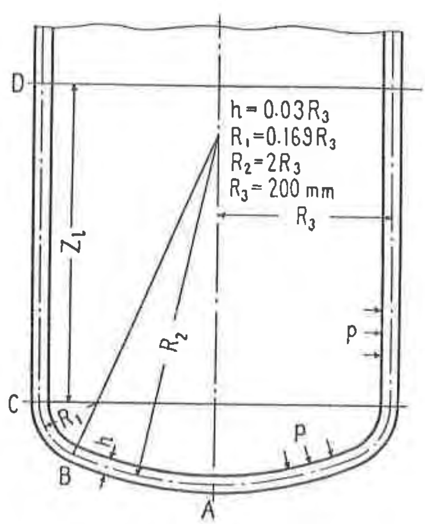

Fig. 2 Torispherical head of pressure vessels.

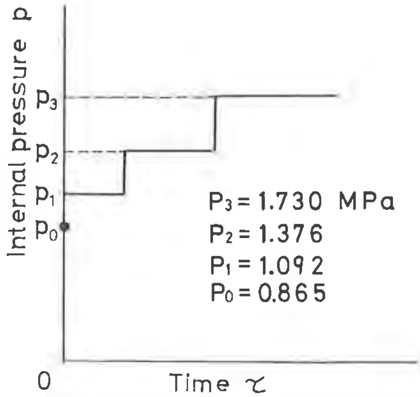

Fig. 3 Loading.

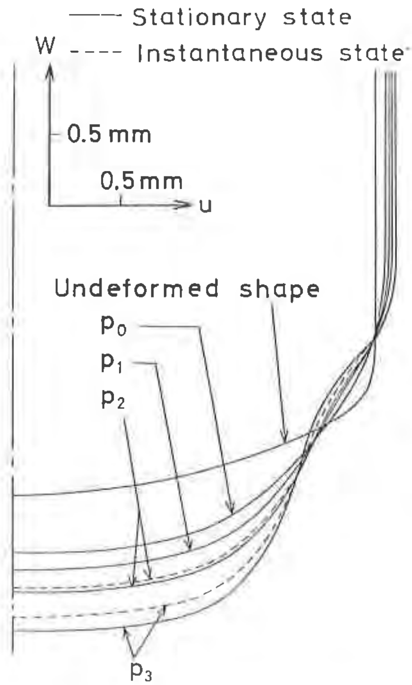

Fig. 4 Deformations.

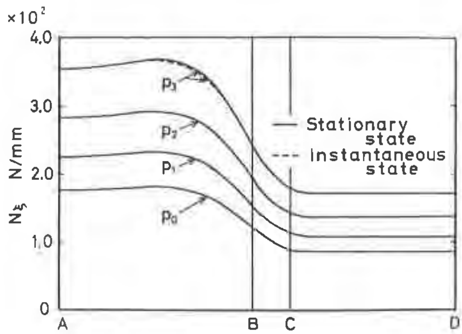

Fig. 5 Meridional normal force.

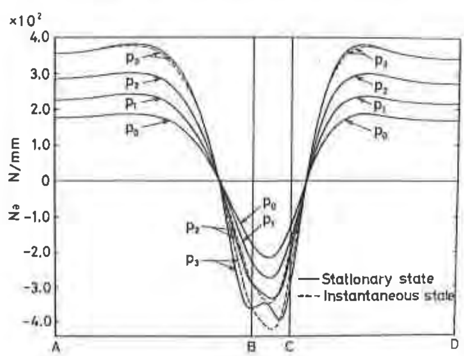

Fig. 6 Circumferential normal force.

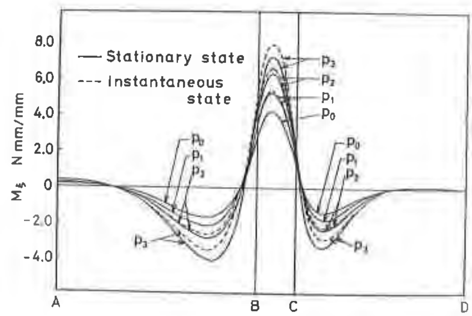

Fig. 7 Meridional bending moment.

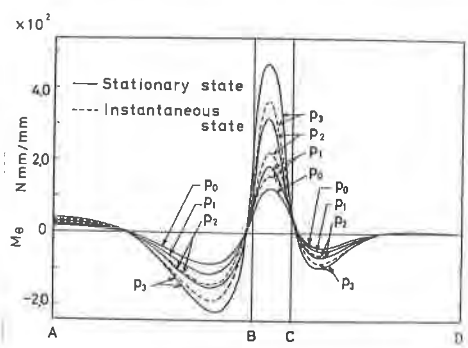

Fig. 8 Circumferential bending moment.

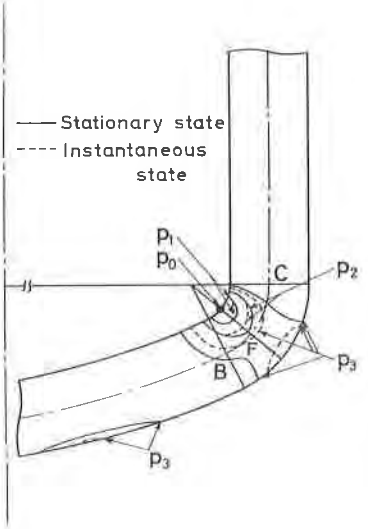

Fig. 9 Progression of yield.

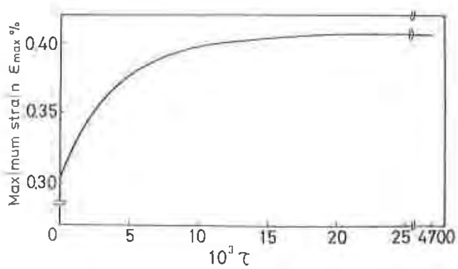

Fig.11 Variations of maximum strain with time.

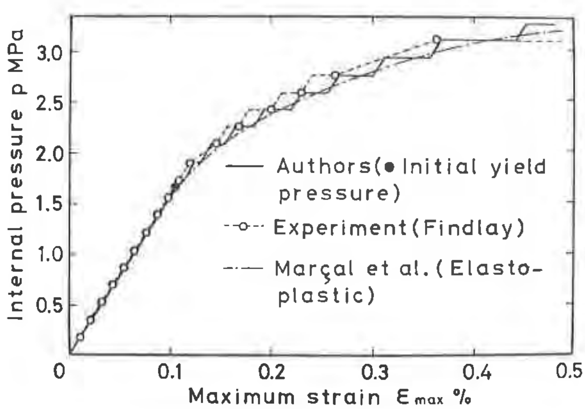

Fig.12 Pressure plotted against maximum strain.

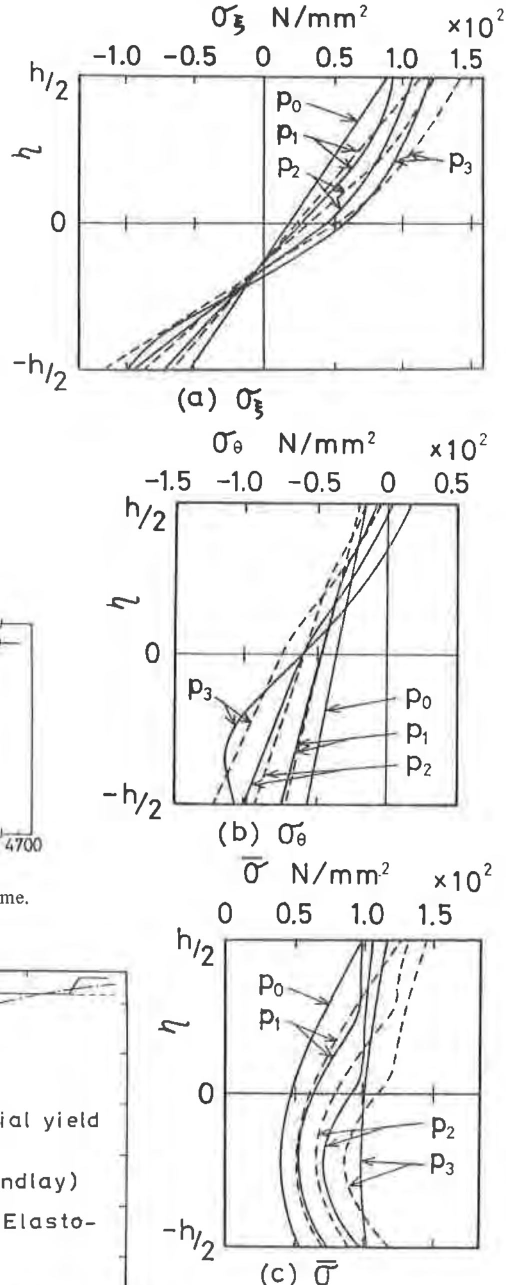

Fig.10 Stress distributions through thickness of shell at point F