ABSTRACT

SUSI, BRYAN. A Methodology for Translating Detonation Wave Effects between One and Two Dimensions. (Under the direction of Dr. Kevin Lyons.)

This research focuses on evaluating empirical methods and implementing a prototype Transitional Airblast Model (TRAM) for facilitating communication between one-dimensional and two-dimensional airblast models. An overview of detonation phenomena is presented, especially concerning detonation waves and accompanying airblast effects. Two existing airblast models are discussed that were designed to predict the effects of a detonation in two separate types of geometries, one-dimensional and two-dimensional. The functionality and behavior of each airblast model will be scrutinized giving particular insight into their performance in applications with both one-dimensional and two-dimensional components. The strengths and deficiencies of the different airblast models will offer motivation for the development of the TRAM prototype. The TRAM prototype consists of two separate methodologies, one for translating one-dimensional airblast propagation to two dimensions, and another for translating two-dimensional airblast propagation to one dimension. The selection of those two methodologies will be presented, along with results of detonation scenarios using both existing airblast models as well as the TRAM prototype. The TRAM prototype performed well for both types of detonation scenarios and is recommended for further development.

A Methodology for Translating Detonation Wave Effects between One and Two Dimensions

by
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BIOGRAPHY

Bryan Thomas Susi was born February 15th, 1983 to Janet and Jesse Susi. He was raised in Erie, PA until the age of 8 and thereafter in St. Louis, MO where he eventually attended DeSmet Jesuit High School. He had always been interested in flying for the military, but that dream had been eclipsed by glasses at the age of 16 so building aircraft for the military seemed the obvious next choice. During high school he made the decision to pursue that interest, and he applied to the University of Iowa for Mechanical Engineering. While at Iowa, Bryan naturally tended towards the fields of thermal and fluid sciences, especially when he was introduced to Computation Fluid Dynamics late in his curriculum. Taking his interest in aircraft, and natural inclination toward CFD with him, he chose to go to graduate school at North Carolina State University for a Master's degree in Aerospace Engineering. Upon graduation from NC State, Bryan will stay in the Raleigh area working for a research and development company.

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I owe a debt of gratitude to a great many people for helping me along the way through my thesis research but a few deserve special recognition.

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To compliment the quality of the technical research in this paper, I most certainly owe thanks to Janet MacKenzie for taking extra time out of her schedule on numerous occasions to make sure that this document looked and read as great as possible.

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LIST OF SYMBOLS, ABBREVIATIONS, OR NOMENCLATURE

Abbreviations and Nomenclature:

Interface	Opening between a tube and chamber
TD	Tube Diameters
TRAM	Transitional Airblast Model
NPH	Needham, Potter, Hikida
BMP	Britt, McMahon, Patterson
EMI	Ernst Mach Institute

Variable Definitions:

$\Re$	Universal Gas Constant
P	Pressure
P_d	Dynamic Pressure
T	Temperature
θ	Angle
Φ	Needham, Potter, Hikida angular offset ratio
λ	Scaling Factor
ρ	Density
h	Enthalpy

e	Internal Energy
$\dot{q}$	Heat Addition
$\bar{u}$	Velocity Vector of Cartesian Components $u\hat{i} + v\hat{j} + w\hat{k}$
E_0	Explosive Energy Density
W	Unaltered Charge Mass
W_{red}	Reduced Charge Mass
F	Charge Mass Ratio
f	Body Forces
Ψ	Equation of State Variation Ratio
X	Scaled Distance
R	Range of Charge
H_C	Chamber Height
D	Tube Diameter
A	Cross-Sectional Area

Arbitrary Constants:

α, β, Γ, τ, ζ

Mathematical Symbols:

∇	Gradient

∂	Partial Derivative Operator

A Methodology for Translating Detonation Wave Effects between One and Two Dimensions

1. Introduction

A thorough understanding of the physical phenomena associated with detonations is desirable for many reasons, and is therefore the subject of much research. While high fidelity computational fluid dynamics can accurately capture the physical behavior resulting from a detonation, the considerable time required and complexity associated with the process can detract from the advantages of using such a powerful tool. Conversely analytical expressions for the governing equations of a detonation exist for only the most simplistic, and uninteresting situations. Empirical relationships offer the most attractive compromise. These relationships are generally engineering level algorithms that approximate the airblast environment due to a detonation either from a simplified application of physical first principles, or a relationship developed from test data.

Tailoring the empirical models to suit a specific need is common in the field of airblast simulation. Generally the application where airblast effects are important will dictate how the empirical model will act. Take for instance a ballistics study of an explosion in a gun barrel. The detonation effects are constrained to one dimension; therefore an accurate simulation of a detonation in a gun barrel could be achieved with an airblast model that predicts airblast propagation in only one dimension. A very different example would be that of an explosion in a chemical plant or similar facility. Prediction of the airblast effects inside that facility could potentially dictate certain design requirements and safety protocols which both require that the airblast prediction be accurate throughout the entire facility. In this case however there are likely to be more spaces where the one-dimensional propagation assumption breaks down such as large volume rooms. For this type of simulation a two-dimensional airblast model that predicts airblast expansion in two dimensions would be necessary. Throughout the course of this study these two different types of airblast prediction models will be studied closely, termed throughout the rest of this paper as the two *types* of airblast models.

Introduction

The premise of the research presented in this paper is that there are many situations where there is a need to use both types of airblast models in the same facility where it would be more accurate to apply both airblast models. Take for example the simulation of an explosion in a mine. Being able to accurately predict the airblast effects of an accidental explosion in a mine is very important, especially from a safety standpoint. Being able to judge where to place obstructions to contain an incident could save not only equipment and property, but more importantly lives. The only problem is that facilities like mines may have sections that are conducive to one-dimensional flow like mine shafts, and other areas that would be more accurately characterized by two-dimensional flow like large caverns or staging areas.

To maintain usage of existing airblast prediction models, communication of airblast information between one-dimensional and two-dimensional propagation is necessary. This research focuses on evaluating empirical methods and implementing a prototype Transitional Airblast Model (TRAM) that will facilitate communication between existing one-dimensional and two-dimensional airblast models. This will entail an investigation of the differences in the two types of airblast methodologies, evaluation of a transition methodology, and a conceptual assessment of the prototype's airblast prediction accuracy for airblast that transitions between one-dimensional and two-dimensional propagation.

Leveraging the existing airblast models instead of replacing them is advantageous for several reasons. The airblast models are designed to operate accurately in specific domains. It would be beneficial to utilize the strengths of both types of models, without sacrificing anything by eliminating their weaknesses. Consider airblast models as falling into one of two categories; prediction of airblast that propagates in one dimension, or prediction of airblast that propagates in two dimensions. The airblast models which currently exist for one-dimensional propagation accurately capture the airblast environment for a one-dimensional type of propagation only. This model does not lend itself well to predicting airblast in a chamber simply because the phenomena would be much different and dominated by a different type of airblast behavior than the one-dimensional model could predict. That is precisely why a second type of airblast model exists for airblast predictions in chamber like structures. This concept is clarified by Figure 1, which shows the two types of geometries in conjunction with the type of behavior predicted by the respective airblast models.

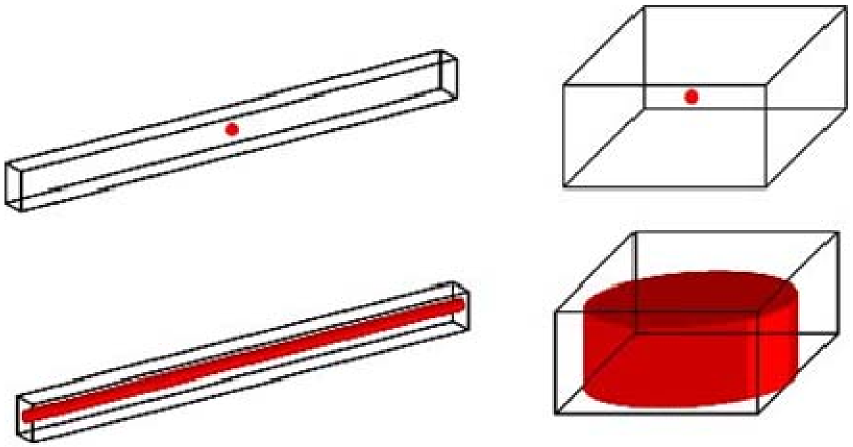


Figure 1) One-Dimensional and Two-Dimensional Airblast Model Behaviors

The approach for the transition between the two types of airblast models is to determine a consistent and accurate method to translate airblast information from one model into a quantity that is understood by the other. In this manner the existing airblast models are allowed to function independently, though not entirely uncoupled allowing the use of their individual strengths without their respective drawbacks. The nature of the TRAM prototype is illustrated as Figure 2, where it is obvious that the one- and two- dimensional airblast models will still operate in their native geometries, but will now use the TRAM prototype as an interface to communicate for hybrid environments.

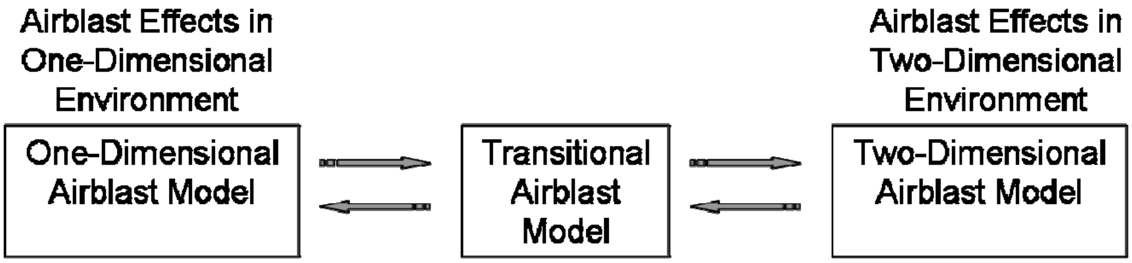


Figure 2) TRAM Architecture in Hybrid Facility

The methodology for the TRAM prototype consists of two distinct scenarios; one process for treating one-dimensional airblast expanding into two-dimensional propagation, and another process for treating two-dimensional airblast that is being confined to one-dimensional propagation. For the sake of generality, the type of geometry which forces one-dimensional airblast propagation will be known throughout the paper as a tube while the two-dimensional environment will be referred to as a chamber.

2. Airblast Phenomenology

2.1. Detonation

An explosion is characterized by a rapid release of energy, and is defined broadly enough so as to apply to a large range of circumstances. A closed volume of liquid will explode if kept under a heat source long enough to vaporize, as will a fuel-air mixture in a car's engine or even flour in a mill if ignited. All of these events can be represented as a general equivalent source of energy release if appropriately handled. If the proper source characterization is used, any rapid release of meaningful size can be represented by a scaled equivalent explosive charge. This analogy makes it convenient to discuss in a very general manner the effects of any of the previously mentioned energy release events in the context of a detonation. Within this research, the source of the energy release is immaterial; so long as it is an energetic enough release to propagate a detonation wave. Out of convenience, the phenomenology of the energy release event in this paper is most aptly represented by the detonation of an equivalent charge. This allows for the detonation event that propagates airblast to be referred to herein as a single physical entity.

For an ignition event to commence for any flammable medium, a catalyst will induce a chemical reaction that proceeds very quickly from that ignition point. Reactants will dissociate into highly dense gas products that will seek to attain the lowest energetic state possible. In doing so the gas will expand rapidly, changing the properties of the local ambient environment it encounters. If the reaction proceeds fast enough, the expansion of gas from the energy release increases the density and temperature, which forces the pressure to rise substantially in a small amount of time. The result is a deflagration to detonation transition with a detonation wave propagating away from the location of the burst. (Walters, Zukas, 1999) This wave loses strength as the distance from the source increases due to energy dissipation through heat loss and geometrical divergence which lowers the density and pressure of the air behind the shock front. The entirety of this process is the characteristic of an ideal detonation wave shown as Figure 3. (Kuo, 2005) The figure below also illustrates an important distinction between the shockwave, and detonation wave. The structure of the entire process is how the detonation wave is defined, which consists of a shockwave that precedes a deflagration wave. It is after the shockwave in the deflagration region where the pressure and density both drop and temperature rises in this region due to the exothermic burning of un-reacted material.

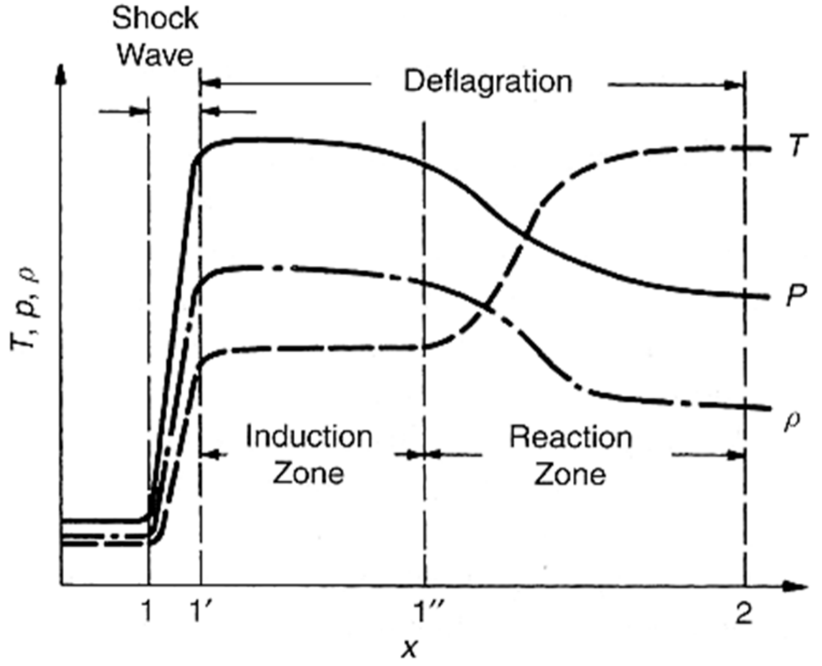


Figure 3) Ideal Detonation Wave Structure (Kuo, 2005)

2.2. Shock Pressure

The chemical reaction that constitutes an explosion is a very complex, highly coupled process. However the airblast effects caused by that reaction are governed by the extensively studied equations of fluid mechanics, closed by an equation of state. Comparison to the field of compressible fluid dynamics is usually discouraged however this is generally just to avoid the application of acoustic theory to blast dynamics. (Baker, 1973) Like any compressible fluid flow, the effects of detonation of must preserve the following physical principles, conservation of mass, conservation of momentum, and conservation of energy.

These principles are mathematically represented as the following three equations where pressure and density are denoted by P and ρ, e and $\dot{q}$ represent internal energy and heat flux respectively, f represents body forces of indiscriminate type, and the vector $\bar{u}$ is the fluid velocity.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \bar{u}\right) = 0 \qquad \text{Eq. 1}$$

$$\frac{\partial\left(\rho \bar{u}\right)}{\partial t} + \nabla \cdot \left(\rho \bar{u} \cdot \bar{u}\right) + \nabla P - \rho f = 0 \qquad \text{Eq. 2}$$

$$\frac{\partial}{\partial t}\left[\rho\left(e + \frac{\left(\bar{u}\right)^2}{2}\right)\right] + \nabla \cdot \left[\rho\left(e + \frac{\left(\bar{u}\right)^2}{2}\right)\bar{u}\right] + \nabla \cdot \left(P\bar{u}\right) - \rho\dot{q} - \rho\left(f \cdot \bar{u}\right) = 0 \qquad \text{Eq. 3}$$

The system is closed by an equation of state that couples the fluid dynamics equations and thermodynamics. The thermodynamic relationship between state variables in real flow can be generalized by Eq. 4 where a value of Ψ=1 corresponds to the ideal gas law. The pressure and density are denoted as the same in Eq. 4 as in the conservation equations and *T*, and $\Re$ represent the temperature and the universal gas constant.

$$\frac{P}{\rho \Re T} = \Psi P(\rho, \Re, T) \qquad \text{Eq. 4}$$

The detonation wave of Figure 3 causes the particles of air to move with it as it travels where the speed of the air particles is known as local particle velocity. The cushion of air is compressively "pushed" and by enforcement of the equation of state, the pressure and temperature of the air increase accordingly. The detonation wave expansion from the explosive charge reaches a point where the air is so compressed that the pressure and temperature increase in a nearly discontinuous transition, more colloquially known as a shockwave. The mathematical expressions noted above break-down at these discontinuities due to the impact that temperature has on the pressure and density gradients, and its absence from any of the stated relations. (Baker, 1973)

To quantify the effect that a shockwave has between the thermodynamic states, assuming no body forces on the fluid, and neglecting any heat transfer from the shockwave, the governing equations can be simplified to one dimension. These one-dimensional equivalents are known as normal shock relations, or jump conditions borrowing from the discontinuous nature of the state variable changes. Subscripts 1 and 2 denote states on different sides of the shockwave, the density and pressure have been defined, *u* is the velocity in the x-direction and *h* is the enthalpy.

$$\rho_1 u_1 = \rho_2 u_2 \qquad \text{Eq. 5}$$

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2 \qquad \text{Eq. 6}$$

$$h_1 + \frac{1}{2} u_1^2 = h_2 + \frac{1}{2} u_2^2 \qquad \text{Eq. 7}$$

The state variable relationship between states before and after the shockwave can be obtained by using the three equations to eliminate the velocity terms. Rearranging terms produces explicit expressions relating the thermodynamic states on either side of the shock called the Rankine-Hugoniot relations, as shown in Eq. 8. (Liepmann, Roshko, 1957)

$$\frac{\rho_1}{\rho_2} = \frac{1 + \frac{\gamma+1}{\gamma-1}\frac{P_2}{P_2}}{\frac{P_2}{P_2} + \frac{\gamma+1}{\gamma-1}} = \frac{u_1}{u_2}$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_2} \frac{\frac{P_2}{P_2} + \frac{\gamma+1}{\gamma-1}}{1 + \frac{\gamma+1}{\gamma-1}\frac{P_2}{P_2}} \qquad \text{Eq. 8}$$

For the purposes of this research, the most interesting type detonation is a short duration, or pulse detonation. The immense pressure release due to a detonation is the driver for fluid and particle transport, and has a very profound effect concerning damage and lethality prediction. An example of a waveform for such an event is shown in Figure 4, where the short duration peak is the direct result of the detonation wave, termed shock pressure, and the subsequent lower pressure that is still above the ambient conditions is termed the quasi-static pressure. (Binggeli, 1985)

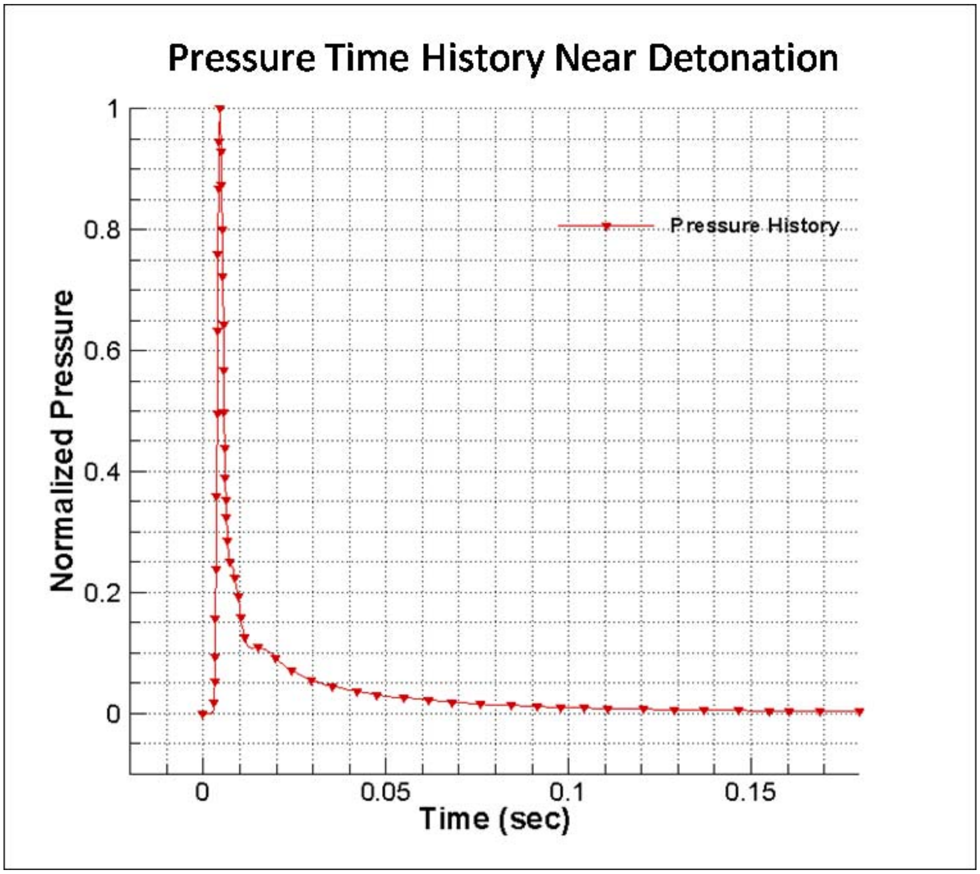


Figure 4) Detonation in a Facility

The research presented in this paper considers only the contributions of shock pressure and neglects the quasi-static contribution because coupling the quasi-static effects with the shock pressure is a highly complex undertaking. Fidelity gained by including the dual contributions is negated by the fact that the vast majority of the hybrid facility would be unaffected by the quasi-static portion, yet still affected by the shock pressure portion. Quasi-static pressure in a confined environment is not miniscule by any means and generally should not be discounted near the detonation, but quasi-static pressure does not propagate like shock pressure does. Its inclusion provides increased accuracy in such a small portion of the overall calculation that the cost to benefit ratio was deemed too high. A brief phenomenological explanation of quasi-static pressure will be included for completeness, but effects such as airblast jetting or vortex formation that are associated with the long duration pressure pulse will not be addressed. For reference, the short duration pulse is the shock pressure dominated portion of Figure 4, between 0 and 0.015 sec, where the long duration is the decay between 0.015 sec and just after 0.15 sec.

2.3. Quasi-Static Pressure

It has been observed that a substantial portion of the explosive potential of a detonated charge is dissipated by the mass transfer of the expanding shockwave, but not the entirety. There is a small amount of energy which is dissipated in what is known as afterburn. (Walters, Zukas, 1999) The process of the detonation reaction includes the expansion of gases which was previously described to be the driver for the shockwave. As the shockwave propagates further away from the point of detonation, the pressure drops behind it, allowing the gases more freedom to expand. The density of these gases will still be much higher than the ambient conditions meaning that the gas will expand within the confines of the environment until equilibrium is established. The equations of state must still be satisfied therefore in a fixed volume environment the pressure will have to increase. The corresponding pressure is called quasi-static pressure and is an important damage mechanism very near the detonation and within confined geometries.

There is inherent difficulty in simulating quasi-static pressure since it is coupled with shock pressure and the dynamic pressure. The expanding detonation wave will “pull” some of the reacting gases with it in a process that basically transports the quasi-static pressure. (Britt, 2008) Such would be the case for the hybrid tube-chamber setup being considered in this research. Often an assumption is made that the detonation chamber must fill with quasi-static pressure before the quasi-static pressure can flow into adjacent rooms or tubes. Adopting that assumption and applying it to a model consisting of a chamber connected to a tube, where the tube volume is so much larger than the chamber portion would completely diminish the contribution of quasi-static pressure as compared to the shock pressure. As a result the quasi-static pressure contribution was not considered independently in this research. The total pressure at the interface may in fact include quasi-static pressure as calculated by the airblast model, but the individual contribution was not considered separately.

2.4. Dynamic Pressure

Pressure is obviously a scalar quantity; the force imposed by pressure is not. The force is contingent upon the velocity of the flow, a vector quantity. The dynamic pressure, P_d, shown as Eq. 9, warrants mention because the Needham-Potter-Hikida empirical method uses it as a parameter as described in Section 4.1.2. In the following equation, $\bar{u}$ is the fluid velocity vector and ρ is the fluid density.

$$P_d = \frac{1}{2}\rho\left(\bar{u}\right)^2 \qquad \text{Eq. 9}$$

The importance of the dynamic pressure to the airblast environment prediction is its relation to the kinetic energy of the airblast. The flow of the detonation carries with it momentum, as does any fluid flow. The shockwave is small enough to be less affected by momentum than the fluid the shock is pulling behind it. This implies that for the scenario of a detonation in a tube that the fluid exhausted from the entrance will be directional. The means to quantify this directional bias is the dynamic pressure.

2.5. Detonation in a Tube

Upon the instigation of airblast, the one-dimensional airblast prediction model used in this study begins to calculate the environment effects experienced in the tube due to the sudden increase in energy. The “airblast environment”, as it is called, consists of the calculation of the thermodynamic state variables along the length of the tube. Physically, what will happen at an interface between a tube and a chamber is that the state variables will be subjected to a sudden expansion in volume. This represents a major challenge for the one-dimensional airblast model when predicting one of the effects of a detonation, which is the detonation wave. Though fundamentally different, a confined detonation wave in a tube is comparable in movement to a planar shockwave, or normal shock. The one-dimensional movement of the detonation wave is easily predicted, but at the interface the wave will be forced to propagate in two dimensions into the larger volume. Due to energy and heat losses to the ambient chamber air, and a geometric degradation of the shock strength as the range from the interface increases, a drop in pressure will be incurred. This geometric expansion is shown in Figure 5 where the detonation process is shown in several time intervals. Initially the detonation wave will expand in a spherical fashion until it encounters the tube boundaries, which is very quick. The effects of the reflections of the expanding wave force a normal shockwave that travels along the tube length, pictured in the middle of the three figures. When the shockwave encounters the abrupt area expansion, the wave will be diffracted around the corners to form traveling wall shocks, and then continue to propagate in two or three dimensions. (Whitham, 1957)

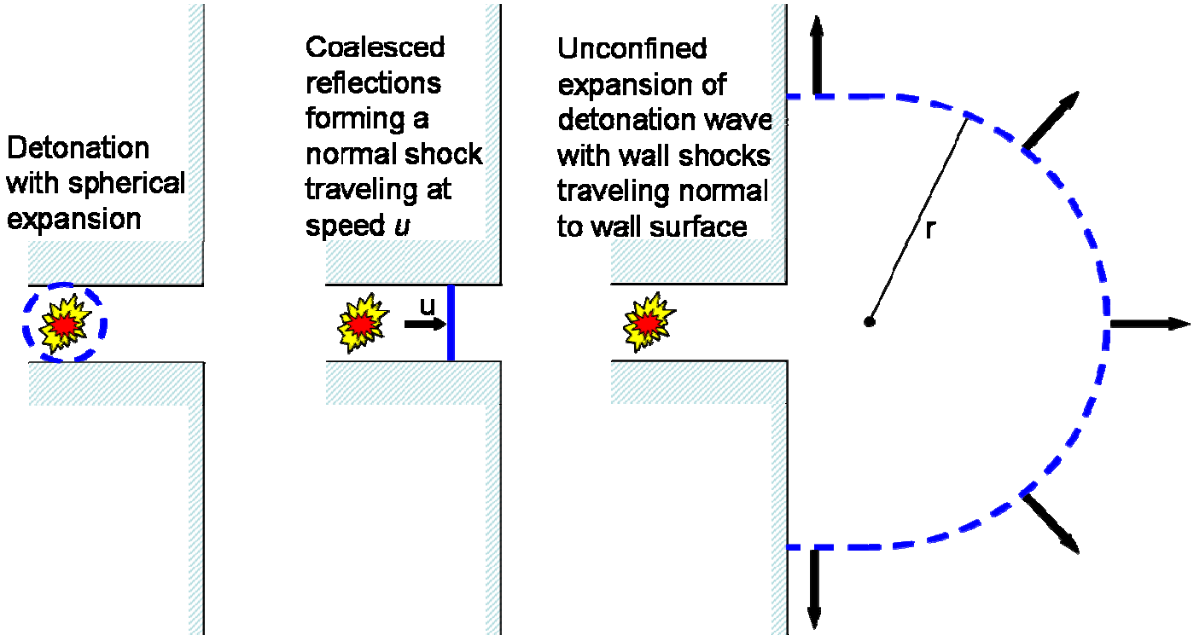


Figure 5) Detonation in a Tube

There is a reduction built into the one-dimensional airblast model to account for a volumetric expansion but the pressure propagation will still be one-dimensional. Intuition aside, Figure 5 illustrates why this is unrealistic. The TRAM prototype will use an empirical relationship to approximate the magnitude of the pressure loss due to the increased volume, and translate that reduced pressure into an effective charge that can be detonated by the two-dimensional airblast model which will simulate two-dimensional propagation. An overview of the TRAM translation for a detonation in a tube is shown in Figure 6.

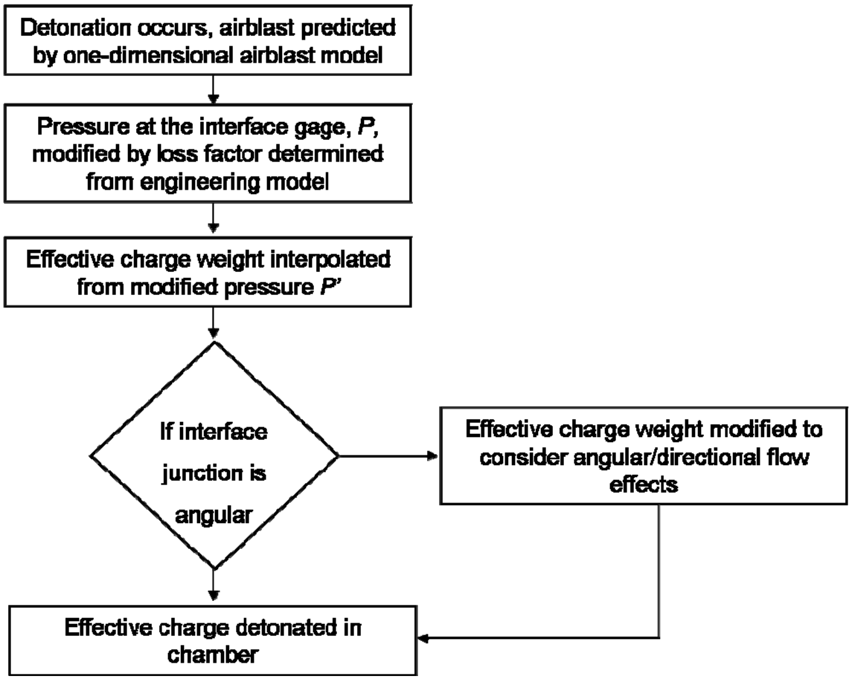


Figure 6) Tube Detonation Flow Chart

2.6. Detonation in a Chamber

The process for handling a detonation in a chamber is different from the tube counterpart and slightly more involved. A detonation in a tube is constrained such that the associated physical phenomena are more easily predicted. A multitude of different variables exist in a chamber burst that must be accounted for including variation in charge location, reflection effects, and varying charge masses. A typical burst and the ensuing airblast are shown in Figure 7 to help illustrate the additional complexity. (Anet, Binggeli, 1989)

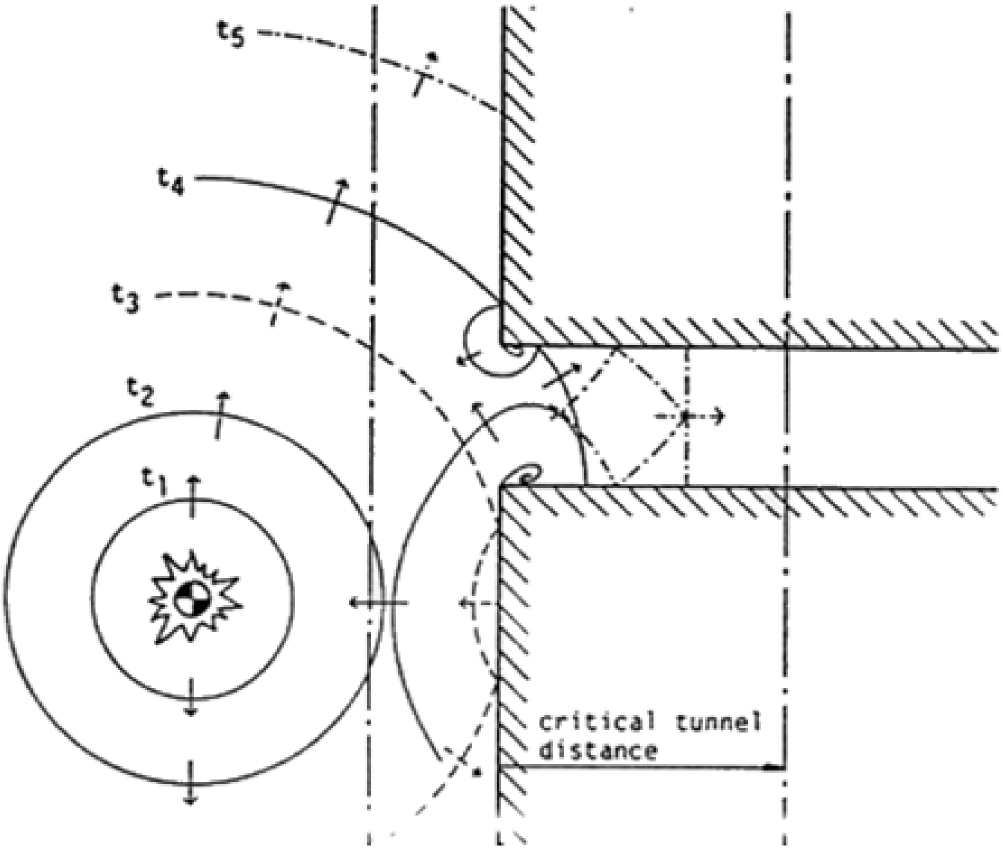


Figure 7) Detonation in a Chamber (Anet, Binggeli, 1989)

As the illustration shows, the unperturbed detonation wave will expand in a cylindrical or spherical fashion. At the wall the blast wave will reflect off of the surface and influence the local environment, specifically the pressure and local particle velocity. The reflections off of the walls undoubtedly have large effects on the pressure inside the chamber, but the main concern is the impact that the reflections have at the tube entrance. The flow at that corner will separate causing a low pressure region to form inducing very strong vortices caused by the increased local rotational velocity and high temperature gradients. These low pressure vortices will bend the detonation wave around the corner as seen in the example above. The bend invariably weakens the shock strength, which in turn slows the particle velocity. The lower velocity coupled with the superposition of wave reflections raises the pressure considerably. Classical compressible fluid theory predicts a reflected pressure increase of anywhere between 2 and 8 times the incident pressure for oblique reflections in air that is approximated as a calorically perfect, ideal gas. Detonation wave reflection pressure increases of up to 20 times the incident pressure have been observed owing predominantly to the non-ideal behavior of air at extreme temperature gradients. (Baker, 1973) The TRAM prototype will have to model this pressure amplification to correctly predict the inputs to give the one-dimensional airblast model so that the airblast effects in the rest of the tube can be obtained.

Once again, the chosen method of the TRAM prototype will be to use an empirical relationship to approximate the pressure change, only this time it will be an increase instead of reduction. Both of the empirical methods under consideration for the pressure amplification take into account the original charge mass and the location of the charge relative to the tube entrance. As shown in Figure 8, the pressure increase is initially calculated based off of the charge location and size. The original source from the chamber is then reduced to a scaled charge that will create the desired pressure when moved into the tube entrance. The one-dimensional airblast model will then use that effective charge to predict the airblast within the one-dimensional portion of the geometry.

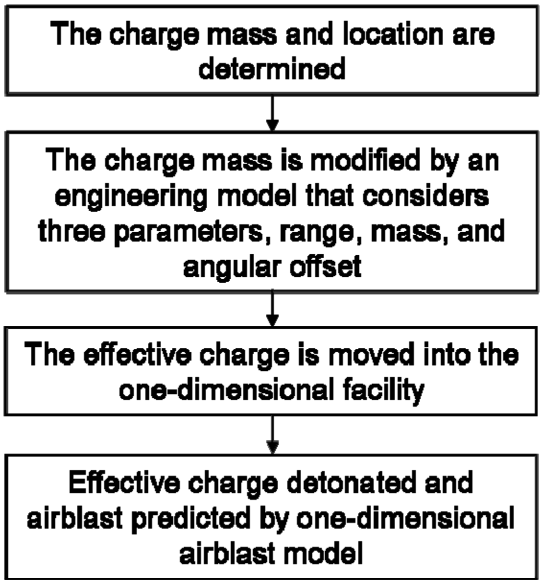


Figure 8) Chamber Detonation Flow Chart

2.7. Blast Scaling

Much of the research conducted within the combustion community or any associated field is either a direct result of live experiments, or validated against them. Many of the relationships governing blast effects were developed by the repetition of experiments with similar test setups, changing only individual parameters at a time to gauge what contributed the greatest impact. Full scale live experiments are not generally feasible for many reasons and so it is common practice to scale these effects down to a reasonable level and conduct them in miniature. Much like the scaling of parameters for traditional fluid dynamics research and tests, the physical parameters of a detonation can be scaled.

Not only does the ability to scale detonation effects facilitate more efficient experimentation, but the portability of a charge that is central to this research is based upon the scaling principle. The ability to take the pressure at a point which is a given distance from an initial equivalent charge, and achieve the same pressure from a different charge at a different location is essential to the TRAM prototype. The principle behind how that objective is achieved by equivalent charges of different masses at different distances from the point of interest is most clearly shown by Figure 9. (Baker, 1973) Note that pressure is the only quantity retained while both the range and the mass have been altered by a value λ. The negative phase, as denoted by T is also operated on by λ but the negative phase is not directly applicable to the TRAM methodology like the charge mass and range are. The mass is not directly operated on either by the illustration in Figure 9, but instead the equivalent charge diameter. This is important to keep in mind when scaling generic detonations to equivalent charges.

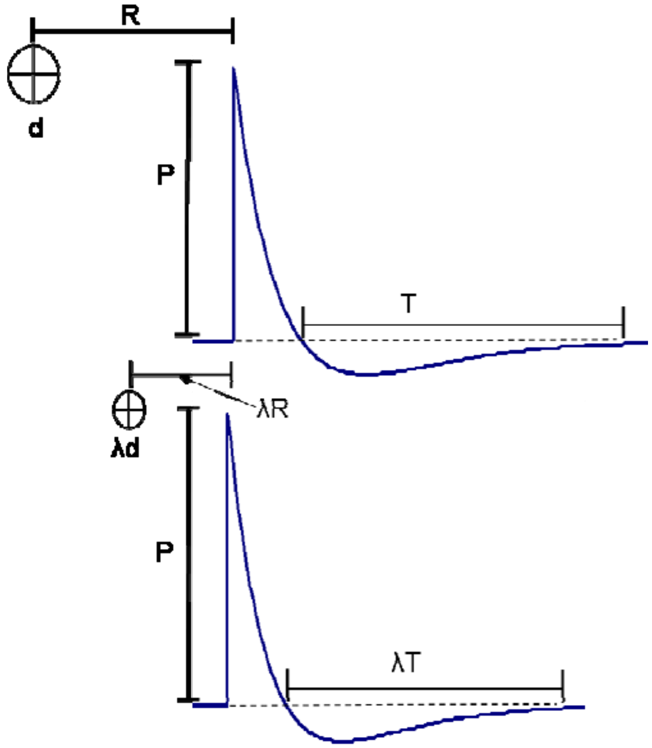


Figure 9) Blast Scaling

Scaling parameters are attainable by many means. Often individual investigators will develop scaling parameters to fit their particular needs. This was the case with several of the researchers involved in developing the empirical methods considered in this research. Since blast scaling is so important to the TRAM methodology and extremely prevalent in this paper, an overview of the scaling parameters incorporated into the empirical methods and airblast models is presented.

2.7.1. Hopkinson Scaling

Hopkinson scaling is usually presented as the most basic scaling method, sometimes referred to as "cube-root" scaling. This rule asserts that two charges of the same shape and composition, but of different sizes will produce identical blast waves at distances proportional to their masses. The scaled distance developed by Hopkinson is shown as Eq. 10 where R is the distance from the center of the charge to the point of interest and W is either the weight of the charge or sometimes the total energy of the explosive mass. This distinction stems from the fact that volumes of work have been completed using W as the weight of a standard effective charge, but the energy is a more realistic, universal physical parameter so many authors choose to represent it that way. (Baker, 1973) The representation using energy allows for the aforementioned practice of characterizing any explosion as an equivalent charge, which extends analogies to a single detonation to represent an explosive event. This would allow the scaling of a fuel-air detonation for instance. In either case, the total energy of an explosive is proportional to the charge mass so as long as the use is consistent, either definition is acceptable.

$$X = \frac{R}{W^{1/3}} \qquad \text{Eq. 10}$$

The two-dimensional airblast model used for the calculations in this research applies Hopkinson scaling in two forms. Eq. 10 is used explicitly for spherical expansion which is always applied to points of interest that are very near the detonation. Eq. 11 is used if the expansion is cylindrical, where H_c represents the height of confinement and α is an arbitrary constant. (Hikida, Potter, Needham, 1999)

$$X = \left(\alpha \frac{R^2 H_C}{W} \right)^{1/3} \qquad \text{Eq. 11}$$

Imposing a constraint on the expansion in the Cartesian Z direction, by a ceiling and floor for example, limits the vertical propagation. The reflections off of the ceiling and floor coalesce into a traveling mach stem along these surfaces, while the middle portion of the sphere propagates unabated. This yields an axi-symmetric cylindrical expansion which can be predicted as a function of Eq. 11.

Portability of a charge is achievable since it is now known that an equivalent charge of a given initial weight detonated in a chamber will induce a known pressure at the tube-chamber interface. That pressure needs to be maintained, but the charge mass and distance can vary as much as needed. In order to propagate pressure through the opening into the next chamber or tube, an effective charge will be used to create the same pressure at the opening. The distance and charge mass will have changed, but the pressure at the point of interest will be conserved. The initial charge mass is scaled using an equation such as Eq. 10. The end result is the new weight of the charge located at the interface. Using this method, the pressure at the interface will be continuous, but now the tube or next room will have a source from which to calculate airblast.

2.7.2. BMP Scaling

Investigators often wonder what other aspects of a certain test may affect the scaling of their explosives, and many times offer their own custom scaled distances. One of the methods used within this research dealt with live, small scale tests of charges detonated near tube entrances, performed for Air Force Research Laboratories, AFRL. The authors of the paper, Britt, McMahon, and Patterson (BMP), proposed a variation of Hopkinson's scaling that will be referred to as the BMP scaled distance. The BMP scaled distance accounts for the area of the opening as denoted in Eq. 12, in addition to the range and mass of the charge. (Britt, McMahon, Patterson, 2004)

$$X = \frac{W^{1/3} R}{Area} \qquad \text{Eq. 12}$$

2.7.3. EMI Scaling

Researchers at the Ernst-Mach Institute (EMI) were working on an analytical expression for the evaluation of pressure in a tube from a burst outside of it, much like the work of Britt, McMahon, and Patterson. Their tests were also small scale detonations which made a scaled distance necessary. The principle researcher, G. Scheklinski-Glück, obtained the expression by fitting a curve through data points as plotted against the scaled distance. The method varies from the BMP method, and as a result the scaled distance deviates as well. It will be discussed in subsequent sections that the EMI methodology uses a pressure ratio between gages located outside of the tube to those inside, to build an empirical relationship, which is different than how the BMP researchers approached the problem. There is no detriment to the approach used by the EMI researcher; it simply means that the charge masses were not part of the range scaling. Eq. 13 is the Ernst-Mach Institute scaled distance, or EMI scaled distance where D is the hydraulic diameter of the tube and again R is the range of the charge from the opening. (Scheklinski-Glück, 1993)

$$X = \frac{R}{D} \qquad \text{Eq. 13}$$

3. Airblast Prediction Methods

3.1. One-dimensional Model

For many applications airblast algorithms are required to be fast running, which includes the airblast models presented in this work. Such a constraint prevents the vast majority of methods that rely on the numerical solutions to the governing equations since these algorithms are generally very long running. Simplifying the governing equations to a single dimension drastically reduces the prohibitive run times, and since a detonation in a tube is inherently a one-dimensional expansion, the numerical solution to the governing equations for such a burst is plausible. The governing equations shown previously as Eq. 1, Eq. 2, and Eq. 3 can be simplified into their one-dimensional equivalents and solved with a numerical scheme. The one-dimensional governing equations are shown as Eq. 14, Eq. 15, and Eq. 16.

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0 \qquad \text{Eq. 14}$$

$$\frac{\partial (\rho u)}{\partial t} + \nabla\left(\rho u \bar{u}\right) + \frac{\partial P}{\partial x} - \rho f = 0 \qquad \text{Eq. 15}$$

$$\frac{\partial}{\partial t}\left[\rho\left(e + \frac{u^2}{2}\right)\right] + \frac{\partial}{\partial x}\left[\rho\left(e + \frac{u^2}{2}\right)u\right] + \frac{\partial}{\partial x}(Pu) - \rho\dot{q} - \rho f u = 0 \qquad \text{Eq. 16}$$

Since the thermodynamics of real fluids usually deviates from the ideal gas law, alternate equations of state conforming to Eq. 4 have been developed. The one-dimensional airblast model uses the Jones-Wilkins-Lee (JWL) equation of state, which is shown as Eq. 17. (Burton, Lund, Mandell, 1998)

$$P = A\left(1-\frac{\omega R_1}{\upsilon}\right)e^{(-R_1\omega)} + B\left(1-\frac{\omega R_2}{\upsilon}\right)e^{(-R_2\omega)} + \frac{\omega E_0}{\upsilon} \qquad \text{Eq. 17}$$

The JWL equation of state is commonly used within the blast effects prediction community because of the direct inclusion of the explosive energy density in the last term. The individual components of the JWL equation of state are described in Table 1.

Table 1) JWL Equation of State Notation

A, B, C, R_1, R_2	Experimentally determined constants
υ	Specific volume
ω	Specific density to ambient density ratio
E_0	High explosive specific energy density
P	Pressure

Numerical solutions to partial differential equations (PDE's) are obtained by solving the equations at discrete points of the equation's variables. Using the conservation equations as an example, the two variables being solved for are the distance *x,* and the time *t.* A numerical value is obtained at each discrete value of *x* for every time-step *t.* The collection of points in space where the PDE's solution is evaluated is commonly referred to as a mesh, or grid. For single dimension numerical solutions, the grid can be thought of as a continuous line of discrete points; the accuracy of the solution is contingent on the distance between those steps. A comparison between a solution calculated with a fine mesh and a solution with a coarse mesh is shown as Figure 10.

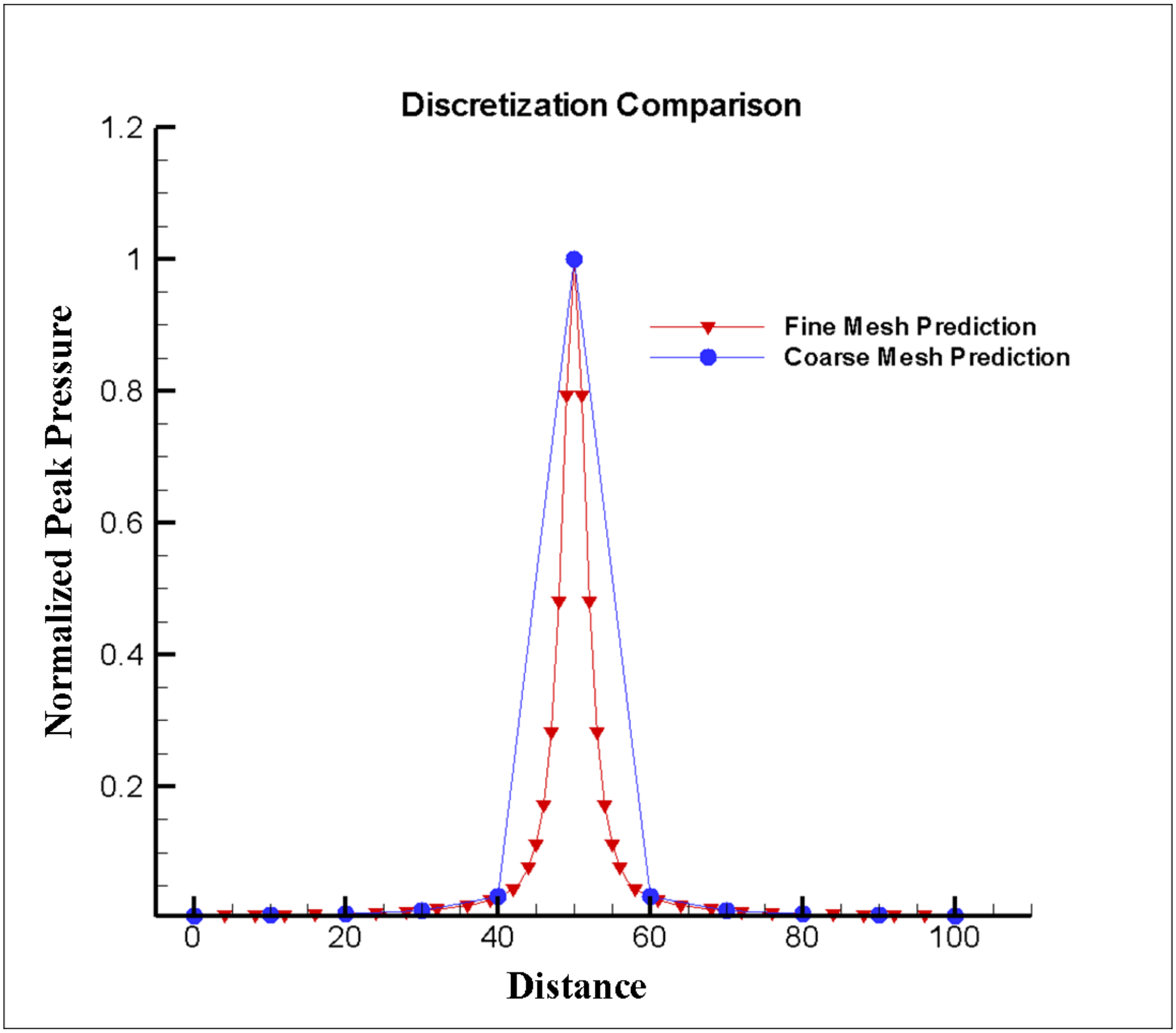


Figure 10) Comparison of a Numerical Solution over Coarse and Fine Meshes

For an airblast effects prediction in a tube using the one-dimensional airblast model in this study, the simplification to one-dimensional propagation in conjunction with a course discretization of the mesh enables a numerical solution within the given time constraint. The reason that this method is so much more desirable than a strictly empirical engineering model is that variables such as velocity do not have to be sacrificed and physical phenomena like reflections and time accurate pressure gradients are possible. Numerical solutions offer greater accuracy than any empirical method while still preserving physical authenticity.

Recall the mine facility example discussed previously. The facility will consist of many interconnected mine shafts that are all much greater in length than they are in width, and the single dimension simplification would be realistic. There are certain places however that such an assumption breaks down. The flow phenomena very near the detonation for instance, as well as near shaft discontinuities such as junctions and abrupt expansions. Near these areas, the one-dimensional airblast model can either specially treat the extenuating circumstances, or accept that as an engineering level algorithm there will be some inaccuracies. The detonation proximity and discontinuity treatment will be used as examples to illustrate some benefits and consequences of the different ways to handle the multi-dimensional flow phenomena in a one-dimensional solution.

A considerable amount of detail has been given to the effects of a highly explosive material detonation, and how complex it can be. The expansion will be spherical for a small amount of time, but the mine shaft geometry is such that the walls, ceiling, and floor will cause reflections of the blast wave almost immediately. These reflections will intersect, interact, and considerably amplify the pressure. Since the numerical scheme only considers reflections off of surfaces in a single dimension the majority of these interactions are missed, and have to be otherwise accounted for. A specific model for the detonation was developed based on three-dimensional high fidelity calculations of separate detonations. The model determines how much energy the high fidelity calculations predicted would be released by the detonation based upon certain input parameters such as charge weight and explosive composition. The two cells of the mesh where the burst occurred are flooded with the predicted energy and the calculation commences. At this point, the numerical solution is able to run its course and solve the governing equations. By inclusion of the energy equation in Eq. 16, the steep energy gradient is progressively smoothed out as the energy dissipates throughout the cells of the mesh. This transient process is illustrated by Figure 11, which shows the energy decay along the mesh length at each time-step of the calculation. The node distance refers to the shaft or tube length, where the burst point was located mid way through the total length. The initial energy is quickly dissipated initially and then more gradually so as the calculation progresses. Had the solution been run to a much longer time, the energy would have eventually reached equilibrium again at the ambient conditions. The ripples propagating in both directions along the mesh length are the detonation waves, preceded by normal shocks.

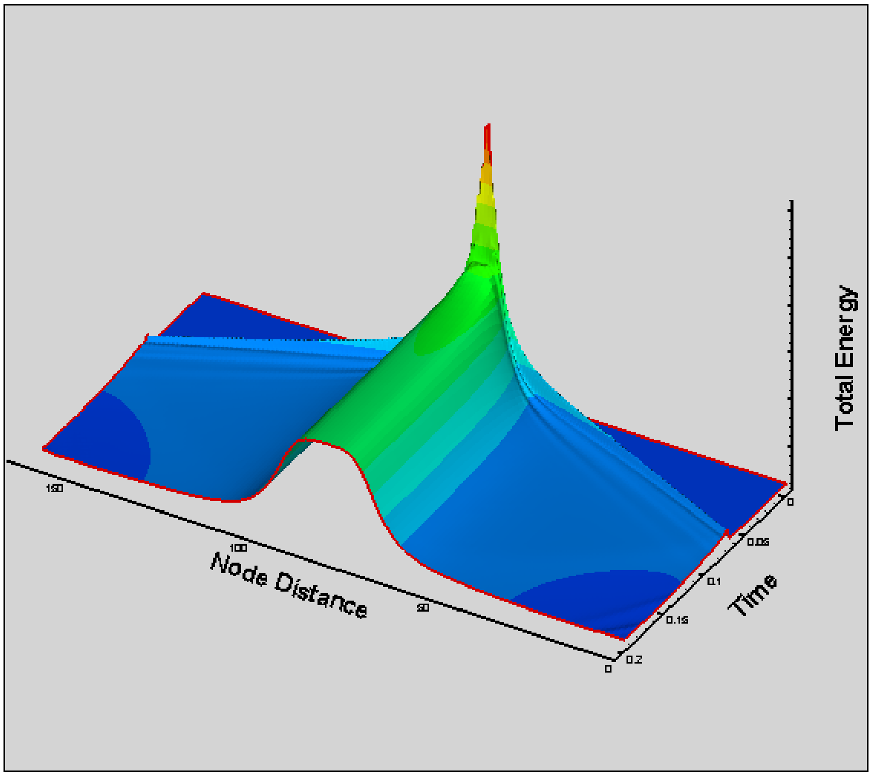


Figure 11) Transient Energy Decay in 1-D Airblast Model

At a discontinuity, the means to handle the one-dimensional airblast model's shortcoming is not so elegant. Take an abrupt area expansion for example, shown in Figure 12. If a test is modeled as a long tube connected to a chamber, the interface between these two distinctly different geometries is effectively a discontinuity. The second section, the larger volume portion, would physically be more accurate if modeled with two-dimensional airblast propagation. Unfortunately the mesh is one-dimensional and so these cells are evaluated just like those in the first segment, the long skinny one. The only treatment afforded this different geometry is that the state variables are adjusted in a volume averaged manner. This allows for the intuitive pressure decrease upon expansion, however as shown by Figure 12, the solution can be inaccurate or misleading. The figure implies that the pressure is uniform across the width of the large segment. This is not a bad assumption for relatively slender segments, but if the pressure at the upper or lower boundaries of the wider segment is of importance the one-dimensional airblast model should not be implicitly trusted. Effects of shock diffraction and reflection are not handled accurately in this type of region either, reflections off of surfaces normal to the one-dimensional propagation are modeled, but not along the upper or lower surface of the chamber.

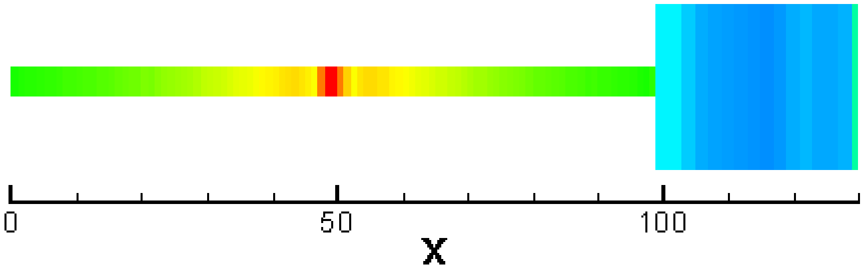


Figure 12) 1-D Airblast Model Behavior at a Discontinuity

3.2. Two-dimensional Model

Imposing the one-dimensional flow assumption on a tube like geometry is natural since the boundary conditions physically enforce such behavior. A confined space with a large aspect ratio, such as a room or warehouse type of geometry cannot take advantage of such convenient assumptions, and can therefore not be solved numerically within the time constraint placed on the airblast models in this research. Predicting the airblast environment in a chamber is therefore accomplished using slightly less realistic and accurate empirical methods. These methods are generally based on relationships derived from test data or high fidelity simulations, and thus lack physical realism in the sense of an accurate solution to all of the thermodynamic variables.

To clarify this point, consider a detonation in the air, far from any reflection planes such as walls or the ground. The unconstrained expansion resulting from such a detonation, termed a free-field detonation, propagates in a predictable fashion. A high fidelity calculation based on physical first principles would solve the governing equations to evaluate the pressure propagation in a physically accurate, and time accurate manner. This approach is computationally intensive, requires an expert analyst, and a large amount of time. If such resources are not available then high fidelity calculations are not viable and another means of airblast effects approximation is necessary. The second approach is generally how empirical models are developed, correctly approximating the detonation behavior, without any physical understanding of why.

The two-dimensional airblast model used in this research is known as the Kingery-Bulmash (KB) method and was built from data recorded at live detonation tests. (Bulmash, Kingery, 1984) There are certain drawbacks to using such a method, however it is a very established airblast model within the blast effects simulation community and extremely fast running. For a detonation in a single chamber the KB method is able to predict the shock pressure as a polynomial function of the scaled distance, where the scaling is governed by the two Hopkinson scaled distances as mentioned in Section 2.7.1. This facilitates quick calculations of the two-dimensional pressure distribution in a chamber as shown by Figure 13

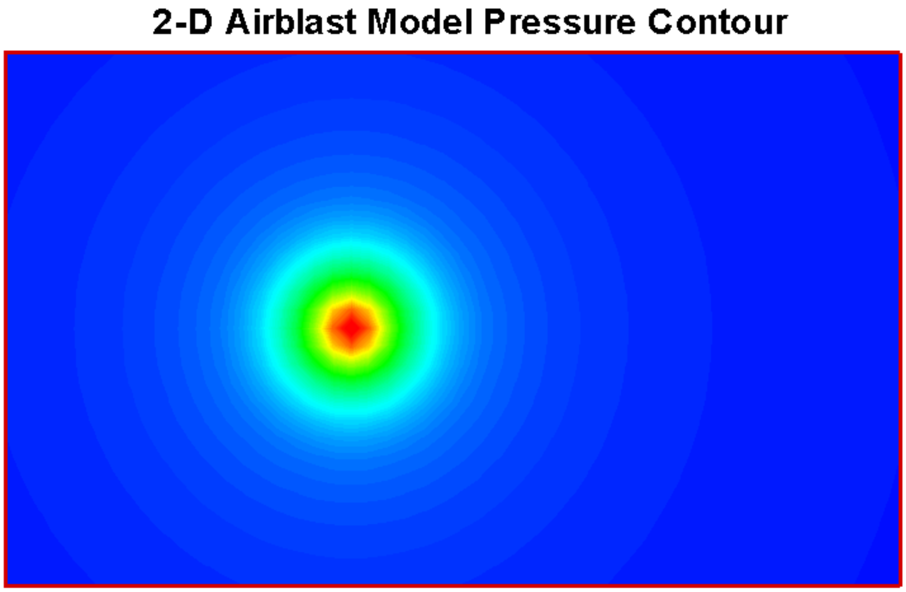


Figure 13) Two-Dimensional Airblast Model Pressure Prediction

Like all empirical methods developed from test data, the approximation is limited by the range of test data and test environment. The KB method is only considered accurate within a finite range, however this range is fairly extensive is not often the limiting factor of this method. The test environment was a free-field ignition event which allowed unperturbed detonation wave propagation in all three dimensions, obviously not the case for a detonation in a chamber. The predicted energy dissipation is very quick as a result. Using the second scaled distance shown by Eq. 11 helps to assuage this quick decay by confining the detonation to two dimensions instead of three. Should the burst be confined in two of the three dimensions however, a tube for instance, the KB method will over-predict the pressure decay as compared to the one-dimensional airblast model. This is very clearly shown in Figure 14, which is a comparison between the existing one-dimensional and two-dimensional airblast models used in this research.

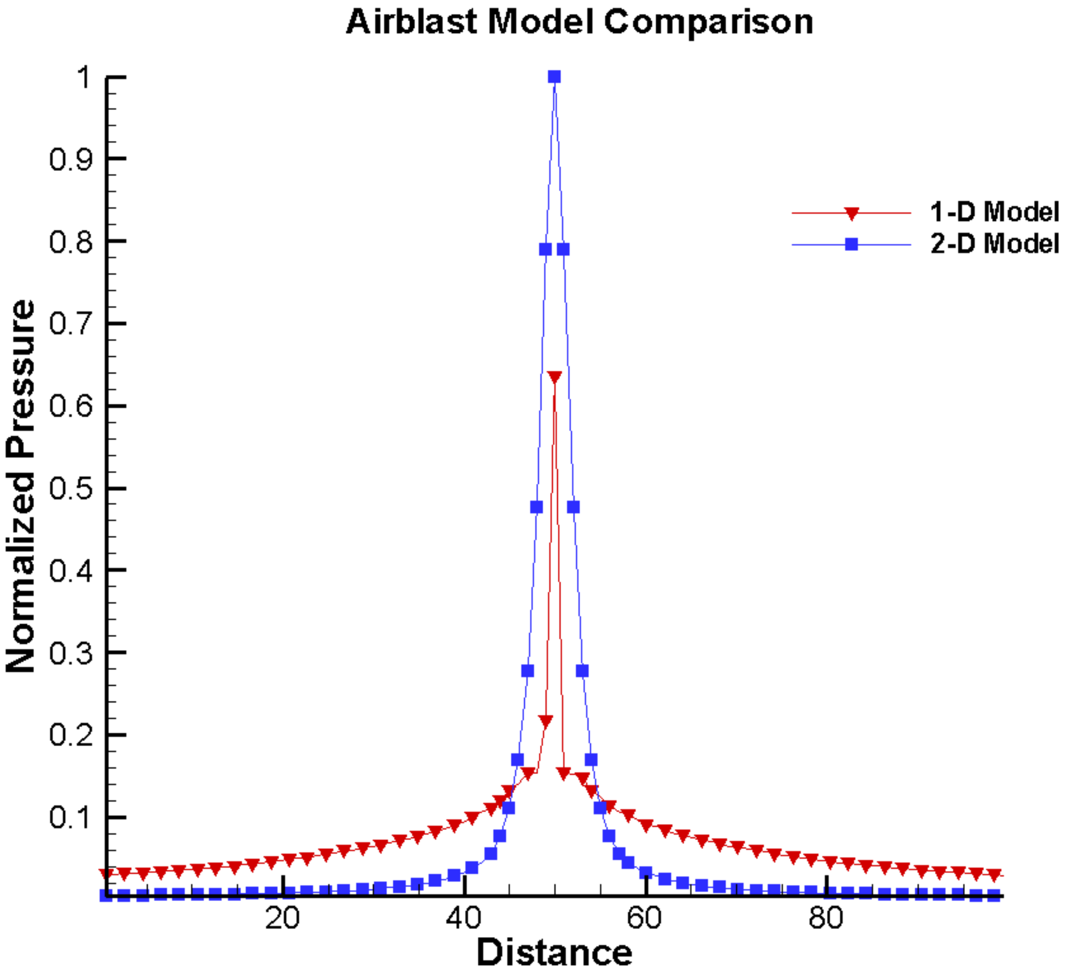


Figure 14) Comparison of Pressure Decay Predicted by 1-D and 2-D Airblast Models

3.3. Charge Mass Interpolation

The two airblast models exist for a specific purpose, which is to take an effective charge of a known mass, and convert it into an energy source that dissipates via pressure propagation. Going from a pressure back to a known explosive charge is functionality that did not previously exist, but was very necessary for the current research. Both of the empirical methods under consideration for modeling a detonation from a tube into a chamber need to transform the pressure at the interface into an effective charge. Each of the two accredited airblast models already implemented has a different process for approximating this energy release and resulting pressure behavior. The process of mapping the output of each airblast prediction model back into an input without any *a priori* knowledge is what is referred to herein as charge mass interpolation.

3.3.1. One-Dimensional Airblast Model Charge Interpolation

Fortunately, the use of the one-dimensional airblast model charge interpolation was very limited. It was initially investigated as a means to dampen out pressure spikes for an equivalent charge detonation just inside a tube entrance. Through the course of study however it was determined that the pressure spikes may not be undesirable as previously thought, negating the need to interpolate a charge mass from a numerical solution to a set of partial differential equations. The process for interpolating the mass was an uncomplicated curve fit that was quick and easy, yet likely prone to error and contingent upon the tube size. Calculations were run with charge masses incremented in a logarithmic progression. The resulting pressure trend was very well behaved, and a curve fit was applied. The curve fit was composed of three expressions that were all quadratic functions. Application of the quadratic formula and selection of the positive root allowed for the energy needed that would cause a known pressure to be solved for. This energy is then translatable into an effective charge, of a known weight. As shown in Figure 15 pressure-charge weight relationship, as well as data points generated independently using the one-dimensional airblast model.

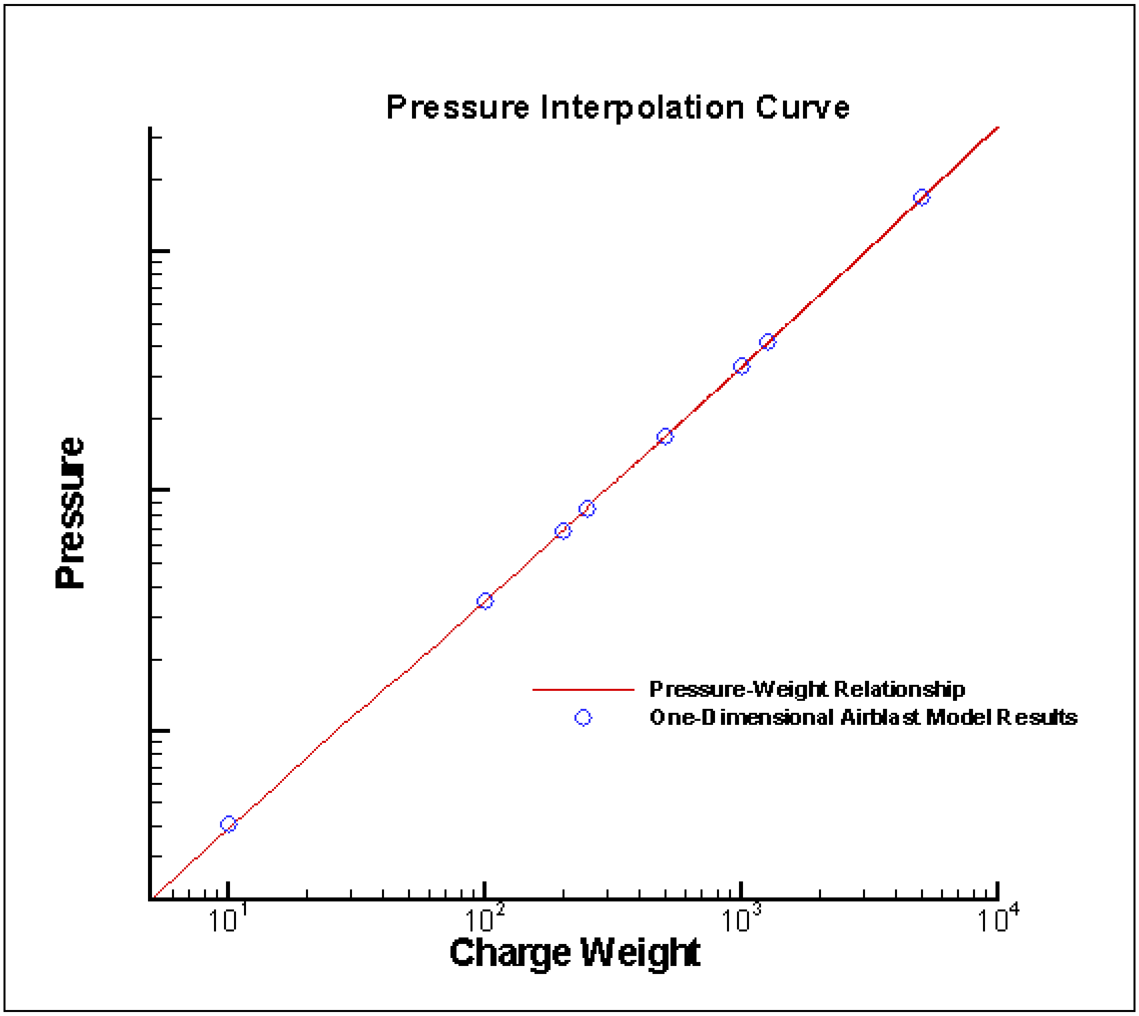


Figure 15) One-Dimensional Airblast Model Charge Weight Interpolation Relationship

3.3.2. Two-Dimensional Airblast Model Charge Interpolation

For shock pressure, the KB curve fit is an eighth order polynomial that is a function of scaled range. Analytically solving for the roots of an eighth order polynomial is unrealistic, meaning that an alternate means for using the KB pressure expression backwards was necessary. A brute force numerical interpolation function was written in C++ to approximate the charge mass that would create a known pressure. The KB polynomial is shown as Figure 16.

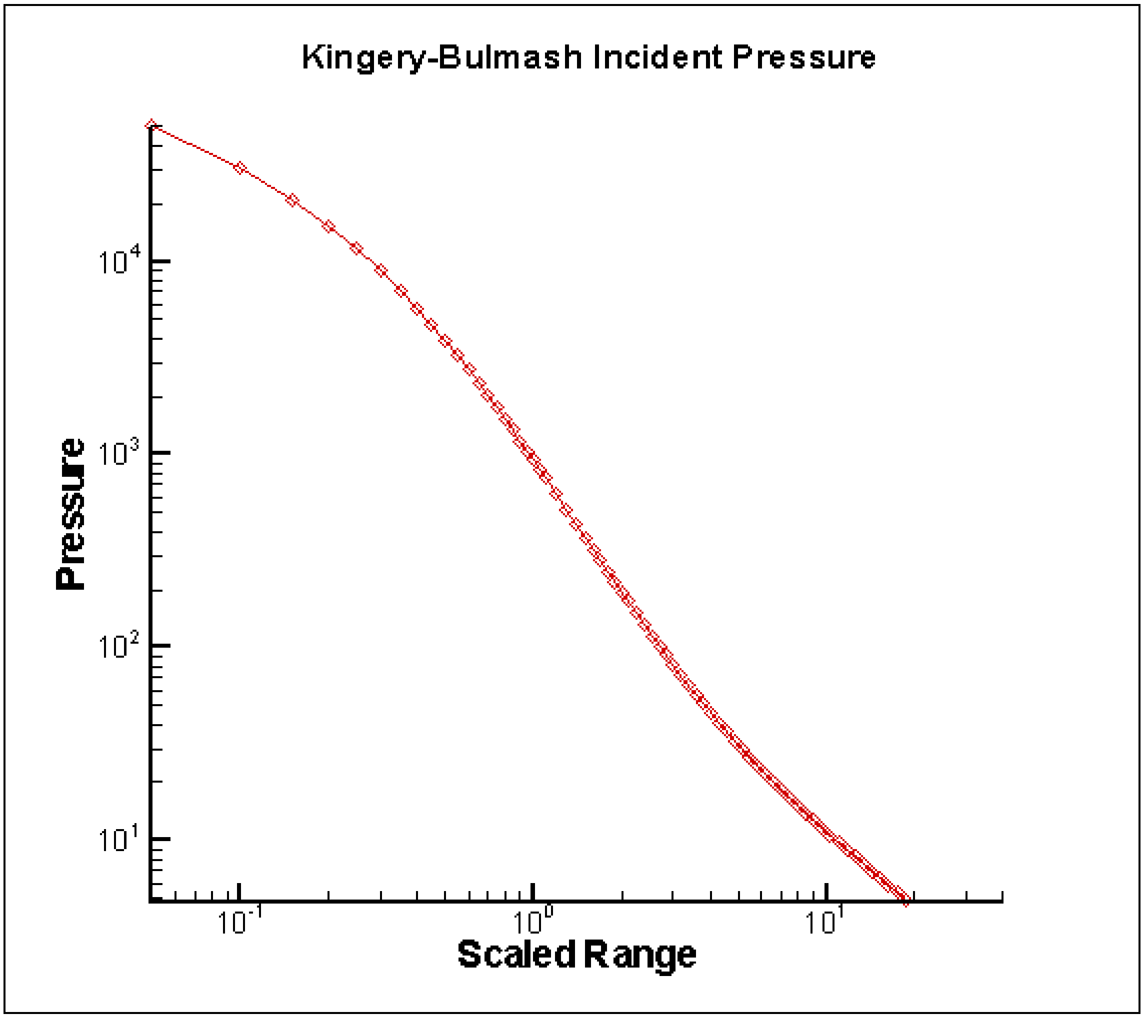


Figure 16) Kingery-Bulmash Pressure Prediction

The interpolation function took advantage that the KB method is only defined over a finite range of data points. The scaled range is limited by a minimum and maximum value between which the solution is predictable and well behaved. This helped to ensure that the correct root of the polynomial was chosen, since an n^{th} order polynomial has *n* roots, and for this application, only one was correct.

The inputs into the charge mass interpolation function were the pressure and the range of the charge from where that pressure was measured. To obtain an equivalent charge mass from the inputs, the numerical solver started at the minimum scaled range and solved for the pressure at discrete scaled ranges. When the pressure obtained for a scaled range was within a reasonable tolerance of the input pressure, the scaled range functions of Eq. 10 and Eq. 11 were used with the input range to solve for the charge mass.

4. Empirical Methods

To most accurately represent the two scenarios discussed in this paper, a detonation in a tube or in a chamber, the airblast phenomena of the respective transitions were studied. Understanding of the physics of the detonation wave amplification and attenuation enabled the selection of four empirical models that were investigated for inclusion into the TRAM prototype.

A certain amount of error in engineering level algorithms is unavoidable since the methods are generally approximations. Since the TRAM prototype will be coupling two engineering level algorithms, with two additional engineering level algorithms the potential for the exacerbation of error was considerable. To minimize the inaccuracy of the final airblast prediction of the coupled algorithms, the choice to communicate between the two airblast models via effective charges was elected. The concepts of blast scaling and charge interpolation are heavily represented within the TRAM methodology, and this section will explain how the various empirical models translate information by way of effective explosive charges using those two concepts.

4.1. Detonation in a Tube

Physically, the only bedrock requirement for this detonation scenario is that the pressure reduction due to the expansion must be modeled as well as the subsequent detonation wave divergence. That pressure loss will be built into an effectively weighted charge governed by one of the methods described in this section. The two-dimensional airblast model will use that effectively weighted charge as an input.

One of the methods imposes the pressure loss by reducing the weight of the effective charge directly thereby limiting the available energy released by the munitions detonation. The other empirical model actually predicts the pressure reduction directly. This loss is manifested by interpolating a charge from a reduced pressure.

4.1.1. Binggeli

Though blast scaling and small scale tests are excellent methods of obtaining empirical relationships to describe blast effects, E. Binggeli took advantage of a much more conventional means of testing and used a shock tube apparatus at the NC Laboratory in Spiez Switzerland. Given that there is no explosive detonation, obviously charge mass, shape, location, and all of the other explosive specific parameters will be absent from this method. The empirical method derived from Binggeli's research revolves around the geometric ratio between the tube and the chamber. This lent very well to inclusion in TRAM since the physical information about the model is readily available. The test setup is that of a standard shock tube arrangement with the addition of a dump tank at the end with gages that recorded pressure along the length of the apparatus. A sketch of the test model is shown as Figure 17.

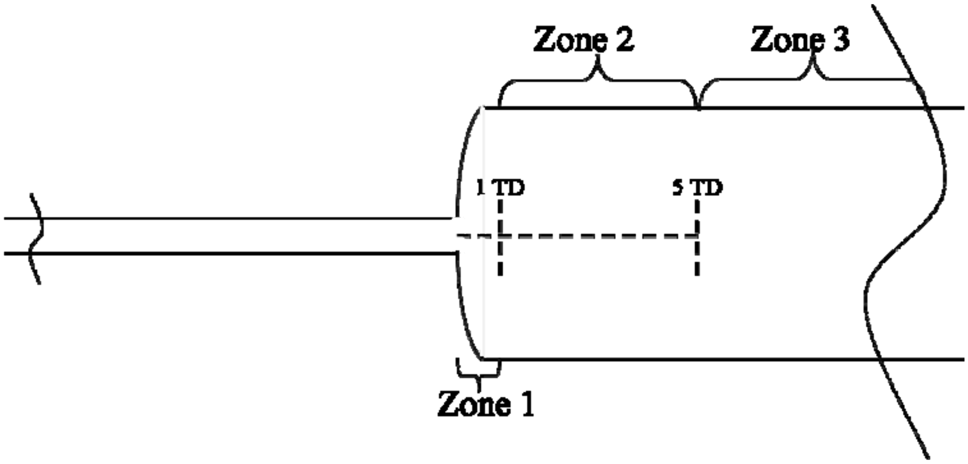


Figure 17) Binggeli Test Setup

The three zones labeled in Figure 17 refer to the areas where Binggeli noticed distinctive behavior. Zone 1 is the area with the most dominant multi-dimensional effects where the expansion of the shockwave dominates the pressure field. Zone 2 is where the shockwave attenuation stabilizes, and then Zone 3 is where the wave settles into one-dimensional propagation as a normal shock respectively. The results from zone 3 are extraneous considering that a meaningful pressure for charge weight interpolation can be obtained much closer to the interface. Due to the highly transient phenomena that dominates the first zone, the empirical relationship was developed using pressure recorded at instrumentation gages in zone 2, meaning that this zone is where it will be most accurate.

The empirical relationship that governs the pressure ratio across the interface was developed as a function of the cross-sectional area ratio between the tube and chamber cross-sections. This relationship is shown in Eq. 18 where A is used to denote cross-sectional area and α and β are numerical constants. The function is then plotted in Figure 18. (Binggeli, 1985)

$$\frac{P_{Chamber}}{P_{Tunnel}} = \alpha \left(\frac{A_{Chamber}}{A_{Tunnel}} \right)^{\beta} \qquad \text{Eq. 18}$$

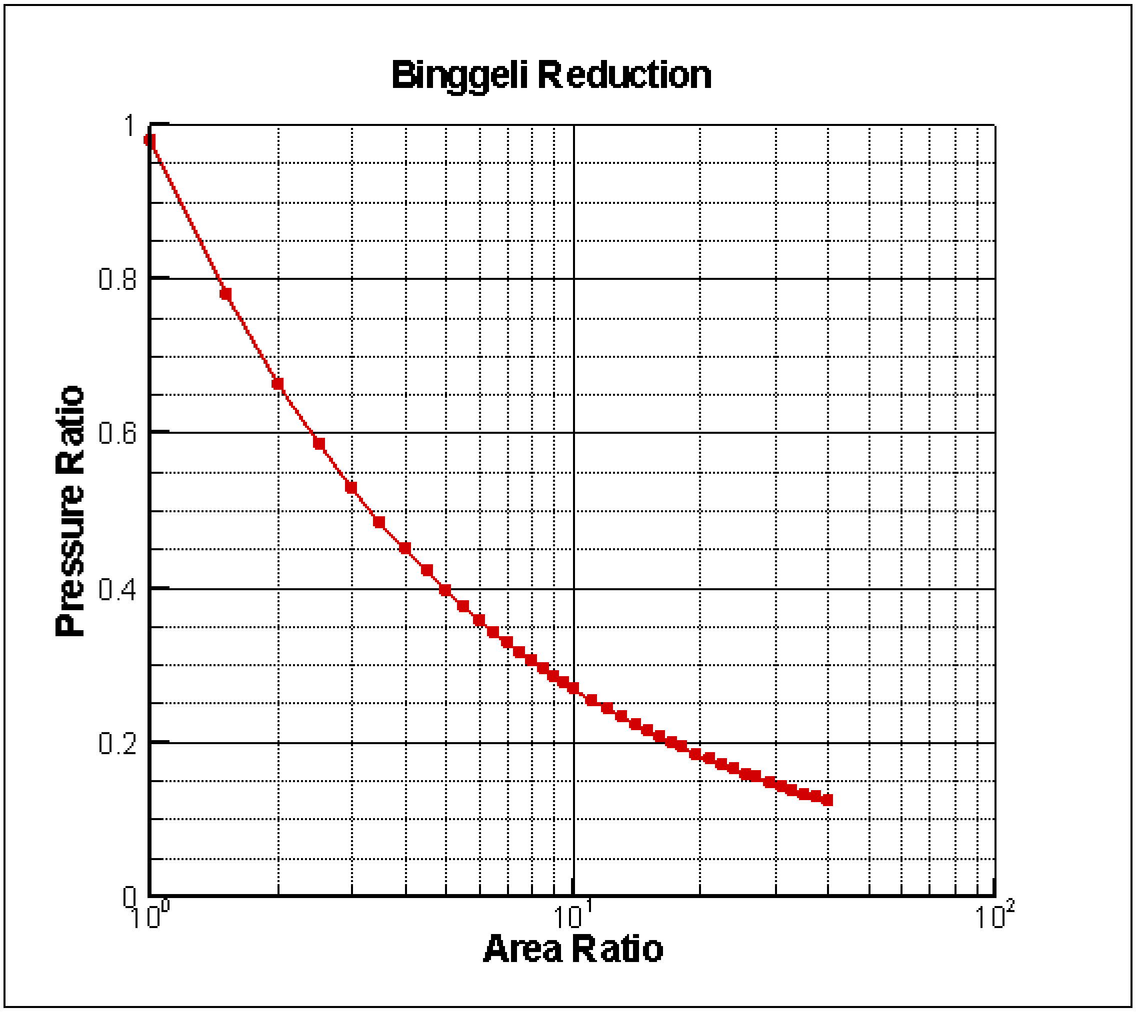


Figure 18) Binggeli Pressure Reduction Model

In order to implement this empirical method in terms of both accredited airblast models in this study, the pressure at the last gage of the tube before the interface is recorded and defined as the tube pressure. The model information is already available to the TRAM prototype so calculation of the area ratio is trivial. Using the Binggeli reduction model of Eq. 18, the chamber pressure is calculated at a point one tube diameter (TD) away from the interface inside the chamber. The effective charge weight placed at the tube-chamber interface can now be approximated using the KB interpolation methods outlined in Section 3.3.2. This interpolated explosive charge is such that the correct pressure as defined by the Binggeli reduction model can be achieved by the detonation of the charge at the interface by the two-dimensional airblast model that is native to chamber bursts. Table 2 summarizes the individual steps of the Binggeli process.

Table 2) Steps of the Binggeli Reduction as Implemented into TRAM

1)	One-dimensional airblast model predicts in-tube effects of detonation in a tube
2)	Eq. 18 is applied to the pressure calculated at the last gage on the tube side of the interface
3)	The reduced pressure is turned into an effective charge via interpolation as discussed in Section 3.3.2
4)	The effective charge is placed on the chamber side of the interface
5)	The chamber airblast effects are predicted by the two-dimensional airblast model from the effective charge

The most significant drawback of this method is that there is no representation of blast effects. For future model development if quasi-static pressure is to be considered, this model may fail to accurately represent the total blast environment. There is also a limitation imposed by virtue of the fact that the model was developed using a series of similarly structured tests. The area ratios were obviously changed, but the tube portion of the test set-up always joined the chamber at the same location and angle. Applying the pressure reduction developed by Binggeli to a geometry that it has not been designed for may have unintended consequences or inaccuracies.

4.1.2. Potter-Hikida-Needham

The two-dimensional airblast model in this study is based off of a data fit to experimental test data of free-field ignition events. As Section 3.2 explained, the pressure decay measured by the test data could be predicted by a function of the charge weight and distance to the charge location. This prediction has been proven effective as long as the point of interest for a pressure value has line-of-sight to the charge detonation location. Areas inside of a structure obstructed by surfaces necessitated extra considerations. To account for detonation wave diffraction through an opening and into another chamber a method was developed by the team of Needham, Potter, and Hikida (NPH).

What makes this method important for consideration in the TRAM prototype is that the means by which the shock pressure is transported from chamber to chamber throughout a facility using the two-dimensional airblast model is with effective charges. These charges have been reduced to account for the energy lost by the shockwave sustained by diffracting through an opening. The propagation of shock pressure by the NPH model is shown by Figure 19. The blue circles represent points of interest where the pressure measurement is desired. In order to propagate pressure to all six points, a charge of reduced mass is moved into the opening of each chamber this way line-of-sight can be established to all charges. The weight is reduced according to the NPH model at each opening, where each subsequent detonation is less energetic than the one preceding it. *M* refers to equivalent charge mass, *R* is the range of the center of the charge to the center of the opening shown in Figure 19, and θ is the angle as used in Eq. 20.

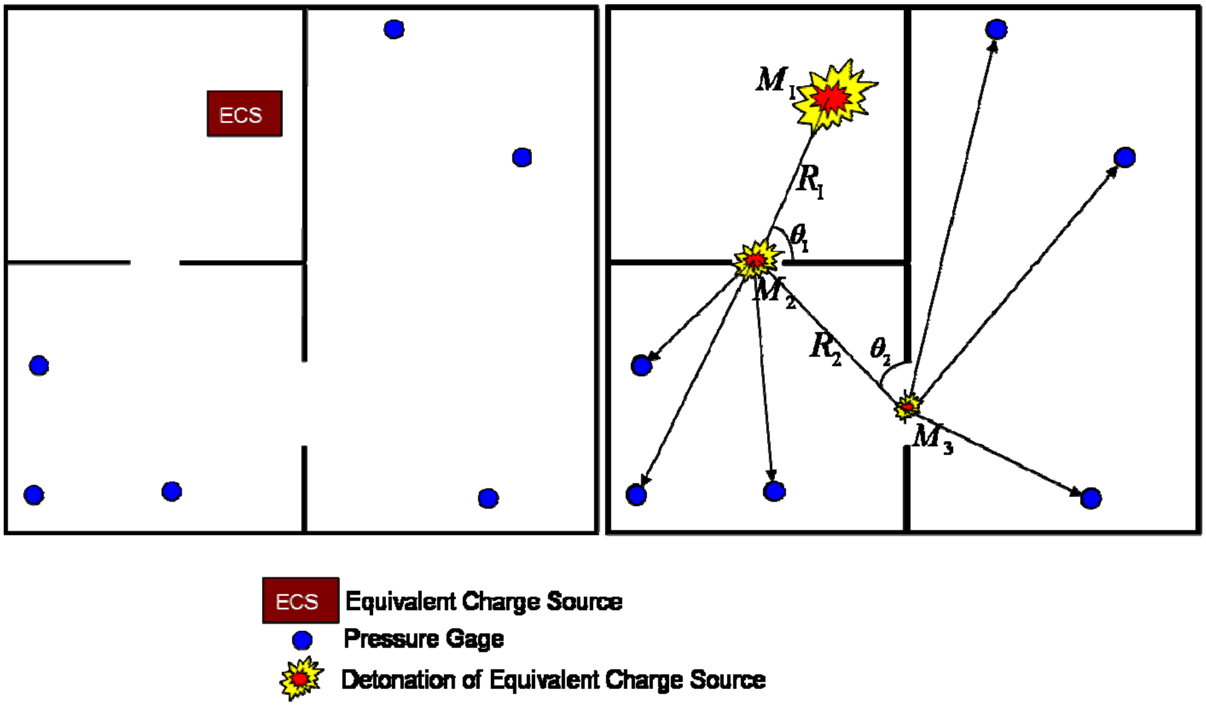


Figure 19) NPH Model Shock Propagation between Chambers

Development of this method did not come directly from live test data, but high fidelity numerical solutions. The authors used a second order accurate hydrodynamics code to evaluate the amount of energy that would be lost to detonation wave diffraction as a function of the opening area, the charge range and angular offset relative to the opening. Contrary to what might be expected, this empirical method is not being considered as a candidate for the TRAM prototype for a detonation in a chamber, despite the fact that the two-dimensional airblast model already uses it. The physical environment at the tube-chamber interface needs to be considered. For a room to room burst for which the method was designed, the detonation wave will be diffracted across the opening losing some of its energy but will not undergo the reflections that amplify the pressure inside of a tube from a chamber burst. For propagation out of a tube and into a chamber, the detonation wave will lose energy as described in Section 5, and this loss will be imposed by the NPH method.

Like the Binggeli model, the pressure at the last gage of the tube before the interface is integral to defining the loss incurred by expansion into the chamber. In contrast to the previous method though, the NPH model imposes a loss on an equivalent charge, not on a pressure measurement directly. This means that the tube pressure as calculated by the one-dimensional airblast model on the tube side of the interface will not be reduced, but used to interpolate an effectively energetic source. The NPH method then utilizes the next two equations to model the loss of energy on that interpolated equivalent charge for pressure propagation through an arbitrary opening in a wall. A is the area of the opening and R is the range of center of the charge to the door opening. (Hikida, Potter, Needham, 1999)

$$W_{red} = \frac{WA}{2\pi R^2}\Phi \qquad \text{Eq. 19}$$

$$\Phi = \frac{P + P_d \cos(\theta)}{P + P_d} \qquad \text{Eq. 20}$$

The only dependence on direction in this model is the result of Eq. 20, which is essentially a weighted ratio denoted as Φ. P is the calculated overpressure at the opening on the detonation side, and P_d is the dynamic pressure on the opening side. Figure 20 illustrates the effect of the angle θ in a potential test layout which would affect the pressure field in the chamber. Losses that would be suffered due to diffraction that does not symmetrically propagate out of the tube are taken into account using this term.

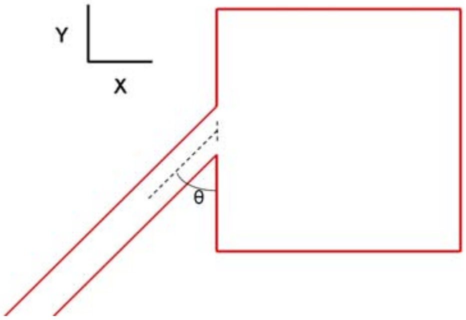


Figure 20) Offset Tube-Chamber Interface

Though the NPH methodology has successfully applied this model for a detonation in a chamber, work was necessary to ensure that the model would still work on the test setup of present concern, which is the propagation from a tube into a chamber. The model imposes a loss of energy via an equivalent charge, but the loss was designed for a different situation. Whereas this may be convenient, it may not be as accurate as a method developed specifically for airblast effects across an abrupt expansion. Table 3 demonstrates the NPH method in a step by step progression.

Table 3) Steps of the NPH Method as Implemented in TRAM

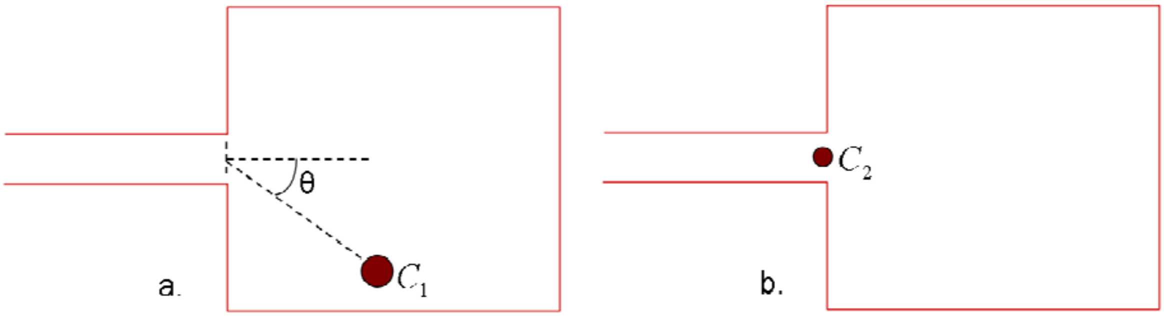
1)	One-dimensional airblast model predicts in-tube effects of a detonation in a tube
2)	The tube pressure at the last point before the interface is turned into an effective charge via interpolation as discussed in Section 3.3.2
3)	Eq. 19 is applied to the interpolated charge mass, including any angular effects from the dynamic pressure as dictated by Eq. 20
4)	The effective charge is placed on the chamber side of the interface
5)	The chamber airblast effects are predicted by the two-dimensional airblast model from the effective charge

4.1.3. Detonation in a Tube Methods Comparison

The two methods proposed for enforcing the pressure reduction across the tube-chamber interface model the reduction in two different ways. This makes a direct comparison between the two impossible. Figure 18 was included to gauge the behavior of the Binggeli model which is easy to predict, since it is only a function of the chamber cross-sectional area as compared to the tube cross-sectional area. The NPH model does not allow for such a comparison. Not only is the cross-sectional area of the chamber absent, but the method is a function of the incident pressure as well as the dynamic pressure. The quantification of the effects of both models will be included in Section 5.1 when the results of airblast predictions for a detonation in a tube are presented.

4.2. Detonation in a Chamber

This second detonation scenario must accomplish the transition of multi-dimensional airblast effects into a single dimension where a one-dimensional numerical method will propagate those effects throughout the rest of the tube geometry. The methods must determine a relationship to quantify the pressure amplification that happens when an expanding detonation wave is confined into a single dimension. Both of the empirical methods presented in this section apply the same general methodology for the airblast translation that is intended for inclusion into the TRAM prototype. A charge at a known location is manipulated to match a prescribed condition in the tube entrance and reduced accordingly, that reduced charge is then detonated within the tube. Figure 21 illustrates the theory, which is actually quite common within the airblast prediction community. The two methods as described in this section are distinguished by different charge reductions and varying reliance on charge location parameters such as angular offset and range.



a) Initial Explosive Charge ***b) Reduced Explosive Charge***

Figure 21) Effective Charge Placement Illustration

4.2.1. Scheklinski-Glück

A researcher named Scheklinski-Glück at the Ernst Mach Institute in Switzerland used a series of small scale tests to develop a relationship to predict the pressure time history at a specific location inside of a long tube that would be induced by the detonation of a charge a known location relative to the tube entrance. This method will be referred to as the EMI method.

It is immediately obvious upon inspection of the test set-up used by Scheklinski-Glück as illustrated in Figure 22 that both the range from the tube entrance and angular offset from the entrance plane were of significance. The charges were detonated between one and five TD away from the entrance, on radial lines conforming to angular offsets of 0°, 30°, 45°, 60°, and 90°. The 90° test cases measured the effects of side-on pressure in the tube, which is not currently modeled in the TRAM prototype, therefore this test series was not considered.

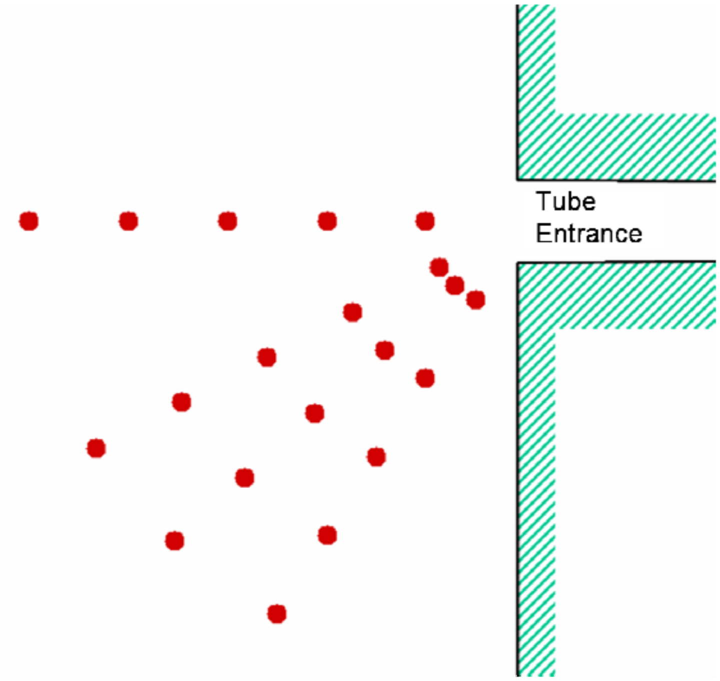


Figure 22) EMI Test Setup

The first gage inside the tube was thirty TD's away, signifying that the area just inside of the entrance was not of imminent importance. This distinction meant that the EMI methodology could decouple the in-tube airblast effects from the free-field airblast effects, specifically citing the detonation wave behavior. (Scheklinski-Glück, 1993) Much like the airblast prediction model of Section 3.1, the EMI methodology treated the detonation wave propagation down the tube in a one-dimensional manner. Previous EMI research had developed an empirical relationship concerning a detonation in a tube and the decay of the pressure as the wave traveled along the tube length. This led to the familiar approach of turning the outside detonation at a known angular offset and distance into a charge of reduced mass that was moved into the tube and then detonated according to the existing empirical one-dimensional airblast model.

The reduced mass charge that is moved into the tube is calculated using Eq. 21 where the pressure ratio $\frac{P_{out}}{P_{in}}$ is calculated using Eq. 22. X is defined in Section 2.7.3 as Eq. 13 and D denotes the tube diameter. (Scheklinski-Glück, 1993) In both of the equations, the symbols α, β, τ, and Γ represent numerical constants.

$$W_{red} = W\left(\frac{P_{out}}{P_{in}}\right)^{\Gamma} \qquad \text{Eq. 21}$$

$$\frac{P_{out}}{P_{in}} = \alpha\left(X^{\beta}\right)e^{\tau\theta} \qquad \begin{array}{c} D \le X \le 5D \\ 0^{\circ} \le \theta \le 60^{\circ} \end{array} \qquad \text{Eq. 22}$$

One of the benefits of using such a method that acts directly on the charge weight is that there is no interpolation necessary. Initially there was concern about how the one-dimensional versus the two-dimensional airblast models handled the energy release from a detonation. Inspection of the results as presented in Section 5 suggests that the discrepancy is immaterial as the weight reduction yields intuitive pressure amplification.

While the EMI method holds great merit, it is not without disadvantages. Since it was developed from test data the accuracy outside of the data bounds is not known. This limits this empirical models applicability in two important respects. One is that predictions for excessively large charges could potentially be inaccurate, or at least necessitate special treatment. The second important limit of this method is the rather constricting detonation location. A realistic detonation will likely not occur on the tube-chamber centerline, so immediately the usage is limited to the angular offset range. Should the charge detonate beyond 60 degrees off centerline, the accuracy of this model cannot be guaranteed.

4.2.2. Britt-McMahon-Patterson

The U.S. Army Engineer Research and Development Center (ERDC) conducted a series of small scale experiments to aid in the development of a relationship that characterizes the behavior of airblast effects inside of a tube created by the detonation of an equivalent charge outside of the entrance. The research was very similar in nature to that of Scheklinski-Glück in that the pressure inside of the tube was simulated by placing an effective charge in the entrance that was derived from the external detonation. Britt, McMahon, and Patterson identified several areas where they felt the EMI tests fell short including test gages closer than thirty TD’s down-tube, and a scaled range that is a function of the charge mass, as denoted by Eq. 12. (Britt, McMahon, Patterson, 2004)

Empirical Methods

After conducting the 1:17.8 scale tests, an empirical method was developed to create a reduced mass effective charge at the tube entrance from a charge located outside. The relationship derived could not be validated against the test data since the test data is what the empirical method was built from. Therefore high fidelity hydrocode simulations were performed that mimicked the tests and the empirical relationship agreed very well. (Britt, McMahon, Patterson, 2004) The relationship is shown by Eq. 23 as a function of *X,* the BMP scaled distance shown as Eq. 12. W and W_{red} are the charge mass and reduced charge mass respectively. (Britt, McMahon, Patterson, 2004) In, Eq. 23 the symbols α, β, and Γ represent numerical constants.

$$W_{red} = \frac{W}{1.25}\left(e^{\alpha X} + \Gamma e^{\beta X}\right) \qquad \text{Eq. 23}$$

In the presented form, Eq. 23 only applies to detonations on the tube-chamber centerline axis. Additional treatment is necessary for off-axis charge locations. Define the following parameters:

$$r_{\min} = \frac{1}{2}D - r_{\text{charge}}$$ Where r_{charge} is the charge radius

$$D = 2\left(\frac{A}{\pi}\right)^{2}$$ Where A is the tube cross-sectional area

$$F = \frac{W_{red}}{W}$$ Common notation for charge reduction ratio

For angles relative to the centerline, *F* would be calculated by Eq. 23 and then post-treated according to the rules in Table 4. The distance outside of the entrance is denoted as *R, θ* is the offset angle as illustrated in Figure 21, and is measured in radians. The effective charge is then calculated using Eq. 24. (Britt, 2008) For each of the relations in the following table the Greek letters α, β, Γ, τ, and ζ all represent different numerical constants.

Table 4) Angular Reduction of Effective Charge

Distance	Angle (rad)	Reduction
$r_{\min} \le R \le D$	$\theta \le 1.3$	$F = 1 - \alpha \dfrac{R - r_{\min}}{D - r_{\min}}$
$R \ge D$	$\theta \le 1.3$	$F = 1 - \alpha\theta$
$R \le 5D$	$\theta > 1.3$	$F = \dfrac{1}{1.25}\left(e^{\alpha X} + \Gamma e^{\beta X}\right) - \xi \dfrac{(\theta - \tau)(5D - R)}{D}$

$$W_{red}(\theta) = FW_{red}(0) \qquad \text{Eq. 24}$$

There is only one potential drawback to using the BMP method over the EMI method, and the drawback is only applicable for detonations that are offset from the interface. The rules of Table 4 for the angular offset treatment are not the result of ERDC testing, which was composed of tests that were performed on the interface centerline. It is instead an application of the actual EMI test data for offset charges in conjunction with other tests performed by various researchers, and then smoothed at the interface. (Britt, McMahon, Patterson, 2004) This detail is included more in the interest of full disclosure, not an attempt to discredit the BMP method.

4.3. Detonation in a Chamber Empirical Methods Comparison

Unlike the two methods for a detonation in a tube, the BMP and EMI methods are similar enough to be compared directly to estimate how they will affect the airblast prediction. Figure 23 shows the BMP and EMI methods for a standard detonation along the tube-chamber centerline for various ranges. The effects of angular offset will be shown in Section 5.2.

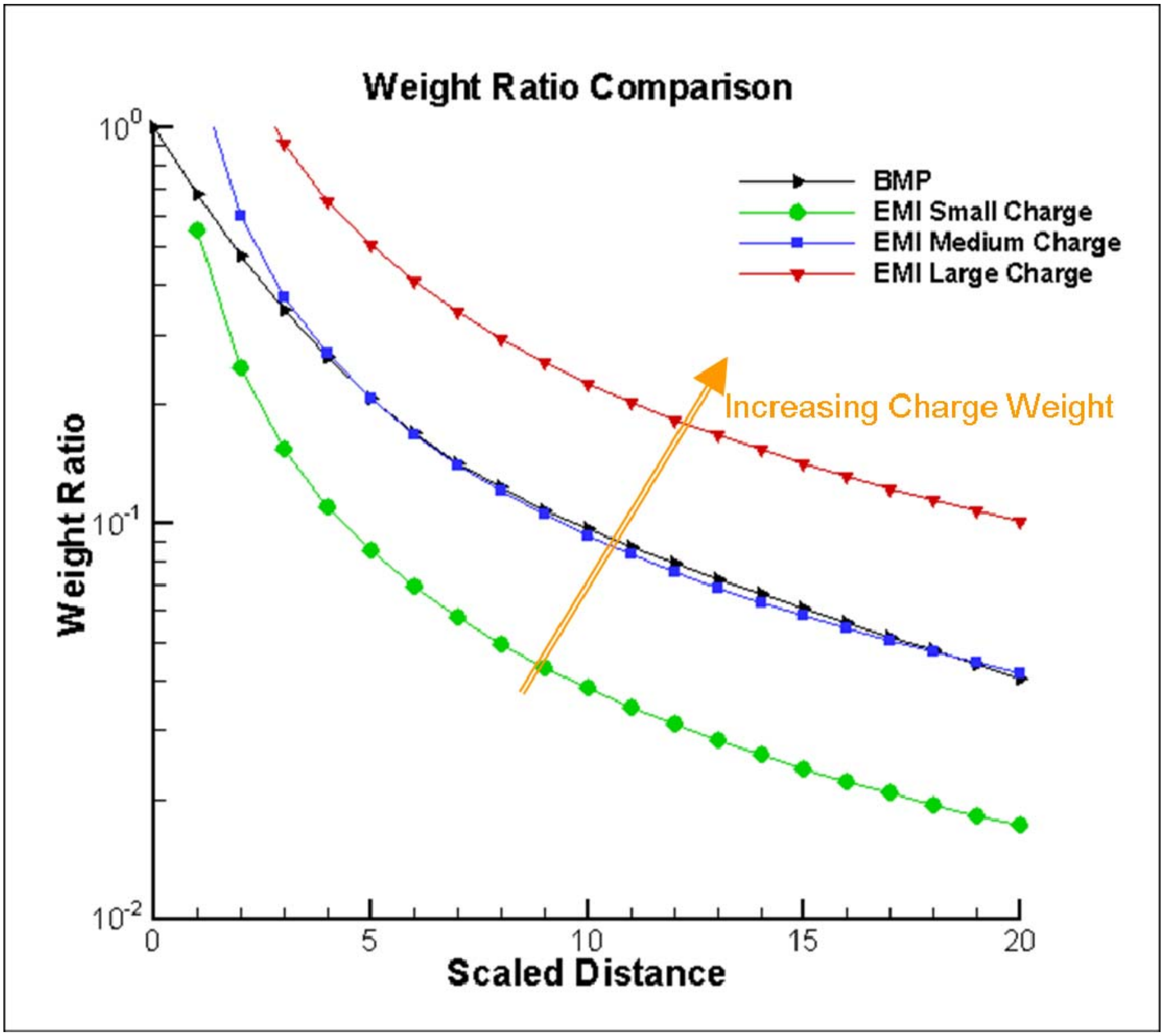


Figure 23) Comparison of BMP and EMI Models

The general behavior of each model is very similar in that an exponential decay is enforced for increasing scaled range. A major difference between the two however is the fact that the BMP method uses a scaled distance that is a function of charge weight, while EMI does not. This is why the plot in Figure 23 shows three curves for the EMI method, and only one for BMP. The trend for the EMI method is that the weight ratio increases per scaled range for an increased charge weight, as denoted by the arrow in the plot.

5. Results

A test setup as shown in Figure 24 was used for both detonation scenarios. The two existing airblast models were each run to predict airblast in the hybrid geometry to illustrate their current capabilities. The appropriate TRAM methods were then applied to illustrate the effect on the overall solution the new functionality would add. Ideally the TRAM data should match the one-dimensional airblast model prediction in the tube, and the two-dimensional airblast model in the chamber.

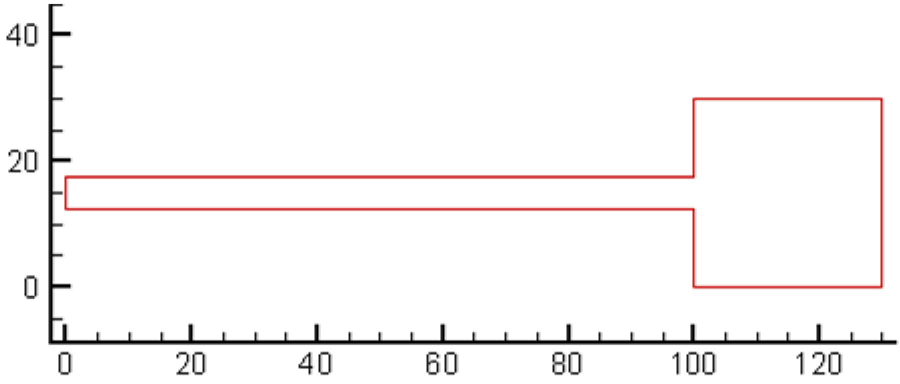


Figure 24) TRAM Prototype Test Layout

5.1. Detonation in a Tube

A representative set of plots is included in this section that exhibits the general behavior of the Binggeli and NPH reduction methods. The parameters varied were the size of the charge that detonated in the tube, and the cross-sectional area ratio between the tube and chamber.

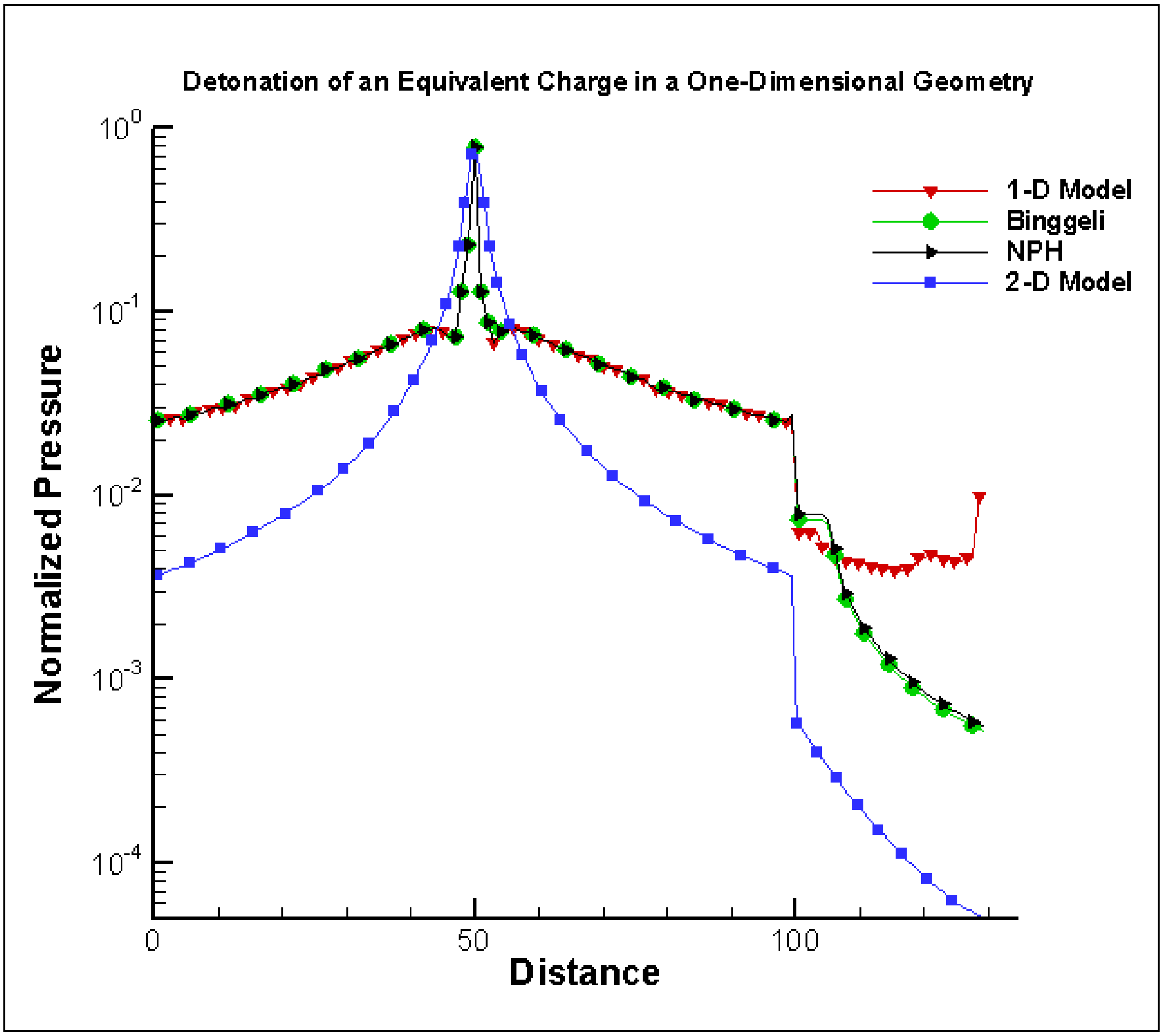


Figure 25) Detonation in a Tube Pressure Plot

The particular simulation shown in Figure 25 was the pressure reduction of a detonation of TNT at 10 TD's down the tube. The most striking disparity of the plot overall is the drastic difference between the one-dimensional model prediction in the tube versus the two-dimensional model in the tube. The very rapid decay of pressure predicted by the two-dimensional model makes sense considering the type of burst for which it was designed. For a detonation in a tube, the expansion is confined to one dimension, which leaves only one direction for energy dissipation. This is in contrast to the two-dimensional model, which assumes that the burst is not confined and will lose energy in two or three dimensions. This poor prediction is exactly why a model such as TRAM is needed.

Aside from the expectedly poor performance of the two-dimensional airblast model in the one-dimensional environment, Figure 26 shows a close-up of the area of interest, from the interface into the chamber. The increase in pressure at the end of the one-dimensional model's prediction is actually a reflection off of the chamber wall.

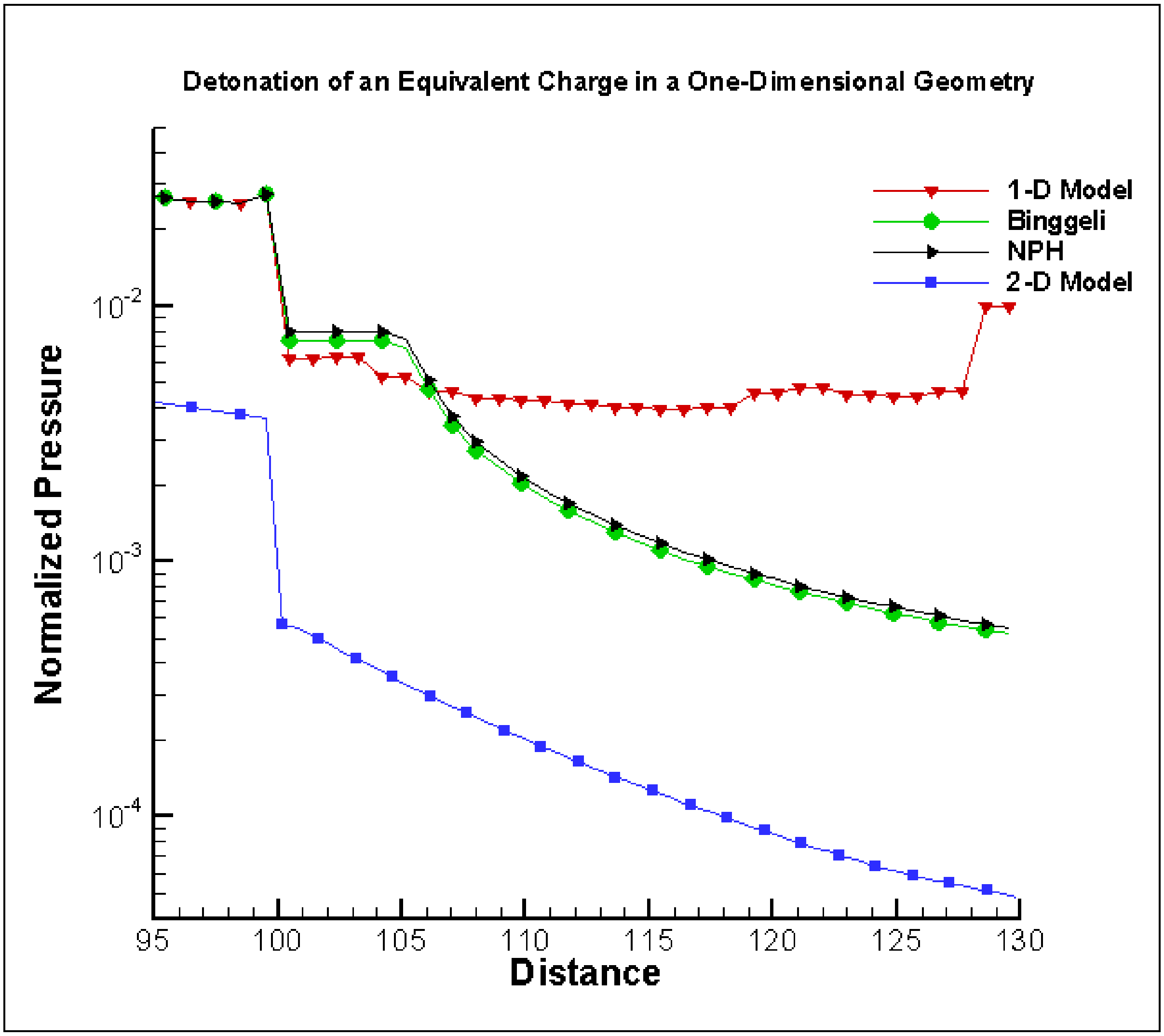


Figure 26) Semi-Log Plot of Interface and Chamber of a Detonation in a Tube

The TRAM prototype implementation with either the NPH or Binggeli model performed better than the one-dimensional model across the interface. This judgment stems from the comparison of TRAM's pressure decay to the two-dimensional model pressure decay.

An important thing to consider concerning the actual pressure values in the previous plots is that the overall pressure at the interface from the two-dimensional model is low to start with due to poor performance in predicting the pressure propagation along the tube. Both of the empirical models closely estimated the drop in pressure due to the expansion just as well as the two-dimensional model. The percentage reductions for the test case presented in the figures above are summarized in Table 5. The initial reduction refers to the drop across the interface, chamber decay refers to the pressure lost in due to increasing distance from the charge, and the total percent reduction is a measure of the total reduction incurred between the last tube pressure and the last chamber pressure. The percent *increase* in the chamber decay for the one-dimensional model is due to the reflection off of the chamber wall.

Table 5) Pressure Reduction by Percentage

	Initial % Reduction	**Chamber Decay % Reduction**	**Total % Reduction**
1-D Model	77.19594112	-59.80195538	63.558668
2-D Model	84.36692506	91.4600551	98.66494401
Binggeli	73.09989996	90.31983849	97.39602687
NPH	71.03615835	90.48652916	97.24453337

To graphically represent the point being made by Table 5, another set of comparisons are included where the pressure at the interface is uniform for both the one-dimensional model and the two-dimensional model. In order to match the interface pressure, a much larger charge was detonated in the tube for the two-dimensional calculation as shown in Figure 27.

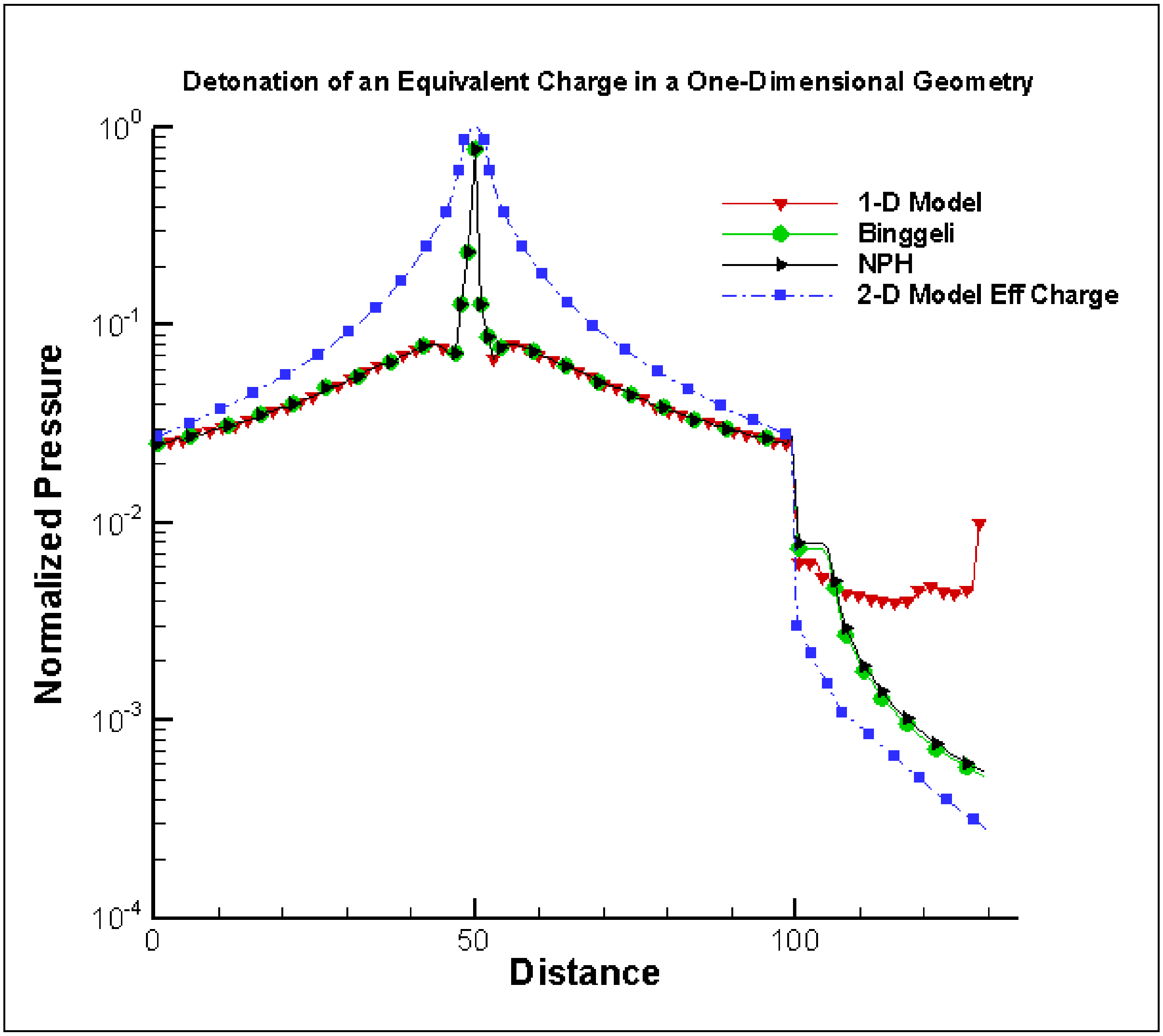


Figure 27) Increased Charge Weight used for Interface Pressure Matching Condition

Figure 28 is a closer view of the area affected by the pressure shift, which is the area at the interface and into the chamber. The pressures predicted by the two-dimensional airblast model are much closer than the previous figures would indicate. When plotted on a standard *Y* axis, the two lines representing the pressure decay of the TRAM prototype cleanly collapses on the line predicted by the accredited two-dimensional airblast model, shown by the figure below.

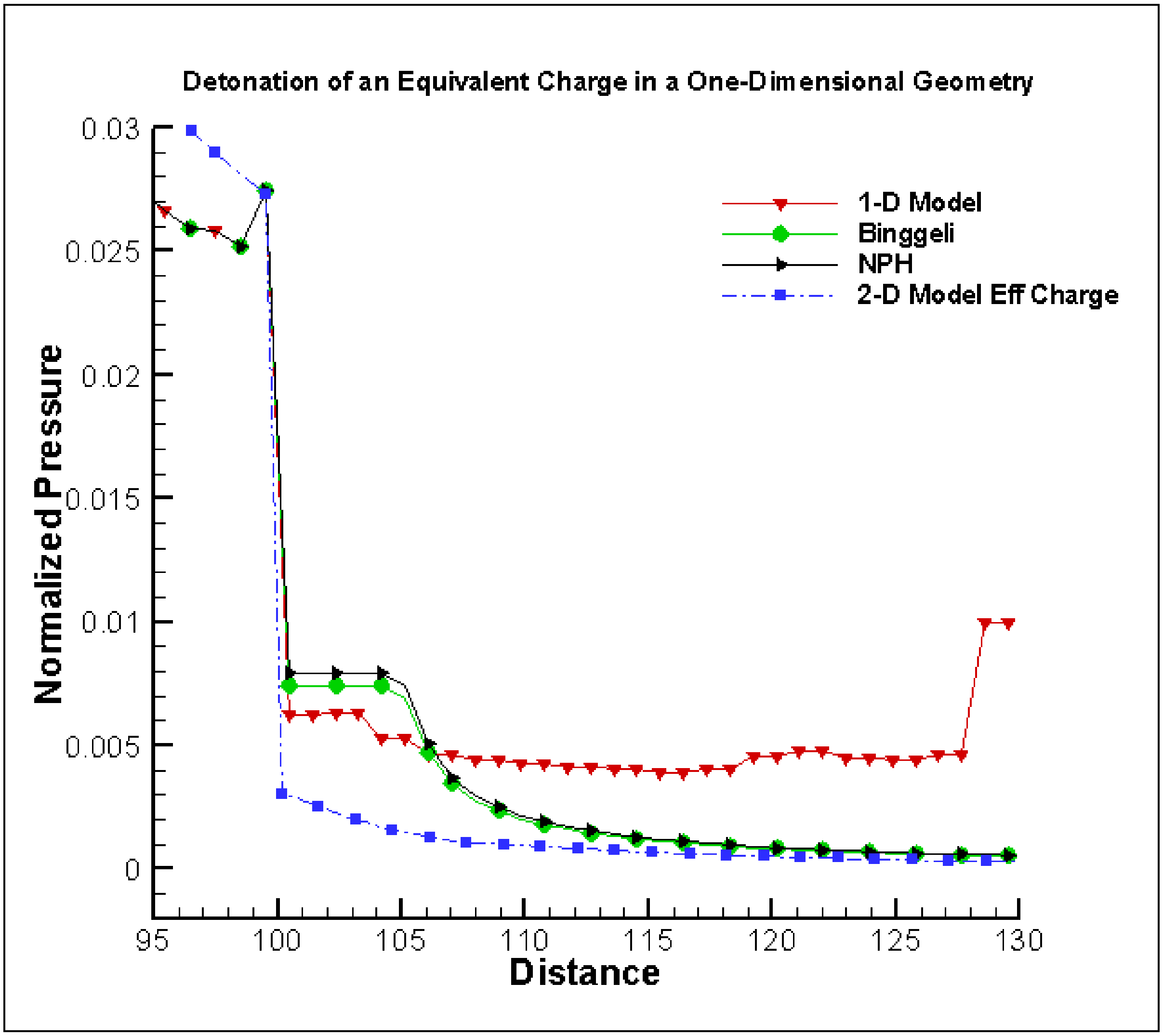


Figure 28) Pressure Decay Comparison on a Normal Axis

Beyond simply matching the pressure decay in the chamber better, there is another obvious reason that the one-dimensional model is inferior past the interface, which is the fact that it does not vary along the width of the chamber. A superior overall solution is obtained using the TRAM prototype where the one-dimensional model predicted the airblast effects in the tube, while one of the empirical models reduced the weight for the two-dimensional model to take over at the interface. In addition to more closely matching the native model, the two empirical models both allow for pressure propagation in two dimensions as illustrated in Figure 29. The contours in the figure are of normalized incident pressure resulting from a detonation inside of the tube midway along the length in the x-direction.

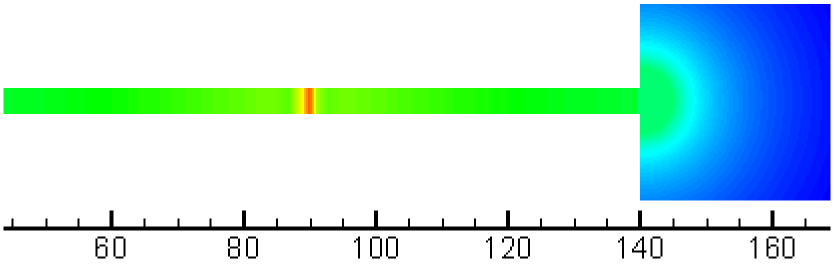


Figure 29) Hybrid Model Normalized Pressure

5.2. Detonation in a Chamber

The same approach for evaluating the quality of the results for a detonation in a tube was utilized to measure the performance of the TRAM prototype for a detonation in a chamber. The empirical models were plotted alongside the one- and two-dimensional airblast models to verify that the empirical models acted like the native airblast model in each type of geometry. A test series of detonations along the tube-chamber centerline of the same test model used in the previous section was conducted at various distances away from the interface, as well as at several angular offsets.

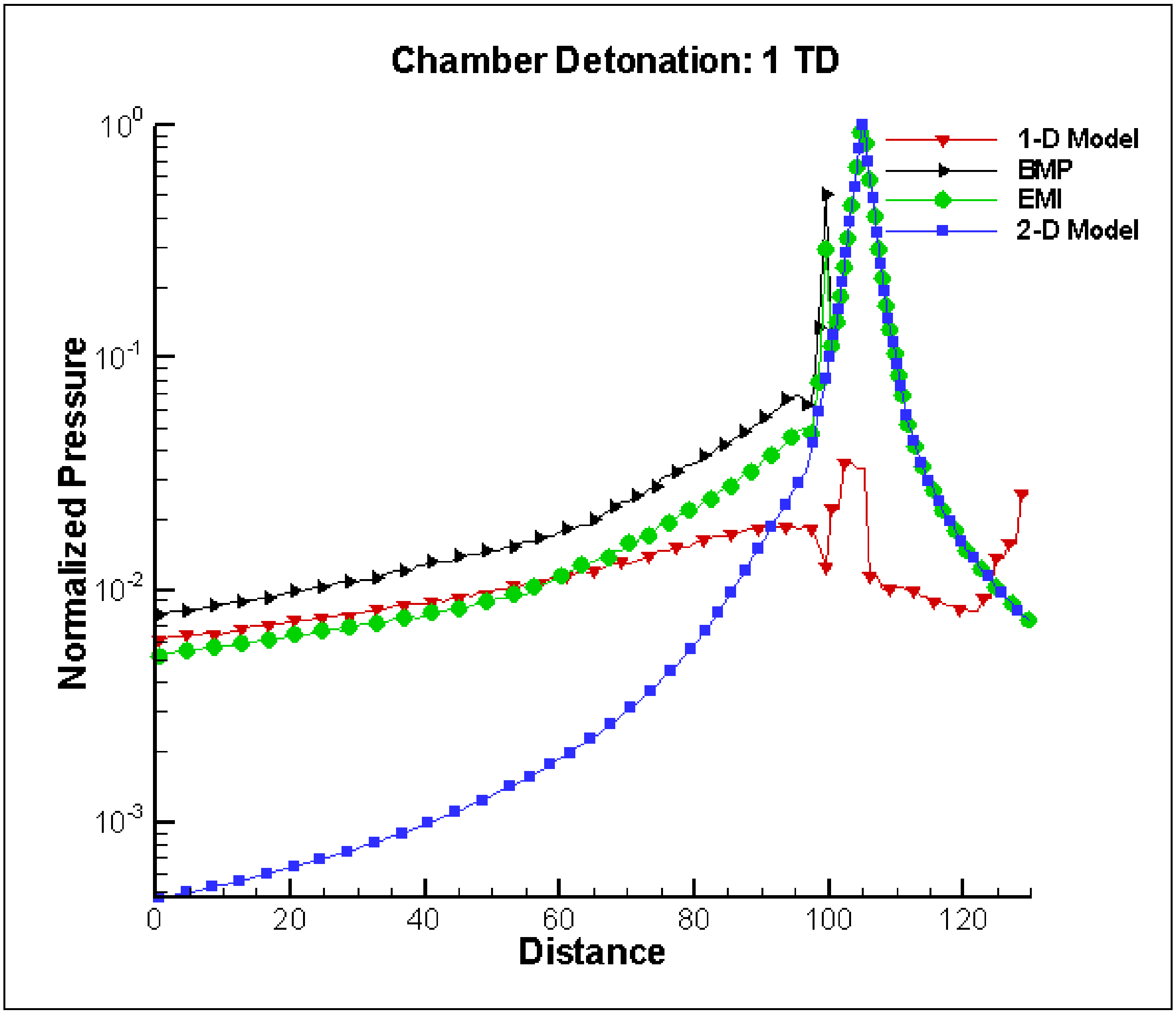


Figure 30) Detonation in a Chamber, 1 TD from the Interface

Though it is difficult to tell, the detonations of the two empirical models and the two-dimensional model are coincident in Figure 30, the detonation occurs at one TD from the interface. The one-dimensional peak pressure is substantially low in this region for two reasons in particular. The energy released by the detonation is being dissipated in one dimension, so the pressure propagation will not energetically spike and decay, but linger and slowly dissipate. In addition, the pressure caused by the energy release will be lower in this region due to the segment's width. The pressure will be volume averaged which lowers the initial pressure value. The figure also clearly shows that the two-dimensional model's decay is not in any manner changed across the interface. This is another reason that a transitional methodology such as TRAM between the one-dimensional and two-dimensional propagation is necessary. Figure 31 shows the effect of moving the charge farther away from the interface, and the impact this has on the pressure decay along the rest of the tube.

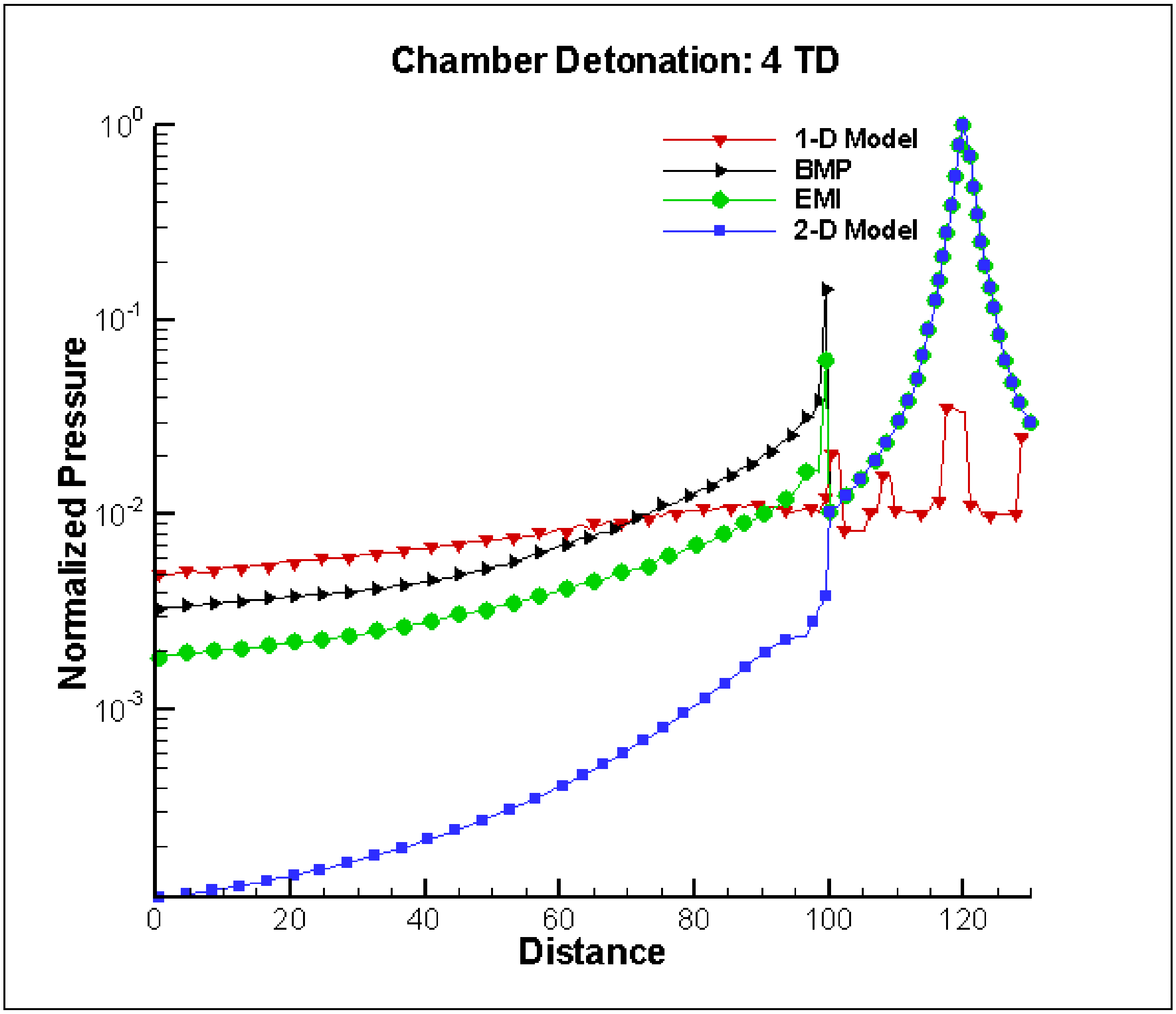


Figure 31) Detonation in a Chamber, 4 TD from the Interface

A change across the interface for the two-dimensional model is now evident, and the agreement between the two empirical models and the one-dimensional model is still good through the majority of the tube. A pattern of very high pressure very near the interface on the tube side is clear in both Figure 30 and Figure 31 which is not modeled by the one-dimensional model. This spike is the manifestation of the empirical methods trying to model the pressure amplification due to the physics associated with this type of burst. The fact that this pressure spike is absent is probably a shortcoming of the one-dimensional model versus an error in the empirical models. The spike intuitively belongs given the pressure amplification; however the magnitude of the spike cannot be verified without test data of some kind.

Both the EMI method and BMP method have built in considerations for an explosive charge being detonated at a point in the chamber that does not fall on the tube-chamber centerline. The trends shown in Figure 32 are intuitively correct considering the physics associated with a detonation in a chamber. The more the charge is offset from the interface, the less energy from the detonation wave that will enter, due to a lesser planar area of the opening. A detonation wave will in fact enter the tube through the opening, but the wave diffraction and bending costs energy. The four plots in Figure 32 show this trend, as the angular offset increases, the predicted energy decreases which is manifested in a further reduced effective charge. The range in all four plots is the same, all detonations were 2 TD's away from the interface.

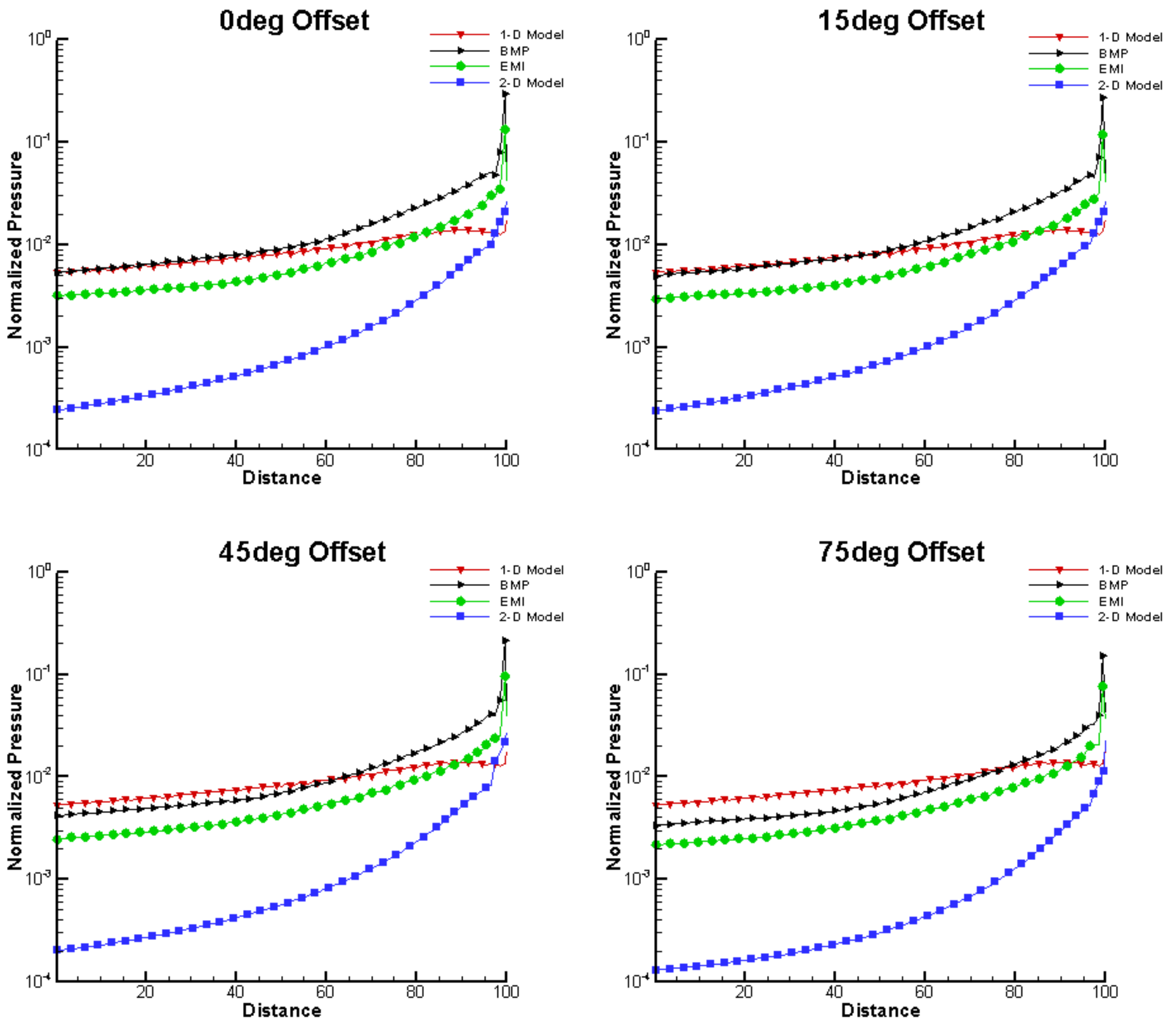


Figure 32) Effects of an Off-Centered Detonation in a Chamber

6. Conclusions

6.1. Recommendation for prototype

6.1.1. Detonation in a Tube

The two models under consideration for inclusion in the TRAM prototype for a scenario of a detonation in a tube were the Binggeli model, and the NPH model. The requirements of the TRAM method for this scenario were to reduce the pressure across the interface, facilitate pressure propagation in two dimensions, and approximate the two-dimensional airblast model results in the chamber portion of the test geometry as closely as possible. Based on those criteria the Binggeli method is the preferred choice for inclusion in TRAM. Both NPH and Binggeli meet all of the requirements, and are each an improvement over the existing dual airblast method of either one-dimensional airblast prediction or two-dimensional airblast prediction. Binggeli more closely approximates the two-dimensional airblast model decay and is superior in methodology since the pressure from the tube is acted on directly. This is opposed to the NPH method which requires a weight interpolation to then impose a reduction. Despite the discrepancy however, both Binggeli and NPH proved to be strikingly similar in solution, and feasible options.

6.1.2. Detonation in a Chamber

For this scenario, a detonation in a chamber, the EMI and BMP methods were reviewed and evaluated. Much like Section 6.1.1, the two empirical methods performed quite similarly, and both approximated the one-dimensional model closely in the tube portion of the hybrid geometry. For this scenario, the requirements for the TRAM prototype were to take a detonation in a chamber and transfer the airblast effects into the tube for the one-dimensional model to use. The BMP model is able to do this in a repeatable fashion with the same relationship, as opposed to the EMI method which varies with the charge size. This is one reason that leads to the conclusion that the BMP method is better suited for inclusion into TRAM. The second reason is that a realistic detonation will generally not occur perfectly on the tube-chamber interface. Therefore the behavior shown in Figure 32 should be heavily considered, which shows the BMP method consistently closer to the one-dimensional airblast model results for each of the angular offsets.

6.2. Future work considerations

The work on the TRAM prototype is by no means finished. There is still a myriad of improvements that can, and should, be made to extend the functionality of the final product. First and foremost, comparisons to test data should be made to validate the chosen methods of Binggeli and BMP.

A great deal of explanation in Section 2 justified the omission of quasi-static pressure effects, and this is one of the areas where the current TRAM prototype is at its weakest. The phenomena of quasi-static pressure are fundamentally more complex than that of shock pressure and the omission, while justified, is an assumption that hampers the TRAM prototype accuracy, especially for small chambers. This topic should be studied and models for calculating the quasi-static effects should be evaluated or even created if none exist.

In addition to the limits imposed by neglecting quasi-static pressure, the two-dimensional airblast model chosen for use in this research is not necessarily the most accurate choice. While the model is still used in many applications concerning blast effect prediction and mitigation, the model neglects shockwave reflection, and the resulting pressure amplification. Extending a model that would predict the shockwave interaction within the chamber where a detonation occurs would benefit both the one-dimensional expansion scenario as well as the two-dimensional confinement scenario.

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