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APPLICATION OF FINITE ELEMENT ELASTIC-PLASTIC STRESS ANALYSIS TO NOTCHED FATIGUE SPECIMEN BEHAVIOR

D.F. MOWBRAY, J.E. McCONNELEE,

General Electric Company,
Materials and Processes Laboratory, Schenectady, New York, U.S.A.

ABSTRACT

Finite element elastic-plastic stress analyses are performed on several notched fatigue specimen geometries used in a test program to determine the notched fatigue strengths of three common low strength structural steels. The analyses are for monotonically applied loading, with the cyclic nature of fatigue loading considered by using "cyclic" stress-strain curves as effective stress-strain relationships. The calculated notch root strains are found to agree well with experimental measurements. Further, a good correlation of the fatigue crack initiation life data for smooth and notched specimens of these steels is found to result when the controlling variable is considered to be effective stress-amplitude at the specimen surface.

NOMENCLATURE

x, y, z	= rectangular coordinates
r, θ, z	= cylindrical coordinates
ϵ_x, ϵ_z	= components of strain-amplitude in x- and z-directions for cantilever bend specimen
$\epsilon_\theta, \epsilon_z$	= components of strain-amplitude in θ- and z-directions for grooved cylinder specimens
$\sigma_r, \sigma_\theta, \sigma_z$	= components of stress-amplitude in r-, θ- and z-directions for grooved cylinder specimens
$\sigma_1, \sigma_2, \sigma_3$	= principal stress-amplitudes
$\bar{\sigma}$	= effective stress amplitude = $\sqrt{2}/2 \{ (\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 \}^{1/2}$
K_f	= fatigue strength reduction factor
K_t	= theoretical elastic stress concentration factor
N	= number of cycles of reversed loading
λ	= load parameter in plasticity analysis

Introduction

Predicting the fatigue life of machine parts with discontinuities or notches is, and has been over the years, one of the major problems in mechanical design engineering [1,2,3]. Consequently, extensive efforts have been applied in developing analytical methods for predicting notch fatigue behavior. Early attempts at defining the fatigue strength reduction factor (K_f) employed the results of elastic stress analysis. K_f was defined as either the theoretical elastic stress concentration factor (K_t) [4] or some fraction of this value through critical depth concepts [5,6]. In general, elastic analysis has not proven satisfactory, and plasticity calculations have been introduced in hopes that the plastic stress or plastic strain concentration factor will define K_f. It appears on the basis of several recent articles [2,7,8,9,10] that concentration factors derived from elastic-plastic analysis may indeed define K_f. For example, Manson and Hirschberg [7] defined K_f for a given notch configuration subjected to strain cycling as the plastic strain concentration factor. They were quite successful in predicting the fatigue life of notched laboratory specimens from smooth bar data with this definition, when coupled with the following two important observations: (1) the concentration factor should be expected to control the crack initiation life stage of fatigue failure only, and (2) for materials whose stress-strain properties are altered by cyclic loading, it is more appropriate in a monotonic loading analysis to use the "cyclic" stress-strain curve than the uniaxial tensile test curve (hereafter referred to as the "monotonic" loading curve). Others [9,10] have employed these same observations and applied the plastic stress concentration factor to load cycling applications with success.

The elastic-plastic analyses performed by the above authors have utilized either the modified "Stowell" [11] or "Neuber" [12] formulas for calculating the elastic-plastic stress- and strain-concentration factors. Although these formulas greatly simplify the stress analysis task, their application to complicated structures is obviously limited. As an improved means of stress analysis, it has been proposed [13] that the powerful finite element method of structural analysis be utilized. In its ultimate application, the finite element method should be able to accurately treat both notched laboratory specimens and complicated machine parts. The work in reference 13 includes a finite element analysis of a notched plate specimen used by other investigators [14] to study strain redistribution as a function of fatigue cycles. The specimen material exhibited considerable cyclic strain-softening so that it was appropriate to use "cyclic" stress-strain curves as the effective stress-strain relationships in the elastic-plastic analysis. The calculated strains were found to correlate well with the measured strains. Fatigue life was not determined in the test program so that the life prediction aspect of the overall problem could not be examined.

More recently, Krempl [15] has completed a test program which provides an excellent body of data for further examining the finite element stress analysis method and the concept of the "cyclic" stress-strain curve. He tested smooth and notched specimens of three low strength steels: low alloy (2 1/4 Cr - 1 Mo), plain carbon and 304 stainless. The notched specimens included axially loaded grooved cylinders having K_t values of 1.48, 1.97 and 3.46, and a cantilever bend specimen having a K_t of 1.98. For all of these tests, the loading was completely reversed; the smooth specimen tests were strain-controlled and the notched specimen tests, load-controlled. Crack initiation life was determined in both the smooth and notched specimen tests. Also, "cyclic" stress-strain properties were obtained from the smooth specimen tests, while notch root strains were measured as a function of cycles in most of the notched specimen tests.

The present authors have analyzed the elastic-plastic response of these notched specimens with a finite element computer program, employing the "cyclic" stress-strain curves as the effective stress-strain relationships. This paper presents the results of these analyses as well as correlations of the fatigue crack initiation life data.

Specimens and Stress-Strain Properties

The finite element solutions obtained are for the grooved cylinder specimens shown in Figure 1 and the notched bend specimen in Figure 2. "Cyclic" stress-strain curves obtained from smooth specimen strain cycling tests are shown in Figures 3 through 5 for the three steels. Shown for comparison in these figures are the "monotonic" curves obtained from a normal tensile test. The low alloy and plain carbon steels exhibited cyclic hardening only, and achieved a stabilized state in ~ 10 cycles. Thus, the shakedown or N=10 curves are the only "cyclic" curves shown for these two steels. The cyclic behavior of the stainless steel was considerably more complicated, with both hardening and softening occurring, and stabilization life being dependent upon the cyclic strain-amplitude. Figure 5 shows "cyclic" stress-strain curves for several numbers of cycles as well as the stabilized state. There is resultant softening below ~ 0.3% strain-amplitude and resultant hardening above this value. Stabilization took ~ 1000 cycles at strain-amplitudes below 0.6% and from 10 to 100 cycles at higher values, so that the stabilized curve for stainless steel does not correspond to a unique number of cycles.

The stress analyses of plain carbon and low alloy steel notched specimens were made using the "cyclic" stress-strain curve for the stabilized (or N=10) condition. Because of the rapid shakedown of these alloys, the stabilized curves should be representative of the stress-strain properties for the major portion of the notched fatigue life. The N=10 curve was used in analyzing the notched specimens of stainless steel. This life was selected because the major part of the change in stress-strain response was complete, but it was prior to the formation of fatigue cracks in any of the smooth and notched specimen tests.

Details of the Stress Analyses

The grooved cylinder and cantilever bend specimens were treated in the analyses as axisymmetric and plane strain bodies, respectively. In the case of the bend specimen, the calculated strains are compared with strains measured at the center of the bar where the state of stress corresponds very closely to plane strain (zero transverse strain).

Because of the double symmetry present in the grooved cylinder problems, it was necessary to consider only one quadrant of the critical section in each finite element analysis, as illustrated in Figure 1. Figure 6 shows an example of the finite element representations for grooved cylinders. The elements are constant stress triangular and quadrilateral (obtained by combining four triangles) types [16].

Since the bending problem possesses no symmetry, the full critical section had to be modeled. From some initial elastic solutions, it was found that some economy in the number of nodes and elements required could be achieved by breaking the structure along the line CD in Figure 2, and replacing the single point cantilever loading by equivalent node point axial and transverse loads. The grid employed was similar in construction to that shown in Figure 6, only with all four quadrants of the critical section modeled.

Solutions were obtained for the specimens subjected to monotonically increasing loads. In the case of the grooved cylinders, the loading was achieved by applying boundary pressures along the line CD in Figure 6 and restraining nodal movement in the z-direction along the line AB. For the bend specimen, the loading was applied as x- and y-direction forces at the nodes along the line CD in Figure 2. The x-direction forces were equivalent to the moment produced when the bend specimen (Figure 2) is cut along CD. Similarly, the y-direction forces were equivalent to the shearing force at this section. The node points along the lines AB and EF were restrained from moving in the x- and y-directions to simulate the clamping.

The method of plasticity analysis employed in the computer program incorporates the Prandtl-Reuss stress-strain relations, the von Mises' yield criterion, and the incremental theory of plasticity [17]. The stepwise calculation procedure proceeds as follows:

1. The elastic solution is calculated for some arbitrary value of a load parameter, λ (for proportional loading).
2. The value of λ (λ_o) required to just yield the most highly stressed element is calculated and the loads, stresses and strains calculated in Step 1 are multiplied by the ratio λ_o/λ_i, where λ_i is the initial value of λ used in Step 1.
3. The stiffness of the yielded element is recalculated using the plastic stress-strain relations, and the overall stiffness matrix is modified accordingly.
4. A small trial increment in λ ($\Delta\lambda$) is selected and the equilibrium equations (with the updated stiffness matrix) are solved for the corresponding displacement, stress and strain increments.
5. The calculated stress increments plus the state of stress at the end of the previous load increment are used to determine the actual increment in λ ($\Delta\lambda^*$) required to just yield the next element. The previously calculated stress and strain increments are then multiplied by $\Delta\lambda^*/\Delta\lambda$ to obtain the stress and strain increments for the present load increment.

6. The stiffnesses of all elements in the plastic range are recalculated and the overall stiffness matrix is modified.
7. The foregoing steps starting at 4 are continued until the maximum load is obtained.

Notch Root Strains

Comparisons of the calculated and measured notch root strains as a function of average net-section stress-amplitude are shown in Figures 7 through 10. The calculated strains are indicated by the lines and the experimental quantities by the data points. The round symbols indicate strains measured upon loading up to the fatigue test load range. This was accomplished by first cycling a specimen several times at 50% of the final load range, and then several more times at 75% of this value. These strains do not correspond to the cyclic condition depicted by the stress-strain curve, but they do represent a net-section stress-notch root strain condition affected by cyclic loading. The square symbols indicate strains after 10 cycles of loading, which also is the shakedown condition for the low alloy and plain carbon steels.

Since the notched fatigue specimens were all tested in the load-control mode, both the stress and strain at the notch root should vary with cycles. For cyclic hardening conditions, the stress should increase and the strain decrease. The inverse should occur for softening conditions. These characteristics can be observed in the experimental strain data; i.e., the square symbols in Figures 7 through 10 fall below the round symbols for a given strain-amplitude, except for the stainless steel in its softening range.

The calculated strains agree very favorably with the measured strains for all three steels and all notched specimen geometries. For the grooved cylinder specimens (Figures 7 through 9), the only appreciable deviation between computation and experiment is at the very highest net-section stress levels examined. Deviation at these levels may be due to exceeding the limits of small deformation strain theory. The calculations indicate that plastic flow is not entirely contained at the high stress levels, but is spread across entire parts of the critical section. A typical example of the size and shape of the elastic-plastic boundary as a function of net-section stress level is shown in Figure 11.

The analysis of the cantilever bending specimen utilized the simplifying assumption of plane strain in which the strain in the direction transverse to loading is set equal to zero. The good agreement between the calculated axial strains and those measured at the bar midsection (Figure 10) indicate that it is a satisfactory assumption for the loading range examined. Although the experiments indicate that transverse strains do develop at the bar midsection, they are less than 10% of the axial strain.

Correlations of Fatigue Crack Initiation Life Data

In attempting to correlate the smooth and notched fatigue specimen crack initiation life data reported by Krempl [15], various stress and strain quantities were considered as controlling variables. It was found, as have others [2,9,10] for notched specimen results from load controlled tests, that the best correlations result from considering stress as the controlling variable.

Both the smooth and notched specimens in the cited tests [15] experienced redistributions of stress and/or strain upon being cycled. However, for the low alloy and plain carbon steels, shakedown occurred in relatively few cycles, so a stabilized state of strain and stress existed in the specimens for a high percentage of their crack initiation life.

The rapid shakedown in these two steels means that all of the tests were in essence conducted under controlled stress conditions. Thus, the only calculations required for a fatigue analysis are those obtained for the shakedown condition. For the stainless steel, the persistent changes in stress-strain properties with cycles means that in the strictest sense, a data correlation should include stress analyses at a number of different cycles and consideration of cumulative damage. Since all of the required analyses were not completed, the N=10 curve was considered to represent average or half-life properties, and a correlation attempted with stresses calculated for this condition.

Good correlations result for all three steels when either the notch root axial stress-amplitude or effective stress-amplitude are considered as the controlling variable. When the axial stress is plotted, there is a trend for the notched specimen results to lie to the right of the smooth specimen results on an S-N diagram. This trend may be due to a biaxial tension strengthening effect [18]. When the effective stress is plotted, there is a trend for the notched specimen results to lie to the left of the smooth specimen S-N curve. However, less scatter results in the notched data when plotted in terms of effective stress-amplitude, so the data are shown in Figures 12 through 14 in terms of this variable. In these plots, the stress-amplitude values for smooth specimens are those measured at shakedown for the alloy and plain carbon steels, and at half-life for the stainless steel.

It is of interest to note that, if the fatigue strength correlations are based on the concept of a critically stressed volume of material near the notch root, the calculations indicate that the above mentioned trends are accentuated. This is illustrated by example in Figure 15 where plots of minimum section stress-distribution are shown for one of the material and notch geometry combinations. Note that the axial stress-amplitude initially rises to higher values at small distances in from the notch root, whereas the effective stress-amplitude steadily decreases.

Summary and Conclusions

Using the "cyclic" stress-strain curves of three low strength steels as effective stress-strain relationships in a finite element program, monotonic loading elastic-plastic analyses were made of several notched fatigue specimen geometries. The calculated surface strains were compared with strains measured at comparable lives during fatigue test experiments in which completely reversed loading was imposed. The measured and calculated strains compared very favorably.

Because of the good agreement between the calculated and measured total strains, it was assumed that accurate calculations were also made of resultant stresses. With this assumption the calculated results were examined for a single stress or strain variable which would correlate the available fatigue crack initiation life data. It was found that for the notched specimens, which were tested under load-control, the fatigue data could best be correlated on the basis of effective stress-amplitude at the surface. Further, good correlations of crack initiation life resulted between the notched and smooth specimen data when the available strain-controlled smooth specimen data were expressed in terms of shakedown effective stress-amplitude.

In conclusion, it can be stated that the finite element stress analyses technique represents a most promising tool for the treatment of notch effects in fatigue, and should be further explored whenever possible. Also, the calculations demonstrate further the utility of applying a monotonic loading analysis to a cyclic loading problem through use of the "cyclic" stress-strain curve.

ACKNOWLEDGEMENT

The authors would like to thank Professor E. Krempl of Rensselaer Polytechnic Institute for his consultation and willingness to provide data during the course of this investigation.

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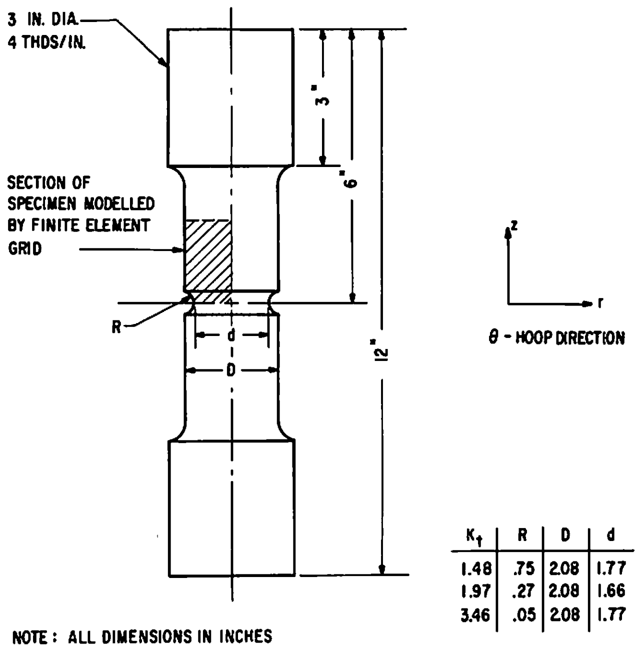
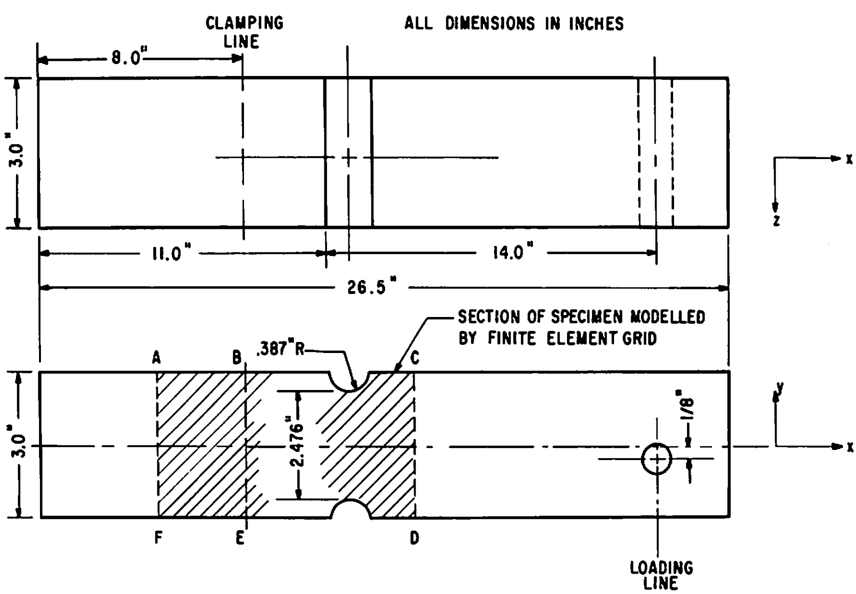
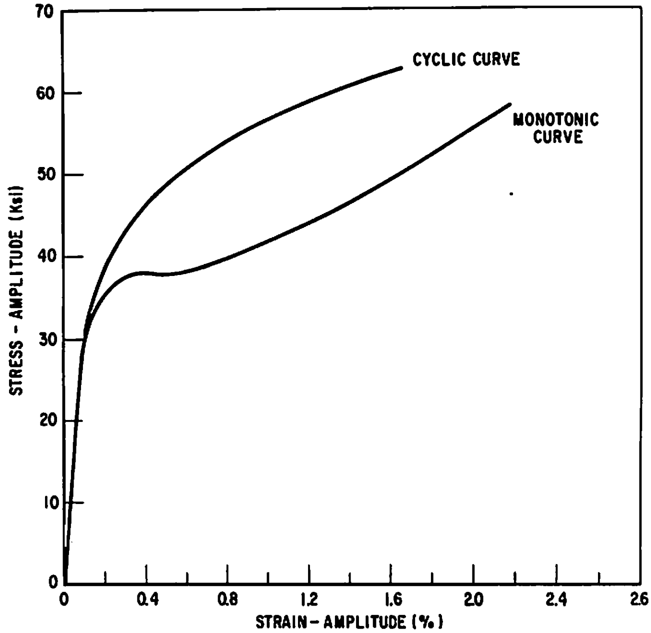


K_t	R	D	d
1.48	.75	2.08	1.77
1.97	.27	2.08	1.66
3.46	.05	2.08	1.77

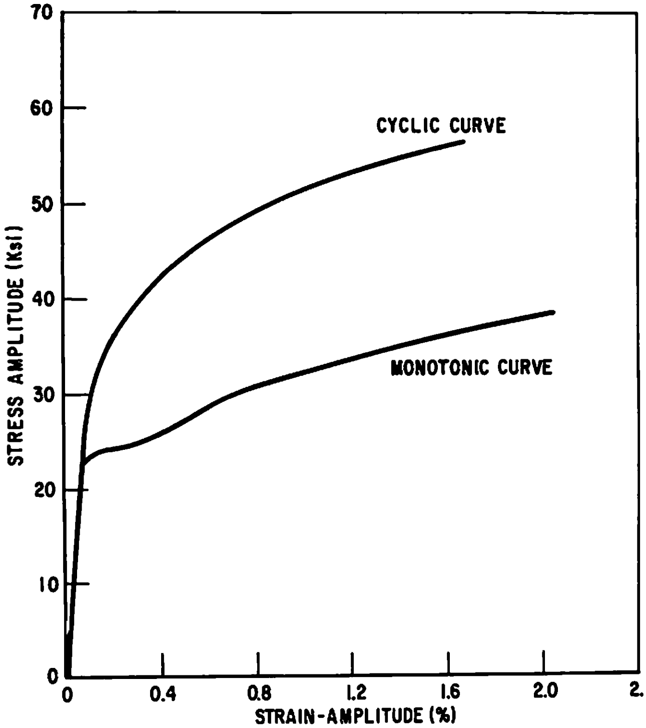
1. Grooved Cylinder Fatigue Specimens



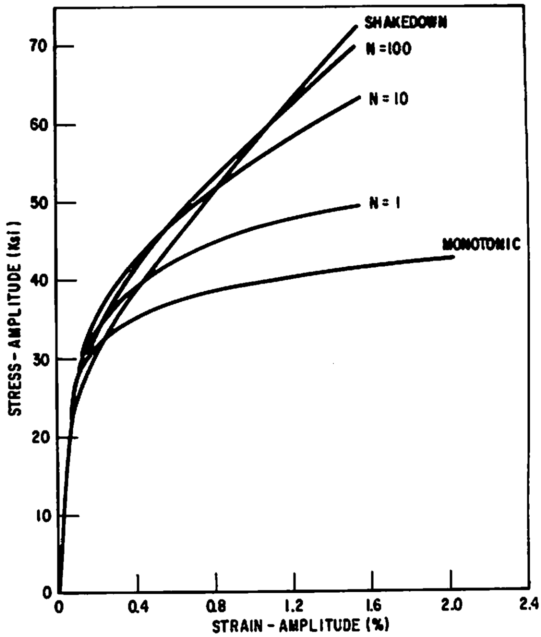
2. Notched Bend Specimen



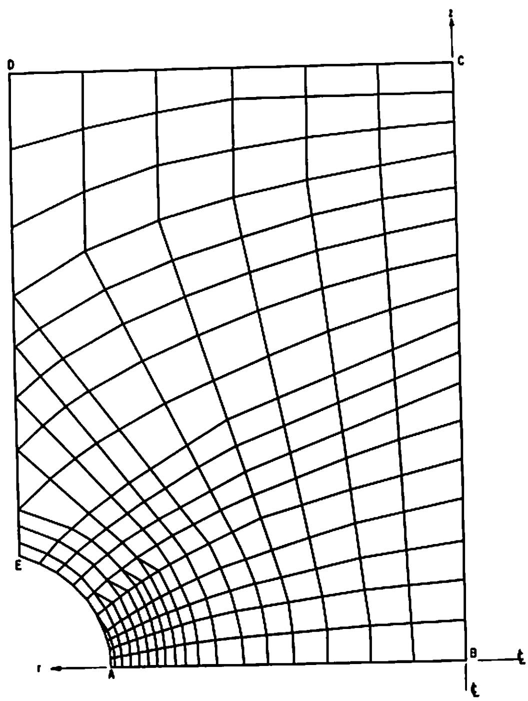
3. Stress-Strain Diagrams for 2 1/4 Cr - 1 Mo Alloy Steel



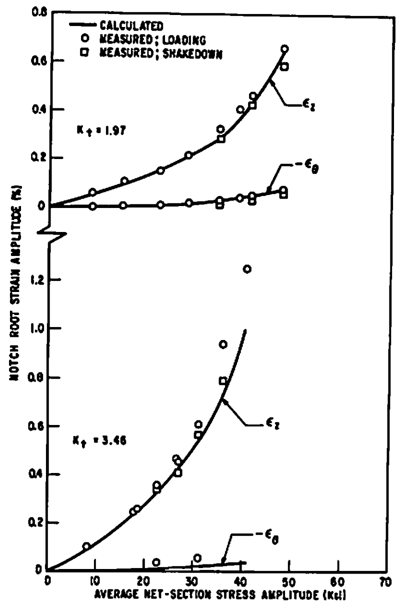
4. Stress-Strain Diagrams for Plain Carbon Steel



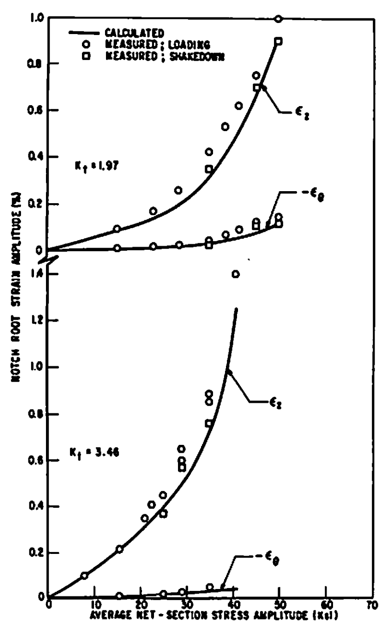
5. Stress-Strain Diagrams for 304 Stainless Steel



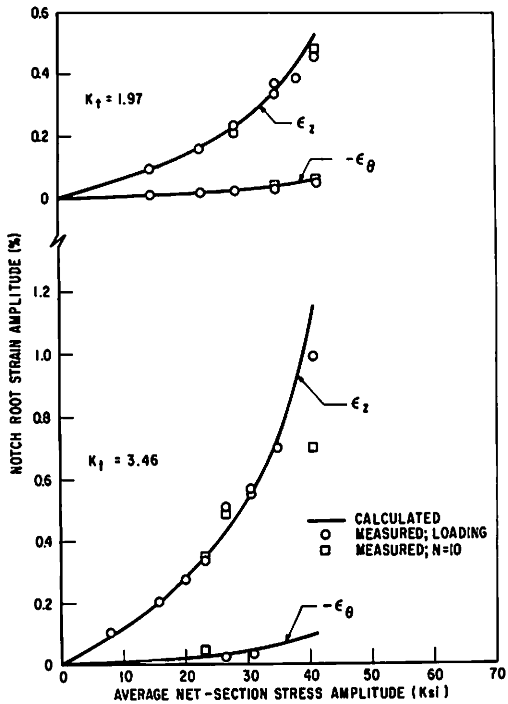
6. Finite Element Model; Grooved Cylinder Specimen, $K_t = 1.97$



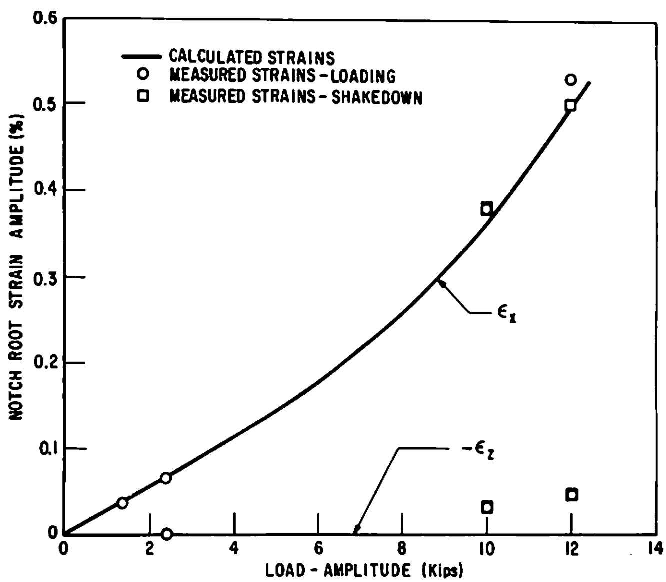
7. Comparisons of Calculated and Measured Strains in Grooved Cylinder Specimens; 2 1/4 Cr - 1 Mo Alloy Steel



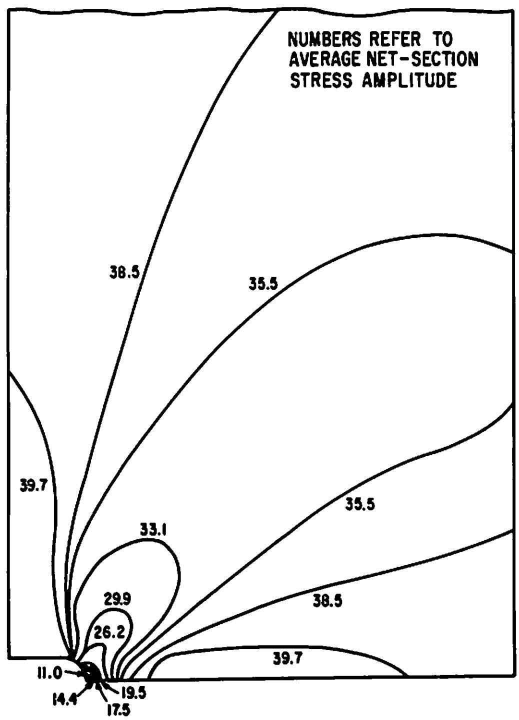
8. Comparisons of Calculated and Measured Strains in Grooved Cylinder Specimens; Plain Carbon Steel



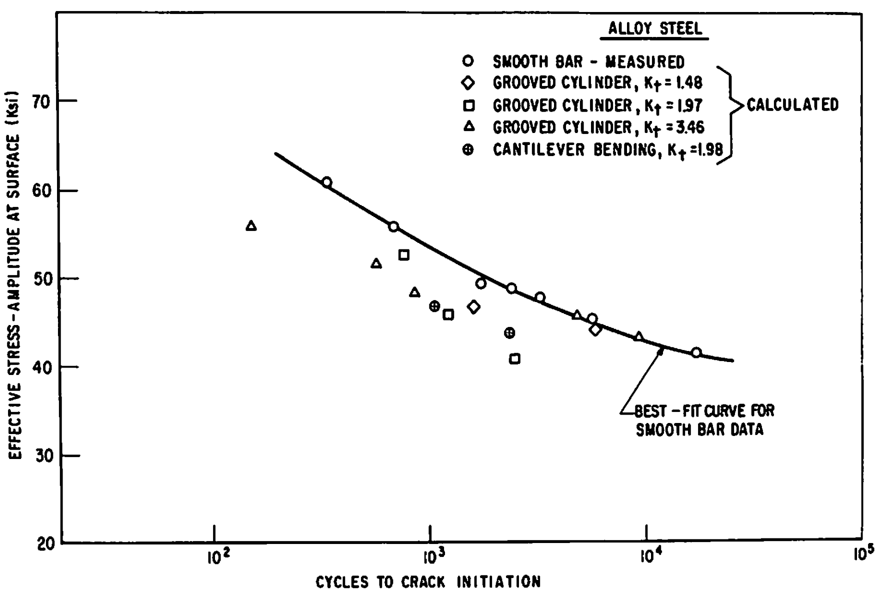
9. Comparisons of Calculated and Measured Strains in Grooved Cylinder Specimens; 304 Stainless Steel



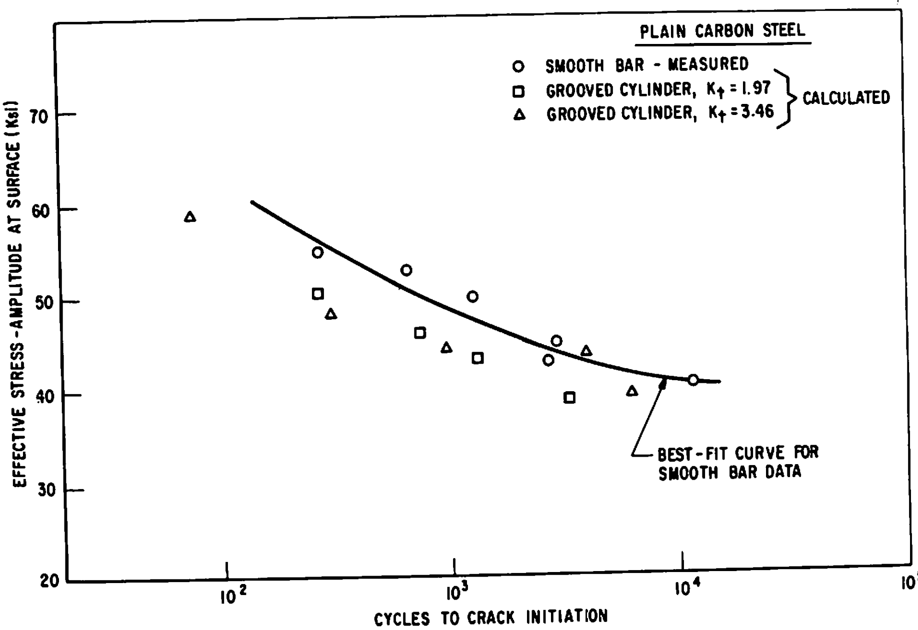
10. Comparison of Calculated and Measured Strains in the Notched Bend Specimen; 2 1/4 Cr - 1 Mo Alloy Steel



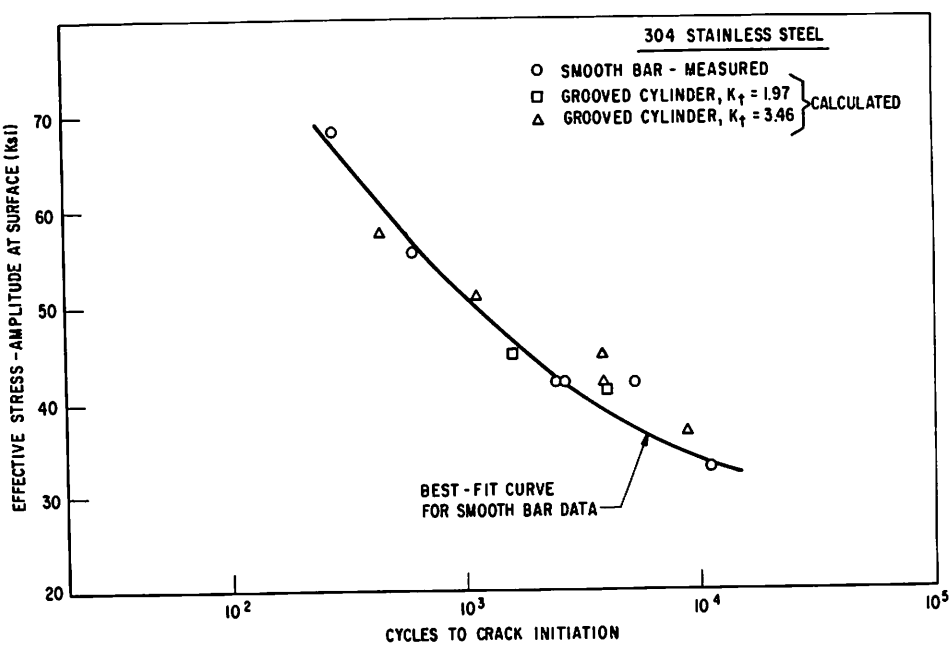
11. Progression of Elastic-Plastic Boundaries in the K_t = 3.46 Grooved Cylinder Specimen; 2 1/4 Cr - 1 Mo Alloy Steel



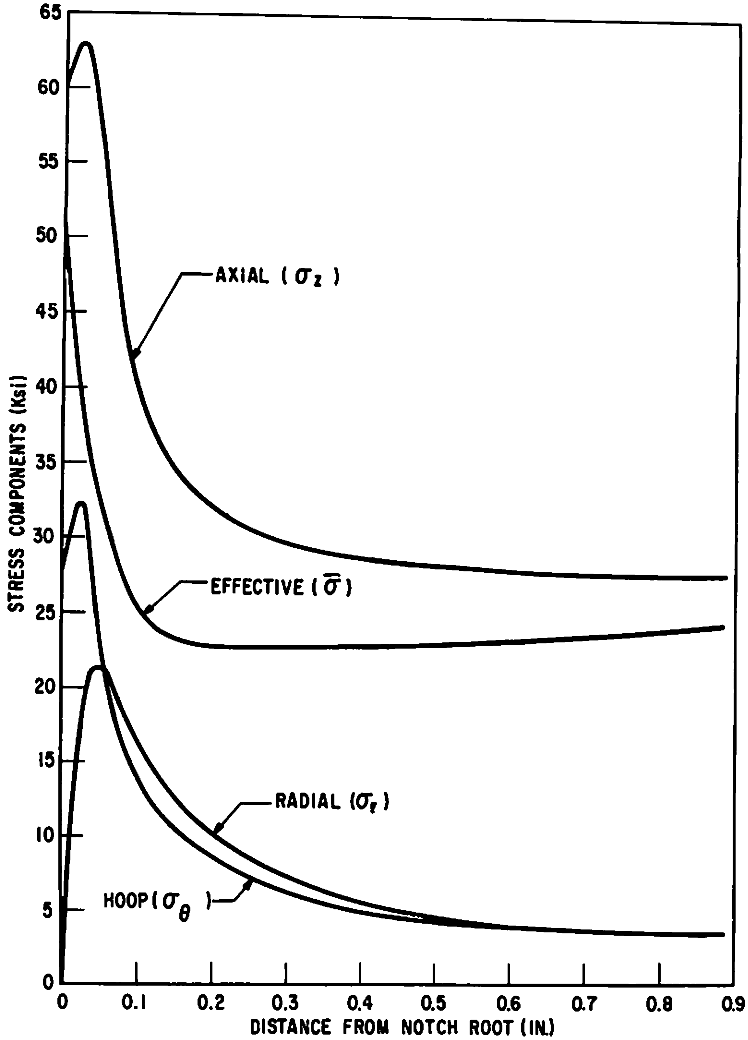
12. Correlation of Crack Initiation Life Data; 2 1/4 Cr - 1 Mo Alloy Steel



13. Correlation of Crack Initiation Life Data; Plain Carbon Steel



14. Correlation of Crack Initiation Life Data; 304 Stainless Steel



15. Example of Elastic-Plastic Stress Distributions; Grooved Cylinder, K_t = 3.46, 2 1/4 Cr - 1 Mo Alloy Steel, Average Net-section Stress Amplitude = 35.5 Ksi

DISCUSSION

Q D. R. J. OWEN, U. K.

I would like to enquire whether the authors encountered any numerical instability in the stresses near the notch root. Such instabilities often occur when strains become large in elasto-plastic problems, and we at Swansea have performed extensive assessments of the Finite Element Analysis of notched specimens of various geometries in bending. Completely satisfactory results were only achieved by use of isoparametric elements.

A D. F. MOWBRAY, U. S. A.

Numerical instabilities were not encountered in these analyses. The strains in these cases were not large, and this may be the reason why the numerical instabilities you mention were not observed.

Q Y. R. RASHID, U. S. A.

What type of hardening law have you used ? If isotropic hardening was used, how did you account for the Bauschinger effect ?

A E. KREMPL, U. S. A. p. p. D. F. MOWBRAY, U. S. A.

The Von Mises yield condition and Prandtl-Reuss stress-strain relations were used. No unloading calculation was necessary since the cyclic stress-strain diagram was considered to be representative of the fatigue condition. The final good correlation of the fatigue data appears to justify the approach fully.

Q D. COSTES, France

If I understand rightly, you present an approximate solution; you assume that deformations in each cycle are proportional, which is not really the case.
Did you try to effectively cycle N times the specimen in your calculations ?

A E. KREMPL, U. S. A. p. p. D. F. MOWBRAY, U. S. A.

Yes, the solution is approximate; the aim was to find a correlation of uniaxial and notched bar fatigue data. The results shown in Figs. (12) and (14) appear to justify the approximations in the calculation. No cycling was done in the calculations used in the paper. Work towards this end is in progress.

C D. F. MOWBRAY, U. S. A.

The excellent correlations found between the calculated and measured notch root strains (Figs. 7 through 10) fully justify the simplifications of the calculational method.