Variational Velocity Formulation for Contact with Friction Problems, and Applications

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INTRODUCTION

Conditions involving contact between two deformable solids, or between a solid and another medium, constitute an important class of problems in nuclear engineering applications. The region of contact between the two bodies changes continuously during the loading, and is not known a priori. In addition, the pressure of friction between the two bodies creates varying regions of stick and slip. The underlying difficulty is that frictional contact problems in solid mechanics are inherently non linear.

We propose a variational formulation and the corresponding numerical methods for solving unilateral contact problems involving friction, for complex quasistatic loadings.

This work is an extension of the formulation given by Duvaut-Lions for static or monotone loading cases and of the numerical methods that we used to solve this problem. We use a variational principle written in terms of velocities. After discretization we have an incremental formulation which is path dependant. This is suitable for dealing with problems under complex loadings and especially when the sliding direction changes during the process.

The mathematical formulation is set as an implicit variational inequation including an undifferentiable term given in a convex subset.

The numerical algorithms are based on overrelaxation methods with projections coupled with a diagonal process. Its efficiency has already been presented in test examples in (Raous et al, 1988).

Here, several physical or industrial applications are presented : structure assembling in nuclear engineering, a blankholder model in metal forming processes, stress wave progression on the contact area of a solid sliding on a rigid plane obstacle.

THE SIGNORINI'S PROBLEM WITH FRICTION

We first set a unilateral problem between an elastic solid and a rigid obstacle for the static case under a small transformation hypothesis. It is a Signorini problem including friction (Duvaut, Lions, 1976). Let $\Omega \subset \mathbb{R}^N$ (N = 2 ou 3) be an open bounded-domain representing the interior of the body. The sufficiently smooth (e.g. Lipchitz continuous) boundary Γ of Ω contains three open subsets Γ_D, Γ_L, and Γ_C such that : $\Gamma = \Gamma_D \cup \Gamma_L \cup \Gamma_C$, Γ_L (resp. Γ_D) is the part of Γ on which the loads f (resp. the displacements) are prescribed and Γ_C is the (candidate) contact surface on which the body may come in contact with the rigid obstacle. We denote u the displacement field and F the density contact force vector. On Γ_C the trace of u and F are written in a local referential (n,t) where n is the exterior normal unit vector to Γ_C :

$$u = u_N n + u_T \qquad F = F_N n + F_T \tag{1}$$

Where index N indicates normal components and index T tangential components. Let us note that F is unknown. The unilateral conditions lead to the following complementary conditions on Γ_C :

$$u_N \leqslant d \qquad F_N \leqslant 0 \qquad (u_N - d).F_N = 0 \tag{2}$$

Where d is the initial distance between the solid and the obstacle, or a movement of the obstacle. The case ($u_N = d$, $F_N \leqslant 0$) characterizes the contact zone and the case ($u_N < d$, $F_N = 0$) corresponds to the separate part. Together with the equilibrium equations and the constitutive law, these conditions lead to the well known Signorini's Problem.

The Coulomb's friction law can be written (Duvaut, Lions, 1976) :

$$|F_T| \leqslant \mu \, |F_N| \tag{3}$$

$$\text{with} \qquad |F_T| < \mu \, |F_N| \Rightarrow \dot{u}_T = 0 \tag{4}$$

$$|F_T| = \mu \, |F_N| \Rightarrow \dot{u}_T = - \lambda F_T \qquad \lambda \geqslant 0 \tag{5}$$

The dot denotes the time relative derivation and μ is the friction coefficient . In the static case or for monotone quasistatic loadings, a displacement formulation can be used in the equations (4), (5). Nevertheless, in the general case the solution is path dependant and an incremental formulation is used.

INCREMENTAL FORMULATION

Let $(t^0,\ldots,t^N)$ be a partition of the time interval [0,T] we use a forward finite difference approximation of $\dot{u}$ by :

$$\dot{u}(t^{k+1}) = (u(t^{k+1}) - u(t^k))/(t^{k+1} - t^k) = \Delta u^{k+1}/\Delta t^{k+1} \tag{6}$$

The Coulomb's friction law is now written under the following form :

$$k = 1,\ldots,N \quad \left|F_T^k\right| \leqslant \mu \left|F_N^k\right| \tag{7}$$

$$\text{with :} \qquad \left|F_T^k\right| < \mu \left|F_N^k\right| \Rightarrow \Delta u^k = 0 \tag{8}$$

$$\left|F_T^k\right| = \mu \left|F_N^k\right| \Rightarrow \Delta u^k = - \lambda F_T^k \qquad \lambda \geqslant 0 \tag{9}$$

The variational formulation leads to a quasi-variational inequation containing an undifferentiable term :

<u>Problem Pl</u> : For $k = 1,\ldots,N$, let u^0 be the initial displacement, $f^{k-1} = (f_1^{k-1}, f_2^{k-1})$ the forces and $\Delta f^k = (\Delta f_1^k, \Delta f_2^k)$ the force increments , f^k and $\Delta f^k \in (L^2(\Omega))^3 \times (L^2(\Gamma_L))^3$, find $\Delta u^k \in \mathbb{K}^k$ with

$\mathbb{K}^k = \left\{ v \in (H^1(\Omega))^3 \; ; v = 0 \text{ on } \Gamma_D ; \; u_N^{k-1} + v_N \leqslant d \text{ on } \Gamma_C \right\}$ such that :

$$\forall\, v \in \mathbb{K}^k \quad a(u^{k-1}+ \Delta u^k, v-\Delta u^k) - (f^{k-1}+\Delta f^k, v-\Delta u^k)+ j(\Delta u^k, v) - j(\Delta u^k, \Delta u^k) \geqslant 0 \tag{10}$$

$$\text{with : } a(u,v) = \int_\Omega [Dv]^T K\, [Du]\, d\Omega \tag{11}$$

$$(f,v) = \int_\Omega f_1^T v\, d\Omega + \int_{\Gamma_L} f_2^T v\, d\Gamma \tag{12}$$

$$j(\Delta u, v) = \int_{\Gamma_C} \mu\, |F_N(\Delta u)|\, |v_T|\, d\Gamma \tag{13}$$

D is the symmetrical gradient mapping and K the elasticity matrix. This problem is an implicit variational inequation because of the term $j(\Delta u^k, v)$ in which F_N depends on the solution. We avoid the implicit character by using a fixed point method on the sliding limit, and we get a more classical variational inequation (Panagiotopoulos, 1985).

We solve a sequence of problems with a sliding limit g given by the previous iteration.

<u>Problem P2</u> : For k = 1,...,N find g^k fixed point of the application :

$g^k \longmapsto \mu \left| F_N^k(\Delta u^k(g^k)) \right|$ where Δu^k is the solution of the problem P3 depending on g^k

<u>Problem P3</u> : Let u^0, $f^{k-1} = (f_1^{k-1}, f_2^{k-1})$, $\Delta f^k = (\Delta f_1^k, \Delta f_2^k)$ be given as in problem P1. Let $g^k \in L^2(\Gamma_C)$ find $\Delta u^k \in \mathbb{K}^k$ such that :

$$\forall\, v \in \mathbb{K}^k \quad a(\Delta u^k, v-\Delta u^k) - (\Delta f^k, v-\Delta u^k) + (R^{k-1}, v-\Delta u^k) + j(v) - j(\Delta u^k) \geqslant 0 \tag{14}$$

$$\text{where: } (R^{k-1}, v-\Delta u^k) = a(u^{k-1}, v-\Delta u^k) - (f^{k-1}, v-\Delta u^k), \quad j(v) = \int_{\Gamma_C} g^k\, |v_T|\, d\Gamma \tag{15}$$

NUMERICAL METHODS

From a numerical point of view, different classes of algorithms are used: penalization, Newton's method, mathematical programming...(see for example 'Numerical methods in mechanics of contact involving friction', Journal of Theoretical and Applied Mechanics, special issue, supplement n°1 to vol 7 1988) and for frictionless contact problems conjugate gradient methods (May, 1986). Our methods are based on projection techniques coupled with overrelaxed Gauss-Seidel methods. Several convergence acceleration techniques are used : partial resolution during the fixed point process, condensation of the problem to the contact variables only, appropriate matrix storage (skyline, sparse or morse storage).

We extend to the friction case the successive overrelaxed method with projection (SORP) introduced in (Glowinski et al, 1976) for the Signorini's problem. The problem P3 is written under the following equivalent minimization problem with constraints :

<u>Problem P4</u> : let $u^0, f^{k-1} = (f_1^k, f_2^k)$, $\Delta f^k = (\Delta f_1^k, \Delta f_2^k)$ be given as in problem P1. Let $g^k \in L^2(\Gamma_C)$, find $\Delta u^k \in \mathbb{K}^k$ such that : $F(u^k) \leqslant F(v)$ for all $v \in \mathbb{K}^k$

$$\text{with : } F(v) = 1/2\; a(v,v) - (\Delta f^k, v) + (R^{k-1}, v) + j(v) \tag{16}$$

A discretization by finite element is done. The contact variables are treated in local normal tangential coordinates. The convenient rotations are done on the finite element matrix. Let I be the set of contact nodes numbers and N_0 be the number of nodes. The SORP leads to :

$$\text{(i)} \quad (\Delta u_i^k)^{\ell+½} = \frac{1}{a_{ii}} (\Delta f_i^k - R_i^k - \sum_{j<i} a_{ij} (\Delta u_j^k)^{\ell+1} - \sum_{j>i} a_{ij} (\Delta u_j^k)^{\ell} + \varepsilon \; g_i^k) \qquad (17)$$

$$\text{(ii)} \quad (\Delta u_i^k)^{\ell+1} = \omega \; (\Delta u_i^k)^{\ell+½} + (1-\omega) \; (\Delta u_i^k)^{\ell} \qquad (18)$$

(iii) Projection : If $i = 2j-1$ with $j \in I$ (normal component of a contact node) :

$$\text{if } (\Delta u_i^k)^{\ell+1} > d - u_i^{k-1} \text{ then } (\Delta u_i^k)^{\ell+1} = d - u_i^{k-1}$$

ε is defined by : If $i = 2j$ with $j \in I$ (tangential component of a contact node) :

if $(\Delta u_i^k)^{\ell+½} > 0$, $\varepsilon = 1$, if $(\Delta u_i^k)^{\ell+½} < 0$ $\varepsilon = -1$,

otherwise $\varepsilon = 0$

$a_{ij} = a(\beta_i, \beta_j)$ is the general term of the elasticity finite element matrix, $f_i = (f, \beta_i)$ where β_i are the shape functions. ω is the relaxation coefficient. It depends on the number of constraints which are saturated on the final solution.

The efficiency of the above algorithm is studied on two examples provided by the groupe "Validation of computer code" of the french "Groupement de Recherches et d'Etudes COordonnées (GRECO) en Grandes Déformations et Endommagement". These very simple tests were defined and applied to several computer codes developped in universitary or industrial environment. The first example is the contact of a long bar on a plane surface. It has the advantage of being very elementary and that of giving different contact states according to the loadings and the friction coefficients values. The second example is a two body contact problem with an oblique contact zone (Raous et al, 1988). We observe three zones : a non contact zone, a sliding zone and a stick zone.

APPLICATIONS

Bolted Junction

We present a structure assembling problem : a bolted junction under internal pressure loading. This work has been supported by Technicatome (Les Milles - France). It is a four body contact problem. There are three contact areas with different friction coefficients (the materials and the surface states are different).

This problem includes three loading steps:

- firstly, the closure of the bolted junction is applied
- secondly, the pressure is applied on the lower part
- finally, the pressure is applied on the upper part.

Solving this problem shall enable us, among other things, to determine the friction coefficients on the first loading step by comparison with experimental measurements which have been given by Technicatome (see Fig. 1). The use of these coefficients (differents on each contact areas) to compute the other steps of loading give still a good agreement with the experimental datas and allows us to conclude on the validity of the Coulomb's friction law for describing the structure behaviour.

	Experiment 1	Experiment 2
sliding plate upper part	0%	0%
thrust	14%	14%
plate rotation	13%	10%
upper part rotation	3%	8%
deflection	7%	2%

Fig. 1: Numerical error (closure)

Friction modelling for metal forming process (Chabrand et al, 1989)

We focus on the behaviour of the sheet metal under the blankholder during the drawing process. The normal action of the blankholder is introduced by the following condition : $u_N \leqslant d$
The displacement of the blankholder d is adjusted during the process to take into account the conservation of the normal load applied by the press. The values of d are computed when convergence is obtained for the fixed point iteration.

Modelization with this condition avoids the discretization of the blankholder and enables us to take into account the loss of contact which can occur. Fig. 2 shows the forces on an intermediate state along the contact zone. The quotient F_T/μ is plotted to observe how well the Coulomb law is verified along the sliding area. A tangential displacement u_T is given at the right extremity : it corresponds to a tangential force at the exit of the blankholder.

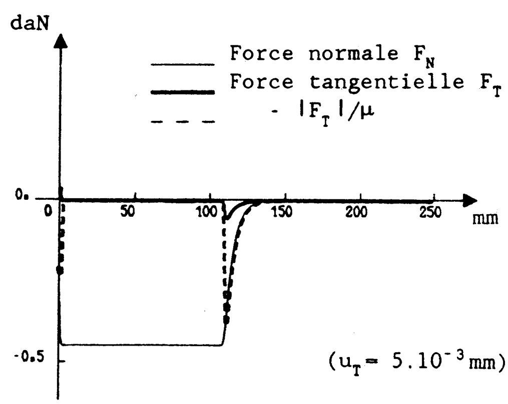

Fig. 2: Contact forces

Stress wave on the contact area of a solid sliding with friction on an obstacle.

We consider a polyurethane solid sliding with friction on an araldite plane. The experience realized by Villechaise (PROGRI et al, 1985) shows the progression of stress waves on the contact area as the tangential forces increase. The topic is to test the aptitude of a Coulomb friction law to describe the progression of the observed stress waves on the contact area (Topin, Raous 1989). Preliminary results obtained by S. Topin will be presented.

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