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METHODS FOR ESTIMATING PENETRATION DEPTHS AND QUANTIFICATION OF THE PROTECTIVE EFFECT OF SOIL IN THE EVENT OF PROJECTILE IMPACT

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ABSTRACT

Critical infrastructures such as nuclear facilities, military installations, and their weapon depots require a particularly high level of security. Due to the catastrophic consequences of damage or destruction of these structures, adequate protection against impact loads such as vehicle collisions, crashing aircraft, or natural disasters must be provided. In the context of concepts for partly embedded SMRs, underground interim storages and buried systems with security-related significance, research is increasingly focusing on impact scenarios involving underground structures.

Current methods for damage simulation rely either on empirical approaches, whose application limits must always be observed due to their empirical nature, or numerical simulations based on the finite element method. Empirical models or formulas are based on impact tests and offer the possibility to calculate or estimate significant failure mechanisms such as penetration depth with only a few input parameters in a short time. In contrast, verified numerical simulations are very time-consuming due to the complex, nonlinear material behaviour of the soil material, but they allow for further investigation of the mechanical effects of dynamic impact loads.

In this paper, existing empirical approaches referenced in guidelines and standards are analysed and evaluated based on current experimental test data on projectile impact into soil. In addition, numerical investigations on projectile impact into soil are carried out. Due to the highly nonlinear material behaviour of the soil, a novel combined solution using the discrete element method (DEM) for the soil and the finite element method (FEM) for the projectile is presented for modelling.

INTRODUCTION

In the design of safety-related structures intended to withstand impact loads, soil barriers emerge as a compelling option. Existing approaches for quantifying the protective effect of the soil primarily rely on empirical formulas and require verification using current experimental data.

With advances in computing capacity, numerical methods, in particular the discrete element method (DEM), are gaining increasing interest. These methods enable more realistic modelling of interactions between the soil and impacting projectiles. By employing numerical approaches, a more detailed analysis can be carried out to evaluate the protective effect under different conditions.

EMPIRICAL FORMULAS

To calculate the penetration depth x of a projectile into the soil, various empirical formulas are recommended according to current guidelines. The status report of the Nuclear Safety Standards Commission (KTA) [1] refers to the approaches of Young (1969, 1997), Kar (1977), Schardin (1954), and Petry presented in Amirikian (1950), which are based on experimental data with non-deformable projectiles.

The input parameters of the empirical penetration formulas include details about the projectile's geometry, such as the diameter d and the impact area A, as well as its mass M and impact speed v. The soil is considered in the calculations, either with an empirical parameter related to the type of soil according to Table 1, or a mechanical characteristic value, depending on the formula used.

Table 1: Material-dependent constants $S, k_{Petry}, k_{Schardin}$

Soil type	S	k_{Petry}	$k_{Schardin} \cdot 10^{-6}$
Clay	5 – 30	0.0732	125 – 180
Sand	2 – 10	0.0367	70
Rock	0.2 – 1	0.002 – 0.005	8.5

Young Formula

In 1969, Young published a penetration formula in the Journal of Soil Mechanics and Foundations, which was developed in connection with the terradynamics test program at Sandia National Laboratories. As the database of impact experiments into the soil increased, the formula underwent multiple adjustments. In 1997, Young published an updated version of the Young/Sandia formula, which can be found in equations (1) and (2) [3]. The use of the formula requires the use of SI units.

If $v < 61 \frac{m}{s}$

$$x = 0.0008 \cdot S \cdot N \cdot K \cdot \left(\frac{M}{A}\right)^{0,7} \cdot ln(1 + 2.5 \cdot v^2 \cdot 10^{-4}) \tag{1}$$

If $v \geq 61 \frac{m}{s}$

$$x = 0.000018 \cdot S \cdot N \cdot K \cdot \left(\frac{M}{A}\right)^{0,7} \cdot (v - 30.5) \tag{2}$$

The term N takes into account the nose shape of the projectile and can be determined according to Young (1997). In equations (1) and (2), the variable S represents the "Soil-Number." This empirical parameter considers soil properties concerning the penetration resistance of the target material. The dimensionless parameter S can be obtained from Table 1. In selecting within the specified parameter range, considerations for soil material, moisture content, and storage density are crucial. The use of the Young formula requires projectile masses greater than 2 kg. For lower projectile masses, an adjustment factor K, as per Equation (3), must be applied.

If $M < 27$ kg

$$K = 0.27 \cdot M^{0,4} \tag{3}$$

For projectile masses greater than 27 kg, the adjustment factor K is equal to 1.

Kar Formula

Another formula was introduced by Kar (1977). With regard to the projectile, different materials are taken into account by the ratio of the moduli of elasticity of the projectile material E_p and steel E_s as well as different projectile geometries by introducing an ideal projectile diameter D. The parameter Y describes the uniaxial static compressive strength of the soil material. The penetration depth according to Kar is calculated using equations (4) and (5). The use of the formula requires the use of SI units.

$$G(x/d) = \frac{\alpha}{Y^{0,5}} \cdot N \cdot \left(\frac{E_p}{E_s}\right)^{1,25} \cdot \frac{M}{d^{1,66} \cdot D^{0,65}} \cdot \left(\frac{v}{1000}\right)^{1.25} \tag{4}$$

$$G(x/d) = \begin{cases} \left(\frac{x}{2d}\right)^2 & f\text{ür } \frac{x}{d} \leq 2.0 \\ \left(\frac{x}{d} - 1\right) & f\text{ür } \frac{x}{d} \geq 2.0 \end{cases} \tag{5}$$

$\alpha = 27183$; Y in [kN/m²]

Petry Formula

The Petry formula was published in 1910. Various soil types are considered through the factor k_{Petry}, as indicated in Table 1. Equation (6) describes Petry's approach for calculating the penetration depth of the projectile into the soil. The use of the formula requires the use of US units.

$$x = \frac{M}{A} \cdot k_{Petry} \cdot \log\left(1 + \frac{v^2}{215000}\right) \tag{6}$$

Schardin Formula

The Schardin formula, developed in 1954, is based on experimental results from bomb drop and ballistic tests. It assumes an approximate proportionality between penetration depth and impact impulse per unit area. The soil is characterized by the material-dependent constant $k_{Schardin}$, which can be extracted from Table 1. The penetration depth according to Schardin is calculated using Equation (7). The use of the formula requires the use of SI units.

$$x = k_{Schardin} \cdot \frac{M \cdot v}{g \cdot A} \tag{7}$$

Application of the Empirical Formulas

The diagram in Figure 1 provides an overview of the calculated penetration depth of projectiles into various soil types as a function of impact velocity. The soil material is differentiated into sand, clay, and rock. The remaining parameters are listed in Table 2.

Table 2. Input parameters of the empirical formulas from Figure 1

Overview (Sand, Clay, Rock)							
d [m]	M [kg]	N [-]	E [N/mm²]	S [-]	Y [kN/m²]	k_{Petry} [-]	$k_{Schardin}$ [-]
0.1	100	0.72	204,000	See Table 1			

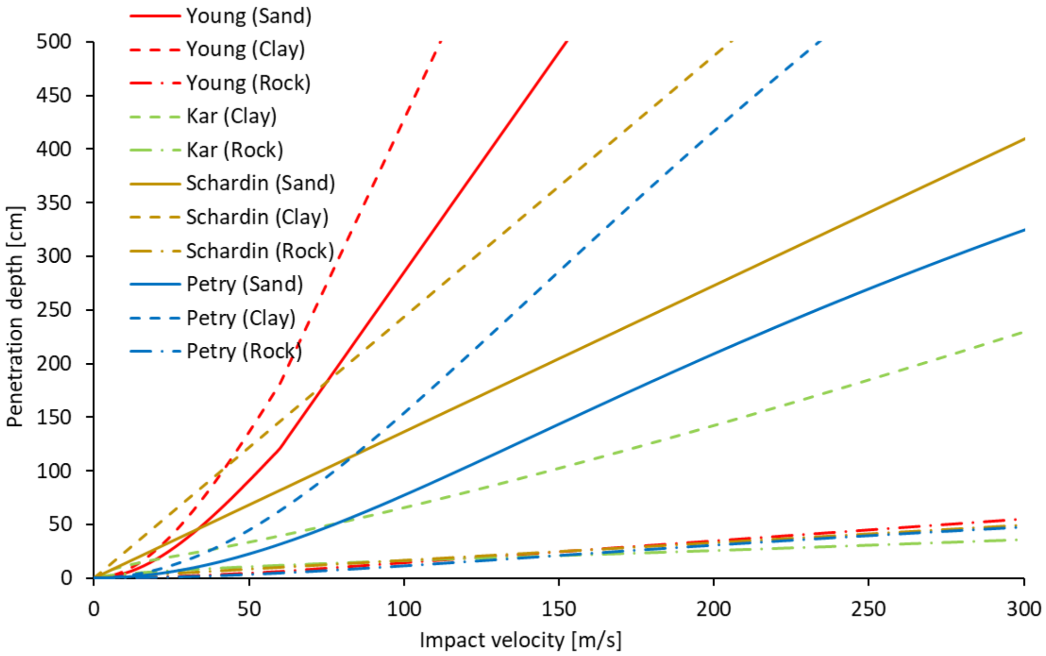

Figure 1. Application of the empirical formulas as a function of the impact velocity

NOVEL NUMERICAL METHOD

In addition to empirical approaches, it is also possible to analyse the mechanical effects of a dynamic impact event into the soil using numerical simulations. A continuum-based simulation method does not accurately represent the mechanical behavior of the soil. Due to the highly nonlinear material response, a novel combined solution using the Discrete Element Method (DEM) for the soil and the Finite Element Method (FEM) for the projectile is introduced in the following (see Figure 2).

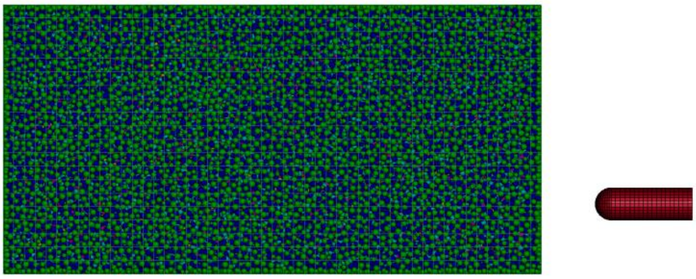

Figure 2. Combined method of DEM and FEM

The Discrete Element Method is a simulation technique based on the principles developed by Cundall and Strack (1979). It enables the modeling of granular material using spheres and disks. In this method, each particle is assigned numerical properties, such as size, elastic modulus, or density. By selecting sufficiently small time step, it can be ensured that external influences propagate only between neighboring elements. The interaction between individual particles and other elements is achieved through spring-damper contact models, following Newton's second axiom. In the simplest case, the total force acting on a particle i with mass m_i, velocity v_i, and displacement u_i is the sum of the gravitational force F_G and the contact force F_C (see Equation (8)). Due to the large number of collisions between particles, the equations of motion are typically integrated using numerically explicit methods.

$$m_i \frac{du_i{}^2}{dt^2} = F_G + \sum F_C \tag{8}$$

In the modeling of soil materials, the size of each discrete element is limited by computational capacity. Therefore, achieving convergence between the actual global material behavior and the scaling of individual elements is essential. Material calibration is crucial in this context.

STUDIES ON SELECTED IMPACT TESTS

Table 3 shows the test setup data of the selected hard impact tests conducted as a part of a joint project involving eleven electric power companies in Japan and published by Koyanagi et al. (2019) and Mihara et al. (2019).

Table 3. Setup of selected impact tests

M_p [kg]	D_p [mm]	v_p [m/s]	Soil type	Soil density [t/m³]	Impact angle [°]
0.1	30	100, 150, 200	Dry sand	1.6	20, 90

For the impact tests with normal impact angle, penetration depth is determined using the presented empirical formulas (see Figure 3). Due to the mass of the projectile, the Young formula is beyond its application limits. Considering the adjustment factor for low projectile masses results in a significant underestimation of the penetration depth when compared to the experimental data. Setting this factor equal to 1 yields to better results. Both Schardin and Petry formula underestimate the penetration depth of the projectile.

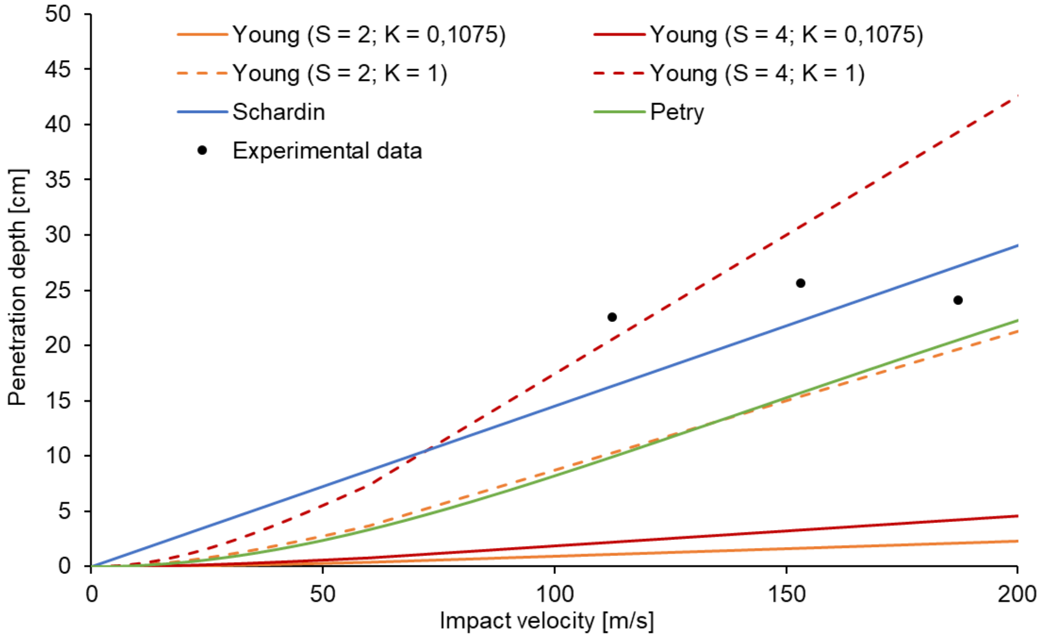

Figure 3. Comparison of penetration depth obtained by empirical formulas and experimental tests

The dynamic numerical simulations are performed using a 3D fully coupled analysis. The projectile is modeled using volume elements, and the sand is modeled with discrete elements contained within a shell element. An elasto-plastic material model is used for the projectile. For the projectile, the average mesh size is set to 5 mm and the radius of the discrete elements in a range of 1 – 2 mm. The discrete elements have been calibrated through various tests to accurately represent both macroscopic and microscopic material properties of the target material.

The numerical simulation results are presented in Table 4. The combined solution of FEM and DEM shows good agreement in both the horizontal and vertical penetration depth. Additionally, the velocity time history of test 1 is used for validation purposes, as illustrated in Figure 4. The simulation results closely align with the experimental data.

Table 4. Experimental and simulation test results for normal impact

		Experimental		**Simulation**	
Test	**v_p [m/s]**	**Horizontal penetration depth [mm]**	**Vertical penetration depth [mm]**	**Horizontal penetration depth [mm]**	**Vertical penetration depth [mm]**
1	187.1	210	55	299	46.4
2	153.1	240	30	284	4.59
3	112.4	210	30	246	34.4

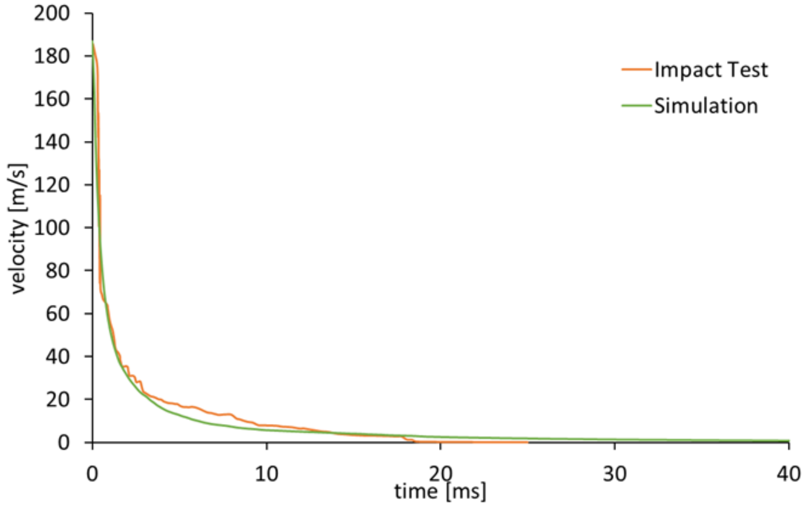

Figure 4. Comparison of residual velocity of projectile of test 1

Table 5 presents the test results of impact tests conducted at a 20° impact angle. The experimental data describe the failure mode of the soil as the projectile skidding over its surface. This behavior is accurately reproduced in the numerical simulations (see Figure 5).

Table 5. Experimental and simulation test results for 20° impact angle

			Experimental	**Simulation**
Test	**v_p [m/s]**	**Impact angle [°]**	**Failure mode**	**Failure mode**
7	184.9	20	Skidding	Skidding
8	110.1	20	Skidding	Skidding

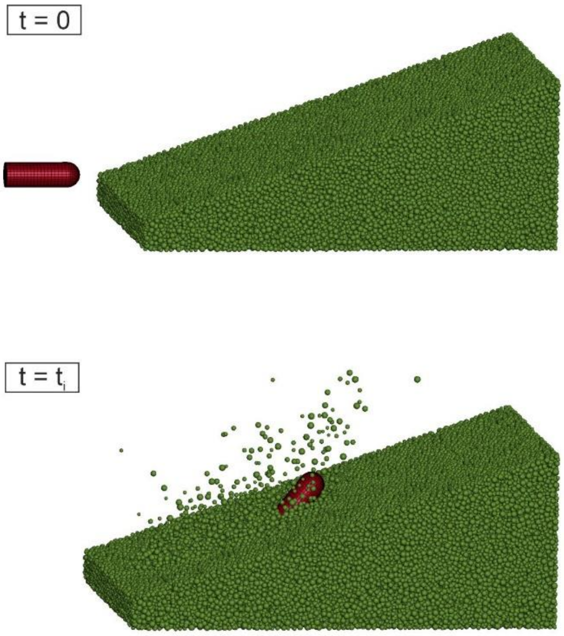

Figure 5. Simulation of test case 7

CONCLUSION

This paper presents empirical and numerical approaches for analysing the protective effect of soil. A comparison of the empirical formulae reveals a scattering of the calculated penetration depths. Subsequent investigations will apply these empirical formulas to a large database, assessing and verifying their applicability and application limits.

Furthermore, a combined numerical solution using DEM and FEM was introduced for estimating penetration depths and investigating the protective effectiveness of the soil. The DEM enables a realistic behaviour of granular material and is therefore very well suited for modelling the soil under impact loads. The numerical simulations show good agreement with the experimental data. To enhance the reliability of the combined numerical solution, its application should be extended to a broader experimental database. Especially the scaling of the discrete elements should be further investigated.

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