The Rate(Time)- Dependent Mechanical Behavior of Modified 9Cr-1Mo Steel - III. Unified Constitutive Model: Extension to Nonisothermal Small Strain Behavior

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1. INTRODUCTION

In the design and life prediction of load or pressure bearing structures for operation at high temperatures, fatigue, creep strain and ratchet strain accumulation must be considered. Methods have been advanced for the case of metallic engineering alloys including frequency modified Coffin-Manson approaches [1-2] linear damage summation [3-4], strain range partitioning [5-6], damage rate approaches [7-8] and coupled continuum damage models [9-10]. In each case, some knowledge of the constitutive behavior of the structural material is necessary if in-service structural performance is to be predicted. For structures which operate under thermal transients, the problem of combined mechanical and thermal loading due to temperature gradients is a further complicating factor. Thus, it is desirable to develop constitutive models for high temperature structural alloys which can predict material response under nonisothermal loading histories within the range of service conditions.

In this paper, constitutive equations are presented for the nonisothermal small strain behavior of modified 9Cr-1Mo steel between $25^oC - 600^oC$. The basic framework uses the superposition of nonlinear kinematic hardening rules as proposed by Chaboche and coworkers [11]. A framework for accounting for isotropic softening is presented wherein this softening can be included in a drag stress, in a yield stress or as a decrease in the saturation level of kinematic hardening or some combination of these; a single isotropic softening equation is used as determined from experiments.

The behavior of modified 9Cr-1Mo steel between $25^oC - 600^oC$ has been investigated by Swindeman [12], and Majors et. al. [13] and many important features have been discussed in Part I. Here, model parameters are determined from available isothermal uniaxial data and some correlations with isothermal and nonisothermal tests are presented.

2. PROPOSED FRAMEWORK

The flow rule is stated as

$$\dot{\underline{\epsilon}}^n = \dot{p}\underline{n} \qquad ; \qquad \dot{p} = G(\frac{\langle \Sigma - \kappa \rangle}{D}, T)\Theta(T) \tag{1}$$

where $\underline{n} = (\underline{s} - \underline{\alpha}/||\underline{s} - \underline{\alpha}||$ and $\Sigma = \sqrt{\frac{3}{2}}||\underline{s} - \underline{\alpha}|| = \sqrt{\frac{3}{2}}[(s_{ij} - \alpha_{ij})(s_{ij} - \alpha_{ij})]^{1/2}$; $\underline{s} = \underline{\sigma} - \frac{1}{3}\sigma_{kk}\underline{I}$ is the deviatoric stress and $\dot{p} = \sqrt{\frac{2}{3}}||\dot{\underline{\epsilon}}^n||$ is the effective inelastic strain rate normalized to tne uniaxial case. For small strains and rotations $\dot{\underline{\epsilon}} = \dot{\underline{\epsilon}}^e + \dot{\underline{\epsilon}}^n$ and $\dot{\underline{\sigma}} = \mathbb{L} : \dot{\underline{\epsilon}}^e$ where $\mathbb{L}$ is the fourth rank tensor of coefficients for isotropic, linear elasticity. The brackets $\langle\ \rangle$ indicate that $\langle \Sigma - \kappa \rangle = 0$ if $\Sigma - \kappa < 0$ and $\langle \Sigma - \kappa \rangle = \Sigma - \kappa$ if $\Sigma - \kappa \geq 0$.

In equations (1), κ is a "yield stress" or "threshold stress", $\underline{\alpha}$ is the second rank "backstress" and D will be called the "dragstress". The absolute temperature is denoted by T.

For kinematic hardening the superposition of nonlinear kinematic hardening rules, including thermal recovery terms [11] is used, i.e.

$$\dot{\underline{\alpha}}_i = C_i[b_i\underline{n} - \underline{\alpha}_i]\dot{p} - R_i\frac{\underline{\alpha}_i}{||\underline{\alpha}_i||} \qquad ; \qquad \underline{\alpha} = \sum_{i=1}^{N_\alpha}\underline{\alpha}_i \tag{2}$$

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For isotropic hardening a similar superposition of nonlinear evolution equations is offered, i.e.

$$\chi_i = \mu_i[\bar{\chi}_i - \chi_i]\dot{p} \quad ; \quad \chi = \sum_{i=1}^{N_\chi} \chi_i \tag{3}$$

This isotropic hardening may be partitioned between the yield stress, drag stress and saturation level of backstress using

$$b_i = b_i^o(T) + \sqrt{\frac{2}{3}}\omega_i\chi \quad ; \quad \kappa = \xi\chi + \kappa_o \quad ; \quad D = 1 + \frac{\beta\chi}{G^{-1}[\frac{\dot{p}_o}{\Theta(T)}]} \tag{4}$$

$$\sum_{i=1}^{N_\alpha} \omega_i + \xi + \beta = 1 \tag{5}$$

To interpret this framework, it is useful to consider the stress to which the model will saturate in a constant strain rate, proportional loading segment. Define a bounding surface radius as

$$R^* = \sqrt{\frac{3}{2}}\sum_{i=1}^{N_\alpha} b_i + \Sigma \tag{6}$$

which at a constant inelastic strain rate $\dot{p}_o$ is

$$R_o^* = \sqrt{\frac{3}{2}}\sum_{i=1}^{N_\alpha} b_i + DG^{-1}(\frac{\dot{p}_o}{\Theta(T)}) + \kappa = \sqrt{\frac{3}{2}}\sum_{i=1}^{N_\alpha} b_i^o(T) + G^{-1}(\frac{\dot{p}_o}{\Theta(T)}) + \kappa_o + \chi \tag{7}$$

where κ_o is the initial value of κ and all other "o" *subscripts* refer to the value of the respective parameter at $\dot{p} = \dot{p}_o$. Such an interpretation is useful for determination of the various temperature dependent functions and constants in the constitutive model. If $\dot{\chi} > 0$ then the material exhibits isotropic hardening; likewise $\dot{\chi} < 0$ is associated with isotropic softening and $\dot{\chi} = 0$ is an isotropically neutral response.

3. MODEL FITTING PROCEDURE

Baseline temperature-and strain rate-dependence was determined by estimating a zero strain ($p = 0$) intercept stress from uniaxial, constant strain rate tests at various strain rates and temperatures [12]; this intercept stress can then be set equal to R^* at $p = 0$. From tests at constant strain rate, the following correlations were determined for R^*

$$R^* = \begin{cases} 574.1e^{101.4/T} & ; \quad T < 723^oK \\ 1582[\ln(1/T) + 7] & ; \quad T \geq 723^oK \end{cases} \tag{8}$$

From tests at constant temperature

$$R^* = \begin{cases} 778\dot{\epsilon}^{0.082} & ; \quad T = 873^oK \\ 752\dot{\epsilon}^{0.054} & ; \quad T = 811^oK \\ 778\dot{\epsilon}^{0.035} & ; \quad T = 773^oK \end{cases} \tag{9}$$

Assuming a multiplicative dependence as in the flow rule, we can write

$$R^* = A(T)\dot{p}^{m(T)}\Theta'(T) \tag{10}$$

then

$$\Theta'(T) = \begin{cases} e^{101.4/T} & ; \quad T < 723^oK \\ \ln(1/T) + 7 & ; \quad T \geq 723^oK \end{cases} \tag{11}$$

The deformation softening parameters were determined by letting χ equal the decrease in flow stress from the intercept stress or the decrease in stress amplitude in cyclic strain-controlled tests. This softening behavior was found to be relatively independent of temperature. To approximately restitute both monotonic and cyclic strain softening $N_\chi = 2$ was found to be adequate so that

$$\chi = R^* - \sigma_f = \chi_1 + \chi_2 \quad ; \quad (N_\chi = 2) \tag{12}$$

$$\chi_1 = \bar{\chi}_1(1 - e^{-\mu_1 p}) \quad ; \quad \dot{\chi}_1 = \mu_1[\bar{\chi}_1 - \chi_1]\dot{p} \tag{13}$$

$$\chi_2 = \bar{\chi}_2(1 - e^{-\mu_2 p}) \quad ; \quad \dot{\chi}_2 = \mu_2[\bar{\chi}_2 - \chi_2]\dot{p} \tag{14}$$

where σ_f is the flow stress or stress amplitude at a certain accumulated inelastic strain. From a monotonic test at $\dot{\epsilon} = 6.7 \times 10^{-5}s^{-1}$, $T = 600^oC$ (873^oK) and from cyclic strain controlled tests at various temperatures [12], $\bar{\chi}_1 = -65MPa$, $\mu_1 = 30$, $\bar{\chi}_2 = -15MPa$, $\mu_2 = 0.3$.

For kinematic hardening, $N_\alpha = 3$ was chosen as this case was found adequate in earlier studies for correlating multiaxial nonproportional test results [14-15]. Additionally, the C_i were assumed to be independent of temperature and the yield stress was taken to be zero so that $\kappa_o = \xi = 0$. There is no evidence that a yield stress of significant magnitude exists for this material, particularly at elevated temperature. Based on the results of the short term relaxation tests of Ruggle's (see Part I), it appears that the flow softening is , to first order, the result of decreasing overstress with accumulated inelastic strain. Thus, the best approximation with the available data is $\omega_i = 0$ so that $\beta = 1$. The attenuated effective overstress is then

$$\frac{\Sigma}{D} = A(T)\dot{p}^{m(T)}\Theta'(T) - \sqrt{\frac{3}{2}}\sum_{i=1}^{3} b_i^o(T) - \chi \tag{15}$$

For relatively high strain rate tests, $R_i \approx 0$. Additionally, a good approximation under isothermal, constant strain rate, monotonic loading is that $\dot{p}$ quickly approaches $\dot{\epsilon}$ so that with $\dot{p} = \dot{\epsilon}$

$$\sigma = \sum_{i=1}^{3} \sqrt{\frac{3}{2}} b_i[1 - e^{-C_i p}] + \Sigma(\dot{p}) \tag{16}$$

for the uniaxial case. With $\omega_i = 0$, $b_i = b_i^o(T)$ so that it was possible to find $b_i^o(T)$ and C_i to fit the isothermal monotonic curves at constant strain rate. This was accomplished by assuming

$$\sqrt{\frac{3}{2}}\sum_{i=1}^{3} b_i^o(T) = B^o\Theta'(T) \tag{17}$$

This is equivalent to self-similar dependence of the saturation level of kinematic hardening and the overstress with temperature at constant strain rate and strain and corresponds to the requirement for temperature path history-independence of kinematic hardening as proposed for rate-independent thermoplasticity models [16-17]. B_o was chosen so that the saturation level of backstress is 60% of R^* at $\dot{p} = 6.7 \times 10^{-5}s^{-1}$ to be consistent with short term relaxation and stress dip tests (Part I). For $T < 723^oK$,

$$\sqrt{\frac{3}{2}}b_1^o = 150 \quad ; \quad \sqrt{\frac{3}{2}}b_2^o = 117.5 \quad ; \quad \sqrt{\frac{3}{2}}b_3^o = \Theta'' + 50\left(\frac{723 - T}{425}\right)^{0.10} \quad ; \quad \Theta'' = 344.5\Theta' - \sqrt{\frac{3}{2}}(b_1^o + b_2^o) \tag{18}$$

and for $T \geq 723^oC$

$$\sqrt{\frac{3}{2}}b_1^o = 105 + 45\left(\frac{873 - T}{150}\right)^{0.2} \quad ; \quad \sqrt{\frac{3}{2}}b_2^o = 0.25\Theta'' + 20\left(\frac{873 - T}{150}\right)^{0.20} \tag{19}$$

$$\sqrt{\frac{3}{2}}b_3^o = 0.75\Theta'' - 20\left(\frac{873 - T}{150}\right)^{0.20} \quad ; \quad \Theta'' = 949.2\Theta' - \sqrt{\frac{3}{2}}b_1^o$$

and $C_1 = 7000$, $C_2 = 500$, and $C_3 = 37.5$. By choosing $D = 1$ corresponding to $\chi = 0$ for which A, m and Θ' were determined (this results in some loss of generality), the overstress-temperature-strain rate relation becomes

$$\frac{\Sigma_1}{\Theta'(T)} = A(T)\dot{p}^{m(T)} - B^o \tag{20}$$

where $\Sigma_1 = \Sigma/D$. We can recorrelate equation (20) over a range of strain rates as

$$\frac{\Sigma_1}{\Theta'} = A'\dot{p}^{m'} \tag{21}$$

Then, from the tests at constant temperature (three strain rates) appropriate functions $m'(T)$ and $A'(T)$ were found. Finally we may write

$$\dot{p} = (\frac{\Sigma_1}{A'\Theta'})^{1/m'} = (\frac{\Sigma}{D})^{1/m'}\Theta \tag{22}$$

where $\Theta = (A'\Theta')^{-1/m'}$. For $T < 723^oK$, $m' = 0.024$ and $A' = 263.2$; for $T \geq 723^oK$, $m'(T) = -1.052\ln\frac{1}{T} - 6.901$ and $A'(T) = 3.905 \times 10^7 \exp(-\frac{7875}{T})$.

Forms for thermal recovery terms were formulated to be compatible with a first order thermodynamics approach [11] as follows:

$$R_i = C_i b_i \frac{(\sqrt{\frac{3}{2}}||\underline{\alpha}_i||)^{1/m'}}{K_i}\Theta \tag{23}$$

where K_1, K_2, K_3 were found by fitting to constant stress creep history responses; $K_1 = 5000$, $K_2 = 75$ and $K_3 = 1$ were found to provide an approximately best fit over three temperatures and a range of stress levels [12].

4. NONISOTHERMAL GENERALIZATION- THE INCLUSION OF TEMPERATURE RATE TERMS

Forms for temperature-rate terms have been postulated for kinematic and isotropic hardening rules of the forms given in equations (2)-(3). Essentially identical forms can be obtained on the basis of a first order thermodynamics approach [11] or on the basis of similarity of bounding and loading surfaces with temperature [16-17] leading to temperature-path-history-independence of $\underline{\alpha}$ (when the R_i are negligible) and χ, i.e.$\partial \underline{m}_i/\partial T = \partial M_i/\partial T = 0$, where $\underline{m}_i = \underline{\alpha}_i/C_i b_i$ and $M_i = \chi_i/\mu_i\bar{\chi}_i$. In either case, the following are obtained:

$$\dot{\underline{\alpha}}_i = C_i[b_i\underline{n} - \underline{\alpha}_i]\dot{p} + \frac{\underline{\alpha}_i}{C_i b_i}\frac{\partial(C_i b_i)}{\partial T}\dot{T} - R_i\frac{\underline{\alpha}_i}{||\underline{\alpha}_i||} \tag{24}$$

$$\dot{\chi}_i = \mu_i[\bar{\chi}_i - \chi_i]\dot{p} + \frac{\chi_i}{\mu_i\bar{\chi}_i}\frac{\partial(\mu_i\bar{\chi}_i)}{\partial T}\dot{T} \tag{25}$$

Clearly, isothermal tests alone are insufficient to prescribe specific forms for these terms, in lieu of restrictive assumptions such as those leading to equations (24)-(25). Extensive nonisothermal testing may, in fact, be required. On the basis of available data, therefore, equations (24)-(25), are explored with reference to nonisothermal test results.

5. CORRELATION OF EXPERIMENTAL RESULTS

Figures (1)-(3) indicate the predictive capability of the model for isothermal and nonisothermal tests. While the monotonic tests shown in Figure 1 were used for the determination of all constants and temperature-dependent functions (except K_1, K_2 and K_3), the cyclic tests shown in Figure 2 were not. The latter results are for a constant total strain rate of $\dot{\epsilon} = 10^{-4}s^{-1}$. Figure 3 shows the results of a two-bar ratchetting test [12] wherein the displacements of two test bars are maintained equal and the average stress in the bars is kept constant at 140 MPa; the temperatures of the two bars are then varied as shown in Figure 3. The average ratchet strain as predicted by the model is slightly conservative. Temperature rate terms as defined in equations (24)-(25) were used in obtaining the model results. In all correlations shown, elastic properties were taken to be independent of temperature. Elastic Poisson's ratio was taken as $\nu = 0.3$ and Young's modulus was taken as $E = 185$GPa. Additionally a constant value of thermal expansion coefficient was used, i.e. $\alpha_T = 12.08 \times 10^{-6o}C^{-1}$.

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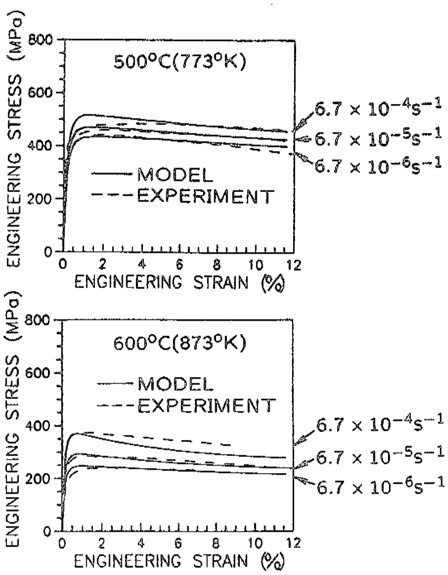

Fig. 1 - Correlation of constant strain rate monotonic tests at two temperatures.

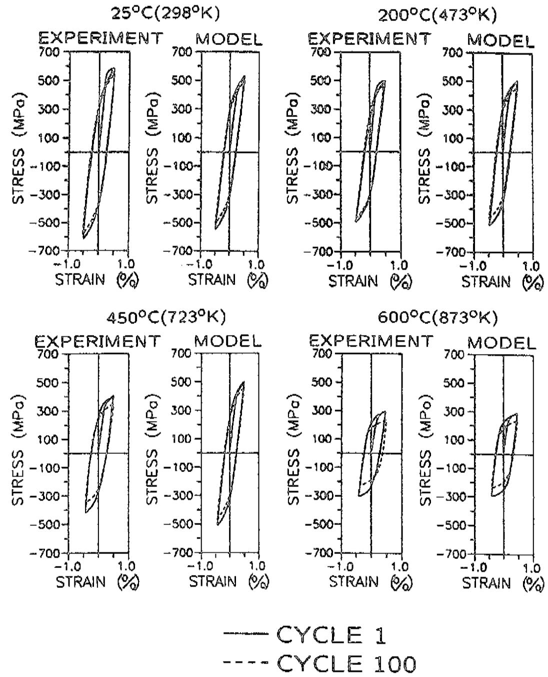

Fig. 2 - Prediction of strain-controlled cyclic response at various temperatures.

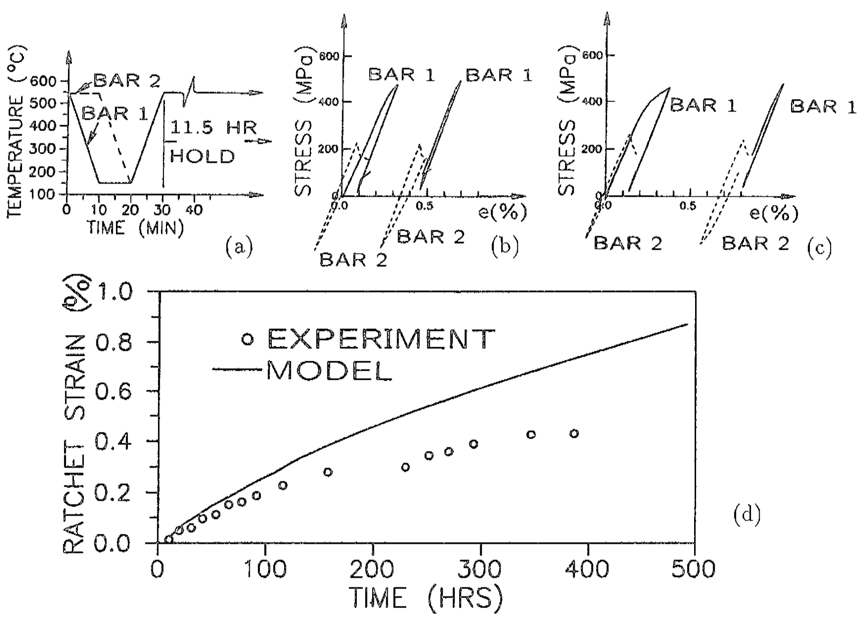

Fig. 3 - Two-bar ratchetting test: (a) temperature histograms, (b) stress vs. mechanical strain for cycles 1 and 30 (experimental), (c) stress vs. mechanical strain for cycles 1 and 30 (model) and (d) ratchet strain accumulation.

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