STABILITY AND TOE PRESSURE CALCULATION OF A REACTOR BUILDING SUBJECT TO SEISMIC DISTURBANCE

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SUMMARY

At the present time, the stability and toe pressure calculation of a reactor building subject to seismic disturbance is still based on the equivalent inertial force method applied statically. Even though it has long been recognized that this method sometimes gives unrealistically high response, especially when uplift is considered, it is still used by the industry due to the lack of better alternatives.

First of all, the equivalent static method fails to admit the fact that the inertial force acts cyclicly instead of constantly in one direction. Secondly, the maximum inertial force used to calculate stability and toe pressure lasts only a small fraction of a second according to the equivalent linear time history analysis. Of course, the validity of the linear analysis is in doubt when the resulting maximum inertial force predicts extensive uplift. Thirdly, when the maximum inertial force dictates a large amount of uplift, the static force method fails to consider the energy required to raise the center of gravity of a relatively rigid building.

In order to obtain a more realistic response, a more sophisticated model is required in the dynamic analysis. One way of doing it is to use the finite element method and represent the soil by either plane strain or 3-D elements. However, most general purpose finite element program cannot handle the problem of separation between soil and foundation mat in the dynamic analysis or the nonlinear force displacement curve for the soil. Even if a special finite element program were written for this purpose, the computation cost could be very high due to the nonlinear nature.

It is the intention of this paper to introduce a numberical scheme to handle this nonlinear dynamic problem with reasonable computation cost. A set of nonlinear differential equations is derived by coupling the modal information of a free-free building model with the compression only nonlinear force displacement curve representing the soil. This formulation has several advantages. These are (1) rigid body large displacement rotational mode is included in the analysis to account for the uplift, (2) the compression only nonlinear force displacement curve can be defined at several control points along the building soil interface, (3) the mode shapes used in the analysis are free-free. This allows for a better estimate of the importance of the building flexibility effects on soil pressure since the rigid body modes and flexible modes are separated, and (4) the boundary restraints imposed on the rigid body or free-free modes are a function of the relative motion between the structure and soil.

The set of nonlinear differential equations are solved using a digital computer code that functions as an analog computer. By using this code, the compression only nonlinear force displacement curve for the soil can easily be handled. A parametric study is conducted to evaluate the importance of uplift, building flexibility and the nonlinear force displacement curve. The results obtained are also compared with the conventional linear dynamic analysis.

1. Introduction

At the present time, the stability and toe pressure calculation of a Reactor Building subject to seismic disturbance is still based on the equivalent inertial force method applied statically. Even though it has long been recognized that this method sometimes gives unrealistically high response especially when uplift is considered, it is still used by the industry due to the lack of better alternatives.

First of all, when the maximum inertial force dictates a large amount of uplift, the static force method fails to consider the energy required to raise the center of gravity of a relatively rigid building. Secondly, the maximum inertial force used to calculate stability and toe pressure lasts only a small fraction of a second according to the equivalent linear time history analysis. Of course, the validity of the linear analysis is in doubt, when the resulting maximum inertial force predicts extensive uplift. Thirdly, the equivalent static method fails to admit the fact that the inertial force acts cyclicly instead of constantly in one direction.

In order to obtain a more realistic response, a more sophisticated model is required in the dynamic analysis. One way of doing it is to use the finite element method and represent the soil by either plane strain or 3-D elements. However, most general purpose finite element programs cannot handle the problem of separation between soil and foundation mat in the dynamic analysis or the nonlinear force displacement relationship of the soil. Even if a special finite element program were written for this purpose, the computation cost could be very high due to the nonlinear nature.

It is the intention of this paper to introduce a numerical scheme to handle this nonlinear dynamic problem with reasonable computation cost. A set of nonlinear differential equations is derived by coupling the modal information of a building model with the compression only nonlinear force displacement curve representing the soil. This formulation has several advantages. These are (1) the detailed analysis accounts for the energy required to raise the structure during uplift, (2) the compression only nonlinear force displacement curve can be defined at several control points along the building soil interface, (3) and, the mode shapes used in the analysis are independent of the rigid body modes. This allows for a better estimate of the importance of the building flexibility effects on soil pressure since the rigid body modes and flexible modes are separated.

The set of nonlinear differential equations are solved using a digital computer code[1] that functions as an analog computer. By using this code, the compression only nonlinear force displacement curve for the soil can easily be handled. A study is conducted to evaluate the importance of uplift, building flexibility and the nonlinear force displacement curve. The results obtained are also compared with the conventional linear dynamic analysis.

2. Derivation of the Equations of Motion

2.1 Conventional Model

This model, as shown in figure 1a, consists of the rigid body horizontal and vertical translation and the rocking mode as well as a number of flexible modes. The influence of the soil on the structure is represented by three independent soil springs located at the interface between the base mat and soil. The soil springs used in the analysis are the conventional values obtained from an elastic half-space analysis as given in reference 2.

The equations of motion were obtained by using Lagrange's equation:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} + \frac{\partial D}{\partial \dot{q}} - Q_q = 0 \qquad \text{eq. (1)}$$

The kinetic and potential energies of the system are:

$$T = \frac{1}{2}\Sigma_p\, m_p\, (\dot{\bar{r}}_p \cdot \dot{\bar{r}}_p)$$

$$U = \frac{1}{2}\sum_i (m_i\, \omega_i^2)\, u_i^2 \qquad \text{eq. (2)}$$

where $\bar{r}_p = (\delta_h + \ell_{p,v}\,\theta + \sum_i u_i\, \xi_{p,i})\bar{I}$

$+ (\delta_v)\bar{J}$

δ_h = Horizontal Rigid Body Displacement of Structure

δ_v = Vertical Rigid Body Displacement of Structure

θ = Rigid Body Rotation of Structure

u_i = Displacement of ith Mode

$\xi_{p,i}$ = Normalized Amplitude of Mass Point p for the ith Mode

m_i = Generalized Mass of ith Mode

ω_i = Frequency of ith Mode

m_p = Mass at Location p

$\ell_{p,v}$ = Vertical Distance to Mass Point p

$\overline{I}$ = Unit Vector Along Horizontal

$\overline{J}$ = Unit Vector Along Vertical

The Generalized Force, Q_q is:

$$Q_q = \sum_p \overline{F}_p \cdot \frac{\partial \overline{r}_p}{\partial q} + \overline{F}_B \cdot \frac{\partial \overline{r}_B}{\partial q} \qquad \text{eq. (3)}$$

where $\overline{F}_p$ = Gravity Force Vector at Point p

$\overline{F}_B$ = Spring and Damping Force Vector at the Center (Located at Base of Structure)

$$\overline{r}_n = (\delta_h - G_h + \sum_i u_i \xi_{n,i})\overline{I} + (\delta_s + \delta_v - G_v)\overline{J}$$

δ_s = Static Vertical Displacement

G_h, G_v = Rigid body displacement in horizontal and vertical directions

The resulting equations of motion for the independent coordinates δ_h, δ_v, θ and u_i are obtainted by carrying out the operations defined in equations 1, 2 and 3. These equations are:

$$\sum_p m_p \ddot{\delta}_h + \sum_p m_p \ell_{p,v} \ddot{\theta} + \sum_p m_p \sum_i \ddot{u}_i \xi_{p,i} + K_h (\delta_h - G_h + \sum_i u_i \xi_{n,i}) + C_h (\dot{\delta}_h - \dot{G}_h + \sum_i \dot{u}_i \xi_{n,i}) = 0$$

$$\sum_p m_p \ddot{\delta}_v + K_v (\delta_v - G_v) + C_v (\dot{\delta}_v - \dot{G}_v) = 0 \qquad \text{eq. (4)}$$

$$\sum_p m_p \ell^2_{p,v} \ddot{\theta} + \sum_p m_p \ell_{p,v} \ddot{\delta}_h + \sum_p m_p \ell_{p,v} \sum_i \ddot{u}_i \xi_{p,i} + K_r \theta + C_r \dot{\theta} = 0$$

$$\sum_p m_p \sum_i \ddot{u}_i \xi^2_{p,i} + \sum_p m_p \sum_i \xi_{p,i} \ddot{\delta}_h + \sum_p m_p \ell_{p,v} \sum_i \xi_{p,i} \ddot{\theta}$$

$$+ K_h (\delta_h - G_h + \sum_i u_i \xi_{n,i}) \xi_{n,i} + C_h (\dot{\delta}_h - \dot{G}_h + \sum_i \dot{u}_i \xi_{n,i})\xi_{n,i}$$

$$+ K_i u_i + C_i \dot{u}_i = 0$$

where $\sum_p m_p = M$

$$\sum_p m_p \ell^2_{p,v} = I$$

$$\sum_p m_p \xi^2_{p,i} = m_i$$

$$K_h = 32 (1-\mu) Gr_o$$

$$K_v = \frac{4 Gr_o}{1-\mu}$$

$$K_r = \frac{8 Gr_o^3}{3(1-\mu)}$$

$$C_h = 2\zeta M\omega_h$$

$$C_v = 2\zeta M\omega_v$$

$$C_r = 2\zeta I\omega_r$$

$$\omega_h = \sqrt{\frac{K_h}{M}}$$

$$\omega_v = \sqrt{\frac{K_v}{M}}$$

$$\omega_r = \sqrt{\frac{K_r}{I}}$$

ζ = Percent Critical Damping

2.2 Detailed Model

The derivation is carried out in the same manner as previously defined. The equations obtained for this model shown in 1b, however, are considerably more complicated because of the large displacement considerations for the rigid body degrees of freedom as well as the addition of a number of soil springs located between the interface of the structure and soil.

For this system the kinetic and potential energies are:

$$T = \frac{1}{2} \sum_p m_p (\dot{\bar{r}}_p \cdot \dot{\bar{r}}_p) \qquad \text{eq. (5)}$$

$$U = \frac{1}{2} \sum_i (m_i \omega_i^2) \dot{u}_i^2$$

where $\bar{r}_p = (\delta_h + \ell_{p,v} \text{SIN}\theta + \sum_i u_i \xi_{p,i}) \bar{I} + (\delta_v + \ell_{p,v} (1-\text{COS}\theta)) \bar{J}$

The generalized force for this model is:

$$Q_q = \sum_p \bar{F}_p \cdot \frac{\partial \bar{r}_p}{\partial q} + \sum_n \bar{F}_n \cdot \frac{\partial \bar{r}_n}{\partial q} \qquad \text{eq. (6)}$$

where the first dot product represents the summation over the number of mass points p, while the second dot product is the summation over n, the number of sets of horizontal and vertical soil springs located along the base of the structure.

Again, as stated in the previous derivation, the $\bar{F}_p$ and $\bar{F}_n$ force vectors consist of the gravity, spring and damping forces. The displacement vectors $\bar{r}_n$ and $\bar{r}_p$ used to calculate the generalized forces are:

$$\bar{r}_n = (\delta_h - G_h - \ell_{n,h} (1-\text{COS}\theta) + \sum_i u_i \xi_{n,i}) \bar{I}$$
$$+ (\delta_s + \delta_v - G_v + \ell_{n,h} \text{SIN}\theta) \bar{J}$$
$$\bar{r}_p = (\delta_h + \ell_{p,v} \text{SIN}\theta + \Sigma_i u_i \xi_{p,i}) \bar{I} + (\delta_s + \delta_v + \ell_{p,v} (1-\text{COS}\theta)) \bar{J}$$

The resulting set of equations is:

$$\Sigma m_p \ddot{\delta}_h + \sum_p m_p \ell_{p,v} \text{COS}\theta\ddot{\theta} - \sum_p m_p \ell_{p,v} \text{SIN}\theta \dot{\theta}^2$$
$$+ \sum_p m_p \sum_i \ddot{u}_i \xi_{p,i} + \sum_n \{K_h [\delta_h - G_h - \ell_{n,h} (1-\text{COS}\theta) + \sum_i u_i \xi_{p,i}] \qquad \text{eq. (7)}$$
$$+ C_h [\dot{\delta}_h - \dot{G}_h - \ell_{n,h} \text{SIN}\theta \dot{\theta} + \sum_i \dot{u}_i \xi_{p,i}]\} = 0$$

$$\sum_p m_p \ddot{\delta}_v + \sum_p m_p \ell_{p,v} \text{SIN}\theta \ddot{\theta} + \sum_p m_p \ell_{p,v} \text{COS}\theta \dot{\theta}^2$$
$$- \sum_p m_p (g) + \sum_n \{K_v [\delta_s + \delta_v - G_v + \ell_{n,h} \text{SIN}\theta]$$
$$+ C_v [\dot{\delta}_v - \dot{G}_v + \ell_{n,h} \text{COS}\theta \dot{\theta}]\} = 0$$

$$
\begin{aligned}
&\sum_p m_p \ell_{p,v}^2 \ddot{\theta} + \sum_p m_p \ell_{p,v} \cos\theta \, \ddot{\delta}_h \\
&+ \sum_p m_p \ell_{p,v} \cos\theta \sum_i \ddot{u}_i \xi_{p,i} \\
&+ \sum_p m_p \ell_{p,v} \sin\theta \, \ddot{\delta}_v \\
&- \sum_p m_p (g) \ell_{p,v} \sin\theta \\
&- \sum_n \{K_h [\delta_h - G_h - \ell_{n,h} (1-\cos\theta) + \sum_i u_i \xi_{p,i}] \\
&+ C_h [\dot{\delta}_h - \dot{G}_h - \ell_{n,h} \sin\theta \, \dot{\theta} + \sum_i \dot{u}_i \xi_{p,i}]\} (-\ell_{n,h} \sin\theta) \\
&+ \sum_n \{K_v [\delta_s + \delta_v - G_v + \ell_{n,h} \sin\theta] + C_v [\dot{\delta}_v - \dot{G}_v + \ell_{n,h} \cos\theta \, \dot{\theta}]\} (\ell_{n,h} \cos\theta) = 0
\end{aligned}
\qquad \text{eq. (7)}
$$

$$
\begin{aligned}
&\sum_p m_p \xi_{p,i}^2 \ddot{u}_i + \sum_p m_p \ddot{\delta}_h \xi_{p,i} + \sum_p m_p \ell_{p,v} \cos\theta \, \ddot{\theta} \, \xi_{p,i} \\
&- \sum_p m_p \ell_{p,v} \sin\theta \, \dot{\theta}^2 \xi_{p,i} + K_i U_i + C_i \dot{U}_i \\
&+ \{K_h (\delta_h - G_h - \ell_{n,h} (1-\cos\theta) + u_i \xi_{n,i}) \\
&+ C_h (\dot{\delta}_h - \dot{G}_h - \ell_{n,h} \sin\theta \, \dot{\theta} + \dot{u}_i \xi_{n,i})\} \xi_{n,i} = 0
\end{aligned}
$$

3. Analysis

Response information was calculated for two soil conditions. These are rock, with a shear modulus of 500,000 psi, and soil, with moduli of 70,000 and 5000 psi. The extremely low value is used to demonstrate the failure of foundation material. The dynamic characteristic of the containment structure used in this study was such that its rigid body modes ranged from about 2 cps to 20 cps while the flexible modes were from about 4 cps to 20 cps.

Six flexible modes were used in this study since it is a qualitative investigation. For a detailed design analysis, obviously all flexible modes between 0 and about 40 cps should be used. The mode shapes and frequencies used herein were obtained by using a simplified beam element dynamic model.

Some approximations had to be made in determining the spring constants for the soil springs located along the soil and rigid mat interface. For the detailed model shown in figure 1, a total of 9 soil springs were used. The assumption used in determining the soil springs was that they are proportional to the force distribution. The force distribution was obtained by using the concave-parabolic pressure distribution and the tributory areas given in figure 2. Then, by scaling the spring rates according to the force distribution such that the relationships

$$K_r = \sum_{i=1}^{9} K_i \ell_i^2 \text{ and } K_v = \sum_{i=1}^{9} K_i$$

were satisfied, the spring rates were finally determined.

The other assumptions on the pressure distribution, e.g. uniform and convex parabola, may be used as well. However, the resulting vertical and rocking springs may not be compatible.

All the response studies were conducted using a percent of critical damping of 0.03 for all rigid body and flexible modes. It should be noted that the modal superposition method used here, where the rigid body modes are independent of the flexible modes, allows the analyst to properly adjust the damping for rigid body and flexible modes independently and eliminates the need for using weighted damping.

The force displacement functions for the soil springs have been coded such that separation between the soil and base mat as well as plastic action of the soil can be accounted for. This is done by having a function generator for each soil spring. The function can be set to operate in any one of the following modes: 1) completely linear 2) elasto-plastic in tension and compression 3) and elasto plastic during compression only. As shown in figure 3, the soil spring is operating elasto-plastically during compression only. For instance, as shown in figure 3, with increasing compression distortion the force will increase up to point 2 at which time it became plastic. The force will remain constant at that level until the distortion changes direction which occurs at point 3. At this point the force decreases until an increase in distortion compression occurs. Then, the force can increase again as indicated.

The horizontal and vertical inputs are the artificially generated time histories which are consistent with the USAEC Regulatory Guide 1.60.

4. Results

When the detailed model has soil springs which are compatible with the conventional models springs and act completely linear, its response compares essentially the same as the response of the conventional model.

The results of the dynamic analysis will be discussed for two catagories. These are (1) results where the separation criteria is that the initial static displacement has to be overcome before separation occurs (2) and results were the separation criteria is defined as a percent of the initial static displacement. This is done to account for the assumed slow rebound phenomenon of soil.

For the first separation criteria, the dynamic results show that separation does not occur for the .36 g earthquake time history input when the containment structure was founded on soils with shear module of 500,000 psi and 70,000 psi. However, using the present static design method of using the maximum moment and shear from a dynamic analysis such as the conventional model, separation is predicted for both soil conditions and gives a maximum toe pressure of 191 psi for the case where the shear modulus is 70,000 psi. Using the results from either the detailed or conventional models, which are essentially the same for these cases, a maximum stress of 130 psi is calculated using the maximum moment and its associated vertical force. When using the maximum vertical force and its associated moment, a stress of 101 psi is obtained.

When a shear modulus of 5000 psi was used, as expected, the dynamic response of the structure greatly exceeded the initial static deflection. Then, when plastic action and separation were included in this case, the structure becomes unstable dynamically.

From the results using the second criteria, it appears that early separation can have a varied effect. For instance, for cases where separation was 1 percent of the original static displacement and with a shear modulus of 70,000 psi, lower toe pressures were calculated compared to those cases were the springs were completely elastic. However, on the other hand, a higher toe pressure was calculated for cases with a separation criteria of 1 percent static deflection and a shear modulus of 500,000 psi compare to cases which where completely elastic. The results also show that the cases with early separation give much higher uplift than those cases with completely linear springs. This larger separation is probably due to the fact that when early separation occurs, less energy is absorbed as indicated by the small amount of area under the force deflection curve as compared to the area under the force deflection curve when the total deflection is taken into account. This also explains why higher toe pressures can occur from early separation. A possible explanation as to why separation can cause lower toe pressures is that the system is non-linear and as separation occurs it tends to detune the system resulting in a lower response.

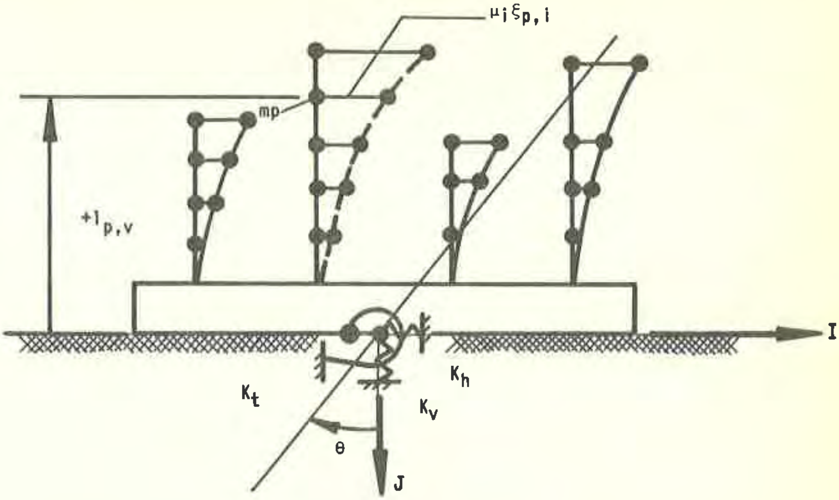
5. Conclusion

A technique is described in this paper which can handle the nonlinear effect of uplift and the extra energy required to uplift a structure in the dynamic analysis. The computer cost of this run is only $40.00 when a 22 sec. duration input is integrated at 0.01 sec. increment by the 4th order Runge Kutta method.

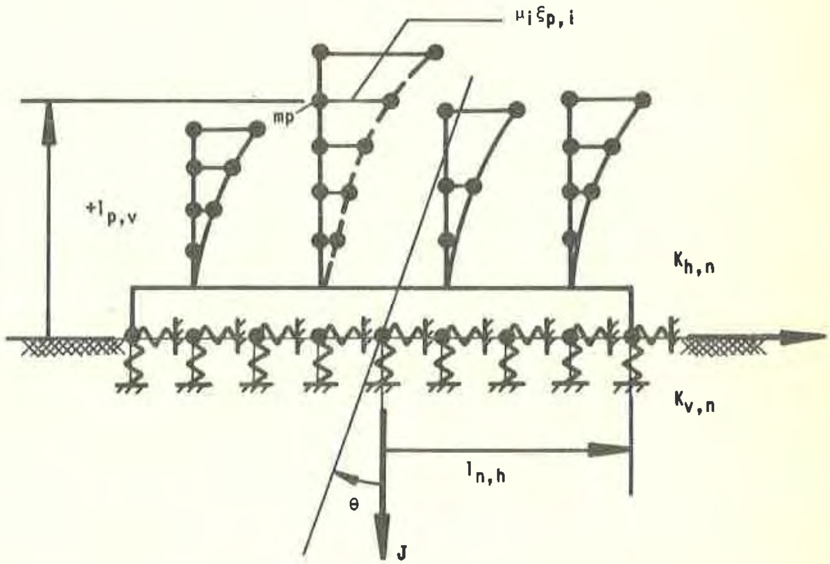
The same analysis can be extended to a three dimensional model to account for the two horizontal and one vertical time histories simultaneous input. If the foundation mat is under water table, the effect of buoyance can be taken into account as well. The nonlinear effect of possible sliding due to seismic disturbance can also be handled by this program.

(1) System/360 Continuous System Modeling Program (360A-CX-16X) User's Manual, IBM Technical Publications Department, White Plains, N.Y.

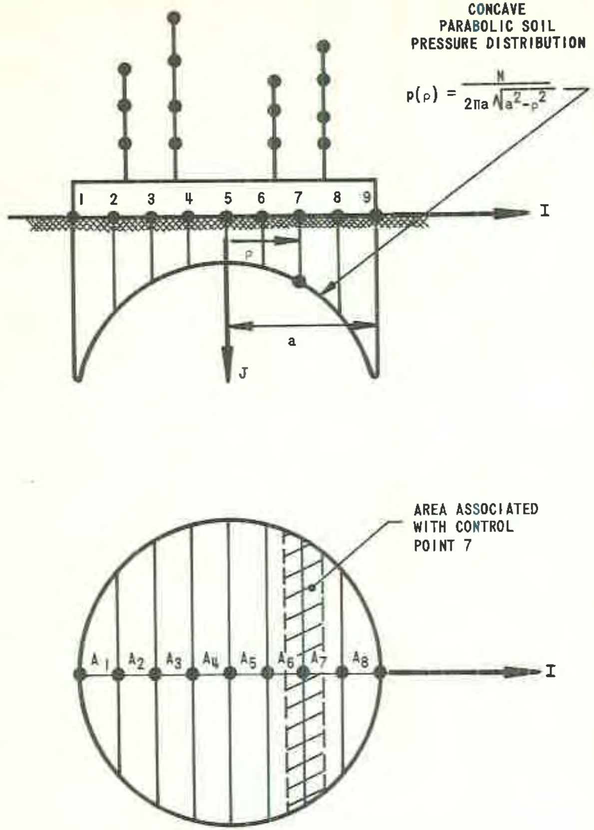
(2) Whitman, R. V., Richart, F. E., "Design Procedures for Dynamically Loaded Foundations," Journal of the Soil Mechanics and Foundations Division, November, 1967.



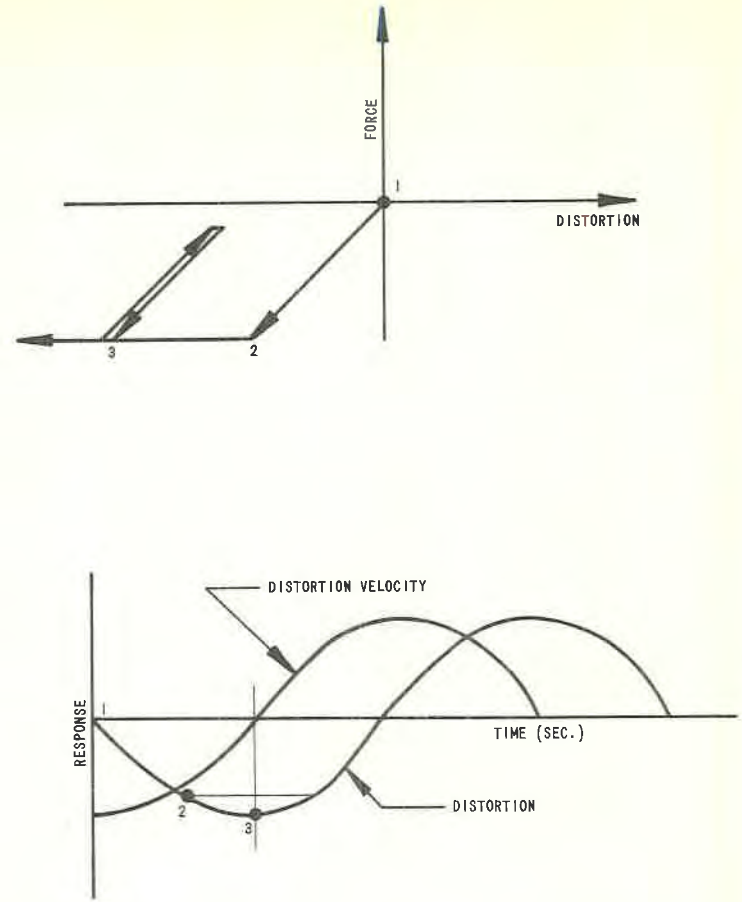
CONVENTIONAL MODEL
FIGURE Ia



DETAILED MODEL
FIGURE Ib



SOIL PRESSURE DISTRIBUTION AND PLAN
FORM SHOWING AREA DISTRIBUTION
FIGURE 2



ELASTO-PLASTIC COMPRESSION ONLY FORCE - DEFLECTION CURVE
FIGURE 3