MODIFICATION OF NEWMARK'S ALGORITHM TO ACHIEVE EFFECTIVE NUMERICAL DISSIPATION

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SUMMARY

The Newmark family of methods is widely used for the direct integration of the matrix equations of structural dynamics. However, it has been noted by several investigators that when numerical dissipation is introduced in the higher modes, the lower modes are effected all too strongly. In this paper we describe a modification of Newmark's algorithm which achieves improved numerical dissipation properties.

In many structural dynamics applications only low mode response is of interest. For these cases the use of implicit unconditionally stable algorithms is generally preferred over conditionally stable algorithms.

Conditionally stable algorithms require that the size of the time step employed be inversely proportional to the highest frequency of the discrete system. In practice this is a severe limitation as accuracy in the lower modes can be attained with time steps which are very large compared with the period of the highest mode.

For unconditionally stable algorithms a time step may be selected independent of stability considerations and thus can result in a substantial saving of computational effort.

In addition to being unconditionally stable, when only low mode response is of interest it is often advantageous for an algorithm to possess some form of numerical dissipation to damp-out any spurious participation of the higher modes. Examples of algorithms commonly used in structural dynamics which possess these properties are Houbolt's method, the Wilson θ-method, Park's method and the Newmark family of methods restricted to parameter values of $\gamma > \frac{1}{2}$ and $\beta \geq (\gamma + \frac{1}{2})^2/4$.

The Newmark family of methods allows the amount of dissipation to be continuously controlled by a parameter other than time step. For example, set $\beta = (\gamma + \frac{1}{2})^2/4$ and $\gamma > \frac{1}{2}$; then the amount of dissipation, for a fixed time step, is increased by increasing γ. On the other hand, the dissipative properties of this family of algorithms is considered to be inferior to the other methods, since the lower modes are affected too strongly. (It seems all of these algorithms adequately damp the highest modes.)

A new method of introducing dissipation into the Newmark scheme is developed. The main results are summarized as follows:

- The new schemes are unconditionally stable when applied to linear problems.
- The amount of dissipation may be continuously controlled by a parameter other than the time step. In particular, no numerical dissipation is possible.
- The low modes are affected very little; in fact second-order accuracy is maintained.
- The new schemes represent only a minor modification of the basic Newmark scheme.

Comparison of important measures of the goodness of the algorithm (such as spectral radius, damping ratio, period elongation, etc.) is made with other popular schemes and the present algorithm is shown to compare favorably.