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Recovery of the initial elastic-plastic properties through demagnetisation-like stress paths

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ABSTRACT The aim of the paper is to suggest, through the study of demagnetization-like stress paths, that it actually exists an opportunity to define a powerful constitutive benchmark in the field of multiaxial cyclic plasticity, pointing out to its hysteretic character. For the benefit of engineers, the origin of this opportunity, the nature of the benchmark and the way to obtain it, are briefly introduced.

Introduction. i) Owing to the complexity of the questions raised through the problems of hardening, rate dependence, fatigue, damage, coupled fields effects, second order effects of ratchet type, it is necessary, at least in principle, to rely on a well founded and physically relevant pattern of rate independent and without hardening multiaxial elastic-plastic behaviour, under random cyclic loading. Reliance is necessary in this particular case (apparently rather theoretical but in fact representative of the basic feature of the almost utterly irreversible global behaviour), and dilemma is therefore removed as to the hierarchy of priorities in the field of constitutive approaches. Accordingly, the basic benchmarks are necessarily those regarding the rate independent behaviour of plastic hysteresis without hardening. Although tensorial, this behaviour is similar to that, considered as vectorial, which is exhibited by ferromagnets, at least under usual conditions.

ii) In these standard conditions, it is a well known fact that demagnetisation processes are quite readily available. Moreover, it is worth noting that the microstructures of the ferro-materials are then almost perfectly restored [9]. Regarding the mechanical case, macroscopic and microscale results are also available: although less numerous and less complete, they suggest similar prospects [12,13].

iii) Consequently, the study of a demagnetization-like process in the mechanical case, seems of special interest when it is performed through a pattern able to describe both the plastic hysteresis and the ferro hysteresis. In spite of the experimental evidence, it can be difficult to admit that the basic mechanical study has to be devoted to processes which may appear as of unloading type, in a broad sense. Fortunately, it can be shown that the relevant demagnetization strategies are, in fact, highly cyclic and out of phase, and therefore highly different from most previsions based on intuition.

The paper is four-fold: firstly, we are presenting some remarks dealing with the question of experimental evidence available in the magnetic case; secondly, we are suggesting hints of similar evidence partly available in mechanical situations of fatigue type; thirdly, the results obtained through numerical simulations, and in the field of multiaxial cyclic plasticity, are introduced: a simple standard form of benchmark may be obtained on the ground of these results; finally, we offer a short reminder as to the discrete-memory thermomechanical pattern which is implemented in the approach, as well as a short reminder regarding the more general pattern of "pure hysteresis in preferred reference frame" which allows the study of second order effects of ratchet type.

1. The heuristic character of experimental evidence in the case of ferrohysteresis.

1.1. Standard tests. The heuristic character of the results obtained through standard tests is well known[1,2]. The behaviour may be interpreted with the aid of symbolic models of "pure" hysteresis. These models are constituted of ordered infinite parallel (for example) succession of couples defined by a spring and a dry-friction slider associated in series [3,4,5,6,7].

1.2. Microstructural basic processes. At the level of domains and wall pinning, the interpretation of the relationship between the field H and the magnetisation M is possible on the ground of the pure hysteresis behaviour introduced above [7]. It is worth noting the importance of this possibility not only at the level of principles but also with regards to the methods and physical relevance of the modelling [8].

1.3. Fatigue-like tests and mesoscale substructures. The detailed features of the substructures may be periodically restored under cyclic field-control loading which are similar to the mechanical stress-control fatigue tests, owing to the fundamental analogy (stress σ-field H; strain ε-magnetisation M or induction B; $\sigma d\varepsilon$- HdB). A typical example is provided in [9], where the topology of a magnetic cellular pattern is studied through the case of an amorphous lattice of bubbles (hexagonal and with many defects). The disordered cellular pattern is obtained, "under the first loading" (cf. Fig. 1, here given as a reminder of [9]), when the field H is applied. Under zero field (point C of Fig.1.b), at the end of the "first unloading", a labyrinth type structure appears, keeping the topology of the cellular pattern. Accordingly, if the field is increased back through a fatigue-like field control process (point B2 of Fig.1.b), the cellular pattern is periodically (almost exactly) recovered.

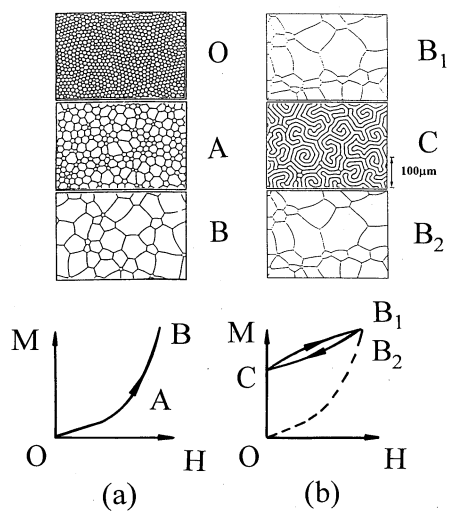

Fig. 1. Fatigue test: the labyrinth structure in a bubble lattice under zero field.

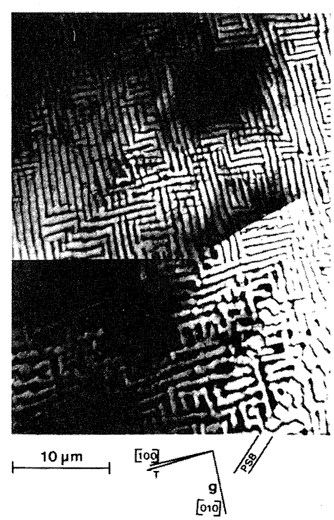

Fig. 2. Fatigue test: the labyrinth structure in polycrystalline copper

2. Some hints available in the mechanical case: fatigue tests and labyrinth-type mesoscale substructures of Krumhansl.

2.1. Standard tests. When performing one temperature (ordinary) and slow ($D=10^{-5}$ to $10^{-4}s^{-1}$) tests, the experimental evidence is that noticed above (§ 1.1): the behaviour of usual metallic alloys is indeed of pure hysteresis type, up to moderate hardening effects [10,11].

2.2. Microscale basic processes. The situation is that described above, § 1.2 (cf.[7,8,10,11]).

2.3. Fatigue tests and mesoscale substructures of labyrinth type. The microanalysis may be performed in the null stress, remnant strain state and this state is similar to that of null field and remnant magnetisation which is involved above (§1.3). It is worth noting that the microstructural pattern is once more of labyrinth type [12,13], as recalled regarding the case of a polycrystalline copper (cf. Fig. 2 , here given as a reminder of [13]).

A provisional conclusion may be as follows: it is physically relevant to admit that the detailed features of the mesoscale substructures of Krumhansl [14] are almost periodically restored under periodic cyclic loading. Accordingly, one is allowed to implement a tensorial (resp. vectorial [7]) constitutive pattern endowed with the following features: i) it results in strictly cyclic order-disorder bursts; ii) it involves a disorder variable I; iii) it allows a "generalized" demagnetization expressed through σ,ε,I (resp.H,M,I); iv) this general assumption is maintained even if the intended theoretical pattern is of utterly unclassical functional form, for example of discrete memory (functional) form.

The pattern implemented in this study is actually of discrete memory (functional) form, and it implies therefore the above assumption (an assumption which may be slightly modified later on in order to express experimental evidence regarding special effects such as ageing or fatigue).

3. An example of demagnetization-like process.

The example is that of the spiral-like Slowly Decreasing Stress (SDC) paths in a "simple" case of cyclic multiaxial plasticity with limit surface of von Mises type and irrotational triaxial kinematics. The case under consideration is called "simple" because it involve functional relationships between two-dimensional stress paths and two-dimensional strain paths. This simplification is obtained through imposing deviatoric stress paths (and also $I_\sigma=0$) and using a basically deviatoric elastic-plastic law involving that the strain rate tensor $\mathbf{D}$ is also deviatoric ($I_D=0$, $\mathbf{D} = \overline{\mathbf{D}}$) when the stress is a deviator ($\sigma = \overline{\sigma}$). The spiral-like SDC stress path is approximate with 32 sides polygonal lines, each side being discretized with 200 steps. The integrations of the homogeneous problems are performed with the aid of a Runge-Kutta-Merson routine. The intensity of the stress deviator Q_σ may be rather close to the von Mises radius Q_0 , provided that: $1-Q_\sigma/Q_0 > 10^{-7}$.

3.1. From the first loading to a relevant remnant state. The irrotational kinematics are defined by the stretches K_1, K_2, K_3, and the strain rate components are: $D_n = \dot{K}_n \;/\; (1+K_n)$, n=1,2,3. The Cauchy stress components are σ_n (n=1,2,3). Along the radial (with constant phase φ_σ) first loading path, the differential-difference deviatoric rate form of the pattern is reduced to:

$$(3.1.a)\quad \begin{cases} \dot{\overline{\sigma}}_n = 2\mu\,\overline{D}_n + \beta_4\,\overline{\Phi}\,\overline{\sigma}_n\,;\beta_4 = \dfrac{-2\mu}{(Q_0^c\,Q_0^{2-c})}\;;\;\omega\omega' = \omega\omega'(\alpha)\,,\;\alpha = \varphi - \varphi_R = 0\;,\omega'(0)=1 \\ dW = \dfrac{2}{(\omega\omega')^2}\overline{\Phi}\,dt\;;\;\overline{\Phi} = \overline{M} = Q_\sigma Q_D\;;\dfrac{d\varphi}{dt}=0\;;\;\omega=1\;;\overline{M} = tr(\overline{\sigma}\,\overline{D}) = \Sigma_1^3\sigma_n D_n \end{cases}$$

$$(3.1.b)\quad \begin{cases} Q_\sigma dQ_\sigma = 2\mu\overline{M}\left[1-\left(\dfrac{Q_\sigma}{Q_0}\right)^c\right]dt\;;\;0<t<t_R\;;\;Q_\sigma^2 = (\omega Q_0)^2\left[1-\exp\left(-\dfrac{\mu W}{S_0^2}\right)\right] \\ W = -\dfrac{S_0^2}{\mu}Ln\left[1-\left(\dfrac{Q_\sigma}{\omega Q_0}\right)^2\right]\;;\;Q_0^2 = 2S_0^2 = \sigma_1^2+\sigma_2^2+\sigma_3^2 \end{cases}$$

where t_R, ω, φ, $\overline{\Phi}$, W, μ, S_0 and c (c>0) are the time parameter of the end of the first loading, the Masing functional (=1 or 2), the phase, the discrete memory form intrinsic dissipation, its "modified" integral along a branch of cycle, the shear modulus, the yield limit and the "elastic-plastic transition" parameter, respectively. The well known case of Prager is obtained if c=2.The implemented remnant state is defined as follows. Firstly, the reached strain is "highly plastic" that is to say much greater than S_0/μ; secondly, a very small unloading is added (a single step of the numerical process of integration). The state so obtained is considered a remnant state. Owing to the basic hints given below in § 4.1, it is possible to imagine that this state is relevant in order to maintain the bearing of the study.

3.2. The reverse rule. Owing to the well known properties of the one-dimensional case (the apex of the symmetric cycles are on the first loading curve), the reverse rule is introduced as follows:

$$(3.2)\begin{cases} \dfrac{Q_{spiral}(\varphi_\sigma)}{Q0} = \tanh\left[\left(\dfrac{2\pi n - \varphi_\sigma}{2\pi n}\right)\tanh^{-1}(x)\right], \quad x = \left[1 - \exp\left(-\dfrac{W}{(S_0^2/\mu)}\right)\right]^{1/c}, \quad c = 2 \\ \dfrac{Q_{spiral}(\varphi_\sigma)}{Q0} = 1 - \exp\left[\left(\dfrac{2\pi n - \varphi_\sigma}{2\pi n}\right)\mathrm{Ln}(1-x)\right], \quad x = \left[1 - \exp\left(-\dfrac{W}{(Q_0^2/\mu)}\right)\right]^{1/c}, \quad c = 1 \\ \dfrac{Q_{spiral}(\varphi_\sigma)}{Q0} = 1 - \exp\left[\left(\dfrac{2\pi n - \varphi_\sigma}{2\pi n}\right)\mathrm{Ln}(1-x^{1/3})\right]^3, \quad x = \left[1 - \exp\left(-\dfrac{W}{(Q_0^2/\mu)}\right)\right]^{1/c}, \quad c = 2/3 \end{cases}$$

and: n=10. The expression chosen for x is generic. In fact, the exact forms are those of the case c=2: approximations are implied in the two other cases, given for illustration. However, the approximate forms are accurate when necessary, that is to say at the beginning of the demagnetization process, very close to the plastic limit. The choice of n is for numerical convenience. It is worth noting that n=10 implies approximately 100 loading-unloading events and that, in the relevant engineering conditions [15], n is rather of the order of 1000. However, the present value of n is not sufficient as to the final part of the demagnetisation processes.

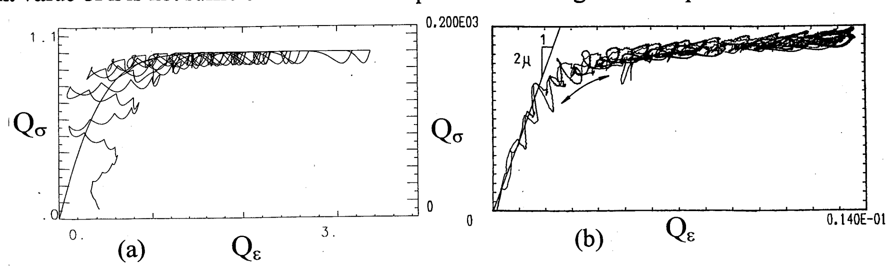

Fig. 3. Qualitative comparison between the experimental evidence and the three parameters tensorial pattern in spiral-like out-of-phase test [10].

3.3. Preliminary study. At the first stage, a set of numerical simulations has been performed in order to get some hints regarding the following questions:

3.3.1. In spite of its simplicity, the pattern results in a (not simple) behaviour which is similar to experimental evidence (Fig. 3). It is worth noting that the pattern implies a complicated discontinuous process (as suggested below, § 4.1) and that, consequently, the existence of an approximately Unique Thermomechanical Neutral (UTN) state is not obvious, nor is the existence of a SDC path allowing its recovery with satisfactory accuracy.

3.3.2. It is necessary to establish a comparison, at least between the piecewise continuous SDC stress paths and the utterly continuous SDC stress paths (Fig 4):both appear as cyclic, but the behaviour of W and I are more compatible with the reverse rule in the case of continuous SDC stress path (continuous sky lines of the W and I spectra).

3.3.3. It is worth noting that intuitive strategies can result in admissible as well as coarsely false demagnetisation (only the last case is illustrated through figure 5).

These results are sufficient to suggest briefly why the case studied further is only that of continuous SDC stress paths which are of (slow-fast) spiral-like type.

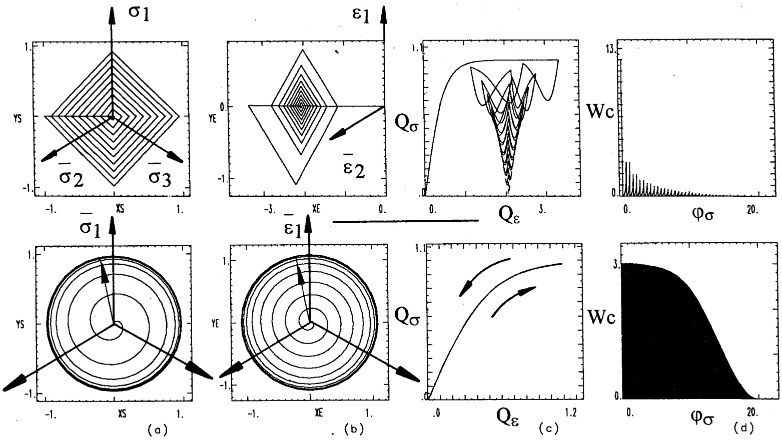

Fig. 4. Comparison between piecewise continuous and utterly continuous stress paths: it is worth noting the sky-line behaviour.

3.4. The main results.

Owing to the remark of § 3.3.1 as to the actual existence and accessibility of the UTN state, the main result is that it exists at least a spiral-like demagnetisation strategy in the "simple" mechanical situation introduced above. The illustration is obtained through a slow-fast spiral-like SDC path, making use of the reverse rule (c=cr=2/3)and with:nw=32;$\xi=10^{-7}$ (Fig.6). The sky line of W is slightly discontinuous during the third turn.

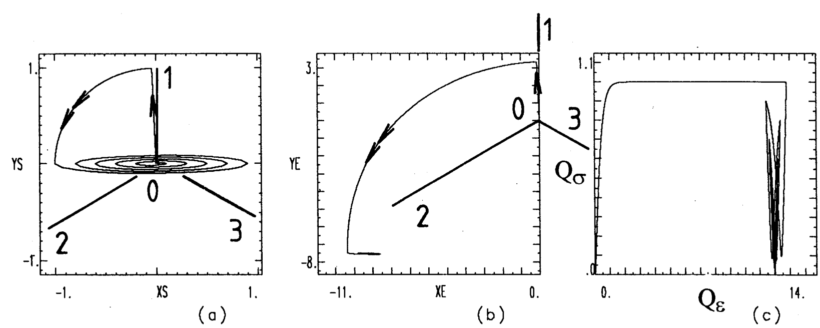

Fig. 5. Unsuccessful demagnetisation as a result of the implementation of a one-dimensional intuition.

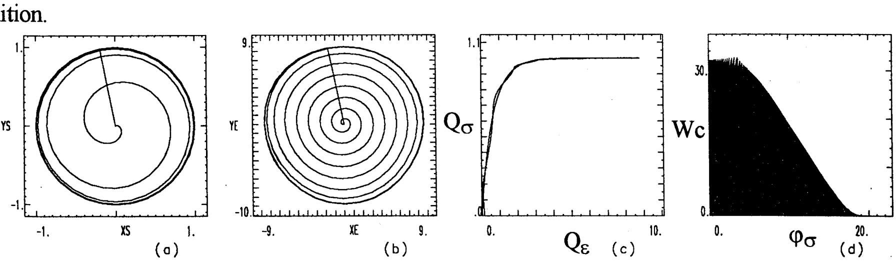

Fig. 6. An example of demagnetisation similar to that introduced on figure 4: the SDC stress path is of slow-fast spiral-like type(b); it is worth noting that the sky-line of W is rather continuous in spite of the fact that the beginning (3 turns) of the stress path is very close to the von Mises plastic limit (c).

4. Short reminder as to the underlying constitutive pattern.

4.1. Heuristic hints: from the notion of plastic limit to the discontinuous process of the wrapping up type, performed by the nest of neutral thermomechanical surfaces.

In order to describe out of phase paths, an orientation parameter: $\alpha=\varphi-\varphi_R$ (cf. eq.(3.1)) is introduced (cf. Fig. 7) in the Ilyushin Stress Space. In a first step, the arrow (Fig.7.a) shows the forbidden transgression of the von Mises plastic limit(MPL) at the end of the first unloading performed up to the circle FU in the direction defined by α (if the Masing rule is followed without precaution, fixing tentatively the radius of FU equal to twice the radius of circle FL); consequently, the relevant yield limit P1P2 (Fig.7.b) regarding the unloading MR-CS must be defined as a function of $\alpha=\varphi_\Delta-\varphi_R$ (and $\alpha=\pi$ in the usual push-pull case). In a second step one recalls the discontinuous sliding process of the thermomechanical neutral loci (Fig.8): along each NP locus $\Phi=0$ and W is constant; during the first loading(see arrow of Fig. 8a) the family of NP locus is located as shown; to the right of the inversion point MR1 the family jumps and remains stationary during the unloading MR1-MR2 (Fig. 8b); the subsequent cases (Fig. 8c,d and Fig.9) are similar; the shaded zones are not accessible without coincidence of the W level.

4.2. Formal features of the pattern when it is, only temporarily in principle, defined in a fixed orthonormal reference frame.

In the three-dimensional case, the rate form (3.1) of σ_n (now n=6) is obtained making use of the general form of Φ. Let us denote by: Q, φ^d, θ_1, θ_2, θ_3 the Ilyushin parameters of the stress deviator $\overline{\sigma}$. One has:

(4.1) $\overline{\sigma}_1=q(2/3)^{1/2}\cos\varphi^d$;...; $\sigma_{n+3}=(Q/\sqrt{2})\cos\theta_n$; n=1,2,3 ; $q^2=\Sigma_1{}^3\overline{\sigma}_n{}^2$;$Q^2=q^2+2\Sigma_4{}^6\sigma_n{}^2$
and then:

(4.2) $$\Phi=\Gamma\,\dot{p}+(\overline{M}+\gamma_4\dot{\varphi}^{d})+(\delta_{41}\dot{\theta}_1+\delta_{42}\dot{\theta}_2+\delta_{43}\dot{\theta}_3)\ ;\ 3p=I\sigma$$

where the scalar functionals Γ, γ_4, δ_{4n} (n=1,2,3) are of discrete memory form. Through (4.2), the interpretation of the notion of Cauchy stress tensor is linked to the notion of plastic limit and associated utter irreversibility. Moreover, the basic interest of shear tests is underlined. One of the simplest way to obtain an illustration of these features may be to perform the local study of the functional relationship between stress and strain [16].

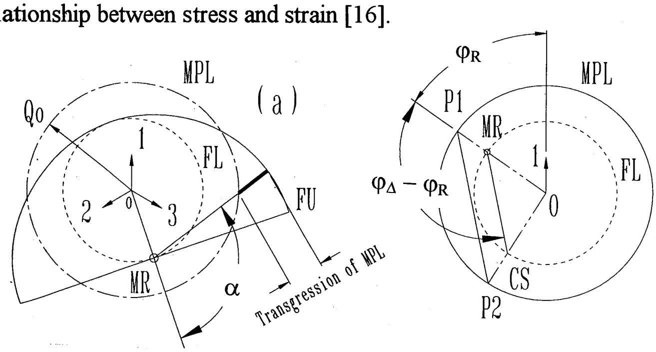

Fig. 7. Towards the thermogeomertrical features of the three-dimensional tensorial generalisation of the constitutive pattern.

4.3. The definition of the Preferred Reference Frame and the final discrete memory pattern of pure hysteresis [7,20].

The main part of the rotation of the orthonormal PRF is that of the principal directions of the strain rate tensor **D**: accordingly, the PRF "follows approximately" the material point with a rate of rotation $\dot{\alpha}_\delta$ which is obtained through the time derivative of:

(4.3) $$\tan(2\,\alpha_\delta)=2D_{23}/(D_2-D_3)\ ;\ J_3D_{23}=-\tau\,D_2+\tau\ ;\ J_n=1+K_n\ ;\ n=1,2,3$$

in the two-dimensional case under consideration. Moreover, a Constitutive Rotation (CR) is added to the previous one. It is a discrete-memory functional α_φ of a Thermomechanical Rotation (TR), namely (Fig.10):

$$\Delta_R{}^t\alpha_\varphi=\alpha_\varphi-\alpha_{\varphi R}=\omega f(\Delta_R{}^t\Omega/\omega)\ ;\ f(\Omega)=\alpha_\chi(1-\exp(-(\Omega/\Omega_\chi)^2))\tanh(\Omega/\Omega_\chi)$$

(4.4) $\dot{\alpha}_\varphi\,\dot{\Omega}\le 0$; $\alpha\lambda$ and $\Omega\chi \approx So/\mu$; $\omega=1,2$

The simplest definition of the TR is as follows: let the spatial distribution of Φ be uniform in the material point, and let the TR be that of the principal direction of inertia of this uniform distribution of Φ; then, in the two-dimensional case, one of the possible forms is:

(4.5) $\tan(2\,\Omega)=2\,(\,2\tau\,J_3\,)/((2\,\tau\,)^2+J_2{}^2-J_3{}^2\,)$

The pattern involves two classes of constitutive differential-difference equations as well as interesting but complicated second order effects.

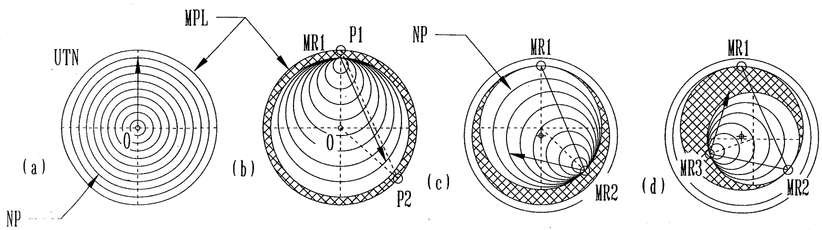

Fig. 8. The discontinuous sliding process of the family of thermomechanical neutral circles.

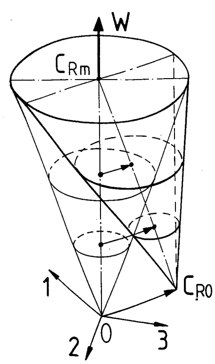

Fig.9. Suggesting the shaking (of the thermomechanical neutral loci) during the demagnetisation towards the UTN.

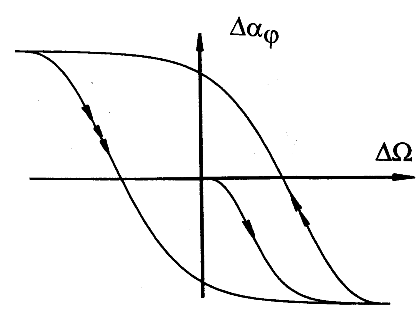

Fig.10. Illustration of the equation (4.4) giving the thermomechanical contribution α_φ to the intensity of the rotation of the PRF.

5. Short concluding remarks.

The pattern has been generalized to the anisotropic case [7,19], to granular media [6,17], to one-way and two-way shape memory effects [18,19] and to ferrohysteresis [7]. It has been also implemented regarding fatigue [21] and finite element simulation [22]. The study of demagnetisation-like processes has been introduced for the first time at the 31st Polish Solid Mechanics Conference (1996): a more comprehensive introduction of the results given in 3.4 has been submitted for publication elsewhere(Arch. Mech., Warsaw). The existence of at least one efficient type of SDC path (of spiral type) is sufficient, up to now, to define a constitutive thermomechanical benchmark. However, owing to the question of plastic anisotropy and to the existence of various possible forms (4.5), the basic theoretical problem of demagnetisation-like processes (stress or strain controlled) must be further studied in the mechanical case.

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