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Damping models in seismic analysis and design

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ABSTRACT

A comparative review is presented of proportional and hysteretic damping models for the seismic analysis and design of large structures and buildings. The emphasis is on passive damping in discrete systems solved by modal superposition. Two applications to the design of buildings to withstand seismic excitation are discussed.

INTRODUCTION

The laws of applied mechanics provide direct methods for computing the mass matrix and stiffeners matrix of a discrete, dynamic mechanical system. The determinations of the damping matrix is usually indirect, relying on the practical and analytical experience of the investigator. Since each investigator has different experiences and different proclivities, the group choices can have a wide variability. A comparative review can, therefore, be useful in narrowing these choices and thereby improving the comparability of dynamic analyses of proposed designs.

This review will necessarily be an abbreviated one; those wishing a more detailed discussion can find it in [1], [2], and [3].

VISCOUS VS. HYSTERETIC

For linear passive systems, the choice of damping model is usually between linear viscosity and linear hysteresis. If one chooses viscosity, the damping force is assumed proportional to velocity and in anti-phase with velocity, i.e..

$$F = -c\dot{x}. \tag{1}$$

where c is the viscous damping coefficient. If one chooses hysteresis, the damping force is assumed proportional to displacement and in anti-phase with velocity, i.e.

$$\underline{F} = -ik\eta\underline{x} \tag{2}$$

where (__) denotes a complex quantity and $i=(-1)^{1/2}$. The constant c in (1) is the viscous damping coefficient and the constant of proportionality in (2) is the product of the stiffness, k, and the material loss factor, η. Equation (1) has general validity but (2) is limited to harmonic response.

PROPORTIONAL DAMPING

For viscous damping, the multi-degree-of-freedom equations of motion have the form

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F(t)\} \tag{3}$$

For an n degree-of-freedom system, [M], [C], and [K] are the nxn mass, damping, and stiffness matrices and {x}, { $\ddot{x}$ }, and { $\ddot{x}$ } are the nx1 displacement, velocity, and acceleration vectors.

Let [Φ] be the modal matrix of the reduced system

$$M]\{\ddot{x}\}+[K]\{x\}=0 \tag{4}$$

where the columns of [Φ] are the eigenvectors of (4). Then (3) can be written

$$[m]\ \{\ddot{q}\}+[\hat{c}]\{\dot{q}\}+[k]\ \{q\}=[\Phi]^T\{F\} \tag{5}$$

where $[m]=[\Phi]^T[M][\Phi]$ and is diagonal; $[k]=[\Phi]^T[K][\Phi]$ and is diagonal; and the modal coordinates $\{q\}$ are defined by $\{x\}=[\Phi]\{q\}$. The diagonal elements of [m] and [k] are, respectively, the modal masses m_i, and the modal stiffnesses, k_i. The matrix $[\hat{c}]=[\Phi]^T[C][\Phi]$is diagonal, if and only if

$$[C][M]^{-1}[K]=[K][M]^{-1}[C]. \tag{6}$$

If (6) is satisfied, (5) may be solved sequentially for the $\{q\}$ and then the physical coordinates recovered from $\{x\}=[\Phi]\{q\}$.

Lacking the condition of (6), diagonalization can be achieved by assuming that the system possesses a specific type of damping that has been designated as "proportional" because it assumes that [C] is proportional to [M] or [K], or both, i.e.

$$[C]=a_o[M]+a_1[K]. \tag{7}$$

This is also called Rayleigh damping.

It is important to note that the eigenvectors of (5) are distinct from those of (4) whereas the eigenvectors of (5) with proportional damping are identical to those of (4). As for a single-degree-of-freedom system, the damped eigenvalues differ from the undamped eigenvalues of (4) by second order terms involving the damping.

If the diagonal elements of a proportional damping matrix are given the form

$$c_i=2\zeta_i\omega_i \tag{8}$$

where ω_i are the undamped eigenvalues and ζ_i are the modal damping ratios, then (7) and (8) and a renormalization of [m] such that [m]=[I], lead to

$$\zeta_i=\frac{a_o}{2\omega_i}+\frac{a_1\omega_i}{2} \qquad i=1,2,....n \tag{9}$$

A generalization of (7) and (9) with n-1 undetermined constants can be found in [4]. Since there are only two unknown constants in (9), one can only specify ζ_1 at two frequencies, such as ω_1 and ω_2.

All other values of ζ_i are fixed by these choices. In particular, if the dampening ratio at ω_1 ζ_1 and that at ω_2 is ζ_2, where $\omega_2 > \omega_1$, then

$$a_o = 2\omega_1\omega_2(\zeta_1\omega_2 - \zeta_2\omega_1)/(\omega_2^2 - \omega_1^2) \tag{10}$$

$$a_1 = 2(\zeta_2\omega_2 - \zeta_1\omega_1)/(\omega_2^2 - \omega_1^2) \tag{11}$$

and one is reduced to a proper choice of ω_2 and ω_1. An obvious choice for ω_1 is the fundamental frequency of the system but a proper choice for ω_2 is more problematic. If the excitation has an upper frequency cut-off, then this cut-off frequency would be a logical choice for ω_2. If possible, it is best to pick ω_1 and ω_2 so that they bracket the minimum value of ζ as well as enclosing the dominant excitation frequencies, as shown in Fig. 1.

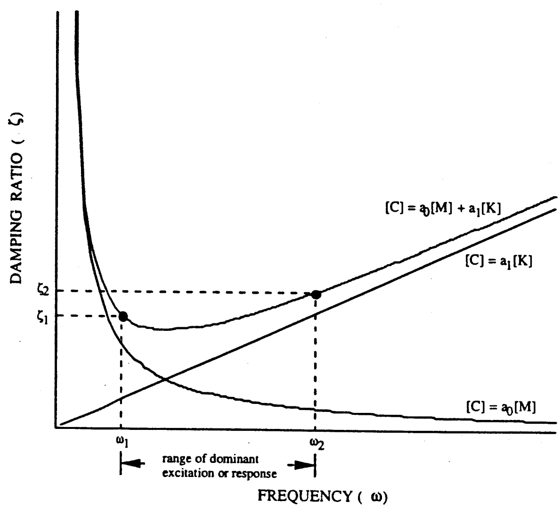

Figure 1. Damping vs. Frequency for Rayleigh Damping

This desirable arrangement ensures that the modes below ω_1 and above ω_2 will be highly damped and that all modes between ω_1 and ω_2 will have damping ratios somewhat less than either ζ_1 or ζ_2 (less than ζ_2 in the case of Fig. 1). Thus if ζ_1 and ζ_2 are chosen conservatively, i.e. less than experience and tests may indicate, then all modes in the range of interest are likely to be conservatively damped. Additional discussion of these aspects of proportional damping can be found in [5].

The analog to (2) for hysteretic damping is

$$[M]\{\ddot{x}\}+i[H]\{x\}+[K]\{x\}=\{\underline{F}\}e^{i\Omega t} \tag{12}$$

where bold face type indicates a phasor $\{\mathbf{x}\}=\{\underline{x}\}e^{i\Omega t}$ Hysteretic damping is provided by the matrix [H]. The eigenvalue problem associated with (12) yields complex eigenvalues $\underline{\lambda}_i^2 = \lambda_i^2(1+i\eta_i)$ and complex eigenvectors, $\{\underline{\varphi}_i\}$ with the properties

$$\{\underline{\varphi}_i\}^T[M]\{\underline{\varphi}_j\} = m_i\delta_{ij} \tag{13}$$

$$\{\underline{\varphi}_i\}^T([K]+i[H])\{\underline{\varphi}_j\} = \underline{k}_i\delta_{ij} \tag{14}$$

where δ_{ij} is the Kronecker delta. Notice that no special restrictions need be placed on [H] in order to achieve completely uncoupled equations of motion in terms of modal coordinates given by $\{q\}=[\underline{\Phi}]^{-1}\{\underline{x}\}$. In this sense, hysteretic damping is a "simpler" model than viscous damping. One may also argue that hysteretic damping is a "better" model of the damping of metals and polymers than is viscous damping, since measurements of the energy dissipated per cycle for metals and polymers show only a slight average dependence on frequency, consistent with the hysteretic damping model, whereas the viscous damping model shows a linear dependence, see [6] and [7].

Similarly, one can propose proportional hysteretic damping, i.e.

$$[H]=a_0[M]+a_1[K]. \tag{15}$$

This has the advantage of making the mode shapes real and, moreover, equal to the undamped mode shapes of (4). The eigenvalues remain complex but simplify somewhat to $\underline{\lambda}_i^2 = \omega_i^2(1+_i\eta_i)$ where ω_i^2 are the eigenvalues of (4). Thus, for the important case of harmonic forcing, the proportional hysteretic damping model has much to recommend it, being "simpler" and "better" than proportional viscous damping. It is therefore surprising how seldom it is used. An early example of its use is given in [8] where its effectiveness is made evident.

SEISMIC APPLICATIONS

The idea of using discrete, flexible connections (i.e. isolators) between a machine or a structure and its supporting foundation to reduce the transmission of vibration or force, or both, has a long history and a highly developed technology, see [9]. The related concept of using isolators at the base of large structures and buildings to protect them against earthquakes is more recent but of growing popularity, see [10].

The concept can be illustrated using the 3-story, steel-frame, shear building model depicted in Fig. 2.

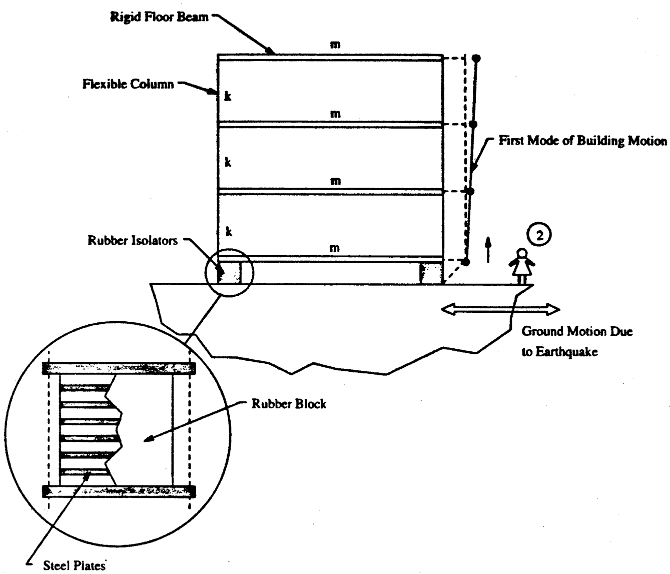

Fig. 2: Model of a steel-frame shear building with base isolation.

The rubber blocks that serve as base isolators typically must support high axial loads (e.g. 1,000 tons) while permitting a low frequency in shear (e.g. 1Hz). A design of growing acceptance is the use of a highly filled natural rubber block (rectangular or circular in cross-section) with embedded horizontal steel plates for enhanced vertical stiffness; see Fig. 2 for a simplified depiction; design details and performance results can be found in [11]. Of course, such rubber isolators are the quintessential example of a system with hysteretic damping and their participation in any mode will likely dominate the damping of that mode. For example, the first (lowest frequency) mode shown in Fig. 1 should exhibit almost pure hystertic damping.

For the model shown in Fig. 2, the distribution of earthquake-induced inertia forces as seen by observer 2 will be uniform from the top to the bottom story. This force distribution thus has almost the same shape as the first mode. Higher modes will involve increasing amounts of frame deformation but will likely still be hysteretically damped. From the above discussion, all these damped modes will be mutually orthogonal. In particular, all modes higher than the first will then be very nearly orthogonal to the distribution of inertia forces and, as such, they will absorb only a minimal amount of energy from the action of these forces and therefore exhibit only a minimal vibration. In other words, the higher mode will be isolated from the earthquake ground motion. This is the essence of base isolation in physical terms and for a simple model. The mathematical explanation, which is easier to generalize to more complex models, can be found in [11].

In [12] it has been suggested that discrete viscous dampers distributed throughout a structure can achieve comparable levels of seismic protection comparable to base isolation but at significantly lower cost. One way of accomplishing this is shown in Fig. 3.

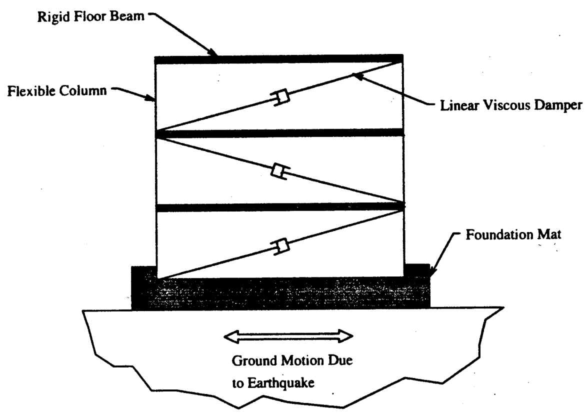

Fig. 3: Model of a steel-frame shear building with inter-story viscous dampers.

Discrete viscous dampers imply a nondiagonal modal damping matrix. The assumption of proportional damping, as per (7), is difficult to justify since in this case the mass and stiffness are distributive but the damping is discrete.

As a result, the modes will not be orthogonal and the seismic energy of an earthquake applied to the structure of Fig. 3 will be distributed among many modes rather than being concentrated in one mode as in the case of the structure of Fig. 2. We see then, that the approach of Fig. 3 is not based on isolation but is based on the augmentation of modal damping; a different approach, but one that has been used with success, see [12]. It would be interesting to investigate the model of Fig. 3 with the discrete viscous dampers replaced with discrete viscoelastic shear dampers, such as those in [13]. Modal orthogonality should be recovered and the seismic energy should become more concentrated in the fundamental mode.

CLOSURE

The premise of this paper is that a comparative review of various damping models will lead to a deeper understanding of how damping affects the dynamic response of discrete mechanical systems and thereby narrow the range of consensus on how to design large structures and buildings to withstand seismic excitation. Only passive damping has been discussed. Ways of using active damping for these purposes are being discussed in the literature, e.g. see [14], and after more experience has accumulated with the associated technology, a similar comparative review of active damping would be useful.

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