Computation of Shrinkage Stresses in Prestressed Concrete Containments

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INTRODUCTION

According to a survey (Irving, 1982), surface cracking on PCRVs and PCCs under the investigations is confined to drying shrinkage and thermal strain effects and no instances of structurally significant cracking has been found. In this paper, the authors use FEM to compute humidity distribution in drying concrete and shrinkage stresses by internal restraint. Since PCC is built segment by segment in several years, a computational model taking into account construction sequence is presented and shrinkage stresses by external restraints are calculated with the model.

DRYING AND DRYING SHRINKAGE OF CONCRETE

The water diffusion equation of drying concrete is

$$\frac{\partial H}{\partial t}=\alpha(H)\nabla^2 H(\underset{\sim}{x},t); \qquad H(\underset{\sim}{x},0)=H_0 \doteqdot 100\% \in V$$

$$H=H_1 \in S_1; \ \frac{\partial H}{\partial n}=0 \in S_2; \ \frac{\partial H}{\partial n}=\beta(H_3-H)\in S_3 \tag{1}$$

where $\underset{\sim}{x}$ is coordinate vector, H relative humidity, α and β diffusion factors, S_1, S_2, S_3 surfaces of V. Using Galerkin's method and Green's intergration formula, we derive

$$[M]\dot{\underset{\sim}{q}}+[K]\underset{\sim}{q}=\underset{\sim}{F}; \quad [M]=\sum_i \iiint_{V_i} \underset{\sim}{N}^T\underset{\sim}{N}dV; \quad \underset{\sim}{F}=\sum_j \iint_{S_j} \alpha\beta H_3 \underset{\sim}{N}^T dS$$

$$[K]=\sum_i \iiint_{V_i} \alpha\nabla\underset{\sim}{N}^T\nabla\underset{\sim}{N}dV+\sum_j \iint_{S_j}\alpha\beta\underset{\sim}{N}^T\underset{\sim}{N}dS \tag{2}$$

Here $\underset{\sim}{N}$ is shape function vector of element V_i, S_j the outer boundary of jth element with the 3rd boundary, q nodal humidity vector. From (2), we further derive

$$([M]/\Delta t+\theta[K])\underset{\sim}{q}^{n+1}=([M]/\Delta t-\gamma[K])\underset{\sim}{q}^n+\theta\underset{\sim}{F}^{n+1}+\gamma\underset{\sim}{F}^n \tag{3}$$

where $\theta \geq 0.5$ and γ =1-0. Only if time step length Δt and effective element size Δx are chosen properly can reasonable numerical solutions be obtained (Ou Yang and Xiao, 1989).

Free shrinkage strain is (Bažant, 1982)

$$\varepsilon_{fs}(\underset{\sim}{x}, t) = \underset{\sim}{\varepsilon}_s^0[1 - H^3(\underset{\sim}{x}, t)] \tag{4}$$

where ε^0_s is drying shrinkage coefficient.

As water diffuses from concrete, it undergoes non-uniform shrinkage, which results in shrinkage stresses even without external restraints. These stresses resulting from restriction to free shrinkage by concrete itself is called shrinkage stresses by internal restraint. If external restraints are involved, shrinkage stresses by external restraints will occur. The real shrinkage stresses are the combination of the above two kinds of shrinkage stresses.

COMPUTATION OF SHRINKAGE STRESSES BY INTERNAL RESTRAINT

Based on the rate-type constitutive model for long-term deformation of concrete given by Bažant and Chern (Bažant and Chern, 1985), we put forward an alternative model. In calculating 1-dimensional shrinkage stresses, we can write

$$\varepsilon_{sh}(t) = [1-b\sigma_{sh}(\underset{\sim}{x}, t)/f_t(t)]\,\varepsilon_{fs}(\underset{\sim}{x}, t) + \int_{t_0}^{t}[1/E(\tau)+C(t,\tau,\underset{\sim}{x})]\frac{\partial \sigma_{sh}}{\partial \tau}d\tau;$$

$$C(t,\tau,\underset{\sim}{x}) = \sum_{i=1}^{K}\phi_i(\tau)\left\{1 - e^{-[\psi(\underset{\sim}{x},t)t-\psi(\underset{\sim}{x},\tau)\tau]/\tau_i}\right\};$$

$$\phi_i(\tau) = d_i + e_i/\tau; \qquad \psi(H) = a+(1-a)H \tag{5}$$

where t_0 is the age when drying starts, $E(\tau)$ is the elastic modulus of concrete, a denotes drying creep effect while b denotes stress-induced shrinkage effect (Wittmann and Roelfstra, 1980; Bažant and Chern, 1985) respectively. Formulation of 3-dimensional problems is easily derived from (5).

Following the idea of the implicit scheme by Zhu (Zhu, 1983), we can derive a recursive formula (OuYang, 1989)

$$\Delta\varepsilon_{sh_{n+1}} = \Delta\varepsilon_{fs_{n+1}} + b\sigma_{sh_n}\varepsilon_{fs_n}/f_{t_n} - b\sigma_{sh_{n+1}}\varepsilon_{fs_{n+1}}/f_{t_{n+1}} + B_{n+1}\Delta\sigma_{sh_{n+1}} + \sum_{i=1}^{K}D^i_{n+1}G^i_n$$

$$B_{n+1} = 0.5[1/E_n + 1/E_{n+1} + C(t_{n+1}, t_n, \underset{\sim}{x})]; \quad D^i_{n+1} = e^{-\psi_n t_n/\tau_i} - e^{-\psi_{n+1}t_{n+1}/\tau_i};$$

$$G^i_n = G^i_{n-1} + P^i_n\Delta\sigma_{sh_n}; \quad P^i_n = 0.5(\phi_{in}\,e^{\psi_n t_n/\tau_i} + \phi_{i_{n-1}}e^{\psi_{n-1}t_{n-1}/\tau_i});$$

$$G^i_0 \equiv 0 \tag{6}$$

Self-equilibrium of shrinkage stresses by internal restraint on any cross section A of concrete members or structures is

$$\int_A \sigma_{sh}(\underset{\sim}{x}, t)dA = 0 \tag{7}$$

On section A, there exists a curve L dividing tensile zone and compressive zone. On line L, shrinkage stress $\sigma_{sh}(\underset{\sim}{x}, t)=0$ and average shrinkage strain ε_{sh} on section A satisfies $\varepsilon_{sh}(t) = \varepsilon_{fs}(\underset{\sim}{x}, t)$, The position of line L should be determined before computation of σ_{sh} . For cross sections in regular shapes like rectangular, ring, circle, it is easy to do so. The followings are the computing process (OuYang, 1989; OuYang and Wu, 1989):

Step 1. Set initial value for iteration: $x_1=\rho_1$, $x_2=\rho_2$. Here ρ_1 and ρ_2 are internal and external radii of a ring, or $\rho_1=0$ and ρ_2 is half of the depth of a rectangular, respectively.

Step 2. Determine the position x=r(t) of line L with 0.618 algorithm. Let $r^K_n = 0.382x_1+0.618x_2$. Here n represents time step and k iteration step. Therefor we get $\varepsilon_{sh}{}^K_n = \varepsilon_{fs}(r^K_n, t_n)$. Substitution of it into (6) gives $\sigma^K_{sh_n}$.

Step 3. Integrate $F_n^K = \int_{\rho_1}^{\rho_2} \sigma_{sh_n}^K(x, t_n)dA$

numerically. If $|F_n^K| < \delta$ (a very small positive number set as a precision), the section A is considered to be in self-equilibrium and σ_{sh} at time t_n has been obtained. Let time t step forward -- n=n+1, k=1, and go back to Step 1. Otherwise, go to Step 4.

Step 4. Case (1): $F_n^K > 0$ means the assumed tensile zone is larger than it should be, so r_n^K should be increased. Let $x_1 = r_n^K$ and begin next iteration step with k=k+1. Go back to Step 2; Case (2): $F_n^K < 0$ means the assumed compressive zone is larger than the accurate so r_n^K should be decreased. Let $x_2 = r_n^K$, k=k+1 and go back to Step 2.

With this algorithm, we can get shrinkage stress distribution at any time.

COMPUTATION OF SHRINKAGE STRESSES BY EXTERNAL RESTRAINTS

For large concrete structures like PCCs, they are built segment by segment in several years. PCCs are not prestressed until they acquire enough strength. We know that maximum shrinkage stress occurs before prestressing.

Some research has indicated that results are quite different whether construction sequence in time dimension is considered or not (Zhong and Wu, 1983). For large structures, more stories or more segments they have, the more different their result of structural analysis are.

Each segment of a PCC has different casting age and drying age, so is its development of creep and drying shrinkage. Only if construction effects are considered can reallistic results be obtained. For this computational model is very complicated, the age-adjusted effective modulus method (Bažant, 1982) is used to compute shrinkage stresses.

Let t_0^i denote the time when the ith segment of a PCC (a tube) is cast, t_2^i the time when it starts to dry and t_1^i the time when it acquires an effective modulus--the average elastic modulus of the ith segment while being cured. We give physical equation as follows:

1. While $t_2^i < t \leq t_0^{i+1}$

$$(\underset{\sim}{\sigma}^i - \underset{\sim}{\sigma}_0^i)/E''(t-t_0^i,\ t_2^i-t_0^i) = [D][\underset{\sim}{\varepsilon}^i - \underset{\sim}{\varepsilon}_0^i - \Delta\underset{\sim}{\varepsilon}_{sh}(t,\ t_b,\ t_2^i)] \tag{8}$$

where t_b is the previous time step and t the current time step, $\underset{\sim}{\sigma}_0^i$ and $\underset{\sim}{\varepsilon}_0^i$ the stress and strain vectors at t_b serving as initial stresses and strains for t, $\Delta\underset{\sim}{\varepsilon}_{sh}$ the absolute shrinkage strain between t_b and t. If i=1, $\underset{\sim}{\sigma}_0^i = \underset{\sim}{\varepsilon}_0^i \equiv 0$.

2. While $t_0^{i+1} < t \leq t_2^{i+1}$

Meantime, the (i=1)th segment (newly cast) is in curing and does not shrink, but it will provide certain restraints to those older segments under it. Imagine it behaves like an elastic body with constant modulus $E(t_1^{i+1}-t_0^{i+1})$, we may write

$$(\underset{\sim}{\sigma}^{i+1} - \underset{\sim}{\sigma}_0^{i+1})/E(t_1^{i+1}-t_0^{i+1}) = [D](\underset{\sim}{\varepsilon}^{i+1} - \underset{\sim}{\varepsilon}_0^{i+1})\ ;$$

$$(\underset{\sim}{\sigma}^i - \underset{\sim}{\sigma}_0^i)/E''(t-t_b,\ t_b-t_0^i) = [D][\underset{\sim}{\varepsilon}^i - \underset{\sim}{\varepsilon}_0^i - \Delta\underset{\sim}{\varepsilon}_{sh}(t,\ t_b,\ t_2^i)] \tag{9}$$

$$\underset{\sim}{\sigma} = \{\sigma_r,\ \sigma_z,\ \tau_{rz},\ \sigma_\theta\}^T\ ; \quad \underset{\sim}{\varepsilon} = \{\varepsilon_r,\ \varepsilon_z,\ \nu_{rz},\ \varepsilon_\theta\}^T\ ;$$

$$[D] = \frac{1}{(1-\mu)(1-2\mu)} = \begin{bmatrix} 1-\mu & \mu & 0 & \mu \\ \mu & 1-\mu & 0 & \mu \\ 0 & 0 & (1-2\mu)/2 & 0 \\ \mu & \mu & 0 & 1-\mu \end{bmatrix} \tag{10}$$

As time t steps forward, the structure grows with non-uniform segments which interact. It can be analyzed as a problem of initial stresses and strains (Zhong and Wu, 1983) with variable stiffness. The age-adjusted effective modulus E" can be obtained (Bažant, 1982).

$$E''(x, y)=\frac{E(y)}{1+K_a(x, y)\phi(x, y)} ; \quad \phi(x, y)=E(y)C(x, y)$$

k_a is the age adjuste coefficient.

NUMERICAL ANALYSIS AND CONCLUSIONS

PCC of Qinshan Nuclear Power Plant (Wang) in China is analyzed in this paper. According to the report of ACI 209 R-82, for concrete used there

$$\varepsilon_{sh}(t, t_0)=0.457\times10^{-4}\frac{t-t_0}{35+t-t_0} ; \quad \phi(t, t_0)=1.806t_0^{-0.118}\frac{(t-t_0)^{0.6}}{(t-t_0)^{0.6}+10} \tag{11}$$

Average curing time of Qinshan PCC is 30 days. As the latest segment starts to dry, another new segment is cast on it. When the cylindrical shell is constructed to the height of about 42 m (the 23rd segment) above the foundation mat, construction is suspended for 10 months to install other equipment. It is found that maximum shrinkage stress occurs in circumferential direction on the surface of the bottom of the 23rd segment. Take $t_0^i=0$, $t_1^i=10$, $t_2^i=30$, and $t_K^i=t_K^{i-1}+30$ (k=1, 2, 3, $i\geq1$). Maximum shrinkage stresses by external restraints on some segments are shown in Table 1

Table 1 Maximum Shrinkage Stresses (MPa)

ith segment	1	3	5	7	9	11	13	15	17	19	21	23
shrinkage stresses	0.548	0.391	0.928	0.293	0.293	0.294	0.294	0.294	0.294	0.294	0.294	0.472

Let $\alpha(H)=\alpha_0(0.3+3.6/\sqrt{t_e})\{0.05+0.95/[1+(4(1-H))^4]\}$,

$\alpha_0=10$ mm²/d, $\beta=0.01$/mm , $\varepsilon_s^\circ=-4\times10^{-4}$, T_s=70%, a=0.5, b=0.4.

We can get shrinkage stresses by internal restraint. Let σ_0 represent shrinkage stress at internal surface of the shell and σ_t shrinkage stress at external surface. Shrinkage stresses by internal restraint of 23rd segment are listed in Table 2.

Table 2 Shrinkage Stresses by Internal Restraint (MPa)

Drying time (Day)	7	15	30	60	90	150	210	270
σ_c	-0.006	-0.012	-0.022	-0.037	-0.052	-0.074	-0.094	-0.114
σ_t	0.529	0.868	1.218	1.573	1.774	2.026	2.184	2.296

Although maximum shrinkage stress by external restraints occurs at the foot of the shell, maximum shrinkage stress by internal restraint does not. The total maximum shrinkage stress occurs at 23rd segment on 10 months after it was cast.

Its value is 2.768 MPa. Considering the tensile strength of concrete at age of 60 days is already 3.088 MPa, we know Qinshan PCC would not crack owing to drying shrinkage.

Although surface cracking will not endanger the whole structures, it serves as the cause for further cracking and it would give people the impression of unsafety.

Due to limitation of space, detailed derivation of formulas, analysis and numerical results are not presented here. For further details, see OuYang (1989) and OuYang and Wu (1988).

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