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ABSTRACT

Many policy decisions about social and political programs are exclusive and interdependent because the underlying issue is not so much the desirability of a specific alternative but rather its relative priority with respect to other competitors. In this regard, the classical example is the budget problem, a situation where a finite amount of money is to be spent on a near infinite range of programs.

One appropriate framework for investigating this type of question in sample surveys or panel interviewer studies is in terms of rank preference data. With this approach, each subject is required to order a set of alternatives partially or completely according to a particular criterion. This paper is concerned with the statistical analysis of such data from a multi-dimensional contingency table point of view. For this purpose, weighted least squares methodology is used both to test various hypotheses of interest as well as to fit linear regression models which provide a descriptive basis for conclusions about the rank preference profiles for one or more sub-populations. Finally, the flexibility and scope of this methodology are illustrated in terms of an example involving the ranking of 7 tax alternatives by 1504 subjects in a United States national sample.

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1. INTRODUCTION

An inherent characteristic of policy decisions is the exclusive nature of the choice. In other words, decision-making generally is structured in a manner such that one or more alternatives are excluded or not attained when others are selected. Although this characteristic of decision-making is widely recognized, the methodology for evaluating the underlying preferences that shape political decisions has often failed to incorporate this exclusion principle into the measurement process. Assuming that what people say and what they do are somehow related, researchers often have measured preferences without structuring the alternatives in a fashion that reflects the constraints found in the decision-making process. One of the most unrealistic situations for measuring policy preferences occurs in many public opinion surveys. For example, in the United States, questions concerning the desirability of certain policies like tax reduction, guaranteed minimum income, and national health care, are often presented independently of each other. The respondent is then required to express his desires in isolation of the other alternatives. This situation is inherently unrealistic for correctly measuring the structure of individual preferences regarding these items; because, in this context, the proper question is not so much "Do you favor or oppose national health care?" but "Which do you prefer more -- having national health care or guaranteeing minimum income to people?"

This latter type of question gives rise to ordered preference data which are typical to the behavioral sciences. For example, one such method of presentation is "paired comparisons," which requires the subject to indicate preferences within various possible pairs of alternatives. Where appropriately applied, this approach can provide valuable data. However, this method has not gained wide application in mass sample surveys or panel interviewer studies because of the relatively large number of comparisons required even from a small number of alternatives together with certain difficulties in analysis. Another approach to collecting ordered preference data that avoids the objections raised against paired comparison methods is the collection of fully ordered alternatives, as is commonly used by "unfolding" methods. In this situation, all alternatives (policies) are presented to the subject who is required to rank them according to some criterion like desirability, quality, or risk. Such rank preference data are relatively inexpensive to collect and are especially valuable because of the exclusive nature of the choice task required of the subject. Moreover, this approach can be given added flexibility by permitting tied ranks and partial ranks (e.g., require just three choices in a set of more than three alternatives).

The analysis of rank preference data from a multi-dimensional contingency table point of view is the principal focus of this paper. In particular, weighted least squares methodology will be applied to fit appropriate linear regression models which permit the investigation of hypotheses pertaining to the preference profiles in one or more sub-populations. The advantage of this approach is that it suitably accounts for the inherent multivariate nature of such rank data.

2. METHODOLOGY

One area of policy requiring exclusive choices concerns the use of public money for governmental programs. Wildavsky [1964] describes the budget problem -- "Who gets what the government has to give?" -- as a series of decisions made under conditions including a large number of demands, a finite amount of funds, and limited information. Because the subject of how tax money ought to be spent is inherently exclusive, a question concerning the desirability of various tax and spending alternatives was administered as part of the Southeast Regional Survey I [1969] (hereafter abbreviated SERS - I) to a United States national sample of adults in a manner designed to elicit ranked data. Each respondent was asked to order (descending from 1 to 7) his preferences for the following tax alternatives:

- Education (ED)
- Water and Air Pollution (PL)
- Tax Reduction (TR)
- Anti-Poverty Programs (PV)
- Foreign Aid (FA)
- Guaranteed Minimum Income (GI)
- Health Care (HC)

Respondents were also classified according to their ideology (conservative, liberal, in-between, no ideology), sex (male, female), and criticism of governmental tax policies (no, yes).

2.1. The Categorical Nature Of Rank Preference

The variables under study in this survey involve "categorical" or "non - metric" data since they are measured in terms of either a nominal or ordinal scale. As such, they may be conceptually arrayed in a multi - dimensional contingency table. Since the actual contingency table associated with the data from SERS - I considered in this paper is quite large (16 x 5040), the categorical nature of ranked choice data can best be motivated by considering the hypothetical example presented in Table 1. In this situation, n subjects are presented with L = 3 alternatives to rank -- "tax reduction," "guaranteed minimum income" and "health care." The n subjects are also identified by sex (male, female) with there being n_1 males and n_2 females in the sample ($n_1 + n_2 = n$). The preference orderings of these n subjects may be summarized in an (n x 3) data matrix where the entry R_{igk} is the rank assigned by respondent k of sex i to alternative g.

The rows of the data matrix sum to 6, the sum of the ranks "1", "2", and "3" provided that each subject ranks all alternatives without any ties.[1] In this case, there are exactly ($3! = 6$) ways that the L = 3 alternatives may be ordered. This fact implies that in situations where n is large, there will necessarily be many subjects with identical preference orderings -- that is, exactly similar rows in the data matrix. Thus, the data matrix may be summarized in terms of a contingency table as presented in Table 1. The columns of the table represent the six possible rankings of three alternatives; and the rows represent

[1] The presence of ties causes no real problems. In particular, ties can be analyzed in terms of mid - ranks which can be suitably taken into account by including additional possible preference orders in the contingency table shown in Table 1.

TABLE 1

HYPOTHETICAL TAX PREFERENCE DATA AND RESULTING CONTINGENCY TABLE

Data Matrix

		Alternative		
		1 Tax Reduction (TR)	2 Guaranteed Income (GI)	3 Health Care (HC)
Sex	Subject			
Male	1	R_{111}	R_{121}	R_{131}
Male	2	R_{112}	R_{122}	R_{132}
Male	3	R_{113}	R_{123}	R_{133}
...	...	...	...	...
Male	n_1	R_{11n_1}	R_{12n_1}	R_{13n_1}
Female	1	R_{211}	R_{221}	R_{231}
Female	2	R_{212}	R_{222}	R_{232}
Female	3	R_{213}	R_{223}	R_{233}
...	...	...	...	...
Female	n_2	R_{21n_2}	R_{22n_2}	R_{23n_2}

R_{igk} is the rank assigned alternative g by subject k with sex i ($R_{i1k} + R_{i2k} + R_{i3k} = 6$); and n_1 and n_2 are the number of males and females respectively.

Resulting Contingency Table

Sub-population	Preference order for TR, GI, and HC respectively (123)	(132)	(213)	(231)	(312)	(321)	Total
Male	n_{11}	n_{12}	n_{13}	n_{14}	n_{15}	n_{16}	n_1
Female	n_{21}	n_{22}	n_{23}	n_{24}	n_{25}	n_{26}	n_2

The preference orders assigned the L alternatives are indexed by $j = 1,2,3,4,5,6$. The $\{n_{ij}\}$ represent the numbers of subjects in sub-population i selecting preference order j.

the sub - populations -- in this case, males and females. The entry n_{ij} is the number of subjects from sub - population i who ordered the 3 alternatives in the j-th way. This hypothetical example may be generalized to include further classification of subjects (sub-populations) and larger numbers of alternatives as is the case with the SERS-I data.

A suitable set of statistics for summarizing the average ranking of each of the three alternatives in the two sub - populations are the "mean ranks" $\{\bar{R}_{ig}\}$ defined in (2.1). Because the sum of each row

$$\bar{R}_{ig} = \frac{1}{n_i} \sum_{k=1}^{n_i} R_{igk} \qquad \begin{matrix} i = 1,2 \\ g = 1,2,3 \end{matrix} \qquad (2.1)$$

of the data matrix in Table 1 is the sum of the ranks assigned, any R_{igk} is equal to 6 minus the negative sum of the ranks assigned to the other two alternatives as shown in (2.2). In other words, when (L - 1) choices

$$R_{i3k} = 6 - \sum_{g=1}^{2} R_{igk} \qquad (2.2)$$

have been made, the last choice L is determined. Thus, for L alternatives, there are only (L - 1) choices possible. This constraint leads to the relationship among the mean rank scores $\{\bar{R}_{ig}\}$ given in (2.3). Since

$$\bar{R}_{i3} = 6 - \sum_{g=1}^{2} \bar{R}_{ig} \qquad (2.3)$$

the indexing of the alternatives by the subscript g is assumed to be arbitrary, expressions (2.2) - (2.3) apply regardless of which alternative is placed "last." These relationships also hold for any number of alternatives, where the constant 6 is replaced by the appropriate sum $[L(L + 1)/2]$ of the L assigned ranks.

If the n_i subjects from the i-th sub-population are indifferent with respect to preferences among the L = 3 choices in the sense that all of their possible rank orderings are equally likely, then the mean ranks $\{\overline{R}_{ig}\}$ satisfy the hypothesis H_{RPi} in (2.4) where "E" denotes

$$H_{RPi}: \quad E\{\overline{R}_{i1}\} = E\{\overline{R}_{i2}\} = E\{\overline{R}_{i3}\} = 2 \qquad (2.4)$$

corresponding expected or average values in the corresponding overall populations from which the samples of subject rankings have been obtained Since the ranks assigned to the L alternatives are inherently inter-related in the sense that the position of one alternative in the preference ordering necessarily restricts the positions of other alternatives, the statistical method applied to test H_{RPi} must therefore reflect the underlying dependence of the rank assignments. One appropriate method for this purpose is Friedman's [1937] χ^2-statistic. This procedure is of particular interest for cases where the sample size is small ($n_i \leq 20$) since it permits an exact test of such hypotheses (see Owen [1962]). In large sample situations, however, Friedman's test statistic may represent a conservative approach for testing H_{RPi} because it is based on the assumption that all rank orderings are equally likely within the i-th sub-population and, consequently, that the mean rank scores are equally correlated. This assumption gives rise to possibly large over-estimates of the real standard errors for the mean rank scores in situations where the hypothesis H_{RPi} in (2.4) is not true in the sense of a systematic pattern of preference with certain alternatives tending to be given consistently smaller valued ranks than others.

When respondents are classified into two or more sub-populations as found in Table 1, hypotheses concerning differences among sub-populations are also of interest. These hypotheses are of the form indicated in (2.5);

$$\begin{aligned} H_{S1}&: E\{\bar{R}_{11}\} = E\{\bar{R}_{21}\} \\ H_{S2}&: E\{\bar{R}_{12}\} = E\{\bar{R}_{22}\} \\ H_{S3}&: E\{\bar{R}_{13}\} = E\{\bar{R}_{23}\} \end{aligned} \qquad (2.5)$$

and thus represent comparisons of mean rank scores between males and females for each alternative. These hypotheses are often tested individually in a univariate framework which ignores the underlying multivariate nature of the response data by involving one mean rank function at a time for comparisons between sub-populations. Thus, in a strict sense, it is important to note that the hypothesis H_{RPS} in (2.6) specifies no differences among sub-popula-

$$H_{RPS}: E\{\bar{R}_{11}\} = E\{\bar{R}_{21}\}, E\{\bar{R}_{12}\} = E\{\bar{R}_{22}\}, E\{\bar{R}_{13}\} = E\{\bar{R}_{23}\} \qquad (2.6)$$

tions in an overall multivariate context. Moreover, it also corresponds to the hypothesis of no sub-population x preference interaction since in view of (2.3), it reflects the extent to which different sub-populations order the L alternatives differently. Friedman's test does not allow for the examination of (2.6) because the estimates of the variances and covariances for the mean rank scores which are associated with it are valid only when the null hypotheses $\{H_{RPi}\}$ (of within sub-population indifference) are true. In most situations, however, the researcher must typically work with preference orderings not reflecting such indifference. For these cases, the questions of greatest interest pertain to how these preference orderings differ among the sub-populations of subjects.

2.2 Application of the GSK Approach to the Analysis of Rank Preference Data

Recent developments in the field of applied statistics have provided a framework for more comprehensive analyses of data arrayed in multi-dimensional contingency tables. One general approach, which is suitable for rank preference data, is the method discussed by Grizzle, Starmer, and Koch [1969] (hereafter abbreviated GSK). The GSK method involves the fitting of appropriately formulated linear regression models to various types of functions of the observed proportions in an underlying contingency table. For the hypothetical data in Table 1, the mean rank scores $\{\bar{R}_{ig}\}$ can be generated by the linear transformation of the contingency table frequencies $\{n_{ij}\}$ shown in (2.7). Alternatively, these relationships can be more efficiently written

$$\begin{aligned}
\bar{R}_{i1} &= (n_{i1} + n_{i2} + 2n_{i3} + 2n_{i4} + 3n_{i5} + 3n_{i6})/n_i \\
\bar{R}_{i2} &= (2n_{i1} + 3n_{i2} + n_{i3} + 3n_{i4} + n_{i5} + 2n_{i6})/n_i \qquad (2.7) \\
\bar{R}_{i3} &= (3n_{i1} + 2n_{i2} + 3n_{i3} + n_{i4} + 2n_{i5} + n_{i6})/n_i
\end{aligned}$$

in the matrix notation (2.8) where $\bar{\underset{\sim}{R}}_i$ denotes the vector of mean ranks, $\underset{\sim}{p}_i$

$$\bar{\underset{\sim}{R}}_i = \begin{bmatrix} \bar{R}_{i1} \\ \bar{R}_{i2} \\ \bar{R}_{i3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 & 2 & 3 & 3 \\ 2 & 3 & 1 & 3 & 1 & 2 \\ 3 & 2 & 3 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} p_{i1} \\ p_{i2} \\ p_{i3} \\ p_{i4} \\ p_{i5} \\ p_{i6} \end{bmatrix} = \underset{\sim}{A}\,\underset{\sim}{p}_i \qquad (2.8)$$

is the vector of sample proportions $p_{ij} = (n_{ij}/n_i)$, and $\underset{\sim}{A}$ is the linear transformation operation matrix which determines $\bar{\underset{\sim}{R}}_i$ from $\underset{\sim}{p}_i$. Given the assumption that the responses of the subjects from the respective sub - populations have been statistically generated by an observational (or stochastic) process equivalent to stratified simple random sampling[2], a consistent estimate for the covariance matrix of the vector $\underset{\sim}{p}_i$ is the matrix $\underset{\sim}{V}_i$ in (2.9). As a result, a consistent estimate for the covariance matrix

$$\underset{\sim}{V}_i = \frac{1}{(n_i-1)} \begin{bmatrix} p_{i1}(1-p_{i1}) & -p_{i1}p_{i2} & -p_{i1}p_{i3} & -p_{i1}p_{i4} & -p_{i1}p_{i5} & -p_{i1}p_{i6} \\ -p_{i2}p_{i1} & p_{i2}(1-p_{i2}) & -p_{i2}p_{i3} & -p_{i2}p_{i4} & -p_{i2}p_{i5} & -p_{i2}p_{i6} \\ -p_{i3}p_{i1} & -p_{i3}p_{i2} & p_{i3}(1-p_{i3}) & -p_{i3}p_{i4} & -p_{i3}p_{i5} & -p_{i3}p_{i6} \\ -p_{i4}p_{i1} & -p_{i4}p_{i2} & -p_{i4}p_{i3} & p_{i4}(1-p_{i4}) & -p_{i4}p_{i5} & -p_{i4}p_{i6} \\ -p_{i5}p_{i1} & -p_{i5}p_{i2} & -p_{i5}p_{i3} & -p_{i5}p_{i4} & p_{i5}(1-p_{i5}) & -p_{i5}p_{i6} \\ -p_{i6}p_{i1} & -p_{i6}p_{i2} & -p_{i6}p_{i3} & -p_{i6}p_{i4} & -p_{i6}p_{i5} & p_{i6}(1-p_{i6}) \end{bmatrix} \qquad (2.9)$$

of $\underset{\sim}{\bar{R}}_i$ is the matrix $\underset{\sim}{V}_{Ri}$ in (2.10) where $\underset{\sim}{A}$ is the matrix in (2.8) and

$$\underset{\sim}{V}_{Ri} = \underset{\sim}{A}\underset{\sim}{V}_i\underset{\sim}{A}' \qquad (2.10)$$

$\underset{\sim}{A}'$ is its transpose. Similarly, the overall set of mean rank vectors $\{\underset{\sim}{\bar{R}}_i\}$ across the sub - populations can be represented by the combined vector $\underset{\sim}{\bar{R}}$ in (2.11) for which the corresponding estimated covariance

$$\underset{(6\times 1)}{\underset{\sim}{\bar{R}}} = \begin{bmatrix} \underset{\sim}{\bar{R}}_1 \\ \underset{\sim}{\bar{R}}_2 \end{bmatrix} , \qquad (2.11)$$

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[2] The GSK approach can also be applied to such data obtained from more complex sample survey designs by suitably modifying the structure of the estimated covariance matrices $\{\underset{\sim}{V}_i\}$. For a more complete discussion of such considerations see Koch et. al. [1975].

matrix based on (2.10) is given in (2.12) with $\underset{\sim}{0}_{33}$ being a (3x3) matrix

$$\underset{6x6}{\underset{\sim}{V}_R} = \begin{bmatrix} \underset{\sim}{A}\,\underset{\sim}{V}_1\underset{\sim}{A}' & \underset{\sim}{0}_{33} \\ \underset{\sim}{0}_{33} & \underset{\sim}{A}\,\underset{\sim}{V}_2\underset{\sim}{A}' \end{bmatrix} \qquad (2.12)$$

of 0's.

The variation among the elements of $\bar{\underset{\sim}{R}}$ can be investigated by fitting linear regression models. Such models are of interest here because they provide a framework in terms of which test statistics can be formulated for hypotheses like the $\{H_{RPi}\}$ in (2.4), H_{RPS} in (2.6), as well as others. However, the fitting of such models must be undertaken in a manner which suitably recognizes the fact that the mean ranks in the vectors $\{\bar{\underset{\sim}{R}}_i\}$ satisfy restrictions like (2.3) which induce corresponding singularities into their respective estimated covariance matrices $\underset{\sim}{V}_{Ri}$. For this purpose, the most straightforward approach is to direct the analysis at the centered - reduced score vectors defined in (2.13). These are determined from the corresponding

$$\underset{\sim}{S}_i = \begin{bmatrix} S_{i1} \\ S_{i2} \end{bmatrix} = \begin{bmatrix} \bar{R}_{i1} \\ \bar{R}_{i2} \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix} . \qquad (2.13)$$

mean rank vectors $\bar{\underset{\sim}{R}}_i$ by deleting the last alternative (which, as noted previously, can be done without loss of generality) and subtracting the average rank $[(L + 1)/2] = 2$ given to the L respective alternatives from each of the others. Since the $\{\underset{\sim}{S}_i\}$ are linear transformations of the vectors $\{\bar{\underset{\sim}{R}}_i\}$, consistent estimates for their covariance matrices can be obtained from the corresponding $\{\underset{\sim}{V}_{Ri}\}$ by matrix multiplication operations (2.14).[3] Thus, the overall set

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[3]Although other (L-1) dimensional linear transformations could be used instead of (2.13), the $\{\underset{\sim}{S}_i\}$ have the advantage that first and higher order differences among the $\{\bar{\underset{\sim}{R}}_i\}$ are equal to corresponding differences among the $\{S_i\}$ where the centered-reduced scores for the deleted last alternative are formed as the negative sums of the elements within the $\{\underset{\sim}{S}_i\}$.

$$V_{S1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad V_{R1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad (2.14)$$

of score vectors $\{S_i\}$ and their corresponding estimated covariance matrices $\{V_{Si}\}$ across the sub - populations can be represented by the arrays shown in (2.15) where V_S has been constructed to be

$$S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} , \quad V_S = \begin{bmatrix} V_{S1} & 0_{22} \\ 0_{22} & V_{S2} \end{bmatrix} \qquad (2.15)$$

asymptotically non - singular.[4]

With the previous considerations in mind, linear regression models can be fitted to the vector S by the method of weighted least squares as described in GSK. In general, this phase of the analysis can be formulated in terms of the matrix equation (2.16) where S is a (uxl)

$$E \underset{ux1}{\{S\}} = \underset{uxt}{X} \underset{tx1}{\beta} \qquad (2.16)$$

vector of scores which are functions of sample proportions, X is a pre - specified (uxt) design (or independent variable) matrix of known coefficients with full column rank t, and β is a (txl) vector of unknown parameters. The weighted least squares estimator b for β is given in (2.17). A consistent estimator for the covariance matrix

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[4] For specific samples of finite size, V_S may still occasionally contain singularities. Some methodological strategies for dealing with the computational difficulties which may arise in such situations are discussed in Koch et. al. [1974].

$$\underset{\sim}{b} = (\underset{\sim}{X}' \ \underset{\sim S}{V}^{-1} \ \underset{\sim}{X})^{-1} \ \underset{\sim}{X}' \ \underset{\sim S}{V}^{-1} \ \underset{\sim}{S} \quad . \tag{2.17}$$

of $\underset{\sim}{b}$ is $\underset{\sim b}{V}$ in (2.18). A goodness of fit statistic for assessing

$$\underset{\sim b}{V} = (\underset{\sim}{X}' \ \underset{\sim S}{V}^{-1} \ \underset{\sim}{X})^{-1} \quad . \tag{2.18}$$

the extent to which this model characterizes the score vector $\underset{\sim}{S}$ is the residual sum of squares Q in (2.19) which has approximately a

$$\begin{aligned} Q &= (\underset{\sim}{S} - \underset{\sim\sim}{Xb})' \ \underset{\sim S}{V}^{-1} \ (\underset{\sim}{S} - \underset{\sim\sim}{Xb}) \\ &= \underset{\sim}{S}' \ \underset{\sim S}{V}^{-1} \ \underset{\sim}{S} - \underset{\sim}{b}' \ (\underset{\sim}{X}' \ \underset{\sim S}{V}^{-1} \ \underset{\sim}{X}) \ \underset{\sim}{b} \quad . \end{aligned} \tag{2.19}$$

χ^2 - distribution with degress of freedom D.F. = (u - t) in large samples under the hypothesis that the model fits. If the model does adequately describe the data, tests of linear hypotheses with respect to the parameters comprising $\underset{\sim}{\beta}$ can be undertaken. In particular, for a general hypothesis of the form H_0: $\underset{\sim\sim}{C\beta} = \underset{\sim}{0}$ where $\underset{\sim}{C}$ is a known (d x t) matrix of full rank d $\leq$ t, a suitable test statistic is Q_C in (2.28) which has approximately a χ^2 - distribution with D.F. = d

$$Q_C = \underset{\sim}{b}' \ \underset{\sim}{C}' \ [\underset{\sim}{C} \ (\underset{\sim}{X}'\underset{\sim S}{V}^{-1}\underset{\sim}{X})^{-1} \ \underset{\sim}{C}']^{-1} \ \underset{\sim\sim}{Cb} \tag{2.20}$$

in large samples under H_0. Also, when a model like (2.16) fits, it is useful to determine predicted values $\hat{\underset{\sim}{S}}$ for $\underset{\sim}{S}$ by using the matrix equation (2.21). Consistent estimators for the variances of the predicted

$$\hat{\underset{\sim}{S}} = \underset{\sim}{X} \ \underset{\sim}{b} \tag{2.21}$$

values in $\hat{\underset{\sim}{S}}$ can be obtained from the diagonal elements of the matrix $\underset{\sim}{V}_{\hat{S}}$ in (2.22). The predicted values $\hat{\underset{\sim}{S}}$ represent improved estimators of the

$$\underset{\sim}{V}_{\hat{S}} = \underset{\sim}{X}\,(\underset{\sim}{X}'\underset{\sim}{V}_{S}^{-1}\underset{\sim}{X})^{-1}\,\underset{\sim}{X}' \qquad (2.22)$$

same quantities which are originally estimated by $\underset{\sim}{S}$ since they are based on the data from the entire sample as opposed to its component parts with the gain in precision being reflected by the relative magnitude of the diagonal elements of $\underset{\sim}{V}_{S}$ to those of $\underset{\sim}{V}_{\hat{S}}$. Moreover, the predicted values $\hat{\underset{\sim}{S}}$ are descriptively advantageous to the extent that they often substantially clarify the interpretation of complex relationships like that associated with the effects of ideology, sex, and criticism of governmental tax policies on the ranking of tax alternatives. Finally, it should be noted that models for score vectors $\underset{\sim}{S}$ can be directly transformed into models for mean rank vectors like $\overline{\underset{\sim}{R}}$ in (2.11) by reversing (2.13) and using the restrictions (2.3).

For the hypothetical example which has been considered previously in this section, one model of interest is the <u>identity model</u> in (2.23) where

$$\underset{\sim}{E}\,\{\underset{\sim}{S}\} = \underset{\sim}{E}\begin{bmatrix} S_{11} \\ S_{12} \\ S_{21} \\ S_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \beta_{11} \\ \beta_{12} \\ \beta_{21} \\ \beta_{22} \end{bmatrix} = \underset{\sim}{X}\,\underset{\sim}{\beta} = \underset{\sim}{\beta} \qquad (2.23)$$

β_{11}, β_{12}, β_{21}, β_{22} are parameters which correspond to the expected values for S_{11}, S_{12}, S_{21}, S_{22} respectively. Since this model involves no reduction in dimensionality in the sense that both the score vector $\underset{\sim}{S}$ and the parameter vector $\underset{\sim}{\beta}$ each contain 4 elements, the goodness of fit statistic Q in (2.19) involves no degrees of freedom and hence is of no interest (i.e., Q = 0, D.F. = 0 is degenerate). All other aspects of the analysis of $\underset{\sim}{S}$, however, can be undertaken as indicated in (2.16)-(2.22). In particular, (2.17) and (2.18) imply the results in (2.24). Thus, it is possible to test

$$\underset{\sim}{b} = \underset{\sim}{S} \; , \; \underset{\sim}{V}_b = \underset{\sim}{V}_S \tag{2.24}$$

hypotheses by specifying them in the general form H_0: $\underset{\sim}{C}\,\underset{\sim}{\beta} = \underset{\sim}{0}$ and applying Q_C in (2.20). In this regard, the $\underset{\sim}{C}$-matrix in (2.25) can be used to test

$$\underset{\sim}{C}_{RP1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{2.25}$$

the hypothesis H_{RP1} of equality of average rank preferences for the L = 3 tax alternatives within the male sub-population; and the $\underset{\sim}{C}$-matrix in (2.26)

$$\underset{\sim}{C}_{RP2} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.26}$$

can be similarly used within the female sub-population. The hypotheses H_{S1}, H_{S2}, H_{S3} in (2.5) of no differences between the male and female sub-populations with respect to each of the L = 3 separate tax alternatives can be tested by using the $\underset{\sim}{C}$-matrices in (2.27)-(2.29), where the construc-

$$\underset{\sim}{C}_{S1} = [\;1 \;\; 0 \; -1 \;\; 0] \text{ for tax alternative 1 (TR)} \tag{2.27}$$

$$\underset{\sim}{C}_{S2} = [\;0 \;\; 1 \;\; 0 \; -1] \text{ for tax alternative 2 (GI)} \tag{2.28}$$

$$\underset{\sim}{C}_{S3} = [-1 \; -1 \;\; 1 \;\; 1] \text{ for tax alternative 3 (HC)} \tag{2.29}$$

tion of $\underset{\sim}{C}_{S3}$ follows from the definition of the $\{\underset{\sim}{S}_i\}$ in (2.13) and the restrictions (2.3). In addition, the multivariate hypothesis of no differences between the male and female sub-populations with respect to the overall preference profile for the L = 3 tax alternatives can be tested by using the $\underset{\sim}{C}$-matrix in (2.30).

$$\underset{\sim}{C}_{RPS} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \tag{2.30}$$

Other models may also be of interest for the vector $\underset{\sim}{S}$ depending on the results of the statistical tests for hypotheses like H_{RP1}, H_{RP2}, H_{S1}, H_{S2}, H_{S3}, and H_{RPS}. In particular, if the score vector $\underset{\sim}{S}$ is interpreted as satisfying the hypothesis H_{RPS} in (2.6), then it is appropriate to fit the model in (2.31). Moreover, the goodness of fit statistic Q in (2.19)

$$\underset{\sim}{E}\{\underset{\sim}{S}\} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \underset{\sim}{X}\,\underset{\sim}{\beta} \tag{2.31}$$

for the model (2.31) is identical to the test statistic associated with $\underset{\sim}{C}_{RPS}$ in (2.30) for the model (2.23). Thus, if the latter indicates that $\underset{\sim}{S}$ satisfies H_{RPS}, then so does the former; and this fact is the original basis for considering the model (2.31). Tests of hypotheses may also be undertaken in this framework via (2.20). In particular, the $\underset{\sim}{C}$-matrix in (2.32) can be used to test the hypothesis of equality of average rank

$$\underset{\sim}{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{2.32}$$

preferences (i.e., indifference) for the L = 3 tax alternatives within both the male and female sub-populations which have the same overall preference profiles as a consequence of the model (2.31).

In summary, weighted least squares methods can be used to investigate a broad range of hypotheses pertaining to rank policy preference data and to fit corresponding linear regression models. A more complete discussion of the application of these methods will be given in Section 3 for the example described earlier from SERS-I.

3. ANALYSIS OF TAX ALTERNATIVE PREFERENCE DATA FROM SERS-I

The SERS-I data involve the ranking of seven tax alternatives (ED,PL, TR,PV,FA,GI, and HC) by respondents who have been cross-classified into 16 sub-populations according to ideology x sex x criticism. As noted in Section 2.1, the analysis of the relationship between ideology x sex x criticism and the ranked preference response profile for the seven tax alternatives can be formulated in terms of a contingency table with 16 rows and 7! = 5040 columns. However, it is not necessary to generate this conceptual contingency table if the respective within-sub-population mean rank vectors $\{\bar{\underset{\sim}{R}}_i\}$ for the tax alternatives are regarded as the pertinent preference measures of interest. Alternatively, these quantities and their corresponding estimated covariance matrices $\{\underset{\sim}{V}_{Ri}\}$ can be obtained directly from the observed respondent-wise raw data matrix (analogous to that shown in Table 1) by applying expressions (3.1) - (3.2) where $\underset{\sim}{R}_{ik}$ denotes the vector of ranks

$$\bar{\underset{\sim}{R}}_i = \begin{bmatrix} \bar{R}_{i1} \\ \bar{R}_{i2} \\ \cdots \\ \bar{R}_{i7} \end{bmatrix} = \frac{1}{n_i} \sum_{k=1}^{n_i} \begin{bmatrix} R_{i1k} \\ R_{i2k} \\ \cdots \\ R_{i7k} \end{bmatrix} = \frac{1}{n_i} \sum_{k=1}^{n_i} \underset{\sim}{R}_{ik} \tag{3.1}$$

$$\underset{\sim}{V}_{Ri} = \frac{1}{n_i(n_i-1)} \sum_{k=1}^{n_i} (\underset{\sim}{R}_{ik} - \bar{\underset{\sim}{R}}_i)(\underset{\sim}{R}_{ik} - \bar{\underset{\sim}{R}}_i)' \tag{3.2}$$

corresponding to the k-th respondent from the i-th sub-population. As discussed in Koch et al. [1974], expressions (3.1) - (3.2) represent the most effective procedure for computing the $\{\bar{\underset{\sim}{R}}_i\}$ and the $\{\underset{\sim}{V}_{Ri}\}$ while expressions (2.7) - (2.12) provide the statistical framework for their analysis by linear regression models like (2.16) which are fitted by the weighted least squares methods (2.17) - (2.22).

Strictly speaking, the SERS-I data cannot be rigorously analyzed in this framework because they are based on a complex survey design. As a result, the underlying (16 x 5040) conceptual contingency table contains "weighted frequencies" which reflect certain adjustments of the sample according to the United States national distribution for race, region of the country, and urban vs. rural residence. For this reason, the matrices $\{\underset{\sim}{V}_i\}$ analogous to (2.9) may not necessarily be valid estimators for the covariance matrices of the vectors of sample proportions $\{\underset{\sim}{p}_i\}$; and hence the results of the analyses to be presented in the remainder of this section should be interpreted with some caution. On the other hand, primary emphasis in this paper is directed at those aspects of analysis which involve relationships among variables and/or have a multiple regression flavor. For this type of application, certain empirical results of Kish and Frankel [1970] suggest that the complex sample survey design effect may be small and that the $\{\underset{\sim}{V}_i\}$ may be used as the estimated covariance matrices for the $\{\underset{\sim}{p}_i\}$ in the sense of a heuristic approximation. Thus, these same conclusions would apply to the mean rank vectors $\{\underset{\sim}{\bar{R}}_i\}$ and their corresponding estimated covariance matrices $\{\underset{\sim}{V}_{Ri}\}$ as conceptually determined from the matrix formulations (2.8) and (2.10) respectively. However, these same quantities can be generated directly from the observed respondent-wise raw data (analogous to that shown in Table 1) by applying expressions (3.3) - (3.4) where W_{ik} is the weight associated with the k-th respondent

$$\underset{\sim}{\bar{R}}_i = \frac{1}{N_i} \sum_{k=1}^{n_i} W_{ik} \underset{\sim}{\bar{R}}_{ik} \tag{3.3}$$

$$\underset{\sim}{V}_R = \frac{1}{N_i(N_i-1)} \sum_{k=1}^{n_i} W_{ik}(\underset{\sim}{R}_{ik} - \overline{\underset{\sim}{R}}_i)(\underset{\sim}{R}_{ik} - \overline{\underset{\sim}{R}}_i)' \qquad (3.4)$$

from the i-th sub-population and $N_i = \sum_{k=1}^{n_i} W_{ik}$ is the "weighted sample size" for the i-th sub-population. This latter method was used to compute the $\{\overline{\underset{\sim}{R}}_i\}$ and $\{\underset{\sim}{V}_{Ri}\}$ for the SERS-I data, and the corresponding results for the 16 ideology x sex x criticism sub-populations are given in Tables A.1-A.16 of the Appendix to this paper. With these considerations in mind, the weighted least squares methods (2.17) - (2.22) can now be applied to the overall mean rank vector $\overline{\underset{\sim}{R}}$ and its corresponding estimated covariance matrix $\underset{\sim}{V}_R$ analogous to (2.11) and (2.12). Such analysis will proceed in three basic stages. First of all (Section 3.1), preliminary analyses are undertaken to test hypotheses like the $\{H_{RPi}\}$ in (2.4) and the $\{H_{Sg}\}$ in (2.5) so as to identify the nature and extent of differences among mean ranks

a. for tax alternatives within sub-populations

b. for sub-populations within tax alternatives.

Since the results of this type of analysis suggest significant interaction between preference patterns and ideology, the second stage (Section 3.2) is concerned with the fitting of multivariate linear models in the sense of H_{RPS} in (2.6) to each ideology group separately. These models are then refined by the removal of parameters corresponding to unimportant sources of variation. Then, in Section 3.3, the separate models for each ideology group are unified together to form a final overall model which permits a relatively clear interpretation of the effects of ideology, sex, and criticism on the respective rank preference profiles for the seven tax alternatives.

3.1. Preliminary Analyses

Mean rank scores and corresponding estimated standard errors for the seven tax alternatives within the 16 respective ideology x sex x criticism sub-populations are summarized in Table 2. These quantities are based on the results reported in the Appendix (Table A.1 - A.16) as obtained from expressions (3.3) - (3.4). As noted earlier, the first question of interest is whether the respondents from each specific sub-population are indifferent with respect to their rankings of the seven possible choices in the sense of hypotheses $\{H_{RPi}\}$ analogous to (2.4). Two types of statistics are used to test these hypotheses. One is a "weighted" version of Friedman's [1937] χ^2-statistic which is shown in expression (3.5) where N_i is the "weighted

$$T_i = \frac{12N_i}{L(L+1)} \left\{ \sum_{g=1}^{L=7} \left(\bar{R}_{ig} - \frac{(L+1)}{2}\right)^2 \right\} \tag{3.5}$$

sample size" for the i-th sub-population as defined with respect to (3.3) and (3.4). The other is obtained within the context of the GSK method by fitting the identity model $\underset{\sim}{X} = \underset{\sim}{I}_6$ where $\underset{\sim}{I}_6$ is the 6 x 6 identity matrix to the centered-reduced score vectors like (2.13) and computing test statistics $\{Q_{C,RPi}\}$ which are analogous to (2.25) - (2.26) by applying (2.20) with $\underset{\sim}{C} = \underset{\sim}{I}_6$. Both the $\{T_i\}$ and $\{Q_{C,RPi}\}$ are given in Table 3. Since these statistics approximately have χ^2-distributions with D.F. = 6 in large samples under the respective hypotheses $\{H_{RPi}\}$, these results suggest that there are significant ($\alpha = .01$) differences among the mean ranks for the seven tax alternatives within each of the 16 ideology x sex x criticism sub-populations.[5] Thus, the respondents are not indifferent but systematic in their preference orderings.

On the basis of the preceding conclusions, the analysis is then directed at questions pertaining to the nature and extent of differences

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[5] The discussion in this section will involve the use of many inter-related statistical tests. If desired, these results may be interpreted from a simultaneous inference point of view by using suitable combinations of Bonferroni inequality and Scheffé multiple comparison procedures. For further discussion of this aspect of data analysis, see Koch et al [1975].

TABLE 2

OBSERVED MEAN RANK PREFERENCE PROFILE AND CORRESPONDING STANDARD ERRORS FOR THE SEVEN TAX ALTERNATIVES

Sub-population*				Policy Alternative ED	PL	TR	PV	FA	GI	HC	Weighted Sample Size N_i
1.	Con	Male	No	2.18 (0.13)	3.60 (0.13)	3.01 (0.15)	5.11 (0.12)	6.20 (0.09)	4.65 (0.14)	3.25 (0.10)	155
2.	Con	Male	Yes	2.25 (0.16)	3.56 (0.21)	3.25 (0.24)	4.77 (0.19)	6.37 (0.15)	4.65 (0.22)	3.15 (0.18)	63
3.	Con	Female	No	2.09 (0.12)	3.49 (0.16)	3.05 (0.18)	4.91 (0.13)	6.21 (0.11)	4.68 (0.17)	3.56 (0.11)	121
4.	Con	Female	Yes	2.50 (0.22)	4.04 (0.26)	2.39 (0.22)	4.74 (0.21)	6.56 (0.12)	3.97 (0.29)	3.80 (0.22)	46
5.	Lib	Male	No	2.39 (0.18)	4.17 (0.17)	3.76 (0.24)	4.18 (0.20)	6.26 (0.11)	4.11 (0.20)	3.12 (0.15)	84
6.	Lib	Male	Yes	2.69 (0.28)	3.37 (0.27)	3.45 (0.50)	5.04 (0.26)	6.24 (0.31)	3.79 (0.43)	3.42 (0.29)	24
7.	Lib	Female	No	2.21 (0.17)	4.01 (0.21)	4.74 (0.24)	3.84 (0.18)	6.11 (0.12)	3.66 (0.23)	3.44 (0.17)	76
8.	Lib	Female	Yes	1.97 (0.30)	3.27 (0.39)	3.93 (0.38)	4.59 (0.43)	6.49 (0.17)	4.61 (0.49)	3.14 (0.28)	18
9.	Btwn	Male	No	2.08 (0.11)	3.74 (0.13)	3.73 (0.16)	4.50 (0.12)	6.10 (0.09)	4.42 (0.15)	3.43 (0.10)	169
10.	Btwn	Male	Yes	2.20 (0.17)	3.08 (0.17)	3.60 (0.24)	4.96 (0.17)	6.28 (0.10)	4.81 (0.21)	3.06 (0.13)	75
11.	Btwn	Female	No	2.28 (0.11)	3.82 (0.13)	3.26 (0.13)	4.73 (0.11)	6.20 (0.08)	4.30 (0.15)	3.41 (0.11)	191
12.	Btwn	Female	Yes	2.48 (0.23)	3.34 (0.19)	2.54 (0.24)	5.14 (0.19)	6.67 (0.08)	4.40 (0.24)	3.42 (0.17)	52
13.	None	Male	No	2.76 (0.18)	4.53 (0.16)	3.45 (0.21)	4.63 (0.16)	6.04 (0.12)	3.70 (0.20)	2.88 (0.13)	101
14.	None	Male	Yes	2.71 (0.34)	4.09 (0.24)	3.32 (0.31)	4.51 (0.25)	6.46 (0.19)	3.99 (0.35)	2.92 (0.23)	35
15.	None	Female	No	2.24 (0.11)	4.15 (0.13)	3.21 (0.14)	4.76 (0.12)	6.29 (0.08)	4.24 (0.15)	3.12 (0.09)	176
16.	None	Female	Yes	2.47 (0.28)	4.24 (0.25)	2.23 (0.23)	5.07 (0.19)	6.45 (0.15)	4.16 (0.29)	3.38 (0.16)	42

* The sub-populations are defined by ideology (conservative, liberal, in between, none), sex (male, female), and expressed criticism of taxes (no, yes).

among the mean response profiles $\{\bar{R}_i\}$ for the respective sub-populations in terms of effects corresponding to ideology, sex, and criticism. As noted in Section 2.2, this can be done in either the univariate framework analogous to the hypotheses $\{H_{Sg}\}$ in (2.5) and/or the multivariate framework analogous to the hypotheses H_{RPS} in (2.6). From a conceptual point of view, the multivariate framework is more appropriate because it accounts for the processes by which positive differences between two sub-populations for certain tax alternative are offset by negative differences for others. However, the multivariate approach also requires more extensive computations and hence is more costly. Thus, it is often reasonable to carry out preliminary univariate analyses in order to identify the dominant sources of variation which characterize sub-pcpulation differences. For this purpose, let $\underset{\sim}{Y}_g$ as defined in (3.6) characterizes the vector of mean ranks across the 16

$$\underset{\sim}{Y}_g' = (\bar{R}_{1g\cdot}, \bar{R}_{2g\cdot}, \ldots, \bar{R}_{16,g\cdot}) \tag{3.6}$$

sub-populations for the g-the alternative where g=1,2,3,4,5,6,7. The estimated covariance matrix $\underset{\sim}{V}_{Yg}$ for $\underset{\sim}{Y}_g$ can be constructed by performing suitable sub-matrix operations on the overall estimated covariance matrix $\underset{\sim}{V}_R$. More specifically, the $\{\underset{\sim}{Y}_g\}$ are the respective columns of mean ranks in Table 2 and the $\{\underset{\sim}{V}_{Yg}\}$ are diagonal matrices for which the diagonal elements are the squares of the corresponding columns of standard errors (in parentheses).

Various hypotheses pertaining to each of the within tax alternative mean rank vectors $\underset{\sim}{Y}_g$ can be evaluated via the GSK methodology by fitting the identity model $\underset{\sim}{X} = \underset{\sim}{I}_{16}$ and applying (2.20) to determine Q_C statistics for appropriate $\underset{\sim}{C}$ matrices. In particular, the $\underset{\sim}{C}$ matrix in (3.7) where $\underset{\sim}{1}_{15}$

$$\underset{15\times 16}{\underset{\sim}{C}_S} = [\underset{\sim}{1}_{15} \quad -\underset{\sim}{I}_{15}] \tag{3.7}$$

TABLE 3

FRIEDMAN AND GSK TEST STATISTICS FOR THE HYPOTHESES OF INDIFFERENCE

	Sub-population			Weighted Sample Size	Friedman Statistics T_i D.F.=6	GSK $Q_{C,RPi}$ Statistics D.F.=6
1.	Conservative	Male	No	155	381.95	1017.55
2.	Conservative	Male	Yes	63	150.63	725.99
3.	Conservative	Female	No	121	288.69	779.82
4.	Conservative	Female	Yes	46	117.84	767.67
5.	Liberal	Male	No	84	155.38	693.08
6.	Liberal	Male	Yes	24	45.55	104.57
7.	Liberal	Female	No	76	140.68	613.79
8.	Liberal	Female	Yes	18	47.57	399.44
9.	In-Between	Male	No	169	325.61	815.67
10.	In-Between	Male	Yes	75	191.67	1065.04
11.	In-Between	Female	No	191	381.56	1365.46
12.	In-Between	Female	Yes	52	153.65	1372.83
13.	None	Male	No	101	173.08	598.77
14.	None	Male	Yes	35	72.12	299.02
15.	None	Female	No	176	393.00	1232.43
16.	None	Female	Yes	42	117.53	512.76

is a (15x1) vector of 1's can be used to test hypotheses analogous to the $\{H_{Sg}\}$ in (2.5), and the corresponding Q_C statistics are given in the "over-all total" row of Table 4. Since these results suggest that significant (α = .01) differences exist among the 16 sub-populations for all the tax alternatives except education, attention can then be directed at other hypotheses which facilitate their interpretation. In this context, it is of interest to investigate the extent to which sex and criticism interact with ideology. Thus, the $\underset{\sim}{C}$ matrix in (3.8) is used to test the third order

$$\underset{\sim}{C}_{S,ISC} = \begin{bmatrix} 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 \end{bmatrix} \quad (3.8)$$

ideology x sex x criticism interaction; and the $\underset{\sim}{C}$ matrices in (3.9) - (3.11)

$$\underset{\sim}{C}_{S,I} = \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 \end{bmatrix} \quad (3.9)$$

$$\underset{\sim}{C}_{S,IS} = \begin{bmatrix} 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 \end{bmatrix} \quad (3.10)$$

$$\underset{\sim}{C}_{S,IC} = \begin{bmatrix} 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \end{bmatrix} . \quad (3.11)$$

are similarly used for the unadjusted effects of ideology, ideology x sex interaction, and ideology x criticism interaction respectively. The Q_C statistics corresponding to these hypotheses are also given in Table 4. These results imply that each of the ideology groups should be considered separately since significant differences exist among them with respect to the effects of sex and criticism on the mean rank preferences for the various tax alternatives. Thus, the final set of hypotheses which are tested

TABLE 4

UNIVARIATE Q_C-STATISTICS FOR HYPOTHESES PERTAINING TO IDENTITY MODELS

Source of Variation	D.F.	ED	PL	TR	PV	FA	GI	HC
Conservatives								
Sex	1	0.24	0.96	4.18*	0.50	0.67	2.36	8.83**
Criticism	1	2.13	1.69	1.08	2.40	4.72*	2.80	0.18
Sex x Criticism	1	1.05	2.23	5.08*	0.24	0.57	2.75	1.12
Combined Sub-Total	3	2.70	3.46	8.51*	3.89	6.54	5.02	9.42*
Liberals								
Sex	1	3.49	0.24	4.18*	1.96	0.06	0.26	0.00
Criticism	1	0.02	7.99**	2.44	8.04**	0.83	0.78	0.00
Sex x Criticism	1	1.30	0.01	0.48	0.04	1.13	3.15	1.63
Combined Sub-Total	3	3.65	9.36*	10.92*	15.21**	3.46	4.23	2.30
In-Between								
Sex	1	2.23	1.16	14.41**	1.85	7.09*	1.95	1.73
Criticism	1	0.94	12.81**	4.40*	8.24**	13.06**	1.66	1.88
Sex x Criticism	1	0.06	0.32	2.16	0.02	2.57	0.61	2.15
Combined Sub-Total	3	3.25	15.20**	18.10**	9.70*	25.33**	4.03	6.01
None								
Sex	1	2.44	0.33	8.44**	3.46	0.69	1.82	4.78*
Criticism	1	0.14	0.71	5.80*	0.25	4.12*	0.15	0.86
Sex x Criticism	1	0.35	1.75	3.33	1.34	0.86	0.50	0.50
Combined Sub-Total	3	6.95	4.04	18.50**	4.22	6.20	4.76	6.46
Ideology	3	4.49	36.42**	28.35**	9.04*	0.27	12.77**	10.91*
Ideology x Sex	3	8.36*	2.20	14.29**	6.98	1.48	5.53	3.73
Ideology x Criticism	3	0.59	14.19**	1.58	14.81**	0.56	5.27	2.88
Ideology x Sex x Criticism	3	2.49	1.14	0.46	1.17	3.17	5.71	3.60
Overall Total	15	21.14	75.21**	106.28**	59.81**	42.94**	41.60**	36.74**

* means significant at α = .05, ** means significant at α = .01

at this stage of analysis pertain to the unadjusted effects of sex, the unadjusted effects of criticism, the sex x criticism interaction, and the combined "sub-total" for these sources of variation within each ideology group. For this purpose, the $\underset{\sim}{C}$ matrices in (3.12) - (3.14) respectively

$$\underset{\sim}{C}_{CS,S} = [1\ \ 1\ -1\ -1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0] \qquad (3.12)$$

$$\underset{\sim}{C}_{CS,C} = [1\ -1\ \ 1\ -1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0] \qquad (3.13)$$

$$\underset{\sim}{C}_{CS,SC} = [1\ -1\ -1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0] \qquad (3.14)$$

would be used individually and simultaneously for the "conservative" group; and similarly formed $\underset{\sim}{C}$ matrices would be used for the other ideology groups. From the results given in Table 4 for all of these Q_C statistics, it is apparent that the sex x criticism interaction effects can be regarded as relatively unimportant.

In summary, the analyses presented so far support the following conclusions for the SERS-I data:

a. Significant (α = .01) differences exist among the mean preference ranks for the respective tax alternatives within each of the 16 sub-populations.

b. Significant (α = .01) differences exist among the mean preference ranks for the respective sub-populations within each of the seven tax alternatives except (possibly) Education (ED).

c. The further interpretation of such sub-population differences should recognize that

   1. ideology and sex interact in the sense that sex effects vary among ideology groups

   2. ideology and criticism interact in the sense that crit-

icism effects vary among ideology groups

3. within each separate ideology group, sex and criticism do not interact and hence are additive.

3.2. Multivariate Models

The discussion in Sub-section 3.1 provides the motivation for fitting multivariate linear models with additive (non-interacting) sex and criticism effects to the centered-reduced score vectors $\{\underset{\sim}{S}_i\}$ like (2.13) which are associated with each separate ideology group. These models can be written in terms of the matrix formulation (3.16) where $\underset{\sim}{\mu}_h$ is a (6x1)

$$E\begin{bmatrix} \underset{\sim}{S}_{4(h-1)+1} \\ \underset{\sim}{S}_{4(h-1)+2} \\ \underset{\sim}{S}_{4(h-1)+3} \\ \underset{\sim}{S}_{4(h-1)+4} \end{bmatrix} = \underset{(24x18)}{\underset{\sim}{X}_{1h}} \; \underset{(18x1)}{\underset{\sim}{\beta}_h} = \begin{bmatrix} \underset{\sim}{I}_6 & \underset{\sim}{0}_{66} & \underset{\sim}{0}_{66} \\ \underset{\sim}{I}_6 & \underset{\sim}{0}_{66} & \underset{\sim}{I}_6 \\ \underset{\sim}{I}_6 & \underset{\sim}{I}_6 & \underset{\sim}{0}_{66} \\ \underset{\sim}{I}_6 & \underset{\sim}{I}_6 & \underset{\sim}{I}_6 \end{bmatrix} \begin{bmatrix} \underset{\sim}{\mu}_h \\ \underset{\sim}{\xi}_h \\ \underset{\sim}{\gamma}_h \end{bmatrix} = \begin{bmatrix} \underset{\sim}{\mu}_h \\ \underset{\sim}{\mu}_h + \underset{\sim}{\gamma}_h \\ \underset{\sim}{\mu}_h + \underset{\sim}{\xi}_h \\ \underset{\sim}{\mu}_h + \underset{\sim}{\xi}_h + \underset{\sim}{\gamma}_h \end{bmatrix} \qquad (3.16)$$

vector of parameters corresponding to males with no criticism, $\underset{\sim}{\xi}_h$ is a (6x1) vector of sex effects for females, and $\underset{\sim}{\gamma}_h$ is a (6x1) vector of criticism effects for the h-th ideology group with h=1,2,3,4. In addition, the analyses given here involve $\{\underset{\sim}{S}_i\}$ with "tax reduction (TR)" as the deleted tax alternative.

The validity of the models (3.16) is supported by the non-significance (α = .10) of the residual goodness of fit Q statistics (2.19) which are reported in Table 5. Hence, it is of interest to test various hypotheses pertaining to the parameters $\underset{\sim}{\beta}_h$. In this regard, Table 5 also includes appropriate Q_C statistics based on (2.20) for hypotheses associated with the multivariate effects of sex (H_0: $\underset{\sim}{\xi}_h = \underset{\sim}{0}_6$), criticism ($H_0$: $\underset{\sim}{\gamma}_h = \underset{\sim}{0}_6$), and simultaneous model (H_0: $\underset{\sim}{\xi}_h = \underset{\sim}{0}_6$, $\underset{\sim}{\gamma}_h = \underset{\sim}{0}_6$) sources of variation where

TABLE 5

TEST STATISTICS FOR MULTIVARIATE HYPOTHESES IN THE $\{X_{1h}\}$ MODELS

Source of Variation	Ideology: Conservative D.F.	Conservative Q_c	Liberal D.F.	Liberal Q_c	In-Between D.F.	In-Between Q_c	None D.F.	None Q_c
Sex	6	11.58	6	14.14	6	25.20	6	19.40
Criticism	6	7.00	6	28.99	6	49.05	6	14.37
Model	12	20.24	12	51.07	12	79.94	12	32.46
Residual	6	10.10	6	8.22	6	8.31	6	9.38

TABLE 6

ESTIMATED PARAMETERS FOR THE $\{X_{1h}\}$ MODELS

Ideology	Effect	ED	PL	TR	PV	FA	GI	HC
Conservative	Sex	-0.03	0.05	-0.24	-0.10	0.06	-0.13	0.37
	Criticism	0.15	0.22	-0.20	-0.19	0.25	-0.24	0.01
Liberal	Sex	-0.31	-0.21	0.88	-0.29	-0.08	-0.19	0.20
	Criticism	0.07	-0.78	-0.80	0.99	0.32	0.12	0.08
In-Between	Sex	0.21	0.18	-0.70	0.20	0.26	-0.26	0.12
	Criticism	0.16	-0.58	-0.45	0.47	0.36	0.23	-0.19
None	Sex	-0.43	-0.28	-0.54	0.33	0.15	0.43	0.34
	Criticism	0.03	-0.12	-0.67	0.21	0.26	0.15	0.14

TABLE 7

TEST STATISTICS FOR UNIVARIATE HYPOTHESES IN THE $\{X_{1h}\}$ MODELS

Source of Variation	D.F.	Policy Alternative: ED	PL	TR	PV	FA	GI	HC
Sex in Con	1	0.03	0.09	1.62	0.45	0.32	0.46	7.77
Criticism in Con	1	0.89	1.41	1.08	1.41	4.60	1.46	0.01
Sex in Lib	1	2.22	0.80	8.93	1.58	0.26	0.52	0.99
Criticism in Lib	1	0.08	10.03	6.68	16.19	3.55	0.14	0.12
Sex in Btwn	1	2.30	1.51	16.11	2.11	8.54	2.22	0.90
Criticism in Btwn	1	1.02	13.66	5.08	10.02	16.35	1.42	2.21
Sex in None	1	5.06	2.68	7.26	4.28	1.51	4.04	6.28
Criticism in None	1	0.02	0.39	9.78	1.34	3.84	0.39	0.91

$\underset{\sim}{0}_6$ is a (6x1) vector of 0's. Otherwise, estimated parameters for the sex and criticism effects in the $\{\underset{\sim}{X}_{1h}\}$ models (3.16) are given in Table 6; and corresponding univariate Q_C test statistics are given in Table 7. Finally, predicted values based on the models (3.16) are given in Table 8 for the mean rank preference profiles across the 16 sub-populations together with corresponding standard errors. These results are obtained by first applying (2.21) and (2.22) to the $\{\underset{\sim}{S}_i\}$ and then reversing the score transformations analogous to (2.13). As noted in Section 2.2, the extent to which the standard errors for the predicted values in Table 8 are smaller than the corresponding quantities in Table 2 reflect the gain in estimation precision that is obtained by using the models (3.16).

Since many of the univariate test statistics in Table 7 for the effects of sex and criticism are non-significant ($\alpha = .10$), it is appropriate to refine the models $\{\underset{\sim}{X}_{1h}\}$ in (3.16) by deleting those parameters which are judged to be unimportant. However, this process must be undertaken in a series of carefully formulated steps which are compatible with the following general principles:

1. The multivariate nature of rank preference data implies the existence of inter-relationships among the parameters for the seven tax alternatives which cause the removal of some parameters to affect the behavior of others.
2. It is more reasonable to remove the sex and/or criticism effects from the model (3.16) corresponding to those tax alternatives for which the "combined sub-total" statistics in Table 4 are non-significant ($\alpha = .10$) than for those where they are significant ($\alpha = .05$).
3. It is desirable to maintain some degree of parallelism among the models which are fitted to the respective ideology groups.

TABLE 8

PREDICTED MEAN RANK PREFERENCE PROFILES AND CORRESPONDING STANDARD ERRORS BASED ON $\{x_{1h}\}$ MODELS

Sub-population*				Policy Alternative						
				ED	PL	TR	PV	FA	GI	HC
1.	Con	Male	No	2.15 (0.11)	3.53 (0.13)	3.13 (0.14)	5.07 (0.11)	6.19 (0.08)	4.71 (0.13)	3.22 (0.10)
2.	Con	Male	Yes	2.30 (0.14)	3.76 (0.18)	2.92 (0.19)	4.88 (0.15)	6.44 (0.12)	4.46 (0.18)	3.24 (0.15)
3.	Con	Female	No	2.13 (0.11)	3.59 (0.14)	2.89 (0.16)	4.97 (0.12)	6.25 (0.09)	4.58 (0.16)	3.60 (0.10)
4.	Con	Female	Yes	2.27 (0.16)	3.81 (0.19)	2.69 (0.17)	4.78 (0.16)	6.50 (0.10)	4.34 (0.21)	3.61 (0.16)
5.	Lib	Male	No	2.49 (0.17)	4.18 (0.16)	3.82 (0.22)	4.13 (0.18)	6.22 (0.11)	3.98 (0.19)	3.18 (0.14)
6.	Lib	Male	Yes	2.56 (0.22)	3.40 (0.22)	3.02 (0.32)	5.12 (0.21)	6.54 (0.18)	4.10 (0.32)	3.26 (0.22)
7.	Lib	Female	No	2.18 (0.15)	3.97 (0.19)	4.70 (0.22)	3.84 (0.17)	6.14 (0.11)	3.79 (0.22)	3.38 (0.16)
8.	Lib	Female	Yes	2.25 (0.23)	3.20 (0.26)	3.90 (0.29)	4.83 (0.26)	6.46 (0.15)	3.91 (0.33)	3.46 (0.21)
9.	Btwn	Male	No	2.08 (0.10)	3.67 (0.12)	3.87 (0.15)	4.52 (0.11)	6.00 (0.08)	4.49 (0.14)	3.36 (0.10)
10.	Btwn	Male	Yes	2.24 (0.14)	3.10 (0.14)	3.42 (0.19)	4.99 (0.14)	6.36 (0.08)	4.72 (0.18)	3.17 (0.11)
11.	Btwn	Female	No	2.29 (0.10)	3.85 (0.12)	3.17 (0.12)	4.72 (0.10)	6.27 (0.07)	4.23 (0.14)	3.48 (0.10)
12.	Btwn	Female	Yes	2.45 (0.16)	3.28 (0.15)	2.72 (0.19)	5.19 (0.15)	6.62 (0.07)	4.46 (0.18)	3.29 (0.13)
13.	None	Male	No	2.70 (0.17)	4.48 (0.14)	3.66 (0.18)	4.49 (0.14)	6.11 (0.11)	3.75 (0.18)	2.82 (0.12)
14.	None	Male	Yes	2.73 (0.24)	4.36 (0.18)	2.99 (0.22)	4.69 (0.18)	6.36 (0.14)	3.90 (0.25)	2.97 (0.16)
15.	None	Female	No	2.27 (0.11)	4.20 (0.12)	3.12 (0.13)	4.82 (0.11)	6.25 (0.07)	4.18 (0.14)	3.16 (0.09)
16.	None	Female	Yes	2.30 (0.22)	4.08 (0.19)	2.45 (0.19)	5.02 (0.16)	6.51 (0.12)	4.34 (0.23)	3.31 (0.14)

* The sub-populations are defined by ideology (conservative, liberal, in between, none), sex (male, female), and expressed criticism of taxes (no, yes).

On the basis of the considerations (1) - (3), refined models $\{\underset{\sim}{X}_{2h}\}$ are developed for each ideology group by deleting parameters from the $\{X_{1h}\}$ models which are represented by the "blank" spaces in Table 9. Otherwise, estimated parameters for the sex and criticism effects which are included in the $\{\underset{\sim}{X}_{2h}\}$ models are given in the "non-blank" spaces of Table 9; and corresponding univariate Q_C test statistics are given in Table 10. In addition, Q_C test statistics for the refined multivariate effects of sex and criticism are given in Table 11 together with residual goodness of fit Q statistics which are interpreted as supporting the validity of the $\{\underset{\sim}{X}_{2h}\}$ models. However, it should be noted that the results for the "in-between" ideology group should be evaluated more cautiously because the Q statistic corresponding to it is significant at $\alpha = .10$ although not at $\alpha = .05$. On the other hand, the impact of such concerns will be diminished somewhat as a consequence of further refinements in analysis which are directed at the internal consistencies of results both within and among ideology groups. Finally, Table 12 shows predicted values and corresponding standard errors for the mean rank preference profiles across the 16 subpopulations as based on the $\{\underset{\sim}{X}_{2h}\}$ models; and thus these quantities descriptively reflect the extent to which the sex and criticism sources of variation have important effects within each of the separate ideology groups.

3.3. Final Model

The analyses presented in Sub-sections 3.1 - 3.2 are both comprehensive and straightforward. Thus, for most practical purposes, no additional modifications are required. Nevertheless, the objectives of this paper are primarily concerned with illustrating the full range of model fitting capabilities that are within the scope of the GSK methodology.

TABLE 9

ESTIMATED PARAMETERS FOR THE $\{X_{2h}\}$ MODELS

Ideology	Effect	ED	PL	TR	PV	FA	GI	HC
Conservative	Sex	----	----	-0.36	----	----	----	0.36
	Criticism	----	----	-0.20	----	0.20	----	----
Liberal	Sex	----	-0.36	0.66	-0.30	----	----	----
	Criticism	----	-0.72	-0.72	1.07	0.36	----	----
In-Between	Sex	----	----	-0.50	----	0.25	----	0.25
	Criticism	----	-0.55	-0.30	0.45	0.40	----	----
None	Sex	-0.44	----	-0.45	0.27	----	0.35	0.27
	Criticism	----	----	-0.35	----	0.35	----	----

TABLE 10

TEST STATISTICS FOR UNIVARIATE HYPOTHESES IN THE $\{X_{2h}\}$ MODELS

Source of Variation	D.F.	Policy Alternative						
		ED	PL	TR	PV	FA	GI	HC
Sex in Con	1	----	----	9.83	----	----	----	9.83
Criticism in Con	1	----	----	4.12	----	4.12	----	----
Sex in Lib	1	----	3.58	8.83	2.04	----	----	----
Criticism in Lib	1	----	11.95	6.62	25.75	6.20	----	----
Sex in Btwn	1	----	----	17.50	----	9.69	----	5.40
Criticism in Btwn	1	----	19.17	3.45	10.13	23.02	----	----
Sex in None	1	5.35	----	5.70	3.18	----	3.34	4.32
Criticism in None	1	----	----	9.38	----	9.38	----	----

TABLE 11

TEST STATISTICS FOR MULTIVARIATE HYPOTHESES IN THE $\{X_{2h}\}$ MODELS

Source of Variation	Ideology							
	Conservative		Liberal		In-Between		None	
	D.F.	Q_c	D.F.	Q_c	D.F.	Q_c	D.F.	Q_c
Sex	1	9.83	2	9.11	2	19.00	4	14.83
Criticism	1	4.12	3	30.12	3	44.10	1	9.38
Model	2	15.14	5	45.81	5	67.87	5	23.90
Residual	16	15.20	13	13.47	13	20.38	13	17.94

TABLE 12

PREDICTED MEAN RANK PREFERENCE PROFILES AND CORRESPONDING STANDARD ERRORS BASED ON $\{X_{2h}\}$ MODELS

Sub-population*				Policy Alternative						
				ED	PL	TR	PV	FA	GI	HC
1.	Con	Male	No	2.17 (0.07)	3.61 (0.08)	3.20 (0.11)	4.97 (0.07)	6.22 (0.07)	4.59 (0.09)	3.23 (0.08)
2.	Con	Male	Yes	2.17 (0.07)	3.61 (0.08)	3.00 (0.12)	4.97 (0.07)	6.43 (0.08)	4.59 (0.09)	3.23 (0.08)
3.	Con	Female	No	2.17 (0.07)	3.61 (0.08)	2.84 (0.12)	4.97 (0.07)	6.22 (0.07)	4.59 (0.09)	3.59 (0.09)
4.	Con	Female	Yes	2.17 (0.07)	3.61 (0.08)	2.64 (0.13)	4.97 (0.07)	6.43 (0.08)	4.59 (0.09)	3.59 (0.09)
5.	Lib	Male	No	2.36 (0.10)	4.20 (0.15)	3.90 (0.20)	4.13 (0.17)	6.19 (0.08)	3.95 (0.13)	3.28 (0.10)
6.	Lib	Male	Yes	2.36 (0.10)	3.48 (0.19)	3.18 (0.25)	5.20 (0.18)	6.55 (0.13)	3.95 (0.13)	3.28 (0.10)
7.	Lib	Female	No	2.36 (0.10)	3.84 (0.17)	4.56 (0.20)	3.83 (0.17)	6.19 (0.08)	3.95 (0.13)	3.28 (0.10)
8.	Lib	Female	Yes	2.36 (0.10)	3.12 (0.21)	3.85 (0.28)	4.90 (0.23)	6.55 (0.13)	3.95 (0.13)	3.28 (0.10)
9.	Btwn	Male	No	2.22 (0.07)	3.75 (0.09)	3.73 (0.12)	4.64 (0.08)	5.98 (0.08)	4.44 (0.09)	3.24 (0.08)
10.	Btwn	Male	Yes	2.22 (0.07)	3.20 (0.11)	3.43 (0.16)	5.09 (0.12)	6.38 (0.08)	4.44 (0.09)	3.24 (0.08)
11.	Btwn	Female	No	2.22 (0.07)	3.75 (0.09)	3.23 (0.11)	4.64 (0.08)	6.22 (0.07)	4.44 (0.09)	3.49 (0.08)
12.	Btwn	Female	Yes	2.22 (0.07)	3.20 (0.11)	2.93 (0.15)	5.09 (0.12)	6.63 (0.07)	4.44 (0.09)	3.49 (0.08)
13.	None	Male	No	2.73 (0.16)	4.27 (0.09)	3.47 (0.16)	4.61 (0.12)	6.19 (0.06)	3.82 (0.16)	2.92 (0.11)
14.	None	Male	Yes	2.73 (0.16)	4.27 (0.09)	3.12 (0.18)	4.61 (0.12)	6.54 (0.10)	3.82 (0.16)	2.92 (0.11)
15.	None	Female	No	2.29 (0.10)	4.27 (0.09)	3.01 (0.12)	4.88 (0.10)	6.19 (0.06)	4.16 (0.12)	3.20 (0.08)
16.	None	Female	Yes	2.29 (0.10)	4.27 (0.09)	2.66 (0.14)	4.88 (0.10)	6.54 (0.10)	4.16 (0.12)	3.20 (0.08)

* The sub-populations are defined by ideology (conservative, liberal, in between, none), sex (male, female), and expressed criticism of taxes (no, yes).

Thus, in this sub-section, the models for each ideology will be simplified even further and then linked together in a final overall model. However, although the steps which are involved in these analyses are intuitively reasonable, they are also based on relatively complex statistical and substantive judgments. For this reason, the results which are obtained should be interpreted more in the spirit of descriptive as opposed to inferential statistics.

The underlying principle which motivates the formulation of third generation models $\{\underset{\sim}{X}_{3h}\}$ for each ideology group is the fact that the mean rank preference vectors $\{\bar{\underset{\sim}{R}}_i\}$ satisfy restrictions like (2.3). Thus, when they are analyzed in terms of multivariate models like the $\{\underset{\sim}{X}_{1h}\}$ and $\{\underset{\sim}{X}_{2h}\}$, positive sex or criticism effects for certain tax alternatives are counter-balanced by corresponding negative effects for other tax alternatives. Since these increments and decrements reflect the extent to which such effects vary across the preference profile, it is of interest to test hypotheses which clarify their relationship to one another. In this regard, one appropriate strategy is to identify the extent to which the various increments and decrements are either equal to each other or are multiples of a common baseline. On the basis of these considerations, the $\{\underset{\sim}{X}_{2h}\}$ models were refined to the $\{\underset{\sim}{X}_{3h}\}$ models given in Table 13 by verifying that the parameters for the $\{\underset{\sim}{X}_{2h}\}$ models in Table 8 satisfy the hypotheses (3.17) - (3.20) in the sense of having non-significant (α = .10)

$$\xi_{2,HC,1} = \gamma_{2,FA,1} = (-\xi_{2,TR,1} = -\gamma_{2,TR,1}) \qquad (3.17)$$

$$-\xi_{2,PL,2} = -\xi_{2,PV,2} = -\frac{1}{2}\gamma_{2,PL,2} = \frac{1}{3}\gamma_{2,PV,2} = \gamma_{2,FA,2}$$
$$= (\frac{1}{2}\xi_{2,TR,2} = -\frac{1}{2}\gamma_{2,TR,2}) \qquad (3.18)$$

$$\xi_{2,FA,3} = \xi_{2,HC,3} = -\gamma_{2,PL,3} = \gamma_{2,PV,3} = \gamma_{2,FA,3}$$
$$= (-\frac{1}{2}\,\xi_{2,TR,3} = -\gamma_{2,TR,3}) \qquad (3.19)$$

$$-\xi_{2,ED,4} = \xi_{2,PV,4} = \xi_{2,GI,4} = \xi_{2,HC,4} = \gamma_{2,FA,4}$$
$$= (-\frac{1}{2}\,\xi_{2,TR,4} = -\gamma_{2,TR,4}) \qquad (3.20)$$

Q_C test statistics both individually and simultaneously. Otherwise, it should be noted that the parameters for "tax reduction (TR)" appear in parentheses in (3.17) - (3.20) because it is the tax alternative which is deleted in the definition of the centered reduced score vectors $\{\tilde{S}_i\}$. Estimated parameters $\{\tilde{b}_{3h}\}$ for the $\{\tilde{X}_{3h}\}$ models are also given in Table 13 together with test statistics for the synthesized sex and criticism effects (i.e., Q_C for H_0: $\beta_{37h} = 0$) and for goodness of fit of the model. For the sake of completeness, predicted values based on the $\{\tilde{X}_{3h}\}$ models and corresponding standard errors are given in Table 14.

The $\{\tilde{X}_{3h}\}$ models for the separate ideology groups can be linked together by formulating models for their respective parameter vectors $\{\tilde{\beta}_{3h}\}$. This is accomplished by verifying that the corresponding estimators $\{\tilde{b}_{3h}\}$ satisfy the hypotheses (3.21) - (3.27) in the sense of having non-sig-

$$\beta_{311} = \beta_{312} = \beta_{313} = (\beta_{314} - \beta_{371}) \qquad (3.21)$$

$$\beta_{321} = (\beta_{322} - 2\beta_{371}) = \beta_{323} = (\beta_{324} - 2\beta_{371}) \qquad (3.22)$$

$$\beta_{331} = (\beta_{332} + 2\beta_{371}) = (\beta_{333} + \beta_{371}) = (\beta_{334} + \beta_{371}) \qquad (3.23)$$

$$\beta_{341} = \beta_{342} = (\beta_{343} + \beta_{371}) = \beta_{344} \qquad (3.24)$$

$$\beta_{351} = (\beta_{352} + 2\beta_{371}) = \beta_{353} = (\beta_{354} + 2\beta_{371}) \qquad (3.25)$$

TABLE 13

THIRD GENERATION MODELS $\{\underset{\sim}{X}_{3h}\}$, ESTIMATED PARAMETERS, AND TEST STATISTICS

Sex	Criticism	Policy Alternative	Ideology: Conservative	Ideology: Liberal	Ideology: In-Between	Ideology: None
Male	No	ED	1 0 0 0 0 0 0	1 0 0 0 0 0 0	1 0 0 0 0 0 0	1 0 0 0 0 0 0
Male	No	PL	0 1 0 0 0 0 0	0 1 0 0 0 0 0	0 1 0 0 0 0 0	0 1 0 0 0 0 0
Male	No	PV	0 0 1 0 0 0 0	0 0 1 0 0 0 0	0 0 1 0 0 0 0	0 0 1 0 0 0 0
Male	No	FA	0 0 0 1 0 0 0	0 0 0 1 0 0 0	0 0 0 1 0 0 0	0 0 0 1 0 0 0
Male	No	GI	0 0 0 0 1 0 0	0 0 0 0 1 0 0	0 0 0 0 1 0 0	0 0 0 0 1 0 0
Male	No	HC	0 0 0 0 0 1 0	0 0 0 0 0 1 0	0 0 0 0 0 1 0	0 0 0 0 0 1 0
Male	Yes	ED	1 0 0 0 0 0 0	1 0 0 0 0 0 0	1 0 0 0 0 0 0	1 0 0 0 0 0 0
Male	Yes	PL	0 1 0 0 0 0 0	0 1 0 0 0 0 −2	0 1 0 0 0 0 −1	0 1 0 0 0 0 0
Male	Yes	PV	0 0 1 0 0 0 0	0 0 1 0 0 0 3	0 0 1 0 0 0 1	0 0 1 0 0 0 0
Male	Yes	FA	0 0 0 1 0 0 1	0 0 0 1 0 0 1	0 0 0 1 0 0 1	0 0 0 1 0 0 1
Male	Yes	GI	0 0 0 0 1 0 0	0 0 0 0 1 0 0	0 0 0 0 1 0 0	0 0 0 0 1 0 0
Male	Yes	HC	0 0 0 0 0 1 0	0 0 0 0 0 1 0	0 0 0 0 0 1 0	0 0 0 0 0 1 0
Female	No	ED	1 0 0 0 0 0 0	1 0 0 0 0 0 0	1 0 0 0 0 0 0	1 0 0 0 0 0 −1
Female	No	PL	0 1 0 0 0 0 0	0 1 0 0 0 0 −1	0 1 0 0 0 0 0	0 1 0 0 0 0 0
Female	No	PV	0 0 1 0 0 0 0	0 0 1 0 0 0 −1	0 0 1 0 0 0 0	0 0 1 0 0 0 1
Female	No	FA	0 0 0 1 0 0 0	0 0 0 1 0 0 0	0 0 0 1 0 0 1	0 0 0 1 0 0 0
Female	No	GI	0 0 0 0 1 0 0	0 0 0 0 1 0 0	0 0 0 0 1 0 0	0 0 0 0 1 0 1
Female	No	HC	0 0 0 0 0 1 1	0 0 0 0 0 1 0	0 0 0 0 0 1 1	0 0 0 0 0 1 1
Female	Yes	ED	1 0 0 0 0 0 0	1 0 0 0 0 0 0	1 0 0 0 0 0 0	1 0 0 0 0 0 −1
Female	Yes	PL	0 1 0 0 0 0 0	0 1 0 0 0 0 −3	0 1 0 0 0 0 −1	0 1 0 0 0 0 0
Female	Yes	PV	0 0 1 0 0 0 0	0 0 1 0 0 0 2	0 0 1 0 0 0 1	0 0 1 0 0 0 1
Female	Yes	FA	0 0 0 1 0 0 1	0 0 0 1 0 0 1	0 0 0 1 0 0 2	0 0 0 1 0 0 1
Female	Yes	GI	0 0 0 0 1 0 0	0 0 0 0 1 0 0	0 0 0 0 1 0 0	0 0 0 0 1 0 1
Female	Yes	HC	0 0 0 0 0 1 1	0 0 0 0 0 1 0	0 0 0 0 0 1 1	0 0 0 0 0 1 1
Estimate	b_{31h}		−1.83	−1.64	−1.78	−1.37
Estimate	b_{32h}		−0.39	0.19	−0.33	0.26
Estimate	b_{33h}		0.97	0.16	0.69	0.60
Estimate	b_{34h}		2.20	2.19	1.96	2.20
Estimate	b_{35h}		0.59	−0.05	0.44	−0.14
Estimate	b_{36h}		−0.73	−0.72	−0.80	−1.08
Estimate	b_{37h}		0.27	0.35	0.33	0.30
Model Q_C (H_0: β_{37h} = 0, D.F. = 1)			14.12	45.76	63.02	22.79
Residual Q (D.F.=17)			16.22	13.53	25.23	19.05

TABLE 14

PREDICTED MEAN RANK PREFERENCE PROFILES AND CORRESPONDING STANDARD ERRORS BASED ON $\{X_{3h}\}$ MODELS

Sub-population*				Policy Alternative						
				ED	PL	TR	PV	FA	GI	HC
1.	Con	Male	No	2.17 (0.07)	3.61 (0.08)	3.19 (0.11)	4.97 (0.07)	6.20 (0.06)	4.59 (0.09)	3.27 (0.07)
2.	Con	Male	Yes	2.17 (0.07)	3.61 (0.08)	2.92 (0.09)	4.97 (0.07)	6.47 (0.07)	4.59 (0.09)	3.27 (0.07)
3.	Con	Female	No	2.17 (0.07)	3.61 (0.08)	2.92 (0.09)	4.97 (0.07)	6.20 (0.06)	4.59 (0.09)	3.54 (0.08)
4.	Con	Female	Yes	2.17 (0.07)	3.61 (0.08)	2.65 (0.13)	4.97 (0.07)	6.47 (0.07)	4.59 (0.09)	3.54 (0.08)
5.	Lib	Male	No	2.36 (0.10)	4.19 (0.12)	3.88 (0.14)	4.16 (0.11)	6.19 (0.07)	3.95 (0.13)	3.28 (0.10)
6.	Lib	Male	Yes	2.36 (0.10)	3.49 (0.12)	3.18 (0.17)	5.21 (0.16)	6.54 (0.08)	3.94 (0.13)	3.28 (0.10)
7.	Lib	Female	No	2.36 (0.10)	3.84 (0.11)	4.58 (0.17)	3.81 (0.14)	6.19 (0.07)	3.95 (0.13)	3.28 (0.10)
8.	Lib	Female	Yes	2.36 (0.10)	3.14 (0.14)	3.88 (0.14)	4.86 (0.13)	6.54 (0.08)	3.95 (0.13)	3.28 (0.10)
9.	Btwn	Male	No	2.22 (0.07)	3.67 (0.07)	3.83 (0.11)	4.69 (0.07)	5.96 (0.06)	4.44 (0.09)	3.20 (0.06)
10.	Btwn	Male	Yes	2.22 (0.07)	3.34 (0.08)	3.50 (0.09)	5.02 (0.07)	6.29 (0.04)	4.44 (0.09)	3.20 (0.06)
11.	Btwn	Female	No	2.22 (0.07)	3.67 (0.07)	3.16 (0.09)	4.69 (0.07)	6.29 (0.04)	4.44 (0.09)	3.53 (0.06)
12.	Btwn	Female	Yes	2.22 (0.07)	3.34 (0.08)	2.83 (0.10)	5.02 (0.07)	6.62 (0.06)	4.44 (0.09)	3.53 (0.06)
13.	None	Male	No	2.63 (0.10)	4.26 (0.08)	3.54 (0.14)	4.60 (0.09)	6.20 (0.06)	3.86 (0.11)	2.92 (0.07)
14.	None	Male	Yes	2.63 (0.10)	4.26 (0.08)	3.24 (0.10)	4.60 (0.09)	6.50 (0.07)	3.86 (0.11)	2.92 (0.07)
15.	None	Female	No	2.34 (0.09)	4.26 (0.08)	2.95 (0.10)	4.90 (0.08)	6.20 (0.06)	4.15 (0.10)	3.21 (0.07)
16.	None	Female	Yes	2.34 (0.09)	4.26 (0.08)	2.65 (0.13)	4.90 (0.08)	6.50 (0.07)	4.15 (0.10)	3.21 (0.07)

* The sub-populations are defined by ideology (conservative, liberal, in between, none), sex (male, female), and expressed criticism of taxes (no, yes).

$$\beta_{361} = \beta_{362} = \beta_{363} = (\beta_{364} + \beta_{371}) \tag{3.26}$$

$$\beta_{371} = \beta_{372} = \beta_{373} = \beta_{374} \tag{3.27}$$

nificant (α = .10) Q_C test statistics both individually and simultaneously. The specific model $\underset{\sim}{X}_{F1}$ for the $\{\underset{\sim}{b}_{3h}\}$ which evolves from these considerations is given in Table 15 together with its estimated parameters $\underset{\sim}{b}_{F1}$. These estimators are obtained by applying the GSK methodology to the combined vector $\underset{\sim}{b}_3$ and its estimated covariance matrix $\underset{\sim}{V}_{b3}$ as shown in (3.28).

$$\underset{28x1}{\underset{\sim}{b}_3} = \begin{bmatrix} \underset{\sim}{b}_{31} \\ \underset{\sim}{b}_{32} \\ \underset{\sim}{b}_{33} \\ \underset{\sim}{b}_{34} \end{bmatrix} , \quad \underset{28x28}{\underset{\sim}{V}_{b3}} = \begin{bmatrix} \underset{\sim}{V}_{b_{31}} & \underset{\sim}{0}_{77} & \underset{\sim}{0}_{77} & \underset{\sim}{0}_{77} \\ \underset{\sim}{0}_{77} & \underset{\sim}{V}_{b_{32}} & \underset{\sim}{0}_{77} & \underset{\sim}{0}_{77} \\ \underset{\sim}{0}_{77} & \underset{\sim}{0}_{77} & \underset{\sim}{V}_{b_{33}} & \underset{\sim}{0}_{77} \\ \underset{\sim}{0}_{77} & \underset{\sim}{0}_{77} & \underset{\sim}{0}_{77} & \underset{\sim}{V}_{b_{34}} \end{bmatrix} \tag{3.28}$$

When this is done, the resulting goodness of fit Q statistic (2.19) is <u>identical</u> to the Q_C statistic (2.20) for testing the hypotheses (3.21) - (3.27) simultaneously for the $\{\underset{\sim}{b}_{3h}\}$ in the context of the $\{\underset{\sim}{X}_{3h}\}$ models. As indicated in Table 15, Q = 12.62 with D.F. = 21 is non-significant (α = .10) and thus supports the validity of the model $\underset{\sim}{X}_{F1}$ for $\underset{\sim}{b}_3$. However, the model $\underset{\sim}{X}_{F1}$ for $\underset{\sim}{b}_3$ in combination with the $\{\underset{\sim}{X}_{3h}\}$ is isomorphic to the model $\underset{\sim}{X}_4$ for the $\{\underset{\sim}{S}_i\}$ shown in (3.29) in the sense of reflecting the same relationships and

$$\underset{(96x7)}{\underset{\sim}{X}_4} = \begin{bmatrix} \underset{\sim}{X}_{31} & \underset{\sim}{0}_{24,7} & \underset{\sim}{0}_{24,7} & \underset{\sim}{0}_{24,7} \\ \underset{\sim}{0}_{24,7} & \underset{\sim}{X}_{32} & \underset{\sim}{0}_{24,7} & \underset{\sim}{0}_{24,7} \\ \underset{\sim}{0}_{24,7} & \underset{\sim}{0}_{24,7} & \underset{\sim}{X}_{33} & \underset{\sim}{0}_{24,7} \\ \underset{\sim}{0}_{24,7} & \underset{\sim}{0}_{24,7} & \underset{\sim}{0}_{24,7} & \underset{\sim}{X}_{34} \end{bmatrix} \cdot \underset{(28x7)}{\underset{\sim}{X}_{F1}} \tag{3.29}$$

having the same estimated parameters. In addition, the appropriate goodness of fit statistic for the model $\underset{\sim}{X}_4$ is obtained by simply adding the residual Q for $\underset{\sim}{X}_{F1}$ in Table 15 to the sum of the residual Q's for the

TABLE 15

COMBINED MODELS FOR THIRD GENERATION MODEL PARAMETERS $\{\underset{\sim}{\beta}_{3h}\}$

Ideology	Parameter	$\underset{\sim}{X}_{F1}$ Model	$\underset{\sim}{X}_{F2}$ Model
Conservative	β_{311}	1 0 0 0 0 0 0	1 -3
Conservative	β_{321}	0 1 0 0 0 0 0	1 1
Conservative	β_{331}	0 0 1 0 0 0 0	1 5
Conservative	β_{341}	0 0 0 1 0 0 0	-6 -7
Conservative	β_{351}	0 0 0 0 1 0 0	1 4
Conservative	β_{361}	0 0 0 0 0 1 0	1 0
Conservative	β_{371}	0 0 0 0 0 0 1	0 1
Liberal	β_{312}	1 0 0 0 0 0 0	1 -3
Liberal	β_{322}	0 1 0 0 0 0 2	1 3
Liberal	β_{332}	0 0 1 0 0 0-2	1 3
Liberal	β_{342}	0 0 0 1 0 0 0	-6 -7
Liberal	β_{352}	0 0 0 0 1 0-2	1 2
Liberal	β_{362}	0 0 0 0 0 1 0	1 0
Liberal	β_{372}	0 0 0 0 0 0 1	0 1
In-Between	β_{313}	1 0 0 0 0 0 0	1 -3
In-Between	β_{323}	0 1 0 0 0 0 0	1 1
In-Between	β_{333}	0 0 1 0 0 0-1	1 4
In-Between	β_{343}	0 0 0 1 0 0-1	-6 -8
In-Between	β_{353}	0 0 0 0 1 0 0	1 4
In-Between	β_{363}	0 0 0 0 0 1 0	1 0
In-Between	β_{373}	0 0 0 0 0 0 1	0 1
None	β_{314}	1 0 0 0 0 0 1	1 -2
None	β_{324}	0 1 0 0 0 0 2	1 3
None	β_{334}	0 0 1 0 0 0-1	1 4
None	β_{344}	0 0 0 1 0 0 0	-6 -7
None	β_{354}	0 0 0 0 1 0-2	1 2
None	β_{364}	0 0 0 0 0 1-1	1 -1
None	β_{374}	0 0 0 0 0 0 1	0 1
Estimated parameters for fitted model		$b_{F11} = -1.75$ $b_{F12} = -0.38$ $b_{F13} = 0.94$ $b_{F14} = 2.23$ $b_{F15} = 0.52$ $b_{F16} = -0.76$ $b_{F17} = 0.31$	$b_{F21} = -0.76$ $b_{F22} = 0.34$
Residual goodness of fit statistic		Q(D.F.=21) = 12.62	Q(D.F.=26) = 15.99

$\{\underset{\sim}{X}_{3h}\}$ in Table 13. Since the resulting Q = 86.65 with D.F. = 89 is non-significant (α = .10), it follows that $\underset{\sim}{X}_4$ provides a satisfactory summary characteriztion of the $\{\underset{\sim}{S}_i\}$. With this fact in mind, it is of interest to test the following two hypotheses within the $\underset{\sim}{X}_4$ framework:

1. H_0: $\beta_{F17} = 0$ which is equivalent to the hypothesis of no differences among the multivariate mean rank response profiles for the 16 ideology x sex x criticism sub-populations in the sense of the hypothesis H_{RPS} analogous to (2.6).
2. H_0: $\beta_{F11} = \beta_{F12} = \beta_{F13} = \beta_{F14} = \beta_{F15} = \beta_{F16} = \beta_{F17} = 0$ which is equivalent to the hypothesis of no differences among the mean preference ranks for the respective tax alternatives within each of the 16 ideology x sex x criticism sub-populations in the sense of simultaneous hypotheses $\{H_{RPi}\}$ analogous to (2.4).

For the first of these hypotheses, Q_C = 240.13 with D.F. = 1; and Q_C = 12,277.24 with D.F. = 7 for the other. These quantities may be converted to heuristic weighted least squares measures of the extent to which the model $\underset{\sim}{X}_4$ explains the

1. variation among the mean rank profiles across sub-populations,
2. variation among the mean ranks in an overall sense, both within and across sub-populations

respectively by computing their percentage of corresponding total variation statistics which are obtained by adding the goodness of fit residual Q statistic. Thus, in this context, it can be said that the model $\underset{\sim}{X}_4$ accounts for 73.5 % of the total Q_{RPS} = 326.78 with D.F. = 90 for variation among mean rank profiles across sub-populations and 99.3% of the total Q_{RP} = 12,363.89 (which is also equal to the sum of the $\{Q_{C,RPi}\}$ in Table 3) with D.F. = 96 for variation among the mean ranks in an overall sense.

Finally, as was the case with the previous models, predicted values based on the $\underset{\sim}{X}_4$ model can be generated for the mean rank preference profiles across the 16 sub-populations. Although they are not specifically given in this paper, it can be verified that these predicted values $\{\hat{\underset{\sim}{R}}_{4ig}\}$ satisfy the simultaneous hypotheses (3.30) - (3.34) in the sense that the

$$E\{\hat{R}_{412}\} = 4 + \beta_{F12} = \beta_{F16} + \beta_{F17} + 4 = E\{\hat{R}_{437}\} \tag{3.30}$$

$$\begin{aligned} E\{\hat{R}_{413}\} &= 4 - \beta_{F11} - \beta_{F12} - \beta_{F13} - \beta_{F14} - \beta_{F15} - \beta_{F16} \\ &= \beta_{F16} + 4 = E\{\hat{R}_{417}\} \end{aligned} \tag{3.31}$$

$$\begin{aligned} E\{\hat{R}_{443}\} &= 4 - \beta_{F11} - \beta_{F12} - \beta_{F13} - \beta_{F14} - \beta_{F15} - \beta_{F16} - 2\ \beta_{F17} \\ &= \beta_{F11} + \beta_{F17} + 4 = E\{\hat{R}_{4,13,1}\} \end{aligned} \tag{3.32}$$

$$E\{\hat{R}_{452}\} = 4 + \beta_{F12} + 2\ \beta_{F17} = \beta_{F13} - 2\ \beta_{F17} + 4 = E\{\hat{R}_{454}\} \tag{3.33}$$

$$E\{\hat{R}_{494}\} = 4 + \beta_{F13} - \beta_{F17} = \beta_{F15} + 4 = E\{\hat{R}_{416}\} \tag{3.34}$$

corresponding test statistic $Q_C = 3.37$ with D.F. = 5 is non-significant ($\alpha = .10$). Thus, the model $\underset{\sim}{X}_{F1}$ for $\underset{\sim}{b}_3$ (or equivalently $\underset{\sim}{X}_4$) can be refined to the final model $\underset{\sim}{X}_{F2}$ given in Table 15 by noting that the equations (3.30) - (3.34) can be rewritten as shown in (3.35) - (3.39). As was the

$$\beta_{F11} = \beta_{F16} - 3\ \beta_{F17} \tag{3.35}$$

$$\beta_{F12} = \beta_{F16} + \beta_{F17} \tag{3.36}$$

$$\beta_{F13} = \beta_{F16} + 5\ \beta_{F17} \tag{3.37}$$

$$\beta_{F14} = -6\ \beta_{F16} - 7\ \beta_{F17} \tag{3.38}$$

$$\beta_{F15} = \beta_{F16} + 4\ \beta_{F17} \tag{3.39}$$

case with $\underset{\sim}{X}_{F1}$, estimated parameters $\underset{\sim}{b}_{F2}$ for the model $\underset{\sim}{X}_{F2}$ can be obtained by applying the GSK methodology to $\underset{\sim}{b}_3$ and $\underset{\sim}{V}_{b3}$ in (3.28). These results are also given in Table 15 together with the appropriate residual goodness of fit statistic Q = 15.99 with D.F. = 26 which supports the validity of the $\underset{\sim}{X}_{F2}$ model for $\underset{\sim}{b}_3$ by its non-significance (α = .10). Moreover, the model $\underset{\sim}{X}_{F2}$ in combination with the $\{\underset{\sim}{X}_{3h}\}$ is isomorphic to the model $\underset{\sim}{X}_5$ for the $\{\underset{\sim}{S}_i\}$ shown in (3.40) for which the corresponding residual Q = 90.02

$$\underset{96\times 2}{\underset{\sim}{X}_5} = \begin{bmatrix} \underset{\sim}{X}_{31} & \underset{\sim}{0}_{24,7} & \underset{\sim}{0}_{24,7} & \underset{\sim}{0}_{24,7} \\ \underset{\sim}{0}_{24,7} & \underset{\sim}{X}_{32} & \underset{\sim}{0}_{24,7} & \underset{\sim}{0}_{24,7} \\ \underset{\sim}{0}_{24,7} & \underset{\sim}{0}_{24,7} & \underset{\sim}{X}_{33} & \underset{\sim}{0}_{24,7} \\ \underset{\sim}{0}_{24,7} & \underset{\sim}{0}_{24,7} & \underset{\sim}{0}_{24,7} & \underset{\sim}{X}_{34} \end{bmatrix} \cdot \underset{\sim}{X}_{F2} \qquad (3.40)$$

with D.F. = 94 is similarly non-significant (α = .10). Thus, it follows that $\underset{\sim}{X}_5$ represents a reasonable final overall model for characterizing the variation among the mean ranks $\{ \bar{R}_{ig} \}$ in a complete sense, both within and across sub-populations. In this regard, the model $\underset{\sim}{X}_5$ accounts for 99.3% of such variation since Q_C = 12,273.87 with D.F. = 2 for the hypothesis H_0: $\beta_{F21} = \beta_{F22} = 0$ which is equivalent to the hypothesis of simultaneous indifference in all sub-populations. Predicted values based on the overall final model $\underset{\sim}{X}_5$ in (3.40) are given in Table 16 for the mean rank preference profiles across the 16 sub-populations together with corresponding standard errors. From a descriptive point of view, these final model predicted values are of considerable practical interest because differences among them reflect, for the most part, significant (α = .05) differences among the corresponding observed mean ranks $\{ \bar{R}_{ig} \}$. Thus, they provide an operational basis for the formulation of conclusions regarding the effects of ideology, sex, and criticism on the rank preference profile for the seven tax alternatives. In particular, the respondents in this survey tend to order the tax alternatives in a manner consistent with the extent to which the immediacy of their direct benefits are personally perceived. Thus, in each of the 16 sub-populations, education is the most preferred tax alternative in the sense of having the smallest predicted mean ranks which range from 2.23 to 2.57.

TABLE 16

PREDICTED MEAN RANK PREFERENCE PROFILES AND CORRESPONDING STANDARD ERRORS BASED ON FINAL OVERALL $\underset{\sim}{X}_5$ MODEL

Sub-population*			Policy Alternative						
			ED	PL	TR	PV	FA	GI	HC
Con	Male	No	2.23 (0.03)	3.57 (0.01)	3.24 (0.01)	4.92 (0.03)	6.22 (0.03)	4.58 (0.02)	3.24 (0.01)
Con	Male	Yes	2.23 (0.03)	3.57 (0.01)	2.90 (0.01)	4.92 (0.03)	6.55 (0.03)	4.58 (0.02)	3.24 (0.01)
Con	Female	No	2.23 (0.03)	3.57 (0.01)	2.90 (0.01)	4.92 (0.03)	6.22 (0.03)	4.58 (0.02)	3.57 (0.01)
Con	Female	Yes	2.23 (0.03)	3.57 (0.01)	2.57 (0.02)	4.92 (0.03)	6.55 (0.03)	4.58 (0.02)	3.57 (0.01)
Lib	Male	No	2.23 (0.03)	4.25 (0.01)	3.91 (0.01)	4.25 (0.01)	6.22 (0.03)	3.91 (0.01)	3.24 (0.01)
Lib	Male	Yes	2.23 (0.03)	3.57 (0.01)	3.24 (0.01)	5.25 (0.03)	6.55 (0.03)	3.91 (0.01)	3.24 (0.01)
Lib	Female	No	2.23 (0.03)	3.91 (0.01)	4.58 (0.02)	3.91 (0.01)	6.22 (0.03)	3.91 (0.01)	3.24 (0.01)
Lib	Female	Yes	2.23 (0.03)	3.24 (0.01)	3.91 (0.01)	4.92 (0.03)	6.55 (0.03)	3.91 (0.01)	3.24 (0.01)
Btwn	Male	No	2.23 (0.03)	3.57 (0.01)	3.91 (0.01)	4.58 (0.02)	5.88 (0.03)	4.58 (0.02)	3.24 (0.01)
Btwn	Male	Yes	2.23 (0.03)	3.24 (0.01)	3.57 (0.01)	4.92 (0.03)	6.22 (0.03)	4.58 (0.02)	3.24 (0.01)
Btwn	Female	No	2.23 (0.03)	3.57 (0.01)	3.24 (0.01)	4.58 (0.02)	6.22 (0.03)	4.58 (0.02)	3.57 (0.01)
Btwn	Female	Yes	2.23 (0.03)	3.24 (0.01)	2.90 (0.01)	4.92 (0.03)	6.55 (0.03)	4.58 (0.02)	3.57 (0.01)
None	Male	No	2.57 (0.02)	4.25 (0.01)	3.57 (0.01)	4.58 (0.02)	6.22 (0.03)	3.91 (0.01)	2.90 (0.01)
None	Male	Yes	2.57 (0.02)	4.25 (0.01)	3.24 (0.01)	4.58 (0.02)	6.55 (0.03)	3.91 (0.01)	2.90 (0.01)
None	Female	No	2.23 (0.03)	4.25 (0.01)	2.90 (0.01)	4.92 (0.03)	6.22 (0.03)	4.25 (0.01)	3.24 (0.01)
None	Female	Yes	2.23 (0.03)	4.25 (0.01)	2.57 (0.02)	4.92 (0.03)	6.55 (0.03)	4.25 (0.01)	3.24 (0.01)

* The sub-populations are defined by ideology (conservative, liberal, in between, none), sex (male, female), and expressed criticism of taxes (no, yes).

Depending on ideology, sex, and criticism, Pollution, Tax Reduction, and Health Care compete for second place with predicted mean ranks ranging from 2.57 to 3.24. For example, the "Conservative Ideology" group and "Females with In-Between or No Ideology" have the second most preference for Tax Reduction while the "Liberal Ideology" group and "Males with In-Between or No Ideology" have the second most preference for Health Care. On the other hand, Foreign Aid is the least preferred tax alternative with the largest predicted mean ranks which range from 5.88 to 6.55 followed by Anti-Poverty Programs for which the predicted mean ranks range from 3.91 to 5.25. Finally, the nature of the intermediate preferences within the respective sub-populations can be interpreted as reflecting general indifference since the corresponding predicted mean ranks range from 3.57 to 4.58.

In summary, the rank policy preference data from SERS-I have been analyzed from a comprehensive point of view. Initially, statistical tests were undertaken to verify the existence of differences in preference for seven tax alternatives both within as well as across the 16 ideology x sex x criticism sub-populations. These differences were then subjected to further study through the analysis of a series of linear regression models. This methodological strategy ultimately led to a final set of predicted values which could be used as a descriptive basis for the formulation of specific conclusions about the relationships between the rank preference profile and the respective sub-populations.

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APPENDIX

TABLE A-1

Males With Conservative Ideology Who Expressed No Tax Criticism ($N_1 = 155$)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.1816	3.5966	3.0109	5.1105	6.1992	4.6499	3.2512
Variance-Covariance Matrix	0.0157	0.0001	-0.0058	0.0007	-0.0007	-0.0083	-0.0017
	0.0001	0.0178	-0.0013	-0.0037	-0.0011	-0.0068	-0.0048
	-0.0058	-0.0013	0.0214	-0.0063	-0.0001	-0.0034	-0.0042
	0.0007	-0.0037	-0.0063	0.0134	-0.0010	-0.0016	-0.0013
	-0.0007	-0.0011	-0.0001	-0.0010	0.0082	-0.0032	-0.0018
	-0.0083	-0.0068	-0.0034	-0.0016	-0.0032	0.0199	0.0036
	-0.0017	-0.0048	-0.0042	-0.0013	-0.0018	0.0036	0.0105

TABLE A-2

Males With Conservative Ideology Who Expressed Tax Criticism ($N_1 = 63$)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.2524	3.5581	3.2536	4.7733	6.3683	4.6467	3.1476
Variance-Covariance Matrix	0.0266	0.0028	-0.0150	-0.0054	0.0074	-0.0094	-0.0069
	0.0028	0.0461	-0.0110	-0.0052	0.0042	-0.0198	-0.0171
	-0.0150	-0.0110	0.0592	-0.0140	-0.0053	-0.0074	-0.0063
	-0.0054	-0.0052	-0.0140	0.0368	-0.0100	0.0010	-0.0031
	0.0074	0.0042	-0.0053	-0.0100	0.0222	-0.0143	-0.0042
	-0.0094	-0.0198	-0.0074	0.0010	-0.0143	0.0464	0.0036
	-0.0069	-0.0171	-0.0063	-0.0031	-0.0042	0.0036	0.0341

TABLE A-3

Females With Conservative Ideology Who Expressed No Tax Criticism ($N_1 = 121$)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.0944	3.4947	3.0528	4.9129	6.2066	4.6758	3.5628
Variance-Covariance Matrix	0.0152	-0.0012	-0.0059	0.0015	-0.0002	-0.0088	-0.0005
	-0.0012	0.0247	0.0005	-0.0038	-0.0001	-0.0131	-0.0069
	-0.0059	0.0005	0.0311	-0.0093	-0.0040	-0.0061	-0.0061
	0.0015	-0.0038	-0.0093	0.0159	-0.0011	-0.0013	-0.0019
	-0.0002	-0.0001	-0.0040	-0.0011	0.0114	-0.0044	-0.0013
	-0.0088	-0.0131	-0.0061	-0.0013	-0.0044	0.0294	0.0044
	-0.0005	-0.0069	-0.0061	-0.0019	-0.0013	0.0044	0.0125

TABLE A-4

Females With Conservative Ideology Who Expressed Tax Criticism ($N_1 = 46$)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.4984	4.0449	2.3945	4.7369	6.5569	3.9652	3.8032
Variance-Covariance Matrix	0.0482	0.0154	-0.0202	-0.0169	0.0079	-0.0416	0.0071
	0.0154	0.0663	0.0054	-0.0222	0.0016	-0.0489	-0.0177
	-0.0202	0.0054	0.0479	-0.0008	-0.0036	-0.0004	-0.0281
	-0.0169	-0.0222	-0.0008	0.0436	-0.0030	0.0109	-0.0114
	0.0079	0.0016	-0.0036	-0.0030	0.0152	-0.0129	-0.0052
	-0.0416	-0.0489	-0.0004	0.0109	-0.0129	0.0863	0.0066
	0.0071	-0.0177	-0.0281	-0.0114	-0.0052	0.0066	0.0487

TABLE A-5

Males With Liberal Ideology Who Expressed No Tax Criticism (N_1 = 84)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.3855	4.1743	3.7608	4.1796	6.2644	4.1105	3.1249
	0.0331	-0.0028	-0.0041	-0.0040	0.0011	-0.0185	-0.0046
	-0.0028	0.0279	-0.0021	-0.0073	-0.0016	-0.0095	-0.0044
Variance-	-0.0041	-0.0021	0.0571	-0.0207	-0.0159	-0.0089	-0.0052
Covariance	-0.0040	-0.0073	-0.0207	0.0389	0.0091	-0.0045	-0.0113
Matrix	0.0011	-0.0016	-0.0159	0.0091	0.0130	-0.0026	-0.0029
	-0.0185	-0.0095	-0.0089	-0.0045	-0.0026	0.0392	0.0048
	-0.0046	-0.0044	-0.0052	-0.0113	-0.0029	0.0048	0.0238

TABLE A-6

Males With Liberal Ideology Who Expressed Tax Criticism (N_1 = 24)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.6910	3.3725	3.4518	5.0374	6.2350	3.7903	3.4219
	0.0787	0.0092	-0.0207	-0.0300	0.0006	-0.0153	-0.0225
	0.0092	0.0730	-0.0480	-0.0100	0.0342	-0.0462	-0.0121
Variance-	-0.0207	-0.0480	0.2494	0.0216	-0.0759	-0.0895	-0.0368
Covariance	-0.0300	-0.0100	0.0216	0.0657	0.0097	-0.0271	-0.0298
Matrix	0.0006	0.0342	-0.0759	0.0097	0.0961	-0.0419	-0.0228
	-0.0153	-0.0462	-0.0895	-0.0271	-0.0419	0.1825	0.0376
	-0.0225	-0.0121	-0.0368	-0.0298	-0.0228	0.0376	0.0865

TABLE A-7

Females With Liberal Ideology Who Expressed No Tax Criticism (N_1 = 76)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.2112	4.0107	4.7372	3.8395	6.1061	3.6574	3.4379
	0.0272	-0.0067	-0.0061	0.0024	0.0048	-0.0090	-0.0126
	-0.0067	0.0433	0.0027	-0.0083	-0.0024	-0.0241	-0.0045
Variance-	-0.0061	0.0027	0.0574	-0.0212	-0.0042	-0.0192	-0.0093
Covariance	0.0024	-0.0083	-0.0212	0.0334	0.0004	-0.0018	-0.0049
Matrix	0.0048	-0.0024	-0.0042	0.0004	0.0134	-0.0058	-0.0062
	-0.0090	-0.0241	-0.0192	-0.0018	-0.0058	0.0528	0.0073
	-0.0126	-0.0045	-0.0093	-0.0049	-0.0062	0.0073	0.0304

TABLE A-8

Females With Liberal Ideology Who Expressed Tax Criticism (N_1 = 18)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	1.9686	3.2682	3.9328	4.5854	6.4917	4.6134	3.1399
	0.0911	-0.0408	-0.0250	-0.0033	0.0124	-0.0377	0.0034
	-0.0408	0.1543	-0.0008	-0.0006	-0.0038	-0.0612	-0.0468
Variance-	-0.0250	-0.0008	0.1443	-0.0951	-0.0242	0.0111	-0.0102
Covariance	-0.0033	-0.0006	-0.0951	0.1819	0.0336	-0.1161	-0.0002
Matrix	0.0124	-0.0038	-0.0242	0.0336	0.0304	-0.0323	-0.0162
	-0.0377	-0.0612	0.0111	-0.1161	-0.0323	0.2425	-0.0062
	0.0034	-0.0468	-0.0102	-0.0002	-0.0162	-0.0062	0.0762

TABLE A-9

Males With In Between Ideology Who Expressed No Tax Criticism (N_1 = 169)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.0819	3.7379	3.7306	4.5022	6.1015	4.4182	3.4277
Variance-Covariance Matrix	0.0118	0.0006	-0.0041	0.0003	-0.0010	-0.0064	-0.0011
	0.0006	0.0172	-0.0024	-0.0032	0.0023	-0.0099	-0.0044
	-0.0041	-0.0024	0.0263	-0.0087	-0.0034	-0.0029	-0.0044
	0.0003	-0.0032	-0.0087	0.0156	-0.0009	-0.0009	-0.0021
	-0.0010	0.0023	-0.0034	-0.0009	0.0089	-0.0044	-0.0013
	-0.0064	-0.0099	-0.0029	-0.0009	-0.0044	0.0222	0.0025
	-0.0011	-0.0044	-0.0044	-0.0021	-0.0013	0.0025	0.0109

TABLE A-10

Males With In Between Ideology Who Expressed Tax Criticism (N_1 = 75)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.1987	3.0843	3.6041	4.9577	6.2839	4.8144	3.0569
Variance-Covariance Matrix	0.0294	-0.0026	-0.0190	-0.0006	0.0027	-0.0066	-0.0031
	-0.0026	0.0279	0.0033	0.0015	-0.0031	-0.0184	-0.0086
	-0.0190	0.0033	0.0595	-0.0209	0.0013	-0.0147	-0.0095
	-0.0006	0.0015	-0.0209	0.0275	-0.0038	-0.0042	0.0006
	0.0027	-0.0031	0.0013	-0.0038	0.0100	-0.0060	-0.0011
	-0.0066	-0.0184	-0.0147	-0.0042	-0.0060	0.0454	0.0046
	-0.0031	-0.0086	-0.0095	0.0006	-0.0011	0.0046	0.0171

TABLE A-11

Females With In Between Ideology Who Expressed No Tax Criticism (N_1 = 191)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.2843	3.8182	3.2618	4.7289	6.1976	4.3011	3.4080
Variance-Covariance Matrix	0.0123	-0.0000	-0.0049	-0.0010	0.0021	-0.0045	-0.0038
	-0.0000	0.0162	-0.0008	-0.0032	0.0000	-0.0077	-0.0043
	-0.0049	-0.0008	0.0178	-0.0027	-0.0030	-0.0033	-0.0028
	-0.0010	-0.0032	-0.0027	0.0116	0.0003	-0.0035	-0.0014
	0.0021	0.0000	-0.0030	0.0003	0.0069	-0.0048	-0.0015
	-0.0045	-0.0077	-0.0033	-0.0035	-0.0048	0.0216	0.0024
	-0.0038	-0.0043	-0.0028	-0.0014	-0.0015	0.0024	0.0116

TABLE A-12

Females With In Between Ideology Who Expressed Tax Criticism (N_1 = 52)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.4826	3.3440	2.5420	5.1425	6.6705	4.3978	3.4206
Variance-Covariance Matrix	0.0524	0.0064	-0.0251	-0.0032	0.0016	-0.0213	-0.0108
	0.0064	0.0378	-0.0097	0.0008	0.0008	-0.0208	-0.0155
	-0.0251	-0.0097	0.0589	-0.0122	0.0009	-0.0105	-0.0022
	-0.0032	0.0008	-0.0122	0.0369	0.0006	-0.0114	-0.0114
	0.0016	0.0008	0.0009	0.0006	0.0069	-0.0076	-0.0034
	-0.0213	-0.0208	-0.0105	-0.0114	-0.0076	0.0569	0.0148
	-0.0108	-0.0155	-0.0022	-0.0114	-0.0034	0.0148	0.0286

TABLE A-13

Males With No Ideology Who Expressed No Tax Criticism (N_1 = 101)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.7638	4.5259	3.4535	4.6294	6.0404	3.7036	2.8833
Variance-Covariance Matrix	0.0327	0.0016	-0.0090	0.0001	-0.0011	-0.0173	-0.0070
	0.0016	0.0252	0.0036	-0.0078	-0.0037	-0.0126	-0.0064
	-0.0090	0.0036	0.0450	-0.0174	-0.0047	-0.0096	-0.0078
	0.0001	-0.0078	-0.0174	0.0252	-0.0021	0.0020	-0.0000
	-0.0011	-0.0037	-0.0047	-0.0021	0.0137	-0.0044	0.0023
	-0.0173	-0.0126	-0.0096	0.0020	-0.0044	0.0405	0.0014
	-0.0070	-0.0064	-0.0078	-0.0000	0.0023	0.0014	0.0174

TABLE A-14

Males With No Ideology Who Expressed Tax Criticism (N_1 = 35)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.7118	4.0935	3.3194	4.5072	6.4615	3.9878	2.9187
Variance-Covariance Matrix	0.1154	0.0062	-0.0084	-0.0108	0.0082	-0.0772	-0.0332
	0.0062	0.0554	-0.0338	0.0015	0.0032	-0.0215	-0.0110
	-0.0084	-0.0338	0.0963	0.0055	-0.0238	-0.0167	-0.0189
	-0.0108	0.0015	0.0055	0.0607	-0.0083	-0.0226	-0.0259
	0.0082	0.0032	-0.0238	-0.0083	0.0373	-0.0209	0.0042
	-0.0772	-0.0215	-0.0167	-0.0226	-0.0209	0.1248	0.0342
	-0.0332	-0.0110	-0.0189	-0.0259	0.0042	0.0342	0.0507

TABLE A-15

Females With No Ideology Who Expressed No Tax Criticism (N_1 = 176)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.2366	4.1471	3.2051	4.7599	6.2908	4.2399	3.1208
Variance-Covariance Matrix	0.0131	-0.0007	-0.0028	0.0002	-0.0012	-0.0076	-0.0008
	-0.0007	0.0172	0.0006	-0.0051	-0.0023	-0.0080	-0.0017
	-0.0028	0.0006	0.0190	-0.0057	-0.0019	-0.0043	-0.0048
	0.0002	-0.0051	-0.0057	0.0142	-0.0004	-0.0017	-0.0014
	-0.0012	-0.0023	-0.0019	-0.0004	0.0059	-0.0000	0.0000
	-0.0076	-0.0080	-0.0043	-0.0017	-0.0000	0.0219	-0.0000
	-0.0008	-0.0017	-0.0048	-0.0014	0.0000	-0.0000	0.0089

TABLE A-16

Females With No Ideology Who Expressed Tax Criticism (N_1 = 42)

	ED	PL	TR	PV	FA	GI	HC
Mean Rank Preference Vector	2.4731	4.2437	2.2306	5.0681	6.4474	4.1552	3.3820
Variance-Covariance Matrix	0.0796	0.0030	-0.0216	-0.0128	-0.0185	-0.0185	-0.0110
	0.0030	0.0616	-0.0129	-0.0173	0.0116	-0.0446	-0.0012
	-0.0216	-0.0129	0.0515	0.0096	-0.0044	-0.0071	-0.0151
	-0.0128	-0.0173	0.0096	0.0379	-0.0041	-0.0044	-0.0088
	-0.0185	0.0116	-0.0044	-0.0041	0.0237	-0.0141	0.0059
	-0.0185	-0.0446	-0.0071	-0.0044	-0.0141	0.0842	0.0046
	-0.0110	-0.0012	-0.0151	-0.0088	0.0059	0.0046	0.0256