A global model for reinforced concrete beams under alternate loading

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Key to abbreviations
N = A%C (or A%T) designates a normal force value equal to A% of the maximum compression stress (or tensile stress) of the concrete in the beam studied, multiplied by its cross section.

1. INTRODUCTION

It is important for safety reasons to determine the real behaviour and estimate the safety facilities required compared to the norm for reinforced concrete structures in nuclear reactors subjected to accidental loading (impacts, earthquakes, etc...).

Moreover, it is preferable for reasons of cost, to use methods based on the global concepts normally used in resistance of materials, such as forces and moments. Such methods have already been proposed for loadings which are predominantly bending loads, in particular for beams.

Normal forces remain fairly constant, which is not always the case, particularly during impact.

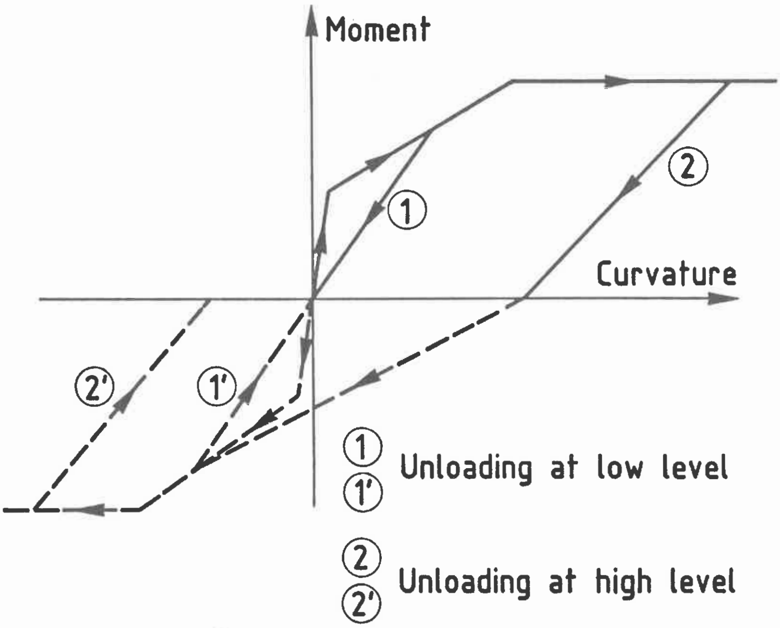
This paper proposes an extension of these models to cases in which the normal force varies considerably. The formulation is elastic-damage-plastic type and the methodology employed is described below.

2. EXISTING MODEL (Ref. [21])

In the past, experimental results obtained for reinforced concrete beams have led to a global model based on a trilinear representation of Moment-Curvature curves (see figure 1). In this model the damage of the concrete is accounted for by means of a modified elastic slope in the case of unloading.

The main limitation of this model is to assume a constant normal force in the beam, which is not valid for complex loading such as impacts, earthquakes, etc. Indeed, for such loadings it is important to choose a criterion based on the moment as well as on the normal force.

Therefore, a new model has been proposed in order to account for both generalized stresses and for the coupling effects between membrane and bending strains in the case of damage or plasticity.



MODELISATION OF MOMENT - CURVATURE CURVES

Figure 1

3. THE NEW MODEL

3.1 Method adopted

We produce global Moment-Curvature behaviour laws for an equivalent homogeneous beam subjected to a given normal force. In order to do this, we use a program [2] which produces these laws on the basis of the characteristics of the concrete and the steel in the reinforced concrete structure studied and its geometry.

This program which works on imposed deformation, also supplies the characteristics of the local state of the structure at each stage of curvature. This enables us to monitor cracking and plastification of the various layers of material.

The concrete and steel in question have relatively simple behaviour laws so as to facilitate the modelling which is described in this document. It should, however, be pointed out that a check has been made to ensure that the general character of the model [3] is not affected.

We looked at the usual practice which led to use steel rates (maximum and minimum) and limit positions for steel layers.

We thus studied four limit situations with respect to normal geometry and steel fixing rates in civil work structures. The results obtained are valid for beams of any height, provided that materials, positions and distributions are considered as homothetic [4].

Figure 2 and 3 show the Moment-Curvature curves for the limit cases studied (light steelwork in the middle). We note a clear reduction in deformation on failure for a corresponding increase in the value of normal force.

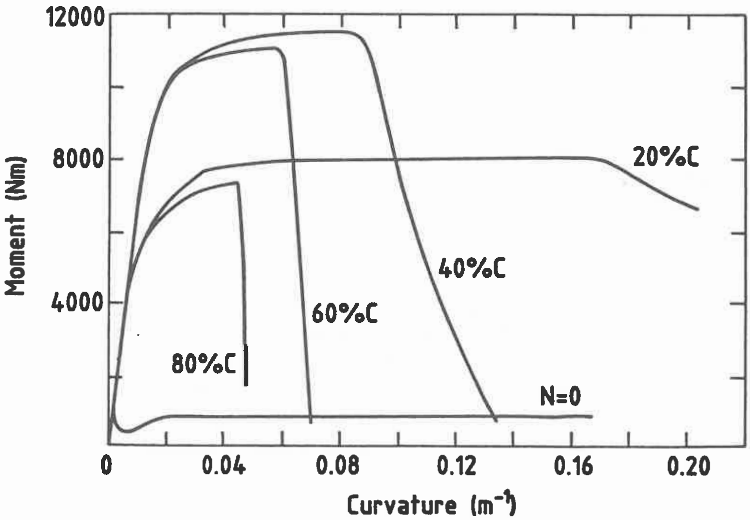


Figure 2

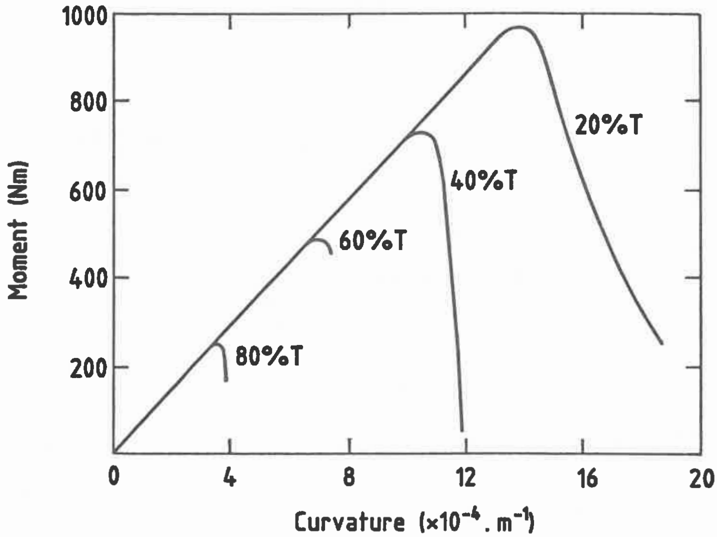


Figure 3

We noted in all the cases studied that :

- the elastic range is limited by the elastic limit of the concrete (either on compression or tensile stress) ;
- the start of plasticity coincides with the start of plastification of the steel under tensile stress ;
- the end of the plasticity coincides with the crushing of the concrete under compression.

We therefore managed to define the initial elasticity range and the ultimate strength range for each of the beams studied. Figure 4 shows these ranges for beams with light reinforcements in the middle.

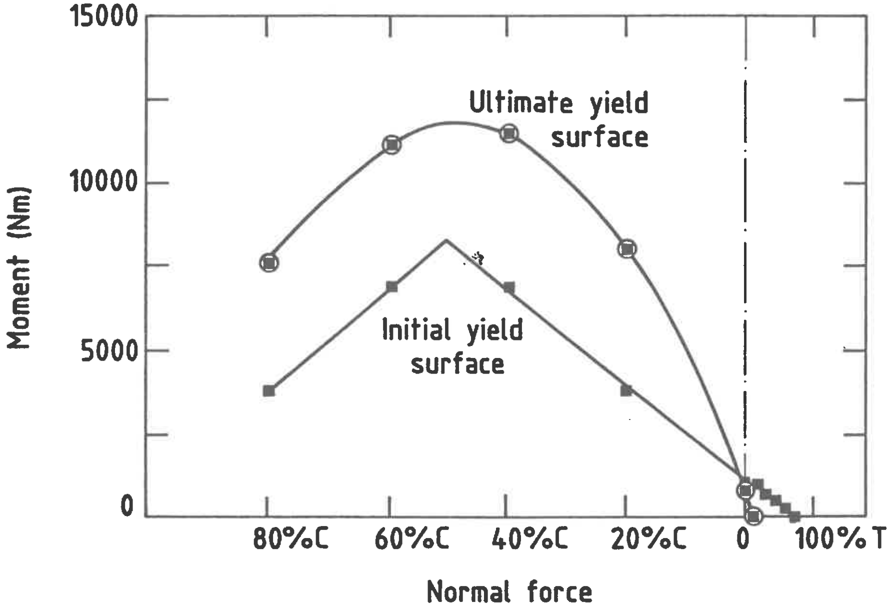


Figure 4

3.2 Search of a criterion

First, we modelled the initial elasticity and ultimate strength ranges. We thus ckecked that a straight line and a parabola defined them fairly well within the range of normal force adopted (40%C < N < 10%T).

$$M = a_1 N + a_2 \quad (3.1)$$

$$M = a_3 N^2 + a_4 N + a_5 \quad (3.2)$$

In fact, in this range of normal force, the elastic limit is always reached at the start of cracking of the tensed concrete fibres. On the other hand, crushing of the concrete under compression only becomes predominant at normal force compression values greater than approximately 45%C.

The various coefficients a_i that we wish to induce and the coefficients necessary for definition of the flow law which we shall describe in the rest of this document, have been produced by a program [5] on the basis of the values supplied by the homogeneization program already mentioned [2].

We have used a non-linear elasticity approach with damage in order to model the predominant phenomenon noted between the elastic limit and ultimate strength - the spread of cracking through concrete under tensile stress.

Changes in the criterion are defined with respect to a damage parameter, for each bending sign, directly associated with the degree of cracking of the concrete. We assume that this cracking does not spreads after the start of plastification of the steel reinforcing bar and we have shown that the error thus made is not very great [4].

The type of modelling used leads to decoupling between the damage and plasticity phases which gave us relatively short computation times.

The solution chosen to obain the global behaviour law in the non-linear elastic phase consist in making the criterion develop in linear fashion ξ. We therefore have :

$$F = M - a_3 \xi N^2 + [a_1 (\xi - 1) - a_4 \xi] N + [a_2 (\xi - 1) a_5 \xi] \leq 0 \quad (3.3)$$

For the limit values of the damage parameter (=0 and =1), we obtain the surfaces (3.1) and (3.2) which limit the damage range.

3.3 Damage and plasticity calculation

We can easily define the secant damage matrix with respect to the height of craking, but we noted [4] that this did not depend biuniquivocally on the damage parameter, owing to the contribution of normal force.

Thus, we noted that the value of normal force which best models the network of "cracking height-damage parameter" curves (see figure 4) is N = 10%C.

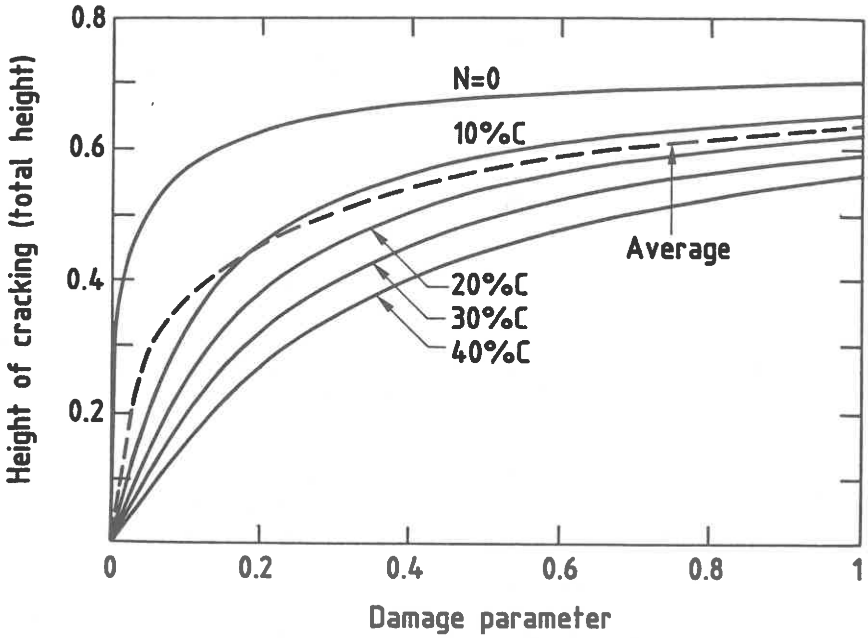


Figure 5

We therefore have :

$$ht = hf\ (\xi,\ N) \tag{3.4}$$

which can be approximated by :

$$hf = hf\ (\xi,\ N = 10\%C)\ \forall\ N \in [40\%C,\ 10\%T] \tag{3.5}$$

For a given deformation couple, the damage calculation method which produces the corresponding force couple, and also the value of ξ which defines the state of the structure, will be :

$$\begin{cases} \begin{bmatrix} N\ (\xi) \\ M\ (\xi) \end{bmatrix} = \begin{bmatrix} DSE_{ij}\ (\xi) \end{bmatrix} \begin{bmatrix} e \\ \chi \end{bmatrix} \\ F\ [M\ (\xi),\ N\ (\xi),\ \xi] = 0 \end{cases} \tag{3.6}$$

When the reinforcement starts to plastify, the modelling used leads to the end of the damage phase. The approach we use for plasticity calculations is perfect elastic-plastic type and the criterion therefore becomes :

$$F_1\ (M,\ N,\ a_i,\ \xi = 1) \leq 0 \tag{3.7}$$

We have noted that we could not consider plastic flow as being associated [4]. Thus, we defined a law based on the results obtained and described above :

$$\dot{\varepsilon}_{\rho} = g(N)\ \dot{\chi}_{\rho} \tag{3.8}$$

with :

$$g(N) = a_6\ N + a_7 \tag{3.9}$$

and we set :

$$\left[\ \dot{\varepsilon}_{\rho}\ \right] = \left[\begin{matrix} g\ (N) \\ 1 \end{matrix}\right] d\lambda \tag{3.10}$$

which, considering the condition of consistency of the criterion, led us to the definition of the tangent plasticity matrix which links to :

$$\left[DTP_{ij}\right] = \left[DSE_{ij}\ (\xi\text{-}1)\right] - \frac{\left[DSE_{ij}\ (\xi=1)\right]\left[\begin{matrix} g(N) \\ 1 \end{matrix}\right]\left[\frac{dF_1}{d\sigma}\right]^T\left[DSE_{ij}\ (\xi=1)\right]}{\left[\frac{dF_1}{d\sigma}\right]^T\left[DSE_{ij}\ (\xi=1)\right]\ \left[\begin{matrix} g(N) \\ 1 \end{matrix}\right]} \tag{3.11}$$

4. NUMERICAL IMPLEMENTATION

This new model was then implemented in a program for calculation by finite elements [6]. The equations (3.6) and (3.11) for the damage phase and the plasticity phase respectively, are solved numerically.

In the first case, we proceed by dichotomy in order to find the value of the parameter ξ, in the second case, given that the plastic tangent matrix depends on the value of the normal force, we used a Eular variable step method, the precision test used is based upon the relative difference between increments in normal force [7].

Having input two damage parameters and two sets of coefficients a_i (one for each bending sign), the problem arose of defining the conditions of transition from one bending range to another. The solution chosen consists in defining the transition when total skin deformation cancels out, which corresponds to complete reclosing of the crack.

We then defined the deformation increment which leads to cancellation of total skin deformation and perform a calculation using the coefficients for the previous range. We then make a second calculation after having changed the previous ones, taking the complement on the step as the deformation increments couple.

Operating tests and comparative calculations with reference experiments showed the influence of the coupling of membrane and bending behaviour due to damage and plasticity.

Figure 6 shows the curves obtained for reinforced concrete beams (of traditional shape), loaded with 4-points bending.

We note that the limit force obtained for longitudinally restrained beams is much greater than that for free beams, due to the presence of a non-negligeable normal force.

5. CONCLUSION

The model described in this paper clearly shows the influence of normal force on the behaviour of reinforced concrete beams at failure.

The correct handling of cracking and plasticity phases enables it to be applied to monotone and cyclic loadings.

Its remains to generalize the application of the model to three dimensional loadings and to validate it by means of representative tests. Moreover, the development of a similar approach for shells should enable calculation of the behaviour of a complete building at reasonable cost.

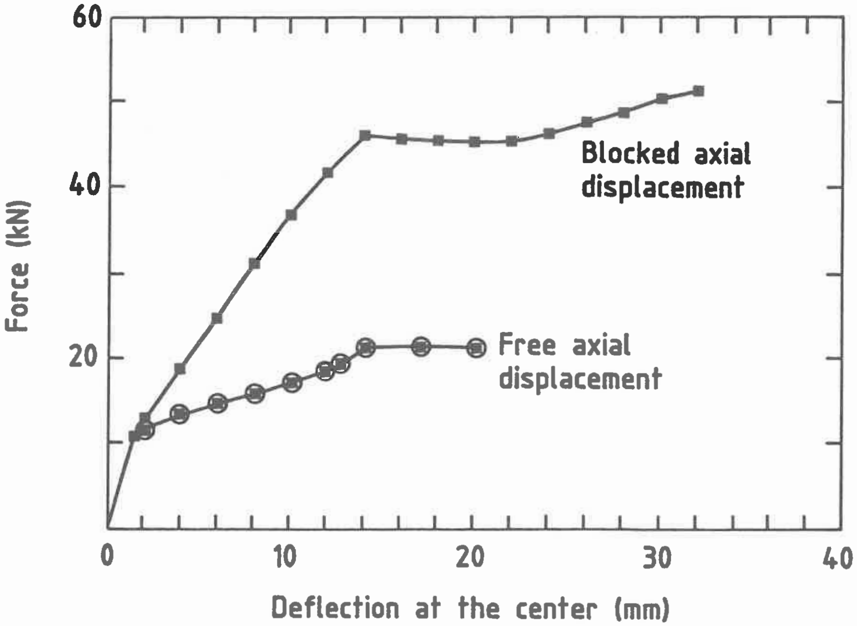


Figure 6

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