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Seismic Analysis of Secondary Systems - An Overview

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ABSTRACT

Analytical methods which can be used for seismic analysis of single and multiple support secondary systems are presented. Some approximate approaches used with multiple support systems are also discussed and their relationships with the rigorous method presented here are described. It is noted that the approximate approaches can lead to responses with large errors. They are expected to give conservative estimates of the design response, but this conservatism is not always assured. The use of these approximate methods is thus discouraged.

1 INTRODUCTION

For the seismic design of the components of a nuclear reactor facility, it is now common to classify various systems as primary and secondary systems. The primary systems are usually the civil structures representing the main building with beams, columns, frames, floors, etc. The secondary systems are usually the mechanical and electrical components which are supported on the primary systems. Although they are called as the secondary systems, they are no less important than the primary systems as they usually are the lifelines or some of the most important links of a nuclear reactor facility. As their malfunction can have catastrophic consequences, they are designed and qualified very carefully for seismic loads which can be expected at the site of the facility.

The secondary systems must be designed for the same ground motion for which the primary systems are designed. The seismic motion experienced by the secondary systems is, however, the motion which filters through the supporting primary structure. It is quite important that this filtered characteristic of the motion be considered in the design of secondary systems.

For the design of nuclear facilities it has been a common practice to define the seismic design motion in terms of the pseudo-acceleration ground response spectra (US Nuclear Regulatory Commission, 1975). Analytical methods have been developed to obtain the design response of a primary system for seismic loading defined in this form. As a natural extension of this practice, it is now common to define the seismic loading for the secondary systems also in terms of the pseudo-acceleration floor response spectra and use them for calculating the seismic design response of the secondary systems. This practice, although still quite popular, can only be justified for some light and single support secondary systems. These limitations are discussed in this paper. Some recently developed

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methods for calculating the design response for different types of secondary systems are also described.

Depending upon the objective, secondary systems can be classified as light or heavy, single or multiple supports, active like rotating machines or inactive with no moving parts, etc. Here we will classify them as the single and multiple support secondary systems. We will also differentiate between heavy and light and tuned and detuned secondary systems. The active secondary systems, such as motors, generators, turbines are also of interest, but they will not be discussed here.

2 FLOOR RESPONSE SPECTRA AND SINGLE SUPPORT SECONDARY SYSTEMS

Small equipment and components, not occupying a large area of the primary system can be considered as single support secondary systems. The motion of the area or the point where they are supported defines the base input for their designs. For the seismic design input defined in terms of ground response spectra, the motion for a secondary system can be defined in terms of a time history or the floor response spectra. To account for the possibilities of encountering different site motions, this input is now commonly defined in terms of floor response spectra. Several analytical methods are now available to obtain such floor response spectra from ground response spectra directly. The use of these methods is simple and straightforward. For example, the expression developed by the writer (Singh, 1975, 1980) for calculating a floor acceleration response spectrum value, which was recently improved to account for the effect of truncating modes, can be stated as follows:

$$R_f^2 = g_s^2 P_o^2 + 2g_s \sum_{j=1}^{r} \rho_j \left[A_j' P_j^2 + B_j' V_j^2 + C_j' P_o^2 + D_j' V_o^2\right]$$
$$+ \sum_{j=1}^{r} \rho_j^2 \left[A_j P_j^2 + B_j^2 V_j^2 + C_j P_o^2 + D_j V_o^2\right] + 2 \sum_{j=1}^{r-1} \sum_{k=j+1}^{r} \rho_j \rho_k \left[A_{jk} P_j^2 + B_{jk} V_j^2\right.$$
$$\left. + C_{jk} P_k^2 + D_{jk} V_k^2 + E_{jk} P_o^2 + F_{jk} V_o^2\right] \qquad (1)$$

where R_f is the pseudo acceleration response spectrum value at the oscillator frequency ω_0 and damping ratio β_0; r is the number of modes considered in the summation; ρ_j is the product of the modal participation factor and modal displacement of the point where the floor spectrum is calculated; $g_s = (1 - \sum_j^r \rho_j)$ is a factor which includes the effect of the truncated modes; P_j and V_j, respectively, are the pseudo-acceleration and relative velocity response spectrum values for the j^{th} mode frequency and damping ratio; P_0 and V_0 are the spectrum values for the oscillator parameters; A_j, B_j etc are the partial fraction coefficients which depend upon the structural modal quantities and oscillator parameters and can be obtained as described by the writer (Singh, 1975 and 1980). The first two terms associated with factor g_s represent the contribution of the truncated high frequency modes These terms were not considered in the initial development of this expression by the writer. In situations where the high frequency modes contribute significantly to the response, inclusion of these terms in the calculation of floor response spectra will provide better results.

For a light secondary system supported at a single point on the primary structure, the floor response spectra can be used as the base input. For this base input, the response of the secondary system can be calculated by using a suitable response spectrum approach. Since floor spectra are usually narrow band inputs, it may also become necessary to have the relative velocity floor response spectra for calculating the secondary system response accurately. Such spectra can also be easily developed as shown by Singh and Burdisso(1987).

If the secondary system is not light compared to the primary system and if the frequencies of the two systems are close to each other, then there can be significant dynamic interaction between the two system. This can affect the response of the secondary system significantly. This interaction effect was ignored in the development of Eq. 1 where it was assumed that the motion is only transmitted up from the primary to secondary system and not backwards. The fact that there is indeed a feed back of motion has prompted some excellent research work whereby the effect of this interaction can now be easily included in the calculation of the secondary oscillator response. See, for example, Kelly and Sackman (1978), Igusa and Der Kiureghian (1985) and Suarez and Singh (1987). The steps of another approach developed by Singh and Suarez (1987), which is applicable to very light to very heavy secondary systems are as follows.

(a) Calculate the eigenvalues of the combined primary system and oscillator by solving the following characteristic equation:

$$\sum_{r=1}^{n} \frac{m_o \nu_i^2}{(p_j^2+2\beta_i \omega_i p_j^2+\omega_i^2)} + \frac{(p_j^2+2\beta_o \omega_o p_j+\omega_o^2)}{(2\beta_o \omega_o p_j^3+\omega_o^2 p_j^2)} = 0 \qquad (2)$$

where p_j = the eigenvalues; ω_o, β_o and m_o, respectively, are the frequency, damping ratio and mass of the oscillator; ϕ_i is the i^{th} modal displacement of the point where the oscillator is attached; $\delta_i = p_j^2 + 2\beta_i\omega_i p_j + \omega_i^2$; ω_i and β_i, respectively are the frequency and damping ratio of the i^{th} mode of the primary system. The equation can be solved by a simple Newton–Raphson scheme. It provides a pair of (n+1) complex and conjugate eigenvalues.

(b) Calculate the product of the combined system participation factor and modal displacement corresponding to the oscillator degree of freedom as:

$$\rho_j = \frac{1}{p_j^2} \sum_{i=1}^{m} \frac{\gamma_i \nu_i}{2(p_j^2+2\omega_i \beta_i p_j+\omega_i^2) D_{ij}} \qquad (3)$$

where $m = n+1$; γ_i is the i^{th} participation factor of the primary system; the product $\nu_m \gamma_m$ corresponding to the oscillator degree of freedom is equal to -1 and; D_{ij} is the normalizing factor defined as:

$$D_{ij} = \beta_o \omega_o \frac{(p_j^2+2\beta_i \omega_i p_j+\omega_i^2)^2}{(2\beta_o \omega_o p_j+\omega_o^2)^2} + \sum_{k=1}^{m} m_o \nu_k^2 (p_j+\omega_k \beta_k) \frac{(p_j^2+2\beta_i \omega_i p_j+\omega_i)^2}{(p_j^2+2\beta_k \omega_k p_k+\omega_k^2)^2} \qquad (4)$$

(c) To calculate a response quantity related to the oscillator degree of freedom, one can now use a response spectrum approach for nonclassically damped system (e.g., Singh, 1980 b; Igusa et al. 1984, Maldonado and Singh, 1991) and to calculate spectrum value, the expression by Singh and Suarez (1987) can be used.

The floor response spectra with interaction, however, have little use if one is interested in calculating the response of a multi degree of freedom secondary system by a response spectrum type of analysis. The problem stems from the fact that it is not simple to account for the dynamic interaction of different modes of the secondary system with the primary system to modify the floor response spectrum value for its use in a response spectrum analysis. If one is interested in including the effect of dynamic interaction in calculating the response of a secondary system, it is best to employ a mode synthesis approach. Details of one such approach are given by Singh and Suarez (1988). The approach to be presented for the analysis of multiple support secondary system can also be used for incorporating the dynamic interaction in response analysis.

3 RESPONSE OF MULTIPLE SUPPORT SECONDARY SYSTEMS

The piping systems in a power plant which usually cover a wide area of a power plant, often traversing from one floor to another of the plant building carrying fluid from one source to another, are classical examples of multiple support secondary systems. Unlike some single support secondary systems which can some time be evaluated by testing, seismic design of piping systems depends solely on analysis. Thus it is imperative that the analytical methods used for calculating the seismic design forces in piping systems be rational with a solid base in the principles of mechanics and vibrations. The author feels that this rationality is often compromised in the current design practice in the power industry.

The complications in the analysis of a piping system arise primarily from the fact that the seismic input to these systems at their supports on primary system are different but correlated. For two random seismic motions, this correlation can not be completely defined by a single attribute like a correlation coefficient, but must be defined by a cross correlation function or a cross spectral density function. Until a few years ago, analytical methods for a spectrum analysis of multi support secondary systems which include the input correlation were not available. Now the methods have become available but only rarely used in the current practice primarily because of lack of dissemination of these later developments and unawareness on the part of practitioners. The papers like this one are, therefore, for the dissemination of such information to the practitioners. With this aim in mind, we will now describe the main steps of a method developed by Burdisso and Singh (1987) for a decoupled response spectrum analysis of multiple support secondary systems. The method is based on the established principles of mechanics and the theory of random vibration of structures. Specific details of the development of the method, the assumptions made in the development and some important analytical expressions are given by Burdisso and Singh (1987) and Singh and Burdisso (1987).

The method divides the total system response into pseudo-static and dynamic parts, as is done in the ASME Boiler and Pressure Vessel Code (1981) for design purposes. The pseudo-static response is due to the differential movement of supports (or anchor movements). This response is self -limiting by nature as it can not lead to a collapse of the system if there is enough ductility in the stressed components. Thus for the pseudo-static stresses, which are also called as the secondary stresses, the ASME Code (1981) permits a higher allowable stress. The dynamic part of the response, on the other hand, is caused by the inertial forces which are induced by the accelerations of the support. The stresses due to the dynamic part are classified as primary stresses in the ASME code. These two response parts are correlated. The numerical results have indicated that this correlation can not be ignored. The procedure to be described includes this correlation explicitly. The basic steps of the method are as follows:

3.1. Calculation of dynamic response

The dynamic part of response, here denoted by R_d, is calculated according to the following expression:

$$R_d^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_j \rho_i \sum_{k=1}^{m} \sum_{l=1}^{m} P_{ik} P_{jl} I_{ijkl} \qquad (5)$$

where n= the number of degrees of freedom, m= the number of supports, ρ_i = the i^{th} modal response of the quantity of interest, I_{ijkl} is quantity which is a function of various floor response spectrum quantities. The expressions to calculate this quantity are given by Burdisso and Singh (1987) for various cases of the subscripts. When k is equal to l, this quantity is concerned only with the motion of a single floor and thus it can be defined in terms of the commonly used pseudo acceleration and relative velocity floor response spectra. Here we refer to these spectra as the auto floor response spectra. When $k \neq l$, this quantity is defined in terms of, what we call, the cross floor response spectra. These spectra can be obtained directly from the ground response spectra by using expressions similar to Eq. 1. These expressions are given by Singh and Burdisso (1987). The quantities P_{ik} are the elements of the participation vector $\{P_i\}$ defined in terms of the matrices of the secondary system as:

$$\{P_i\} = \{\phi_i\}^T [r] = \{\phi_i\}^T [M] [K]^{-1} [K_c] \qquad (6)$$

where $\{\phi_i\}$ is the i^{th} modal vector of the secondary system; [M] and [K] are the mass and stiffness matrices of the secondary system and $[K_c]$ is the cross coupling matrix associated with the support points on the primary system and the unattached degrees of freedom of the secondary system. The k^{th} element of this participation vector is P_{ik} which also represents the i^{th} mode participation factor associated with the motion of k^{th} support.

3.1.1 Approximate approaches

At this point it is relevant to mention various approximations which have been made by practitioners in the nuclear power plant design industry to include (or circumvent) the effect of the correlations between the support accelerations in the calculation of dynamic response, and relate them to Eq. 5. The methods which are commonly used in the industry are (1) the square-root-sum method, (2) absolute-sum method, (3) grouping with square-root-sum method and (4) envelope spectrum method. All these methods are the special cases of Eq.5 and are obtained by deleting certain terms or modifying the summation rule.

Square-root-sum-procedure: In this method it is assumed that the support motions are uncorrelated with each other. In the context of Eq. 5, this implies that the terms with $k \neq l$ are zero. Denoting the response calculated with this assumption as R_s, we can write for this from Eq. 5 as:

$$R_s^2 = \sum_{k=1}^{m} \left(\sum_{i}^{n} \sum_{j}^{n} \rho_i \rho_j P_{ik} P_{jk} I_{ijkk} \right) = \sum_{k=1}^{m} R_{dk}^2 \qquad (7)$$

where R_{dk} is the dynamic response due to the motion of support k. Since the neglected terms could be positive or negative, the response calculated with this assumption can be larger or smaller than the response calculated by Eq. 5.

Absolute-sum-procedure: To ensure that a maximum value of response is obtained no matter what the actual correlations between the support motions are, often the absolute-sum approach is used. That is,

$$R_a = \sum_{k=1}^{m} |R_{dk}| \tag{8}$$

It is obvious that R_a will always be greater than R_d (Eq. 5) and R_s (Eq. 7).

Support grouping procedure: To reduce the amount of the floor spectral information required in evaluating Eq. 5 for piping systems with many supports, Lin and Loceff (1980) have proposed a grouping approach wherein the supports which are suspected to have similar motions are grouped together with a common floor response spectrum input. Thus if all the supports are divided into n_g number of groups, the dynamic response can be obtained from:

$$R_g^2 = \sum_i^n \sum_j^n \rho_i \rho_j \sum_k^{n_g} \sum_l^{n_g} Q_{ik} Q_{jl} I_{ijkl} \tag{9}$$

where the participation factor Q_{ik} for the k^{th} group is the sum of all P_{is} summed up over all s supports in the group. Again to avoid dealing with the subject of correlation between the input motions from various groups, the inputs are often assumed to be uncorrelated. This leads to the square-root-sum combination of the individual grouped responses.

Enveloping Spectrum Procedure: One of the earliest methods used to circumvent the problem of support motion correlation is the envelope spectrum approach. In this approach, the seismic input to the piping is defined in three orthogonal x, y, and z directions. The input spectra in a direction are the envelope of all floor response spectra in that particular direction. Thus if the modal participation factors in the x, y and z directions are γ_{jx}, γ_{jy}, and γ_{jz}, respectively, then the dynamic response calculated by this method, denoted as R_{de}, can be obtained from:

$$R_{de}^2 = \sum_i^n \sum_j^n \rho_i \rho_j \left(\gamma_{ix}\gamma_{jx} I'_{ijx} + \gamma_{iy}\gamma_{jy} I'_{ijy} + \gamma_{iz}\gamma_{jz} I'_{ijz} \right) \tag{10}$$

The participation factors can be defined in the usual manner. I_{ijx} is defined in the same way as I_{ijkk} but with the envelope spectra in x-direction.

It is now relevant to comment on the effects of the above mentioned approximations on the accuracy of the calculated dynamic response. For this, the results obtained by Eq. 5 were considered as the bench-mark results. The results obtained by various other methods were then compared with the bench-mark results for several cases of tuned and detuned piping systems, placed at different locations on the primary system. The piping considered for the numerical results was three dimensional and spanned three different floors. To get several different cases, the piping was placed on the top three, middle three and bottom

Table 1. Maximum Positive and Negative Differences in Percent in the Response Calculated by Eq. 5 and Approximate Procedures for Several Cases of a Piping System.

Method	Dynamic Response Negative Difference	Dynamic Response Positive Difference	Pseudo–Static Response Negative Difference	Pseudo–Static Response Positive Difference
Square–root–sum	–46	42	71	3306
Absolute–sum	5	326	297	5042
Grouping	–26	1090	––	––
Enveloping 1	–9	331	––	––
Enveloping 2	–9	1460	––	––

three floors of the primary system. Also the response values were obtained for bending moments and shear forces at several significant locations in the pipe. The details of the numerical results are presented by Singh and Burdisso (1991). In the cases considered in this reference, the largest and smallest percent differences in the response values obtained by the approximate approaches and the corresponding bench–mark values are shown in Table 1. It is seen that the square–root–sum method underestimated the response by 46% in one case and overestimated it by 42% in another case. The absolute–sum approach, of course, only overestimated the response, but in one case it was as high as 326%. Even the grouping method in which all supports on a floor were grouped together is seen to underestimate the response by 26% in one case. Yet in another case, the overestimation of the response is seen to be unrealistically high. Similar observations are also true in the enveloping method. In Enveloping 1 method, the motions in x– and y–directions were considered separately and combined by the square root sum method, whereas in Enveloping 2 method, the motions in all directions were the same and the input spectra were the envelope of all floor response spectra irrespective of the direction.

3.2 Calculation of the Pseudo–static Response

The pseudo–static response is caused by the differential movements of the supports and its design value can be calculated by the following equation:

$$R_p^2 = \sum_{k=1}^{m} \sum_{l=1}^{m} \eta_k \eta_l U_k U_l \rho_{kl} \tag{11}$$

where U_k is the maximum displacement of support k; η_k is the response due to a unit displacement of support k and; ρ_{kl} is the correlation coefficient of the maximum displacements of supports k and l. The maximum displacement values and the correlation coefficient can be obtained by a simple response spectrum analysis of the primary system.

In industrial practice, however, the correlation between the displacements is again approximated, as it was done in the calculation of the dynamic response. Here also, two types of approximations are made: (1) square–root–sum approximation where the correlation is considered as zero and (2) the absolute–sum method where the supports are forced to move in directions so as to cause an additive effect. The response values obtained by these methods can be written in terms of the quantities in Eq. 11 as follows:

$$R_{ps}^2 = \sum_{k=1}^{m} \eta_k^2 U_k^2 \quad ; \quad R_{pa} = \sum_{k=1}^{m} |\eta_k U_k| \tag{12}$$

The percent differences in the values calculated by these two approximations and by Eq. 11 are also shown in Table 1 along with results of the dynamic response. It is noted that these methods always overestimate the response, and this overestimation in several response cases can be unrealistically high.

3.3 Calculation of the cross response component

This component of the response represents the effect of the correlation between the dynamic and pseudo–static parts. This can be calculated as:

$$R_{pd} = 2 \sum_{i=1}^{n} \rho_i \sum_{k=1}^{m} \sum_{l=1}^{m} \eta_k P_{il} I'_{ikl} \tag{13}$$

where the quantity I'_{ikl} is I_{ijkl} calculated at $\omega_j = 0$. It is a common practice in the industry to ignore this cross response component. However, the numerical results presented by Singh and Burdisso (1991) clearly show that the magnitude of this response component can be even larger than the magnitudes of the dynamic or pseudo–static parts. Also, this part can be positive or negative. Thus ignoring this part can lead to large errors in the calculated response.

3.4 Calculation of total response

The three parts of the response are combined to obtain the total response as:

$$R^2 = R_d^2 + R_p^2 + R_{pd} \tag{14}$$

In the commonly used square–root–sum combination approach, the correlation between the dynamic and pseudo–static part is assumed to be zero. To avoid the possibility of underestimating the response, these two responses are also combined by the absolute–sum approach. However, both these methods are approximate. For the piping cases examined by Singh and Burdisso (1991), it was observed that these approximate method of combining the dynamic and pseudo–static responses can lead to erroneous results. Depending upon how the dynamic and pseudo static parts were calculated, these combinations can lead to an unconservative to an excessively overconservative estimates of the total response. The only way to ensure that a conservative estimate will be obtained all the time is to use the dynamic response obtained by the absolute sum approach and combine this with the pseudo–static response (calculated either by square–root–sum or absolute sum approach) also by the absolute sum approach. This overestimation of forces in the pipe might lead to a design with a very stiff pipe. Stiff pipes are, however, not desirable as they attract larger thermal stresses under normal operating conditions. Thus the use of these approximate procedures for calculating the dynamic, pseudo-static and total responses is not advocated.

4 ANALYSIS OF MULTIPLE SUPPORT SECONDARY SYSTEMS WITH DYNAMIC INTERACTION

The method presented in the previous section is acceptable for the secondary systems which are very light compared to their primary system as in such cases the effect of dynamic interaction and feed-back between the two systems can be ignored. This is an acceptable procedure for most piping systems in the nuclear reactor facilities. However, in some cases it may be necessary, as well as desirable, to include the effect of the dynamic interaction in the calculation of a secondary system response.

To incorporate the dynamic interaction effect in the calculation of secondary system response, one can modify the floor response spectra for the interaction effect before they are used in the analysis. However, this approach involves some questionable assumptions; and it is neither efficient nor accurate. In the writer's opinion, the best way to include the dynamic interaction effect is to obtain the modal properties of the combined primary and secondary system and then use them in a suitable response spectrum analysis to obtain the design response of interest. Here, one can use a perturbation approach as the secondary systems are usually light enough to qualify for its application, or use the mode synthesis approach. In the following paragraphs we present the basic steps of a combined mode synthesis and dynamic condensation procedure for calculating the important eigenproperties of the primary-secondary system, developed by Suarez and Singh, (1991) and Singh and Suarez, (1991). The primary aim of the method will be to obtain a first few modal properties of the combined system from the known modal properties of the two separate systems. If desired, the higher modes can also be obtained. In seismic analysis, however, it is quite sufficient to obtain only a first few modes. The effects of the higher modes which are often truncated can be easily incorporated through recently developed response spectrum methods (Singh and Maldonado, 1991 and Maldonado and Singh, 1991).

It is assumed that the mass normalized modal properties are available for the two systems. The modal properties of the primary system are obtained with the secondary support points assumed unconstrained whereas the modal properties of the secondary system are obtain with their supports fixed. It is also assumed that the masses are lumped at the nodes and thus the mass matrices of the two system are diagonal. The following synthesis and condensation procedure is then used with these modal properties.

(a) <u>Synthesis</u>: Define the combined eigenvalue problem as follows:

$$[K][\Phi] = [\Phi][\Lambda] \tag{15}$$

where $[\Phi]$ and $[\Lambda]$, respectively, are the eigenvector and eigenvalue matrices of the combined system. Matrix $[K]$ for the combined system is synthesized as

$$[K] = \begin{bmatrix} [\Lambda_p + \Phi_b^{p^T} K_{bb} \; \Phi_b^p] & [\Phi_b^p \; K_{bd} \; \Phi_s] \\ [\Phi_s^T \; K_{db} \; \Phi_b^p] & [\Lambda_s] \end{bmatrix} \tag{16}$$

In Eq. 15, $[\Lambda_p]$ and $[\Lambda_s]$ are the diagonal matrices of the eigenvalues of the primary and secondary systems, respectively; $[K_{bb}]$ and $[K_{bd}]$, respectively, are the part of the stiffness matrix of the secondary system associated with the boundary (support) and free degrees of freedom; $[\Phi^p_b]$ is the part of the primary system eigenvector matrix associated with the boundary degrees of freedom and; $[\Phi_s]$ is the eigenvector matrix of the secondary system.

(b) Partitioning of Matrices: To obtain the first k eigenvalues, the matrices and vector in Eq. 15 are partitioned into submatrices and vectors of size k and r=n−k as follows:

$$\begin{bmatrix} K_{kk} & K_{kr} \\ K_{rk} & K_{rr} \end{bmatrix} \begin{bmatrix} \Phi_{kk} & \Phi_{kr} \\ \Phi_{rk} & \Phi_{rr} \end{bmatrix} = \begin{bmatrix} \Phi_{kk} & \Phi_{kr} \\ \Phi_{rk} & \Phi_{rr} \end{bmatrix} \begin{bmatrix} \Lambda_k & 0 \\ 0 & \Lambda_r \end{bmatrix} \tag{17}$$

Before partitioning, it is desirable that the elements of [K] be rearranged in the increasing order of magnitude of the eigenvalues of the two systems.

(c) Condensation: The condensed eigenvalue problem of size k × k is formed as:

$$[K^*]^{(\ell)} [\Phi_{kk}]^{(\ell)} = [M^*]^{(\ell)} [\Phi_{kk}]^{(\ell)} [\Lambda_k]^{(\ell)} \tag{18}$$

where the condensed mass and stiffness matrices in the ℓ^{th} iterative step are:

$$\begin{aligned} [M^*] &= [I + R^{(\ell)^T} R^{(\ell)}] \\ [K^*]^{(\ell)} &= [K_{kk} + R^{(\ell)^T} K_{rk} + K^T_{rk} R^{(\ell)} + R^{(\ell)^T} K_{rr} R^{(\ell)}] \end{aligned} \tag{19}$$

The condensation matrix [R] in the first iterative step is defined according to the static condensation procedure as:

$$[R]^{(1)} = -[K_{rr}^{-1} K_{rk}] \tag{20}$$

The reduced eigenvalue problem is solved by any standard eigenvalue solver. The condensation matrix for the next iterative step is defined in terms of the recently calculated eigenvectors as:

$$[R]^{(\ell+1)} = [K_{rr}^{-1}] [R^{(\ell)} \Phi_{kk}^{(\ell)} \Phi_{kk}^{(\ell)^T} K^{*(\ell)} - K_{rk}] \tag{21}$$

(d) Calculation of full eigenvectors: The above steps are repeated till a convergence in the calculated eigenvalues is achieved. The eigenvectors of the condensed problem obtained in the final iteration are then used to obtain the complete eigenvectors as:

$$[\Phi_k] = \begin{bmatrix} I \\ R \end{bmatrix} [\Phi_{kk}] \tag{22}$$

(e) Response calculation: The calculated eigenproperties are then used in a response spectrum approach (e.g., Singh and Maldonado, 1991) to calculate the desired response.

It is noted that in the calculation of the response with dynamic interaction, one does not need any more information about the primary and secondary systems than what is needed in calculating the interaction free response. Therefore, if it is felt that dynamic interaction can be significant, the use of an approach such as the one presented above is advocated. The incorporation of dynamic interaction usually leads to a reduction in the response, and therefore, it must be an added incentive to adopt such an approach.

5 FUTURE DEVELOPMENTS IN THE SEISMIC ANALYSIS OF SECONDARY SYSTEMS

The previous discussion was primarily focused on the analytical methods which are available for seismic design evaluation of secondary systems. Although, the analytical developments are already well ahead of their implementation in practice, further research on other topics concerning these important systems must still go on. The topics of further interest are: (1) design of secondary systems on yielding primary systems, (2) seismic isolation of primary systems to reduce the intensity of motion on the secondary systems as well as isolation of the secondary systems themselves, (3) interaction between the secondary systems through their common primary system, (4) active and passive control of secondary systems, (5) response reduction of secondary systems by supplementary energy dissipation devices and (6) seismic analysis and design of rotating secondary systems. The writer is aware of active research being done on items (1), (2) and (6), and it is hoped that other topics will also be actively explored.

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7 REFERENCES

American Society of Mechanical Engineers, (1981). Boiler and Pressure Vessel Code Section III, Appendix N, Dynamic Analysis Methods.

Burdisso, R. A. and Singh, M. P., (1987). Multiply Supported Secondary Systems, Part I: Response Spectrum Analysis, *Journal of Earthquake Engineering and Structural Dynamics*, Vol. 15, pp. 53–72.

Igusa, T. and Der Kiureghian, A., Sackman, J. L., (1984). Model Decomposition Method for Stationary Response of Non-classically Damped Systems, *Journal of Earthquake Engineering and Structural Dynamics*, Vol. 12, pp. 121–135.

Igusa, T. and Der Kiureghian, A., (1985). Generation of Floor Response Spectra Including Oscillator-Structure Interaction, *Journal of Earthquake Engineering and Structural Dynamics*, Vol. 13, pp. 661–676.

Kelly, J. M. and Sackman, J. L., (1978). Response Spectra Design Methods for Tuned Equipment-Structure Systems, *J. Sound and Vibration*, Vol. 59(2), pp. 171–179.

Lin, C. W. and Loeff, F., (1980). A New Approach to Compute System Response with Multiple Support Response Spectra Input, *Journal of Nuclear Engineering and Design*, Vol. 60, pp. 347–352.

Maldonado, G. O. and Singh, M. P., (1991). An Improved Response Spectrum Method for Calculating Seismic Design Response; Part 2: Non-Classically Damped Structures, *Jour. of Earthquake Eng. and Str. Dynamics*, Vol. 20, (in press).

Singh, M. P., (1975). Generation of Seismic Floor Spectra, *Journal of the Engineering Mechanics Division*, Vol. 101, pp. 593–607.

Singh, M. P., (1980a). Seismic Design Input for Secondary Systems, *J. Structural Division*, ASCE, Vol. 106, pp. 505–517.

Singh, M. P., (1980b), Seismic Response by SRSS for Nonproportional Damping, *Journal of Engineering Mechanics Division*, ASCE, Vol. 106, pp. 1405–19.

Singh, M. P. and Burdisso, R. A., (1987). Multiply Supported Secondary Systems, Part II: Seismic Inputs, *Journal of Earthquake Engineering and Structural Dynamics*, Vol. 15, pp. 73–90.

Singh, M. P. and Suarez, L. E., (1987). Seismic Response Analysis of Structure–Equipment System with Nonclassical Damping Effects, *Journal of Earthquake Engineering and Structural Dynamics*, Vol. 15, pp. 871–888.

Singh, M. P. and Suarez, L. E., (1988). A Method for Dynamic Coupling with Nonclassical Damping Effects, *J. of Sound and Vib.*, Vol. 2(1),

Singh, M. P. and Burdisso, R. A., (1991). Design Response of Multiple Support Piping Systems, Proc. 9th Symposium on Earthquake Engineering, University of Roorkee, Roorkee, India, Dec 14–16.

Singh, M. P. and Maldonado, G. O., (1991). An Improved Response Spectrum Method for Calculating Seismic Design Response, Part I: Classically Damped Structures, *Jour. of Earthquake Eng. and Str. Dynamics*, Vol. 20, (in press).

Singh, M. P. and Suarez, L. E., (1991). Dynamic Condensation with Synthesis of Substructure Eigenproperties, submitted for possible publication in the Journal of Sound and Vibration.

Suarez, L. E. and Singh, M. P., (1987). Floor Resposne Spectra with Structure–Equipment Interaction Effects by a Mode Synthesis Approach, *Journal of Earthquake Engineering and Structural Dynamics*, Vol. 15(2), pp. 141–158.

Suarez, L. E. and Singh, M. P., (1991). A Dynamic Condensation Method for Structural Eigenvalue Analysis, AIAA Journal (in press).