# The nonlinear response of the complex structural system in nuclear reactors using dynamic substructure method

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## 1 INTRODUCTION

Becauce of the existence of nonlinear characteristics in practical engineering structures, such as large steam turbine-foundation system and ofshore platfotm, it is necessary to predict nonlinear dynamic responses for these very large and complex structural systems subjected extreme load. Due to the limited storage and high executing cost of computers, there are still some difficulties in the analysis for such systems although the traditional finite element methods provide basic available methods to the problems. The dynamic substructure methods, which were developed as a branch of general structural dynamics in the past more then 20 years and have been widely used from aircraft, space vehicles to other mechanical and civil engineering structures, present a powerful method to the analysis of very large structural systems. The key to success is due to the considerable reduction in the number of degrees of freedom while not changing the physical essence of the problems investigated. The dynamic substructure method have been extended to nonliner system and applicated to the analysis of nonlinear dynamic response of an offshore platform by Z.C.Zheng, et al.(1983, 1985a,b,c). In this paper, the method is presented to analyze dynamic responses of the systems contained intrinsic nonlinearities and with nonlinear attachments and nonlinear supports of nuclear structural systems. The efficiency of the method becomes more clear for nonlinear dynamic problems due to the adoption of iterating processes.

For simplicity, the analysis procedure is demonstrated brietly. The generized substructure method of nonlinear systems is similar to linear systems, only the nonlinear terms are treated as pseudo-forces. Interface coordinates are classified into two categories; the connecting interface coordinates which connect with each other derectly in the global system and the linking interface coordinates which link to each other through attachments.

In nuclear engineering, there exist some mechanical nonlinearities such as gap in supporting flanges between the core barrel and pressure vessel. The reactor piping system is taken as an example to illustrate the advantages of the methods.

## 2 PARTITION OF GLOBAL SYSTEM

The first step is to divided the complex structural system into several components (subsystems) according to the properties of the system and the requirements of the analysis when using dynamic substructure methods. The common partition is to disconnect the attachment if they exist in the system. For such models the components and attachments provide a natural way to identify and describe various portions of the substructural system since they have been used in the fabrication abd assembly processes. The generalized substructure method becomes more convenient and efficient by patitioning of the complicated system where the nonlinearities in the systems include intrinsic nonlinearities, nonlinear attachments, and nonlinear boundary.

The global system as shown in fig.1 is assumed to be divided into subsystems G, S, …H with connecting interface GS, …HS and linking interfaces. The boundarg is indicated by B. The coordinates of each component can be divided into three parts: internal coordinates $x_i$ direct connecting interface coordinates $x_j$ and linking interface coordinates $x_k$, such as for subsystem G connected with H.S.

(1) $$x^{(g)} = [\, x_i^{(g)T},\ x_j^{(gs)T},\ x_k^{(gh)T},\ x_k^{(gs)T}\,]^T$$

With consideration of geometric compatibility at the interface, we obtain

(2) $$x_j^{(gs)} = x_j^{(sg)}$$

If $f_j^{(gs)}$ and $f_j^{(sg)}$ denote the interface action and reaction forces at $x_j^{(gs)}$ and $x_j^{(sg)}$ respectively, $f_k^{(gh)}$, $f_k^{(gs)}$, $f_k^{(hg)}$ and $f_k^{(sg)}$ the action and reaction forces of attachments at $x_k^{(gh)}$, $x_k^{(gs)}$, $x_k^{(hg)}$ and $x_k^{(sg)}$ respectively, then they are obey the principle of equiliblium.

(3) $$f_j^{(gs)} = f_j^{(sg)},\quad f_k^{(gh)} = f_k^{(hg)},\quad f_k^{(gs)} = f_k^{(sg)}$$

## 3 ANALYSIS OF SUBSYSTEM

By means of finite element method, the equation of motion of any subsystem G excited by external forxes r(t) can be set up as

(4) $$m^{(g)}\ddot{x}^{(g)} + c^{(g)}\dot{x}^{(g)} + k^{(g)}x^{(g)} = r^{(g)}(t) - f^{(g)}$$

where $f^{(g)} = [\, f_{iN}^{(g)T},\ f_k^{(g)T},\ f_j^{(g)T}\,]$ . $f_{iN}^{(g)}$ represents the nonlinear part of intrinsic forces vector and depend on $x^{(g)}$ and $\dot{x}^{(g)}$. $f_k^{(g)}$ may be classified into linear term and nonlinear term, these forces not only depend on displacements and velocities of component G, but also depend on neighborhood connected components.

From equation (2), taking matrices m, k, we can get a set of different modal transform relation according to different dynamic substruture method such as free interface method by Hou (1969) and fixed interface method by Hurty (1965).

## 4 ASSEMBLAGE OF GLOBAL EQUATION

With consideration geometric compatibility eq.(2) and the eqs.(3) of equlibrium of forces at the interface, assembling the eqs. (2) of all substructures, we obtain the eqation as following

(5) $\tilde{M}\ddot{x} + (\tilde{C}+C_k)\dot{x} + (\tilde{K}+K_k)x = R(t) - F_{iN} - F_{KN}$

where x is the independent displacement vector of global system. Corresponding to x, we can obtain the assembled mass matrix $\tilde{M}$, damping matrix $\tilde{C}$ and stiffness matrix $\tilde{K}$, external force vector R(t). $C_k x + K_k \dot{x}$ and $F_{KN}$ are the linear and nonlinear terms of linking forces of attachments. The modal transform relation of physical coordinates x to modal coordinates q also can be ovtained by assembling all the relations of substructures as

(6) $q = \phi x$

Substituting eq.(6) to eq.(5), we get the reduced governing equation as

(7) $Mq + Cq + Kq = Q(t) + F_N$

where

$$M = \phi^T \tilde{M} \phi,\ C = \phi^T(\tilde{C}+C_k)\phi,\ K = \phi^T(\tilde{K}+K_k)\phi$$

$$Q = \phi^T R,\ F_N = \phi^T(F_{iN}+F_{KN})$$

The nonlinear pseudo-force vector $F_N(q,\dot{q})$ combined intrinsic nonlinear force vector $F_{iN}$ and nonlinear linking force vector $F_{KN}$ is the function of q and q. The global equation (7) has much fewer degrees of freedom than the original equations derived directly from the global system by means of FEM, although it is still nonlinear. It is easier to perform the nonlinear response analysis for the reduced equation by means of numerical method such as Newmark-$\beta$ method. Because of the unknowns con tained in the pseudo-forces of the right terms, the iteration procedure is adopted.

## 5 EXAMPLE

A simplified vessel-piping system as shown in fig.2 was investigated. Naturally, the system was divided into two subsystems, one was the vessel and the other the piping system. Point C in fig.2 is the interface of the subsystems. Taking 168 total DOF and 17 reduced DOF, we calculated the lower order modes and the response of the globle system. Table 1 gives the first six order natural frequencies of the linear system with the interface being both connecting interface and linking interface.

Table 1 The first six order natural frequencies (Hz)

| order | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| connecting interface | 2.044 | 2.620 | 4.268 | 5.615 | 5.978 | 8.669 |
| linking interface | 1.904 | 1.914 | 2.502 | 3.910 | 4.127 | 5.358 |

Fig.5 to fig.8 are response curves which were got by means of Newmark-$\beta$

method ( $\beta$ = 0.25, $\beta$ = 0.5, t = 0.02). For all curves, spring H is linear while the other springs may be linear or nonlinear. The force displacement curve (F-x) of nonlinearity is as shown in fig.3. Fig.4 are exciting forces. The maximum exciting force is 6.5T in fig.5, 12.5T in fig.6, 20.0T in fig.7 and fig.8. The letter $\xi$ in figures represents modal damping ratio.

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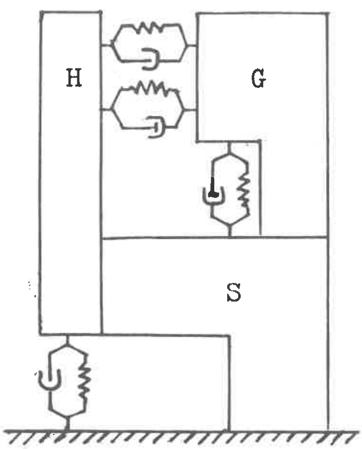


Fig.1 Global system

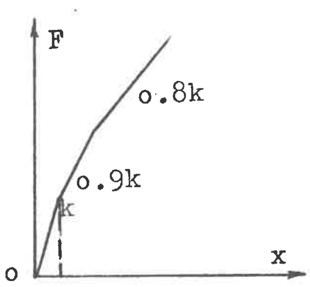


Fig.3
F-x curve of nonlinear spring

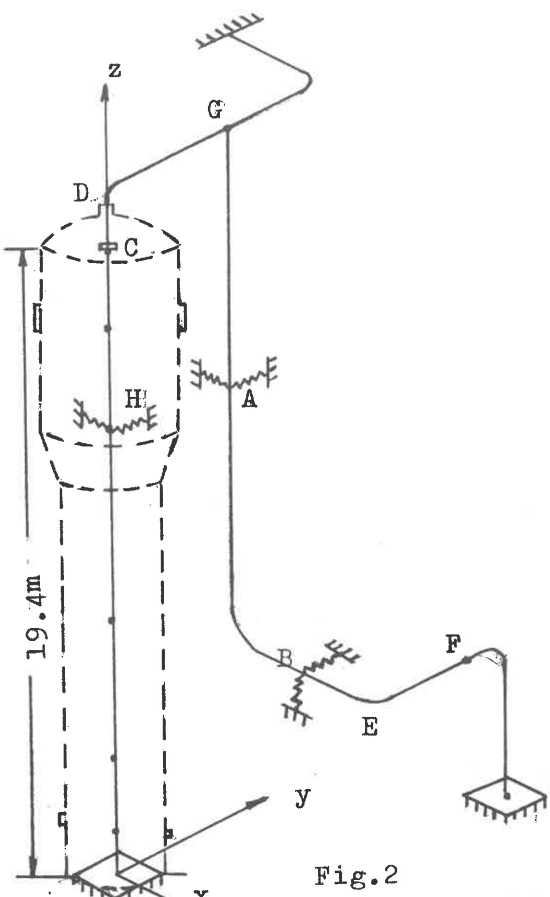


Fig.2
Vessel-piping system

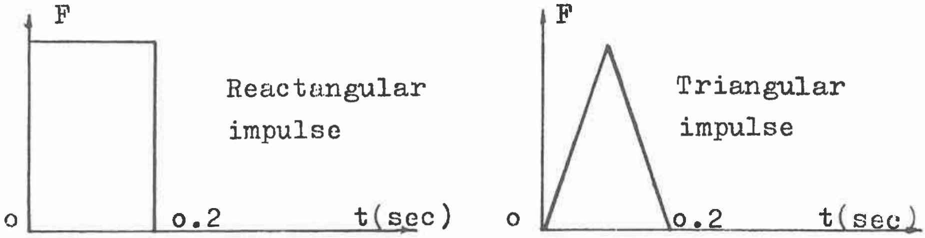


Fig.4

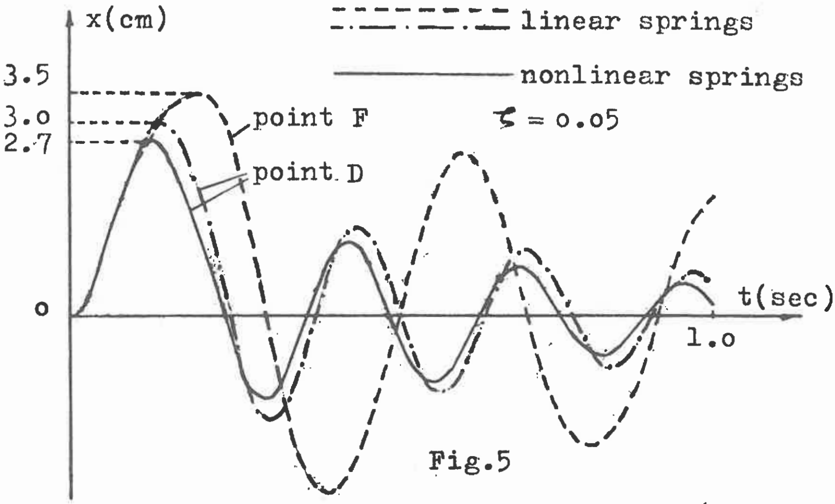


Fig.5

Displacements in the direction of x (Rectangular impulse acts on point A in the direction of x)

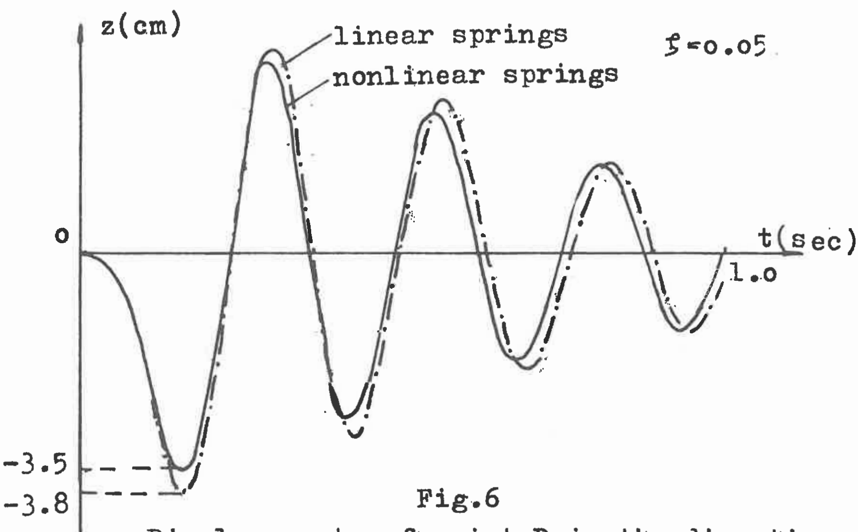


Fig.6

Displacements of point D in the direction of z (Triangular impulse acts on point A in the direction of y )

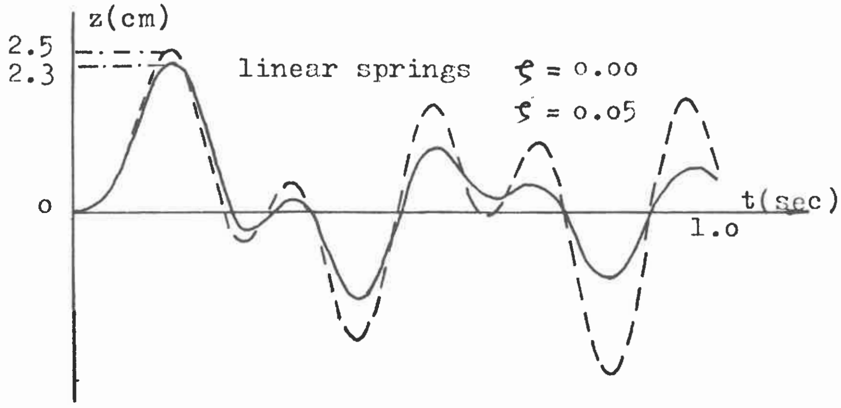


Fig.7

Displacements point E in the direction of z (Triangular impulse acts on point B in the direction of z)

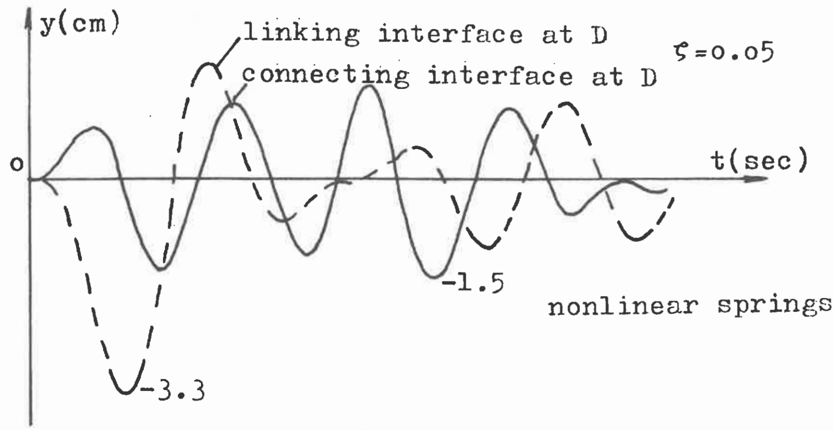


Fig.8

Displacements of point A in the direction of y (Triangular impulse acts on point B in the direction of z)