An examination of the sensitivities resulting from simplified seismic probabilistic risk analysis

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1. INTRODUCTION

In order to carry out a complete seismic probabilistic risk analysis of a complex structure such as a nuclear power plant, it is necessary to define both the site seismic hazard and the response or fragility of the various components of the plant in some detail. The site seismic hazard is generally known to a reasonable degree of confidence, but for various reasons, not least of which is lack of data, it is not always possible to be precise about the fragility, and thus the only alternative is to make certain simplifying assumptions regarding this failure distribution.

This paper descibes an exercise which has been carried out to test the sensitivity of various assumptions. Previously determined site hazard distributions representing low, average and high seismicity (Irving, J., 1985) for the United Kingdom have been chosen for this study. Combining these with various assumed fragility distributions leads to an estimate of the risk of failure due to the occurrence of earthquakes at the particular site.

2. DETERMINATION OF RISK

For the purpose of this exercise, the risk implied as a result of the occurrence of earthquakes, is considered to be equivalent to the probability of failure or loss of function of the overall system. The risk of failure therefore from a number of different earthquake initiating events is calculated by simple summation over all likely events, of the product of the probability of occurrence of the event P_i, and the consequent probability that the structure will fail, G_i, that is:

$$R = \sum_{\text{all } i} P_i \, G_i \tag{1}$$

Earthquakes can range in strength, and this strength can be represented by a choice of parameters such as intensity, peak velocity, peak acceleration, etc. The probability of failure, given the occurrence of an earthquake, or fragility as it is more generally known, is also a function of the strength of the event. The probability of damage, or the risk, is determined by convolving the failure probability distribution with the occurrence probability distribution. This convolution is considerably simplified if both distributions are mapped to the same parameter. The choice of peak ground acceleration, which is certainly the most popular, as the common representation of the earthquake strength, assumes that the fragilities of components include the effects of the transfer functions of the supporting systems and structures. If peak support acceleration is chosen as the common parameter, then it is assumed that the hazard distribution includes the transfer function of the supporting system and structure.

Thus, if P(g) represents the hazard distribution or probability of occurrence at the site, and G(g) represents the fragility distribution, then the risk is given by:

$$R = \int_0^{g'} P(g) \; G(g) \, dg \tag{2}$$

considering earthquakes up to and including peak acceleration g'.

The term represented by P(g) is the probability density function of the more usual earthquake hazard or probability of exceedance of a particular earthquake strength. The term represented by G(g) is the actual fragility of the component, defined as the conditional probability of failure of that component versus an input parameter usually characterised in terms of the peak acceleration. The distribution assumed for G(g) is the subject of further discussion in this paper.

3. FRAGILITY DEFINITIONS

In current seismic probabilistic risk analysis, each component or structure can exist in only two states, that is, either success or failure. Failure in this case also includes loss of function or failure to perform its function. The adoption of this two state approach is dictated to a large extent by the rules of Boolean Algebra coupled with the adoption of fault trees in the analysis.

Thus, the simplest representation of the fragility is by a step function centred about a mean value. The overall risk, as a function of the mean or median value can be calculated for the three levels of seismicity referred to earlier.

However, there are many uncertainties associated with the response or behaviour of components and structures when subjected to an earthquake motion as input. Because of various uncontrollable processes employed in the manufacturing route, no two seemingly identical components would behave precisely in the same manner. The uncertainty associated with this aspect is justifiably considered to be random.

In addition, the loading also is associated with a measure of uncertainty. Although earthquake time histories can be characterised by their peak acceleration values, the dynamic response of a structure in terms of the peak values of acceleration, velocity and displacement will not be the same for different earthquakes with the same peak acceleration values. The main reason for this is that to characterize earthquakes in terms of their peak acceleration, no information as to their frequency content is conveyed in this specification and since the frequency content obviously affects the response of the structure, it is not feasible to predict precisely the displacements and therefore the induced stresses in the components in question. Hence, a further contribution to the uncertainty which again is justifiably considered to be random. Other factors contribute to the uncertainty in a similar manner.

Since fragility has already been defined as a probability which is a function of the fragility parameter taken to be the peak acceleration g, it is appropriate to consider classical probability distributions. The fragility G(g) is related to the fragility density function f(g) by the following expression

$$G(g) = \int_0^g f(g)\,dg \tag{3}$$

The simplest approach is to assume that the input parameter corresponding to failure or the attainment of a limit state is a random variable characterised by a mean value μ and a standard deviation σ, and therefore should be satisfied by a normal distribution. However, the existence of negative peak accelerations is not considered to be possible in this sense and the next logical step therefore is to consider the use of the log-normal distributions with logarithmic mean μ and standard deviation σ.

$$f(g) = \frac{1}{\sqrt{2\pi}\,\sigma\,g} \exp\left\{-\tfrac{1}{2}\left(\frac{\ln g - \mu}{\sigma}\right)^2\right\} \tag{4}$$

It can be shown that the analytical form of the fragility function should result in maximum entropy (or uncertainty) given the present knowledge. If only the mean and standard deviation of the random failure indicator g are known, then the distribution which maximizes entropy is the normal distribution. If the additional knowledge is included that the failure indicator g is essentially a positive number, then the distribution which maximizes entropy is the log-normal.

The two parameters which specify the log-normal distribution are the median peak acceleration A given by exp (μ), and the log- normal standard deviation β_R, given by σ. The median, that is, the peak acceleration for which the fragility is 0.5, describes the central tendency or location of the fragility function. The standard deviation, β_R, describes the slope of the fragility function.

Given the values of A and β_R, the fragility is defined. However, in general, these two parameters are unknown and it is therefore necessary to derive some estimates of these parameters. Estimates can be made from a variety of sources which might include, but are not restricted to, experimental test data, historical data, empirical models, engineering judgement and expert opinions. Uncertainty will be associated with these processes and it is only reasonable that due account should be taken of this modelling uncertainty.

The fragility curve characterized by the median A and the logarithmic standard deviation β_R describing the random uncertainty represents the analysts best estimate for the true fragility of the component. As has just been discussed there is modelling uncertainty in the mediam value A which can be characterized by a logarithmic standard deviation β_U. Thus, the fragility can be represented by a median curve and in addition, a curve either side of the median representing a lower bound and an upper bound, typically 5% and 95%.

In this case it would be possible to make a confidence statement of the type "we are 90% confident that the fragility at the specific peak acceleration of the component falls within these bounds".

The term "High Confidence of Low Probability of Failure" (HCLPF) has come into general usage and is typically defined as about 95% confidence of less than approximately 5% probability of failure. Expressed mathematically, the HCLPF acceleration is:

$$= A \exp \left[-1.65 \left(\beta_U + \beta_R\right)\right] \tag{5}$$

This represents an acceleration value for a component for which we are highly confident that given this level of peak acceleration, there is only a small chance of achieving the limit state or failure.

Since the distribution representing the random and modelling uncertainties are taken as log-normal, it is possible to combine them into a composite fragility curve with a logarithmic standard deviation β_C given by:

$$\beta_C = \left(\beta_U^{\,2} + \beta_R^{\,2}\right)^{\frac{1}{2}} \tag{6}$$

If we now consider the composite fragility curve and take the HCLPF values for low, average and high seismicity to be 0.15g, 0.20g and 0.25g respectively, then for a range of values of β_C, we can calculate appropriate median values and define the fragility curves. The details are shown in Table 1:

β_C \ Seismicity	Low	Average	High
0.4	0.38	0.51	0.63
0.5	0.48	0.64	0.80
0.6	0.61	0.81	1.01
0.7	0.77	1.02	1.28

When convolved with the hazard distribution, the total risk can be calculated from equations (2) and (3).

It can be argued however, that the tails of the log-normal distribution are inappropriate because experience has shown that failure of a component or structure cannot occur below some lower limit g_{min}, and always occurs above some limit g_{max}. This has led to the proposal of the Johnson distribution (Johnson, N.D. 1949) for the fragility density function and is expressed as:

$$f(g) = \frac{(g_{max} - g_{min})}{\sqrt{2\pi}\,\sigma\,(g-g_{max})(g_{max}-g)} \exp\left\{-\tfrac{1}{2}\left(\frac{\ln\left(\frac{g - g_{min}}{g_{max} - g}\right) - \mu}{\sigma}\right)^2\right\} \quad (7)$$

where $g_{min} \leqslant g \leqslant g_{max}$, and 0 elsewhere.

This again can be shown to lead to the maximum entropy condition, given the controlling parameters. Convolution, as before, leads to the calculation of the total risk.

4. DISCUSSION OF RESULTS

The results are displayed graphically and concisely for the three levels of seismicity in Figures 1, 2 and 3. The simple step distribution, as reported shows the sensitivity to the assumed hazard distribution and at any one peak acceleration level, the total risk for high and low seismicity is within a factor of 2 or 0.5 respectively about the average.

The log-normal distribution shows the expected tendency to the step value at the low median levels and the total risk shows only small variation with seismicity, reflecting the choice of the HCLPF values as the design point. However, the total risk appears to be rather insensitive to the median value and is in general greater than 5 x 10^{-6} per annum for median values less than 1.0g.

The Johnson distributions again tend to the log-normal at low median values and also approximate to the step function. The curves labelled (3) and (2) in the figures have the lower limit set at the design point and design point plus 40% respectively. These curves demonstrate that the contribution from the lower tail of the log-normal distribution is considerable and represents a margin in the assessment of risk which is unjustified.

The curves vindicate the policy of designing to the levels suggested and further qualification at the design level plus 40%. An increase in this margin would show minimal returns since the attendant risk curve could not fall below that due to the simple step distribution.

It can be seen that the combination of fragilities through a fault tree must be very sensitive to the shape of the fragility distribution and the effective range of applicability defined by g_{max} and g_{min}. Hence the importance of a correct format for all components in a seismic probabilistic risk assessment.

5. CONCLUSIONS

An exercise to convolve fragility distributions with the seismic hazard at a site has been carried out. The importance of defining a lower limit for the fragility distribution by qualification for example has been highlighted. Armed with this knowledge plus an indication of the median values lead to the suggestion that the step distribution can give a good representation of the overall risk for ranking in preliminary studies. The log-normal distribution is inherently incorrect and only serves to propagate unreal conservatisms through the analysis leading to overestimation of the importance of seismic risk.

REFERENCES

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