A Graph Theory Approach to the Quality Assurance of Welds

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Abstract

Although the quality assurance of a weld is quite important for improving the safety and reliability of a nuclear pressure vessel and/or its piping system, it is not easy to make a mathematical model for it. The greatest difficulty comes from the fact that as the quality of a weld is influenced by a great number of factors, there are not a few "difficult to quantify" factors. But with the remarkable progress of welding technology, their failure logics have been clarified to a considerable degree. Therefore, this paper attempts a logical approach and proposes a new methodology to provide an objective basis for making decisions to secure the quality of a weld. This method is based on the idea that the failure logic of producing a weld defect can be expressed as a network of digraphs without any advanced mathematical knowledge, as long as the causalities are clear between each pair of events, so that we can incorporate "difficult to quantify" factors into analysis without any difficulty. As a network representation provides more visibility than a conventional matrix representation which has often been used in welding, it will substantially reduce the danger of overlooking the important failure modes, especially those of multi-stage causalities. And once the failure logic is expressed as a network, it is easy to re-express it in the form of a matrix and then with the help of graph theory techniques we can ferret out the important failure events which must be prevented to secure the quality of a weld.

The major advantages of this method are: (1) No matter how complicated and large-scaled the problem might be, we can simplify the failure logic into the form of a logic tree in a straightforward manner with the help of a computer, so that we can easily extend the analysis to the problem of securing the structural integrity of a nuclear pressure vessel and/or its piping system; (2) As this method is capable of symbol processing, we can evaluate the importance of each event and make decisions without any knowledge of quantitative data on each event; (3) As a logic tree can be obtained as the final result, the construction of a data base system will be greatly facilitated; and (4) It is easy to switch from qualitative analysis to quantitative analysis, if it is necessary.

1. Introduction

It is widely accepted that the quality assurance of welds is quite important for improving the safety and reliability of a nuclear pressure vessel and/or a piping system. But due to such reasons as (1) Welding phenomenon itself is too much complicated and welding is related to a wide variety of engineering fields, (2) Not only the stage of welding but also the previous stages of manufacturing influences a great deal on the quality of a weld, (3) There are not a few problems left which are still not made clear quantitatively, although their failure logics have been clarified, and (4) There are not a few problems which cannot be quantified easily, such as the influence of the skill of a welding and/or an NDI technician on the quality of a weld, it is not easy to make a mathematical model for the quality assurance of a weld. Further, since such large structures as pressure vessels are produced only in small quantities, the techniques of the conventional reliability engineering which is based on statistics and which has been developed mainly for mass-produced products cannot be applied in a straightforward manner.

This paper points out that graph theory is one of the most effective tool to eliminate these difficulties and proposes a methodology of applying graph theory to the quality assurance of a weld. As the main aim of this paper is to describe the fundamental concept, only simple illustrative example is analyzed.

2. Graph Theory and Conventional Matrix Representation in Welding

Up to now, such a matrix representation as in Fig. 1 has been used frequently in the field of welding to express the causalities, where a row and a column corresponds to cause and effect respectively [1]. Although such a matrix representation is quite useful as long as the problem is relatively small and the relations between events have immediate causalities, it is not easy to understand how a weld defect (used in the broadest sense in this paper) is produced in the case of a multi-stage causality. And especially when we consider the problem of the quality assurance of welds from the standpoint of securing the structural integrity of a pressure vessel and/or a piping system, the scale of the analysis becomes much larger. Therefore, the danger of failing to notice the failure mode corresponding to a multi-stage causality increases exceedingly.

As there are so many factors which affect the quality of a weld in a pressure vessel and/or a piping system, their failure logics are quite complicated. Consequently, the more and more complicated the failure logics are, it becomes more and more difficult to express the failure logic directly in the form of a matrix and the possibility of failing to notice the important failure modes, especially the ones corresponding to multi-stage causalities becomes larger. Further, even if we should be able to represent the failure logic exactly, we cannot easily understand what factor influences the most by merely looking at the matrix.

Now, if we represent two events by circles called nodes, connect the nodes by a line, and put an arrow to it, we can express the causality between these two events as the initial node and the terminal node representing the cause and the effect respectively. In this manner, the same relations of causalities as in Fig. 1 can be expressed as an assembly or a network of directed graphs or digraphs. This representation using digraphs is, therefore, utterly identical with that of a matrix, but it has such advantages as (1) As a graph network representation provides more visibility, the quite complicated relation of causalities can be understood more easily, (2) It is quite easy to construct the whole structure of a network by assembling small modules of digraph networks. Therefore, we can reduce considerably the danger of overlooking the important failure modes, especially those of multi-stage causalities, (3) Once the failure is expressed as a network of digraphs, it is solely a mechanical operation to re-express it in the form of a matrix. As a present-day computer can process a quite large matrix, it is quite easy to find the effective measures for quality assurance by matrix operation, even if the problem, i.e., the matrix is quite large-scaled, and final but most important of all is that (4) No matter how complicated the failure logic might be, it can be expressed as a network of digraphs, as long as the cause and effect relations between pairs of events are clear. Therefore, it is relatively easy to incorporate into analysis such factors as the skill of a welding and/or an NDI technician or experiences of engineers and/or technicians which are difficult to quantify.

The point we should emphasize here is that if we utilize graph theory, we can make a logical approach other than a quantitative approach to the failure analysis for quality assurance. And it must also be added that this approach is not contradictory to the conventional quantitative approaches, but that it is rather a complementary one.

3. Simple Illustrative Example

To discuss the effectiveness of the graph theory approach, let us take up a simple illustrative example of Fig. 2. This illustrative example is a most simplified network representation of the logic of the failure (the state in which required quality cannot be secured.) of the longitudinal welded joint in a pressure vessel. Edge 1 denotes, for example, a failure in the stage of bending due to improper material selection, and Edge 3 denotes, for example, a failure in the state of welding due to improper edge preparation. Of course, when we apply this method to the quality assurance of an actual pressure vessel and/or a piping system, we have to define the content of each event more accurately. But just for the purpose of discussing the effectiveness of the approach, it is much easier and more understandable, and without loss of generality, if we consider a simple network.

The identical relation with that of Fig. 2 can be expressed in the form of a matrix in the following manner. First, we assign nodes and edges to rows and columns respectively. Then with respect to an arbitrary column, we let the elements at the locations of an initial node and a terminal node be 1 and -1 respectively, and let other elements in the column be 0 (Fig.3). Further, if we note that any one of these rows can be expressed as a linear combination of other rows, we can express the identical relation with a one-row-less, or reduced, matrix as in Fig.4 .

Now, let us go back and consider Fig.2 . As one edge in the figure denotes a failure, it means to prevent the failure corresponding to that edge to cut it. The quality assurance of a weld is, therefore, no other than to find the disconnecting sets of edges between Node a and Node e. In graph theory, such a set of edges which cut the connections between a pair of nodes is called a cut set, and various techniques have been developed to obtain cut sets. The effective measures for quality assurance can be determined by obtaining all possible cut sets and by selecting cut sets to which measures can be taken easily.

In the case of Fig.2 , all possible cut sets can be obtained by multiplying the matrix of Fig.4 by the matrix of Fig. 5 from the left hand side. We will explain the meaning of the matrix of Fig.5 briefly. To disconnect Node a and Node e means to divide other Node b - d into two node sets, i.e., the node set A and the node set E which contain Node a and Node e respectively. Now according as Node i belongs to A or E, we assign the value of 1 or 0 to it. Thus, Node a always has the value of 1 and Node e has the value of 0. As we have three other nodes, i.e., Node b - Node d, and each of these three nodes can take the value of 0 or 1, we have 2^3 ways of disconnecting Node a and Node e. These independent 8 ways of disconnecting Node a and Node e are expressed as rows in Fig.**6.** If we multiply the matrix of Fig.4 by the matrix of Fig.5 from the left hand side, we obtain -1's in the elements of the resulting matrix. But the cut sets we are now considering are outgoing (from Node a to Node e) cut sets, so that we may eliminate incoming cut sets. Therefore, by letting these elements of -1 be 0, we can obtain the necessary cut sets as the rows in Fig.6.

Table I shows the edges to be cut. It follows that we can assure the quality of a weld by taking the measures against the failures corresponding to <u>any one</u> of these (1) - (8) cut sets. For example, if we consider the cut set (1), we have to prevent <u>all</u> failures corresponding to Edges 1, 5 and 6.

This content can be expressed as a logic tree shown in Fig.7 In the figure, the symbol i in a circle denotes the event that we prevent the failure corresponding to Edge i, and the symbol T in a rectangle means that the quality of a weld is secured. Once the logic of assuring the quality of a weld can be expressed in this manner using a tree, we can evaluate the importance of each basic event represented by a circle, using a structural importance evaluation technique in Fault Tree Analysis [4], [5], [6].

Here, we will evaluate the importance of each basic event, i.e., the importance of the prevention of the failure corresponding to that edge in terms of the quality assurance of a weld by visual inspection instead of a conventional numerical calculation. It is obvious that the greater the number of an edge is in Table I (or Fig.7), the more important it is to prevent the failure corresponding to that edge to assure the quality. But to take Edge 1 and Edge 9 for example, although they appear 4 times in Table I , Edge 1 appears 3 times in 3-edge cut sets, once in a 4-edge cut set, and Edge 9 appears once in a 3-edge cut set, once in a 4-edge cut set, and twice in 5-edge cut sets. Even if the number is the same in Table I , the more important an edge is, if it appears in a cut set with the smaller number of edges. Therefore, Edge 1 is more important than Edge 9. Thus, we can classify the events according to their importances as shown in Table II. It is observed from the table that Edge 1 and Edge 6 are at the higher level, so that the selection of the material is quite important for quality assurance, as long as the failure logic is such as expressed in Fig. 2. And it is in a sense quite natural that Edge 4 is at the top level, since we are considering the quality of a weld here. It is interesting to note that Edge 7 is at the highest level, if we consider the fact that grooves are becoming narrower and narrower in main welded joints in order to secure a better quality.

It should be noted here that the above process of evaluation is none other than symbol processing. Therefore, the events can be just symbols or sentences, and they do not necessarily have to be expressed quantitatively. Furthermore, as the complicated network of a failure logic is thus simlified into a logic tree, we can greatly facilitate the construction of a data base system, because a logic tree is essential to its construction. And it must also be added that once a logic can be expressed in the form of a logic tree, it is easy to switch from qualitative analysis to quantitative analysis [5], [6]. And the above approach could be used too to pinpoint at which location quantitative data are lacking to carry out a quantitative analysis. Thus it would serve a great deal to collect the more fit-for-the-purpose data.

4. Conclusions

The most important point to be emphasized in this paper is that if we take note that a logical approach is possible to the problem of the quality assurance of a weld and that its failure logic is expressible as a network of digraphs, we can make effective decisions with the aid of graph theory techniques, no matter how complicated the problem might be.

The major advantages of the method proposed herein are as follows;

(1) It is quite consistent with computer processing, so that we can analyze the problem of quality assurance in a straightforward manner no matter how complicated and large-scaled the problem might be. Therefore, we can easily extend it to the problem of securing the structural integrity of a pressure vessel and/or a piping system.

(2) As the process of obtaining a network of digraphs requires only the knowledge of the causality between a pair of events, we can incorporate "not easily quantifiable" factors into analysis without any difficulty, as long as the relation of cause and effect is clear.

(3) As this method is capable of processing symbols, the events do not necessarily have to be quantitative data, but they could be sentences. Therefore, it is expected that it will provide a versatile and quite useful tool for decision-making in the quality assurance problems.

(4) Since the essence of this method is to construct a logic tree, it will greatly facilitate the construction of a data base system.

(5) It is quite easy to switch from qualitative analysis to quantitative analysis, if necessary quantitative data could be obtained.

References

/ 1 / SEJIMA, I, "Managament of Welding - - Concepts and Applications", 75 (1968) Nikkan-Kogyo-Shimbun-Sha (in Japanese)
For graph theory, see references [2],[3] for example.

/ 2 / BUSACKER, R.G., SAATY, T.L., "Finite Graphs and Networks:An Introduction with Applications", (1965) McGraw-Hill, Inc.

/ 3 / WAGNER, H.M., "Principles of Operations Research with Applications to Managerial Decisions" 2nd. ed. (1975) Prentice-Hall, Inc.

/ 4 / BARLOW, R.E., PROSCHAN, F., "Statistical Theory of Reliability and Life Testing" (1975) Holt, Rinehart and Winston, Inc.

/ 5 / FUKUDA, S., "Improvement of the Safety and Reliability of a Welded Structure: an FTA Approach", Proceedings Int. Conf. on Welding Research in the 1980's, Osaka, Japan, October 27 - 29, 1980, Paper P - 16

/ 6 / FUKUDA, S., "An Application of Graph Theory to the Safety and Reliability Analysis of a Pressure Vessel", Proceedings Fourth Int. Conf. on Pressure Vessel Technology , London, England, May 19 - 23, 1980, Paper C12/80

$$\begin{array}{c|cccccc} & A & B & C & D & E & F \\ \hline a & L & & & L & & \\ b & & & L & S & & \\ c & L & L & S & L & L & L \\ d & S & S & & S & L & L \\ e & L & S & & L & L & S \\ f & L & L & S & L & L & L \\ g & L & L & L & L & L & L \\ h & L & S & S & L & L & S \\ i & S & S & L & L & L & L \\ j & S & S & L & L & L & L \\ k & L & S & L & L & L & L \\ l & L & S & & L & L & S \\ m & & & L & & & S \\ n & L & L & L & L & L & L \\ o & S & & L & S & S & S \end{array}$$

Cause events

a=Maintenance of material
b=Material used
c=Maintenance of welding machines
d=Electric power sources and earth
e=Jigs and positioners
f=Instruction and management of welding operators
g=Selection of electrodes
h=Maintenance of electrodes
i=Welding design
j=Size and type of groove
k=Accuracy of fitting
l=Welding position
m=Welding sequence
n=Welding condition
o=Standard practices for welding

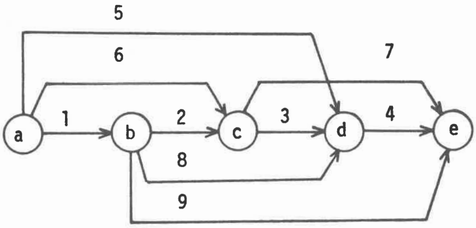
Effect events

A=Undercut
B=Overlap
C=Crack
D=Blowhole
E=Slag inclusion
F=Lack of penetration

* Note

L and S denote large and small influence respectively.

Fig.1 Conventional matrix representation in welding



a=Material selection
b=Bending
c=Edge preparation
d=Welding
e=Defect

Fig.2 Sample network of a failure logic

$$\begin{array}{c|ccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline a & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ b & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ c & 0 & -1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ d & 0 & 0 & -1 & 1 & -1 & 0 & 0 & -1 & 0 \\ e & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & -1 \end{array}$$

Fig.3 Node-edge incidence matrix of Fig.2

$$\begin{array}{c|ccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline a & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ b & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ c & 0 & -1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ d & 0 & 0 & -1 & 1 & -1 & 0 & 0 & -1 & 0 \end{array}$$

Fig.4 Reduced node-edge incidence matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Fig.5 Matrix the rows of which represent independent node sets

$$\begin{array}{c} \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \\ (7) \\ (8) \end{array} \begin{array}{c} \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Fig.6 Cut set matrix

Table I Cut sets of Fig.2

Cut set	Edges
(1)	1 5 6
(2)	1 4 6
(3)	1 3 5 7
(4)	1 4 7
(5)	2 5 6 8 9
(6)	2 4 6 9
(7)	3 5 7 8 9
(8)	4 7 9

Table II Importance of events

↑ Higher	1 4
	6 7
	5 9
	2 3 8

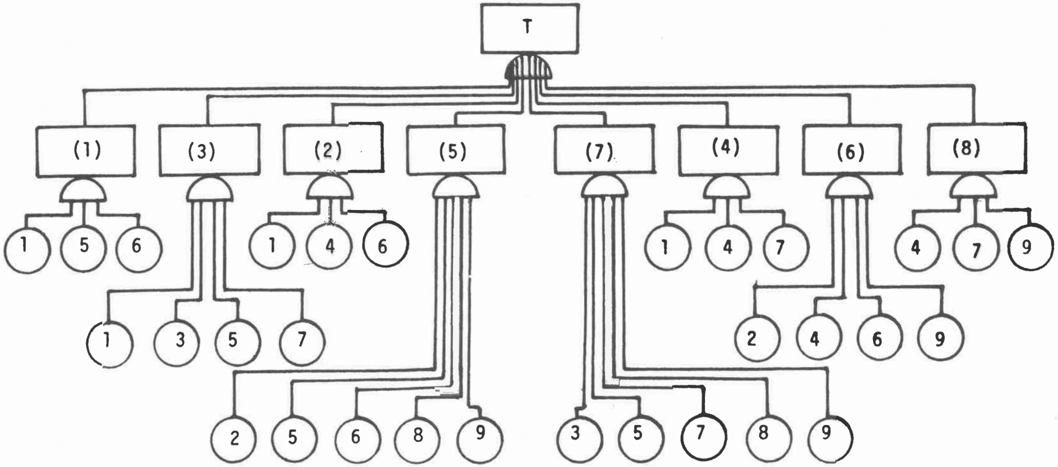


Fig.7 Logic tree of Fig.2