PROBABILISTIC BRITTLE FRACTURE ANALYSIS FOR MAJOR THERMAL TRANSIENTS IN PRESSURE VESSELS

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SUMMARY

Brittle fracture of a pressure vessel during a major thermal transient occurs when the thermal stresses induced by the transient combine with the operating stresses to give a stress intensity at the tip of an existing crack which exceeds the critical stress intensity for that time and position in the vessel. The determination of both the local stress intensity, K_1, and the critical stress intensity, K_{1C}, depends on inferential analysis of various kinds of material data. Hence the question of K_1 exceeding K_{1C} or vice versa is more properly posed as a probability problem involving these stress intensities interpreted as random variables. These random variables, in turn, may be constructed from quantities that are directly or indirectly determined from experiment and thus interpreted as more basic random variables.

The formulation of the probability problem first requires the identification of components of stress intensity that are properly and effectively treated as basic random variables. The second stage of problem development calls for the examination and analysis of available data to construct realistic distribution functions for these random variables and obtain estimates for the parameters in them. In the third and final stage of the problem evaluation of the failure probabilities under specified input conditions is accomplished.

In the initial attack on this problem, only K_{1C}, the critical stress intensity, was treated as a random variable. Available data from the HSST program were studied and it was determined that a three-parameter gamma distribution with temperature dependent parameters gave a satisfactory description of the observed K_{1C} variability. As a demonstration of the simple procedure at this stage, fracture probability during the cold hydro test was estimated as a function of the largest existing crack prior to the test.

In order to treat transients occurring after vessel service of specified duration, irradiation embrittlement is introduced and a random variable defined for the shift in nil-ductility temperature. Again available data are studied to generate satisfactory distributions for ΔRT_{NDT} and obtain estimates of the necessary parameters. Appropriate Monte Carlo techniques for estimating the brittle fracture probability are employed.

In conclusion it is shown how to extend this methodology to incorporate more components of variability, including some in the calculation of K_1.

1. Introduction

In this paper we deal with one phase in the development of statistical methodology for a fracture mechanics analysis of the failure of nuclear steam supply system components. In particular we address brittle fracture in the beltline region of the pressure vessel resulting from a single unscheduled or scheduled transient. Current practice is to use a deterministic approach to decide whether a given transient will cause brittle fracture or not. In this deterministic approach, linear elastic fracture mechanics (LEFM) is used to determine the stress intensity, K_1 (at the assumed crack tip) and a bounding critical stress intensity, K_{1C}, based on test data. Comparing these two quantities we can see if the vessel will fail ($K_{1C}<K_1$) or not. We have introduced a probability structure into the LEFM calculations by considering K_{1C} to be a temperature dependent random variable so that its probability distribution will reflect the variability seen in the K_{1C} data. We have initially taken the local stress intensity K_1 to be a known function of an assumed crack and the particular transient under consideration. Using fracture mechanics notation, the probability of brittle fracture is simply Pr ($K_1>K_{1C}$), that is the probability that the stress intensity K_1 at the tip of the crack exceeds the critical stress intensity K_{1C} at that point.

The first transient we examined was the cold hydro test (a preoperational test condition). A careful study of the available K_{1C} data led to the selection of a three parameter gamma distribution to characterize the variability in observed K_{1C} values for a fixed temperature. After estimation of the parameters in the gamma distribution for the appropriate temperature, the probability of brittle fracture in the cold hydro test assuming a 6:1 semielliptic 1/4 T (a flaw whose depth is 1/4 vessel thickness) flaw exists was estimated.

A number of other transients were then examined and corresponding estimates of brittle fracture calculated. Since these are transients that occur during the operational lifetime of the reactor, it was necessary to introduce a temperature shift in the K_{1C} curve to reflect the embrittlement of the steel due to neutron irradiation. For each of the operational transients thus considered the probability of brittle fracture was less than 2×10^{-10}.

In order to improve the adequacy of our model, a second component of variability was introduced for the magnitude of the irradiation embrittlement. Data for ΔRT_{NDT} were collected and a satisfactory descriptive model was generated; here a normal distribution was used to characterize the variability. A model, with temperature dependent parameters, was developed for the initial K_{1C} values. With the parameters of the K_{1C} and ΔRT_{NDT} distributions as input, a Monte Carlo calculation using importance sampling was run to estimate the probability of brittle fracture.

Section 2 of this paper deals with the earlier form of the problem in which only the initial K_{1C} is considered random. Section 3 is concerned with the improved model in which variability in ΔRT_{NDT} is introduced. A brief appendix describing importance sampling is attached.

2. Model with Deterministic Embrittlement Shift

We want to estimate the probability of brittle fracture of the pressure vessel in the beltline region due to a system pressure-temperature transient. In the LEFM terminology we want to estimate the probability that the stress intensity, K_1, at the crack tip exceeds the fracture toughness (critical stress intensity), K_{1C}, of the material; we want Pr $(K_{1C}<K_1)$. Our starting point for describing how this probability is estimated will be to outline how K_{1C} and K_1 are computed deterministically. We require the following notation

$L \equiv$ age of the pressure vessel in effective full power years (EFPY)

$t \equiv$ elapsed time since transient began

$\phi \equiv$ neutron dose

$x \equiv$ fractional distance of crack tip through the vessel wall

$T \equiv$ temperature of the pressure vessel at crack tip

$RT_{NDT} \equiv$ reference nil ductility transition temperature

$\Delta RT_{NDT} \equiv$ change in RT_{NDT}

To obtain K_{1C}, curves similar to those in Figures 1-4 are used.

Specific values of x, t and L are chosen. Using the values of x and t in Figure 1 yields a value of T and then x and L in Figure 2 gives a value of ϕ. Next this ϕ used in Figure 3 results in ΔRT_{NDT}. This ΔRT_{NDT} must be subtracted from the initial RT_{NDT} (a given value), and the resulting value is then subtracted from T and used in Figure 4 to get K_{1C}.

Under the same conditions the stress intensity is computed. Analysis of the system transient provides the coolant pressure, $p_c(t)$, and coolant temperature, $T_c(t)$, adjacent to the vessel wall. From solution of the transient heat conduction equation, the temperature distribution in the vessel wall, T (x, t), is obtained. Thermal stresses, σ_T (x, t), and pressure stress, $\sigma_p(t)$, are computed and combined to arrive at the total stress distribution , σ (x, t). Finally for an assumed crack, such as a long inside surface crack, the crack tip stress intensity, K_1 (x, t), is determined as a function of crack dimension x. From LEFM considerations in the uniform stress case we have for a crack of depth a ,

$$K_1 = \left(\frac{1.21\ \pi\sigma^2 a}{Q} \right)^{\frac{1}{2}} \qquad \text{eq. (1)}$$

where Q is the crack shape and orientation parameter.

In carrying out such deterministic calculations we ignore the variability (sources of error) in the input quantities, by utilizing boundary values based on judgement. In the probabilistic analysis some of these sources of variability are included to obtain more realistic results. The mechanisms generating variability seem to fall into two categories, namely, variation in material properties and variation from experimental determinations. The curve in Figure 1 is calculated from heat transfer relationships for the transient of interest, each of which has an associated probability of occurrence. The curve in Figure 2 is influenced by the uncertainty of the vessel history reflected in the ϕ value. Uncertainties in ϕ and errors in the experimental determination of ΔRT_{NDT} affect the curve in Figure 3. The curve in Figure 4 is influenced by errors resulting from the experimental determination of K_{1C} and errors in RT_{NDT} and ΔRT_{NDT}. Similarly there are errors introduced in obtaining K_1.

In our probabilistic analysis we will treat K_{1C} as a temperature dependent random variable while we take K_1 to be known as a function of the assumed flaw size and the transient under consideration. For the remainder of this section we will treat the embrittlement shift in K_{1C} as deterministic, but in Section 3 this phenomenon will be treated as random.

The K_{1C} data used are from the HSST* program. There are some large gaps in the data, particularly for temperatures greater than 150°F. One way to get estimates of K_{1C} at temperatures where there are no data is to interpolate with a mathematical model. A model is developed in Section 3 (See Figure 7) but it is not used in this section. With these large gaps in the data the results should be viewed somewhat cautiously. Some conservatisms have been introduced in the analysis by the nature of the data, namely:

1. Most of the data in the temperature range of interest are K_{1CD} values. These aren't valid K_{1C} values per ASTM, E-399 requirements because the specimens used were too small. Since these K_{1CD} values are lower bounds (obtained by equivalent energy methods) for the K_{1C} values, our analysis is conservative.
2. The estimated variability in K_{1C} is inflated because values from locations throughout the thickness of plate have been used rather than only those observations from the depth at which the crack tip lies (the data were grouped so that there would be sufficient information for estimating parameters).

2.1 Cold Hydro

This is a severe preoperational test because it is run at the relatively low temperature RT_{NDT} + 60°F. For this transient and an Appendix G flaw** we calculate K_1=84.4, which we take as known. We want to estimate $Pr(K_{1C}<84.4)$. From probability plots we have concluded that the 3 parameter gamma distribution is the more appropriate (the Gaussian was also considered) to characterize the variability in K_{1C}. The parameters for the gamma distribution are estimated by the standard method of moments. The location parameter, α, deserves mention here. Physically it represents the smallest value that the random variable, K_{1C}, can assume. From a careful inspection of the data we have chosen α=50 and calculate that $Pr(K_{1C}<K_1)=2x10^{-7}$. We feel this estimate is very conservative since it is highly unlikely that the assumed 1/4T flaw in fact is present.

2.2 Operational Transients

The conditions for which these calculations apply are given below:

4 loop plant
beltline region
A533 Grade B Class 1 Plate
Appendix G Flaw
Initial RT_{NDT} = 60°F
Copper Content = 0.15%
32 effective full power years of plant operation

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* Heavy Section Steel Technology Program is a United States Atomic Energy Commission sponsored effort for investigating the effects of flaws, variations of properties, stress raisers and residual stress on the strength and structural reliability of present and contemplated water cooled reactor pressure vessels.

** The flaw suggested in Appendix G of the ASME Boiler and Pressure Vessel Code is to be 1/4T (1/4 thickness of the vessel wall) deep, 1.5T in length, semielliptic in shape and initiating at the inside wall.

In carrying out the calculations for operational transients we must take account of the embrittlement of the vessel steel due to neutron irradiation. From a calculational point of view this is handled by using a corrected value of RT_{NDT} in the K_{1C} curve (Figure 7.) To get this corrected RT_{NDT} requires obtaining ΔRT_{NDT}. This corrected RT_{NDT} is then used in the curve in Figure 7 to estimate the parameters of the K_{1C} distribution. Initial RT_{NDT} for the beltline region is taken as 60°F, the maximum specified value. The value of ΔRT_{NDT} is obtained by entering Figure 5 with x = .25 (corresponding to a 1/4 T flaw) and reading off the value of fluence, $\phi = 1.2 \times 10^{19}$ n/cm^2. This value of ϕ is then used in Figure 6 (copper content = 0.15%) to obtain $\Delta RT_{NDT} = 120$. Thus for operational transients we subtract 180 ($RT_{NDT} = 120 + 60$) from the vessel wall temperature, T, to get the temperature used in getting the K_{1C} distribution. Once to this point the procedure is as for the cold hydro test. For the loss of flow transient with T = 506° and $K_1 = 64.63$ we estimate $Pr(K_{1C}<K_1) = 5 \times 10^{-10}$. The results for other transients* considered are in the same range and so aren't displayed here.

3. Model with Random Embrittlement Shift

We have developed a general model for K_{1C} as a function of T' ($T' = T - RT_{NDT}$) which has the form

$$K_{1C} = b_1 + \frac{b_2(T' + 201)^{b_3}}{b_4 + (T' + 201)^{b_3}} \quad . \qquad \text{eq. (2)}$$

(The constant 201 was added to T' so we wouldn't have negative numbers raised to noninteger powers.) From the data available estimates of the four parameters were obtained. The fitted relationship is (see Figure 7)

$$K_{1C} = 45 + \frac{250\ (T' + 201)^{2.9}}{1.4 \times 10^7 + (T' + 201)^{2.9}} \quad . \qquad \text{eq. (3)}$$

We found the fit to be quite good for T' < 50, a bit low between 100 and 200, and high at 550. The variability in the temperature shift is described by a normal distribution (reflected through T'). In order to incorporate the component of variability in K_{1C} due to other sources such as heterogeneity in vessel material and variation from experimental determinations, we replace the additive coefficient b_1 by a random variable having a gamma distribution with a mean value of 45. We have incorporated Monte Carlo methods with importance sampling in a computer routine IMPTS, to estimate $Pr(K_{1C}<K_1)$. Under the same vessel conditions as used in Section 2.2, we estimate the probability of brittle fracture for a loss of flow to be 2×10^{-10}. Results have been obtained for other transients using IMPTS but since the probabilities are about the same we have omitted them here. It should be noted here that this probability of brittle fracture is conditional upon a 1/4T crack being present. The unconditional probability of brittle fracture is the product of the probability of a 1/4T flaw being present and the conditional probability calculated here. Of course we would have to sum over all flaw sizes to get the total probability of brittle fracture.

* Step load increase, large step decrease, loss of power, loss of load, reactor trip and steady state.

Appendix: Importance Sampling

Consider a random variable X with continuous density f(x). The probability that X will lie below Z is

$$\Pr(X \leq Z) = \int_{-\infty}^{Z} f(x)dx. \qquad \text{eq. (4)}$$

Let $I_Z(X)$ be defined by

$$I_Z(X) = \begin{cases} 1 & X \leq Z \\ 0 & X > Z \end{cases} \qquad \text{eq. (5)}$$

Then

$$\Pr(X \leq Z) = \int_{-\infty}^{\infty} I_Z(x)f(x)dx = E(I_Z) \qquad \text{eq. (6)}$$

where the expectation is taken with respect to the random variable X.

Let g(x) be a positive function subject to

$$\int_{-\infty}^{\infty} g(x)dx = 1 \qquad \text{eq. (7)}$$

so that g(x) can be interpreted as a probability density function.

Now we write

$$\int_{-\infty}^{\infty} I_Z(x)f(x)dx = \int_{-\infty}^{\infty} I_Z(x)\,\frac{f(x)}{g(x)}\,g(x)dx = E(I_Z(x)\,\frac{f(x)}{g(x)}) \qquad \text{eq. (8)}$$

where the expectation is now taken with respect to a random variable whose density is g(x). This new density g(x) is called the importance density as contrasted with f(x), known as the parent density.

The straightforward and importance methods of evaluating $\Pr(X \leq Z)$ are contrasted below.

Straightforward Monte Carlo

1. Draw a number from a population whose density is f(x); call it x_1
2. Evaluate $h_1 = I_Z(x_1)$
3. Repeat steps 1 and 2 for a succession of x_i
4. Calculate $\hat{p} = \frac{1}{m}\Sigma_{i=1}^{m} h_i$

Importance Sampling

1. Draw number from a population whose density is g(x); call it x_1
2. Evaluate $h_1 = I_Z(x_1)\,\frac{f(x_1)}{g(x_1)}$
3. Repeat steps 1 and 2 for a succession of x_i
4. Calculate $\hat{p} = \frac{1}{m}\Sigma_{i=1}^{m} h_i$

The number $\hat{p}$ obtained by following either procedure is an estimate of $\Pr(X \leq Z)$. With the straightforward method, most of the h's will be zero if the probability being estimated is quite small, while the h's from the importance sampling method will have a variety of small values. It can be shown that the variance of $\hat{p}$ from importance sampling can be made much less than the variance of $\hat{p}$ from the straightforward method, when the same m is used for each. In fact, a procedure, known as Spanier's algorithm, has been developed to determine an optimum importance density in the sense of minimum variance; the procedure is iterative.

Although this method can be expanded to problems which involve more than one random variable, we shall illustrate the expansion by considering only two. Let

$$X = \phi(W, Y)$$

be a relation that defines the random variable X in terms of two random variables W and Y, which respectively have densities $f_W(w)$ and $f_Y(y)$. We will denote by f(x) the density of X, although initially that is not given. The probability that X does not exceed Z is given by

$$Pr(X \leq Z) = \int_{-\infty}^{Z} f(x)dx = \int_{-\infty}^{\infty} I_Z(x)f(x)dx = E(I_Z(x)) \qquad \text{eq. (9)}$$

where I_Z is defined as before and the expectation is taken with respect to X. We may replace the single integration in x with an integration over the sample space as

$$\int_{-\infty}^{\infty} I_Z(x)f(x)dx = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_Z(x)\ f_W(w)f_Y(y)dwdy \qquad \text{eq.(10)}$$

again giving the expected value of $I_Z(x)$. Now introduce two importance densities $g_W(w)$ and $g_Y(y)$ and so get

$$Pr(X \leq Z) = \int\int I_Z(x)\ \frac{f_W(w)\cdot f_Y(y)}{g_W(w)\cdot g_Y(y)}\ g_W(w)\cdot g_Y(y)dwdy \qquad \text{eq.(11)}$$

$$= E\left[I_Z(x)\ \frac{f_W(w)\cdot f_Y(y)}{g_W(w)\cdot g_Y(y)}\right] \qquad \text{eq.(12)}$$

where now the expectation is taken relative to the importance densities.

It is not necessary to introduce importance densities for all of the input variables. If an input random variable is not to get an importance density, we take f = g for that one, and the ratio of f/g for it becomes unity.

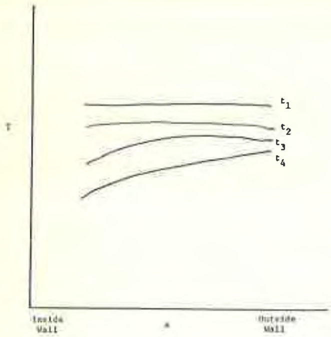

Figure 1. Position vs Temperature at Various Times Into the Transient ($t_{i+1}>t_i$)

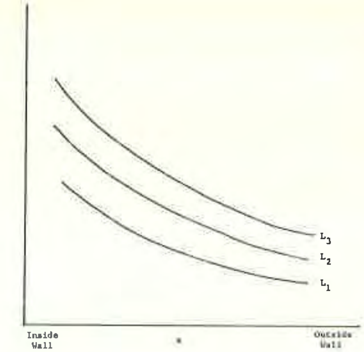

Figure 2. Position vs Fluence for Various Values of L ($L_{i+1}>L_i$)

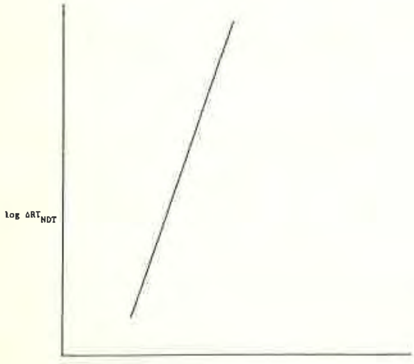

Figure 3. Fluence vs Change in Reference Nil Ductility Transition Temperature.

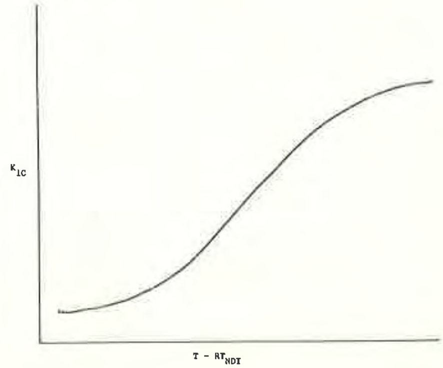

Figure 4. Temperature vs Fracture Toughness

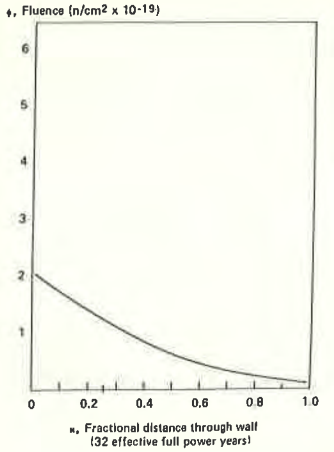

Figure 5. End of Life Fluences for 4-loop plant

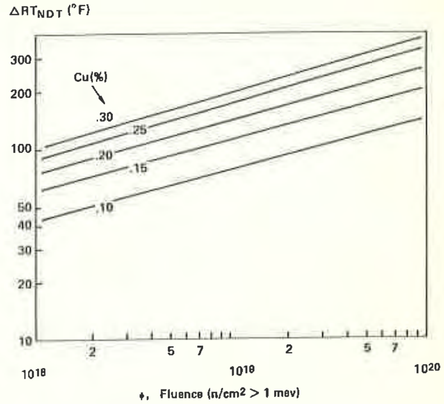

Figure 6. Effect of Fluence and Copper Content on Shift of RT_{NDT} for Reactor Vessel Steels

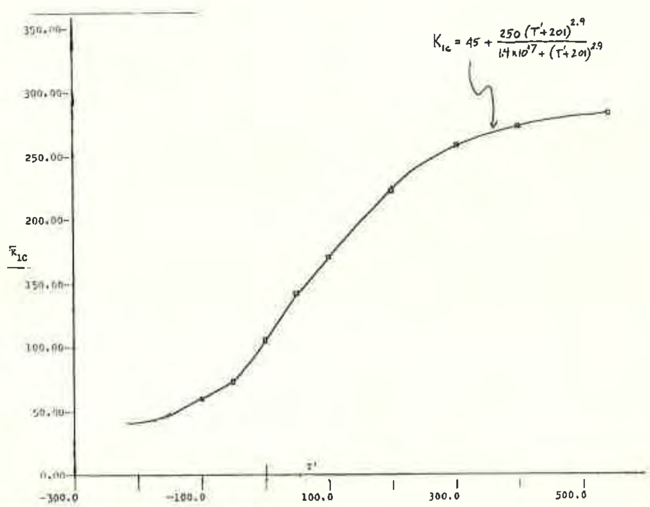

Figure 7. K_{1C} Curve