Including Accidental Torsion in Response-History Analysis of Safety-Related Nuclear Structures

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Introduction

The calculation of seismic response of buildings and safety-related nuclear structures requires consideration of torsion (rotation about a vertical axis). Standards of design practice such as ASCE 7-10 [1] and ASCE 4-98 [2] recognize the importance of torsional contributions to horizontal displacement response and simplified procedures have been proposed to estimate and mitigate these contributions. Two types of torsion are considered: *natural* and *accidental*. Natural torsion is the product of non-coincident centers of mass (CM) and rigidity (CR) at one or more floor levels in a building and can be reduced or eliminated. Natural torsion can be incorporated explicitly in a mathematical model of a building (e.g., Tso, [7]; Goel and Chopra [6]; Basu and Jain [4]). Accidental torsion is used to indirectly account for a) plan distributions of reactive mass that differ from those assumed in design, b) variations in the mechanical properties of structural components in the seismic force-resisting system, c) non-uniform yielding of components in the seismic force-resisting system, and d) torsional ground motion. Accidental torsion is addressed by shifting the center of mass at each floor level by distance equal to 5 percent of the building dimension so as to increase the effect of natural torsion. Although this approach was intended only for use with an Equivalent Lateral Force Procedure, it is common practice to utilize accidental torsion when response-history analysis is performed.

The shifting of CM to consider the effect of accidental torsion alters the dynamic characteristics of the structure in terms of its modal properties and the modal damping ratios when Raleigh damping is used to describe inherent damping.

This paper presents a critical review of the procedures used to include accidental torsion in design when dynamic analysis is performed and proposes an alternate definition of accidental eccentricity for use when torsional ground motion cannot be considered explicitly. A series of single story elastic systems is subjected to translational and torsional components of seismic excitation to study the conventional treatment of accidental torsion. The study shows the limitations of the conventional approach when used with response-history analysis. An alternative definition of accidental eccentricity is proposed and verified by a series of analyses of single story elastic systems and nonlinear seismic isolation systems.

Description of Model

A one story singly symmetric system, composed of a rigid deck of mass m supported on six massless lateral-load-resisting elements, is used for analysis (see Figure 1). The CM of the deck is located at its geometric center and its radius of gyration about a vertical axis passing through the CM is r. Each lateral-load-resisting element has translational stiffness (identical) in the two orthogonal directions but its torsional stiffness is neglected. The system is symmetric about the x axis but has an eccentricity e about the y axis. This system could represent a seismic isolation system supporting a rigid superstructure or a single story singly symmetric building. The system is subjected to translational seismic excitation along the y axis and torsional ground excitation.

The following parameters are defined: i) K_y = total lateral stiffness (which equals the stiffness in the x-direction), ii) $\omega_y = \left(K_y / m\right)^{0.5}$ = uncoupled lateral frequency, iii) $K_{\theta R}$ = torsional stiffness about the center of rigidity (CR), iv) $\omega_\theta = \left(K_{\theta R} / mr^2\right)^{0.5}$ = uncoupled torsional frequency, and v) $\Omega = \left(K_{\theta R} / r^2 K_y\right)^{0.5}$ = ratio of uncoupled torsional frequency to translational frequency.

For a given aspect ratio and location of the elements with respect to the CM, the lateral stiffness of each of the elements may be expressed as

$$K_1 = K_y \left[-\frac{1}{2s_x}\left(\frac{e}{b}\right) - \frac{1}{4}\left(\frac{s_y s_a}{s_x}\right)^2 + \Omega^2 R_a^2 + \frac{1}{s_x^2}\left(\frac{e}{b}\right)^2 \right] \quad K_2 = K_y \left[\frac{1}{2}\left\{ 1 + \left(\frac{s_y s_a}{s_x}\right)^2 \right\} - 2\Omega^2 R_a^2 - \frac{2}{s_x^2}\left(\frac{e}{b}\right)^2 \right]$$
$$K_3 = K_y \left[\frac{1}{2s_x}\left(\frac{e}{b}\right) - \frac{1}{4}\left(\frac{s_y s_a}{s_x}\right)^2 + \Omega^2 R_a^2 + \frac{1}{s_x^2}\left(\frac{e}{b}\right)^2 \right] \tag{1}$$

In Eq (1), $s_x = b^*/b$, $s_y = a^*/a$, $s_a = a/b$ and $R_a = \left[1+s_a^2\right]^{0.5} / \left(2\sqrt{3}s_x\right)$. Assuming a unit mass, K_y in Eq (1) may be replaced by ω_y^2. Given the dimensions of the deck and location of the elements, this elastic system may be uniquely described by three normalized parameters: ω_y, Ω and e/b.

Accidental Eccentricity Using a Conventional Approach

For equivalent lateral force analysis, many seismic codes and standards recommend that the lateral load profile be applied at a distance, the *design eccentricity*, from the CR. The design eccentricity is contributed from both actual (often with a factor) and the accidental eccentricities. Most seismic codes specify the accidental eccentricity as 5-10% of the building dimension normal to the direction of excitation and the effect of accidental torsion should always amplify the design response. This analysis procedure should be limited to structures with regular framing systems.

It is common practice when performing response-history analysis to shift the CM by a distance equal to the accidental eccentricity specified by the seismic codes so as to amplify the maximum translational response. This approach is studied herein and its effect on the displacement demand is examined. For convenience, we denote the two sides with respect to the CR of the model as Side A and Side B (see Figure 1). Since the elements located on Side A are expected to incur more displacement demand, the present study focuses on those elements only. Accordingly, we first shift the CM away from the CR and denote it as Shift 1. The CM is then shifted to both sides in tern and denoted as Shift 2.

Ground Motions Considered

A study of accidental torsion procedures requires direct consideration of torsional ground motion effects. Such histories are not usually recorded and need to be extracted by analysis of earthquake records obtained in dense arrays. One of the events recorded at the Large Scale Seismic Testing (LSST) array in Lotung, Taiwan is considered here for the ground motion inputs. The Lotung-LSST (LLSST) site is a part of the much larger SMART1 array. All fifteen free-surface accelerometers at the LLSST are positioned along a total of three arms at approximately 120 degree intervals. Each arm extends for about 50m and the spacing between the surface stations varies from 3m to 90m. Further details on the site characteristics, instrumentation and recorded seismic events may be obtained from the URL *www.earth.sinica.edu.tw/~smdmc/llsst/llsst.htm*. One of the motions considered here was recorded at the station FA1_1. The translational acceleration histories along directions *x* and *y*, the torsional acceleration and their respective 5-percent damped response spectra are shown in Figure 2. The torsional acceleration history was computed using the SDA approach to be presented in Basu [3]. In the analyses conducted for this study, the EW component is applied in the *y* direction for the system. As the system is symmetric about the *x* axis, the translational acceleration along the NS direction is not included in the analysis.

Analysis Procedure

The steps followed in the analysis and presentation of the results are as follows:

1. Select values for the normalized parameters ω_y, Ω and e/b that uniquely define the elastic system.
2. Apply translational and torsional acceleration histories simultaneously and find the absolute maximum displacement at the furthest element on Side A, U^{fl}.
3. Repeat Step 2 but apply only the translational acceleration history; let the absolute maximum displacement for the same element be $\bar{U}^{fl}$; compute the torsional amplification factor as $R_1 = U^{fl}/\bar{U}^{fl}$.
4. Repeat Steps 2 and 3 but reverse the direction of the torsional acceleration history and compute the torsional amplification factor R_2; set target torsional amplification factor as $R = \max(R_1, R_2)$.
5. a) Shift 1: Shift the CM away from the CR by an offset e_a and analyze the system by applying only the translational acceleration history; let the absolute maximum displacement at the furthest element on Side A be $\bar{\bar{U}}^{fl}$

b) Shift 2: Repeat ‘a’ but shifting the CM in the opposite direction and compare the two values of $\bar{\bar{U}}^{fl}$; record the greater value.

6. Define the torsional amplification factor associated with offset e_a as $R^* = \bar{\bar{U}}^{fl} / \bar{U}^{fl}$. Repeat Step 5 for a range of values of e_a and generate the associated torsional amplification factor. The required accidental eccentricity for the system considered is given by the offset e_a for which $R^* \geq R$.

Results and Discussions

The procedure outlined above is applied to a variety of elastic systems selected by varying the three normalized parameters ω_y , Ω and e/b . In each case, the aspect ratio of the deck and the location of the elements with respect to the CM of the deck are described by $s_x = 1$, $s_y = 1$, and $s_a = 0.5$. Furthermore, the inherent damping of the system is described by Rayleigh damping with 5% of the critical damping ratio in the first and third modes. Analysis of each system is carried out using a state-space procedure and, target and computed torsional amplification factors are compared to calculate the required e_a . Sample results are presented for the case of uncoupled lateral period $T_n = 2\pi/\omega_y$ =1.0 sec and the ratio of uncoupled torsional to lateral frequency Ω = 1.25. Figures 3(a) and 3(b) compare the actual (target) torsional amplification and that achieved with Shift 1 and Shift 2, respectively. The torsional amplification does not have any predictable trend with the increasing accidental eccentricity. One would rather expect the torsional amplification to increase monotonically with increasing accidental eccentricity. The inconsistency illustrated above was observed in every case studied. This is attributed to the change in dynamic characteristics of the system when the CM is shifted. Therefore, shifting the CM for use with response-history analysis is not recommended.

Definition of Accidental Eccentricity for Use in Response-History Analysis

The inconsistencies in the torsional amplification when the CM is shifted in response-history analysis can be eliminated using an approach that can be explained schematically through Figure 4 in terms of application of the inertial force. Figure 4a shows the inertial force and moment acting through the CM of the system when subject to the translational acceleration along the *y*-direction. The inertial force comprises of components $m\ddot{u}_y$ and ma_{gy} . To account for the effect of accidental torsion in the elements located on Side A, only force ma_{gy} is shifted away from the CR by a distance e_a (see Figure 4b). This is equivalent to applying an inertial torsional moment equal to $me_a a_{gy}$ (see Figure 4c). The inertial force and moment shown in Figure 4c can be considered as resulting from a set of equivalent ground motions acting on the original system as shown in Figure 4d.

Accounting for an accidental eccentricity e_a requires a torsional acceleration history, $a_{g\theta} = -\left(e_a/r^2\right)a_{gy}$ to be applied. Denoting e_a/b as the normalized accidental eccentricity, the torsional acceleration history may be expressed as $a_{g\theta} = -(e_a/b)(b/r)^2\left(a_{gy}/b\right)$. For convenience we define the accidental eccentricity through $a_{g\theta} = -\left(\bar{e}_a/b\right)\left(b^*/r\right)^2\left(b/r\right)^2\left(a_{gy}/b\right)$ where b^* is the larger of the two plan dimensions. It is emphasized here that $\bar{e}_a / b$ does not have physical significance. The accidental eccentricity results presented below are for $\bar{e}_a / b$ although it may be denoted as e_a / b .

The systems analyzed previously by shifting the CM are reanalyzed using the alternative definition of accidental eccentricity. The steps in the analysis are identical to those described above except that there is no shifting of the CM by a distance e_a . Instead, the torsional acceleration history defined previously is applied. Figure 5(a) presents the calculated torsional amplification. The torsional amplification increases monotonically with increasing accidental eccentricity. Based on these results, it is now possible to develop recommendations for accidental eccentricity for use in response-history analysis to account for the effects of the torsional ground motion. The recommendations are based on a comparison of calculated torsional amplification and the actual torsional amplification obtained when using the actual torsional ground motion input. Table 1 presents values of the accidental eccentricity as a percentage of the plan dimension normal to the direction of excitation for use in response-history analysis. Note these values are for elastic systems and are based on analysis using one record of torsional ground motion.

Accidental Eccentricity in Nonlinear Isolation Systems

The system shown in Figure 1 is assumed here to represent a rigid structure that is seismically isolated. The isolation system consists of six axisymmetric isolators such that the system is symmetric about *x* axis but has an eccentricity about the *y* axis. The mass of the rigid superstructure is lumped at the CM of the deck, which is also the geometric center. The isolators have the bilinear hysteresis of Figure 6.

The properties of the seismic isolated structure are defined in terms of the yield displacement *Y* (values of 1, 5 and 10 mm), period T_d (values of 2 and 5 sec) based on the post-elastic stiffness K_d and the strength-to-weight ratio Q/W (values in the range of 0.04 to 0.06; see Constantinou et al. [5]). The ratio of uncoupled torsional frequency to lateral frequency (based on the post-elastic stiffness), Ω is taken as 1.0, 1.25 and 1.5. The normalized natural eccentricity (e/b) is assumed in the range of zero to 0.45.

The values of the accidental eccentricity were computed for all systems considered in this study and are reported in Tables 2 through 5. Of these cases, the results on torsional amplification for the case with $T_d = 4\sec$, $\Omega = 1.25$, $Q/W = 0.05$ and $Y = 5mm$ is presented in Figures 5(b) and 5(c), where the two sets of results are based on first neglecting and then including coupled behavior along the two axes. The required accidental eccentricity (i.e., closest to the actual) is nearly independent of the natural eccentricity. Further, it is apparent from the results of Tables 2 through 5 that for a given strength, the required accidental eccentricity depends neither on the period nor on the ratio of the uncoupled torsional frequency to translational frequency. These two observations imply that the required accidental eccentricity does not depend on the post-elastic stiffness and eccentricity of the system provided the strength exceeds a minimum value. It can also be observed from the results of Tables 3 to 5 that the yield displacement has a significant effect on the required accidental eccentricity where systems with a small yield displacement have the least required eccentricity.

Conclusions

The conventional approach of accounting for the effects of accidental torsion by shifting the CM should not be used for response-history analysis due to the resultant change in the dynamic characteristics of the structure. An alternative definition of accidental eccentricity is proposed wherein accidental torsion is accounted for by applying a torsional acceleration history in conjunction with the horizontal excitation. This torsional acceleration history is derived by scaling the horizontal excitation using the proposed accidental eccentricity.

The proposed procedure has been studied for a wide range of single story elastic systems and nonlinear isolation systems. This is found to predict torsional amplification that has the correct trend (increasing torsional response with increasing eccentricity) and for a specific value of the accidental eccentricity to correctly predict the exact torsional response as calculated by use of the actual torsional ground motion.

For the case of nonlinear isolation systems, it is observed that torsional amplification or accidental eccentricity is nearly independent of the post-elastic stiffness and the eccentricity computed based on the post-elastic stiffness, provided the characteristic strength of the isolation system exceeds some threshold limit. Furthermore, the yield displacement of the isolator has a significant influence on the required accidental eccentricity.

Values of the required accidental eccentricity are proposed and tabulated in Tables 1 to 5. However, it must be noted that these values are specific to the ground motions considered in this study. A rigorous verification of these results using a much larger set of ground motions is required before making final recommendations can be made.

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Table 1. Accidental eccentricity (%) for elastic system based on proposed approach

Ω	T_n (sec)				
	0.5	1.0	1.5	3.0	4.0
1	1	1	2	3	3
1.25	1	3	3	3	2
1.5	1	3	2	1	2

Table 2. Accidental eccentricity (%) for nonlinear isolation (orthogonally uncoupled) system based on proposed approach

$Y = 5mm$	$T_d = 4\text{sec}$			$T_d = 5\text{sec}$		
	$Q/W\,(\%)$			$Q/W\,(\%)$		
Ω	4	5	6	4	5	6
1	1	0.5	0.5	1	0.5	0.5
1.25	1	0.5	0.5	1	0.5	0.5
1.5	1	0.5	0.5	1	0.5	0.5

Table 3. Accidental eccentricity (%) for nonlinear isolation (orthogonally coupled) system based on proposed approach

$Y = 5mm$	$T_d = 4\text{sec}$			$T_d = 5\text{sec}$		
	$Q/W\,(\%)$			$Q/W\,(\%)$		
Ω	4	5	6	4	5	6
1	1	0.5	1	1	0.5	1
1.25	1	0.5	1	1	0.5	1
1.5	1	0.5	1	1	0.5	1

Table 4. Accidental eccentricity (%) for nonlinear isolation (orthogonally coupled) system based on proposed approach

$Y = 1mm$	$T_d = 4\text{sec}$			$T_d = 5\text{sec}$		
	$Q/W\,(\%)$			$Q/W\,(\%)$		
Ω	4	5	6	4	5	6
1	0.5	0.5	0.5	0.5	0.5	0.5
1.25	0.5	0.5	0.5	0.5	0.5	0.5
1.5	0.5	0.5	0.5	0.5	0.5	0.5

Table 5. Accidental eccentricity (%) for nonlinear isolation (orthogonally coupled) system based on proposed approach

$Y = 10mm$	$T_d = 4\sec$			$T_d = 5\sec$		
	$Q/W\,(\%)$			$Q/W\,(\%)$		
Ω	4	5	6	4	5	6
1	1	1	1	1	1	1
1.25	1	1	1	1	1	1
1.5	1	1	1	1	1	1

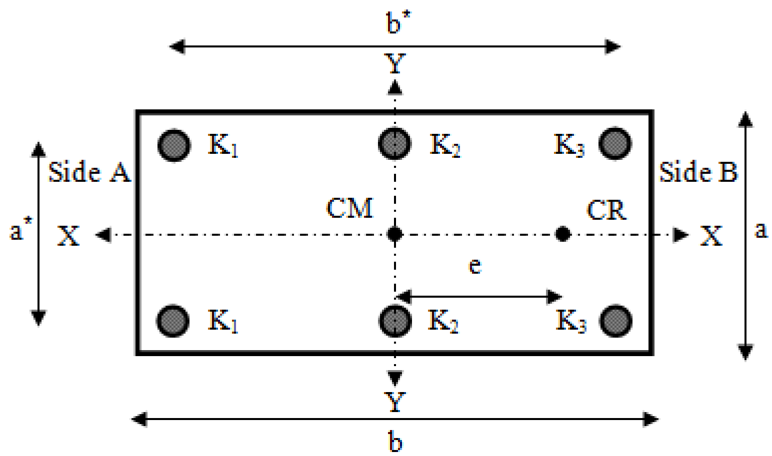
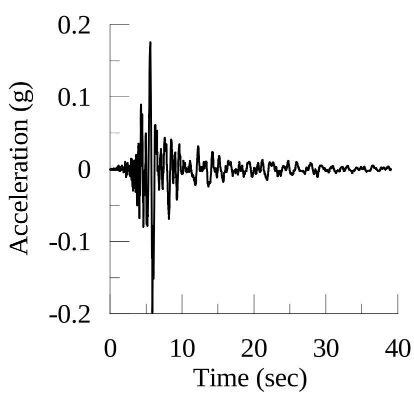
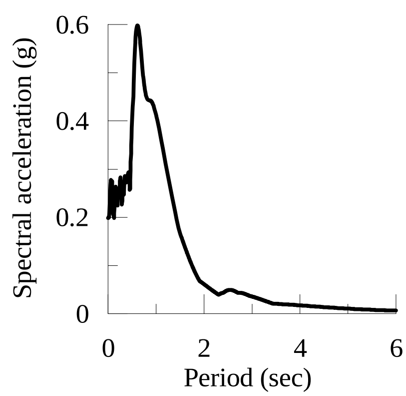


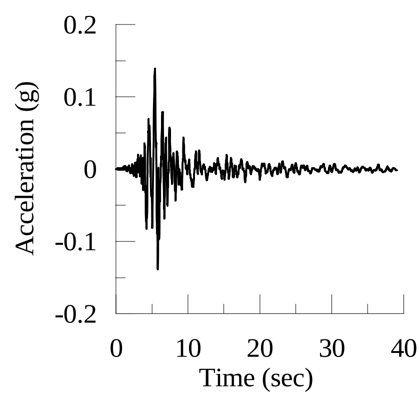
Figure 1: Analytical model



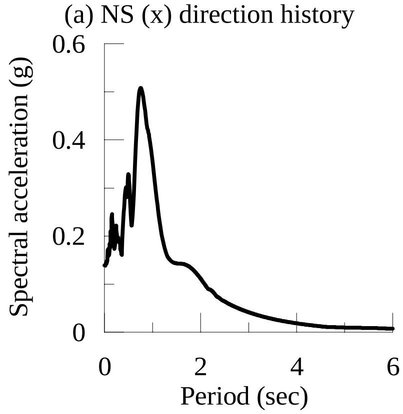
(a) NS (x) direction history



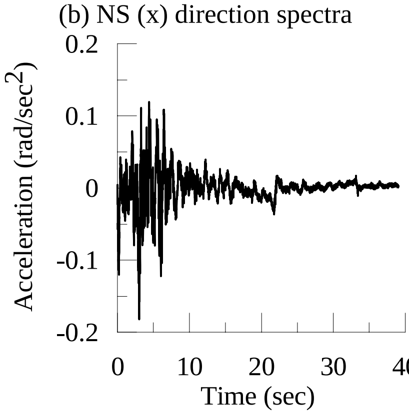
(b) NS (x) direction spectra



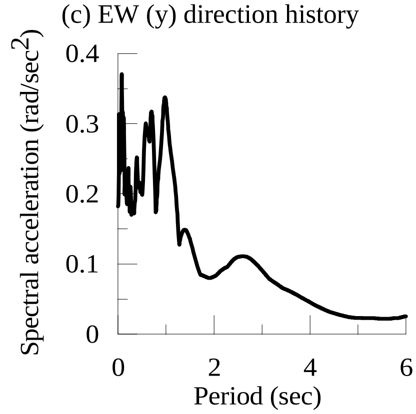
(c) EW (y) direction history



(d) EW (y) direction spectra

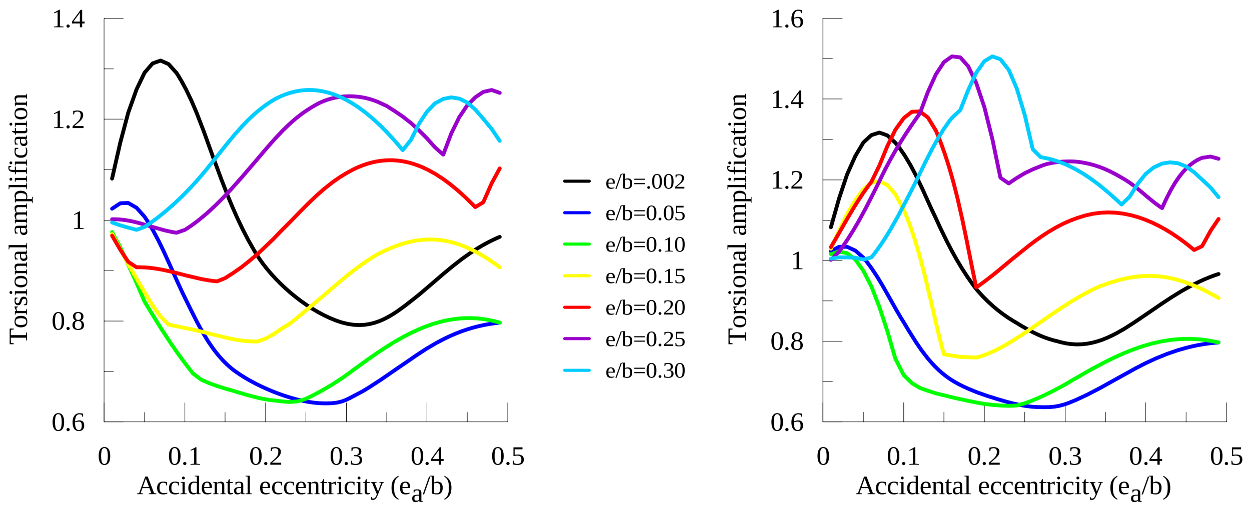


(e) Torsional history



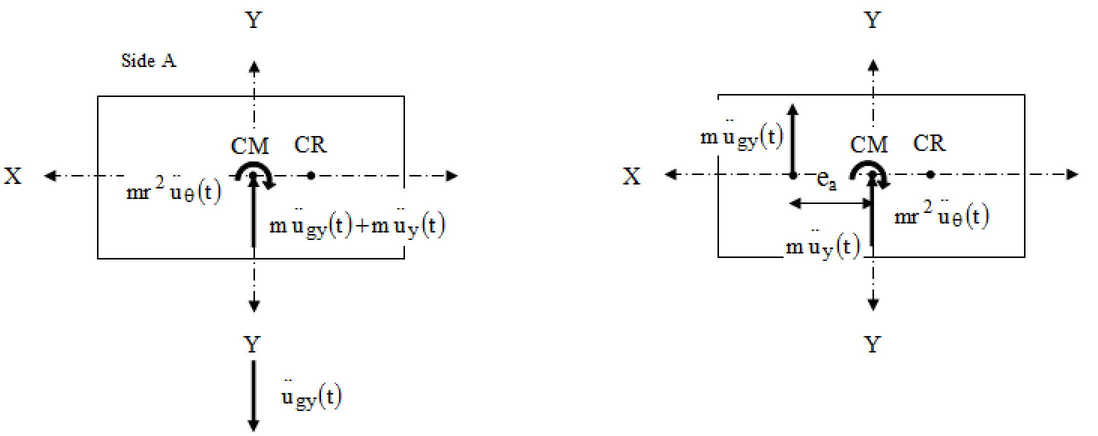
(f) Torsional spectra

Figure 2: Input ground acceleration data



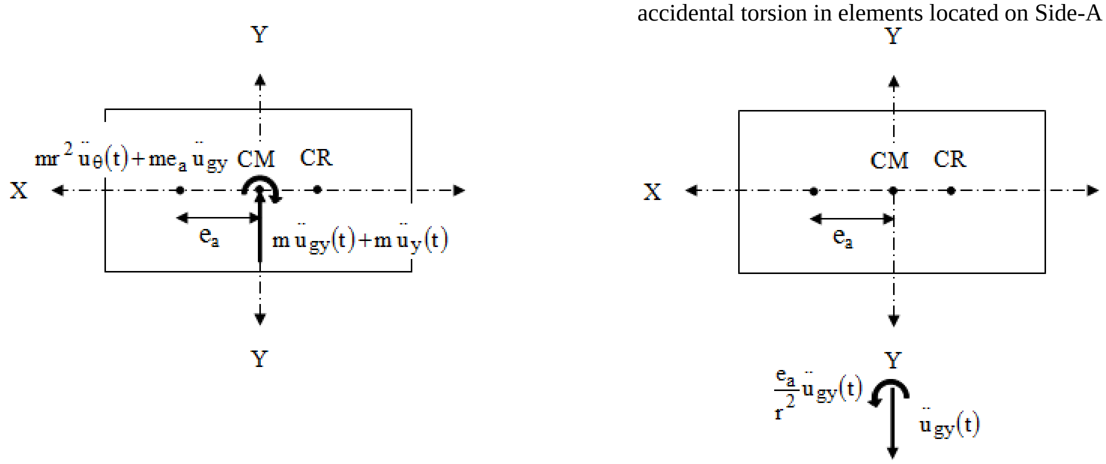
(a) Shift 1 (b) Shift 2

Figure 3: Torsional amplification as function of accidental eccentricity (T_n =1.0 sec and Ω =1.25)



(a) Without accidental torsion

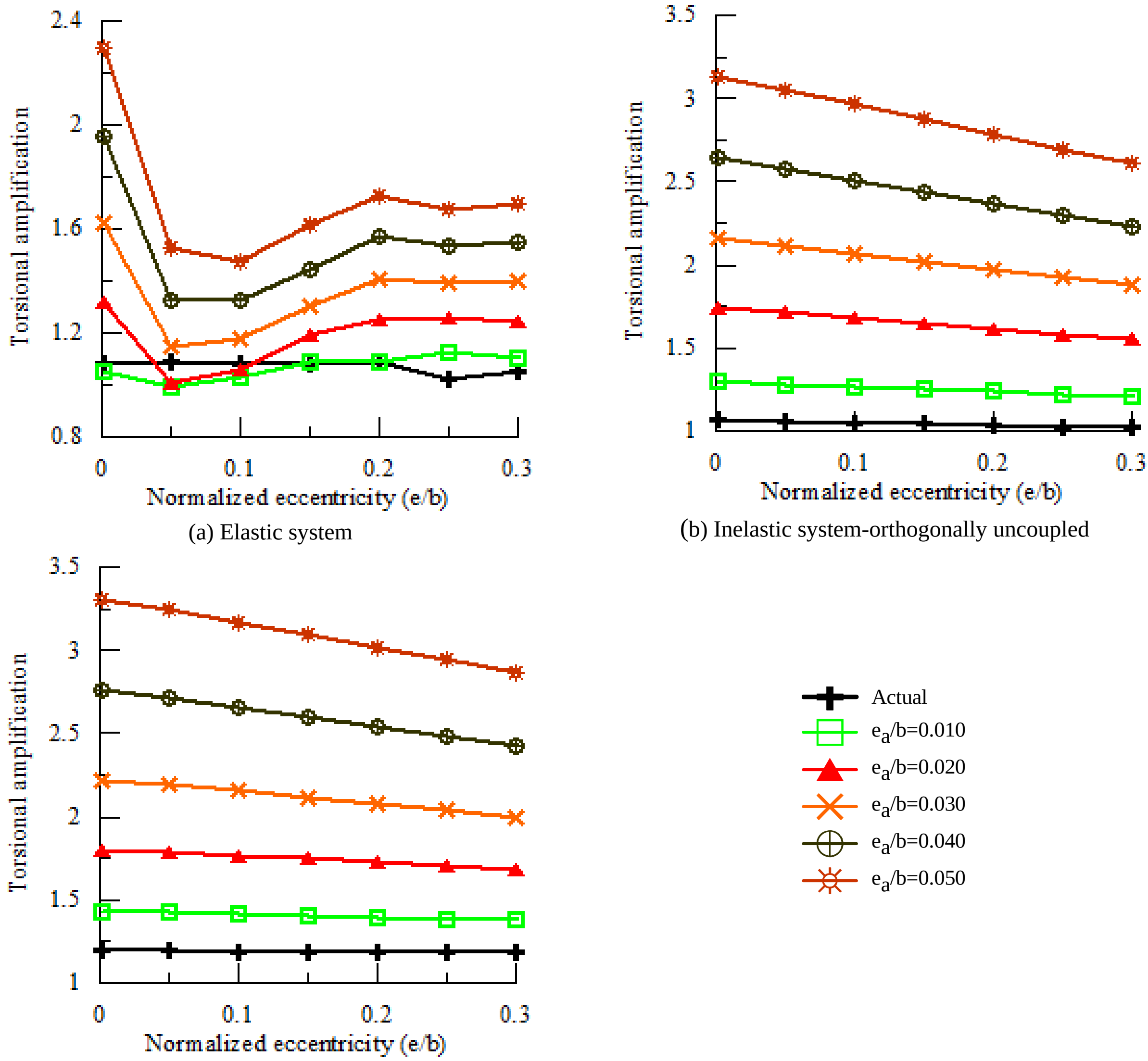
(b) Shifting one part of inertial force to account for accidental torsion in elements located on Side-A



(c) Equivalence of (b)

(d) Equivalent ground motion inputs

Figure 4: Schematic representation of the alternative definition of accidental eccentricity



(a) Elastic system

(b) Inelastic system-orthogonally uncoupled

(c) Inelastic system-orthogonally coupled

(d) Legend

Figure 5: Torsional amplification calculated using the alternate definition of accidental eccentricity

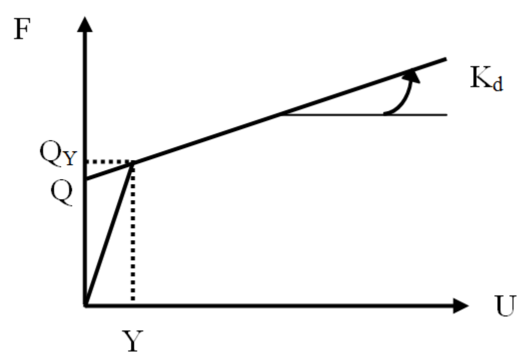


Figure 6: Force-displacement relation for a typical isolator