On the Modelling of Beam Structures with Hardening

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ABSTRACT

An implicit-explicit sheme is proposed for modelling the plasticity with hardening. This sheme is based on an incremental Hencky model (already validate in perfect plasticity) and an extrapolation of some infinitisimal formulae. This approach is justified by the complexity of plastic yield surfaces in beam generalized variables. The application of the method to the beam structures is carried out with the concept of plastically admissible convex for a beam element rather than for a section.

Experimentation on a simple steelwork under cyclic loading is related and experimental and numerical results are compared.

1. Introduction

In a former paper [1], the projection method was introduced for modelling the perfectly plastic behaviour and the application to beam elements was proposed. We now introduce the hardening in this model and again, we focus on the application to beam elements.

The perfectly plastic modelling was validated by comparison with the analytical results of the plastic hinge theory. Yet, the modelling of hardening can't be validated with the aid of such a theory and in order to do that, we have carried out experimentations on a simple structure.

2. The modelling of hardening

2.1 Recalls about the modelling of perfect plasticity

As in a former paper [1], q_0 and Q_0 notice the displacements and the forces in a structure in equilibrium under the external loading F_0. Q_0 is assumed to reached the plastic yield surface.

With the aim of a displacement approach, we have to know the increment of forces ΔQ for a given increment of displacements Δq.

In the case of a perfectly plastic behaviour an accurate method is the following one [1] :

We compute the elastic response of the structure : $\Delta\tilde{Q}$, defined by :

$\Delta q = \Lambda\, \Delta\tilde{Q}$ (Λ is the elastic operator)

We notice :

$\tilde{Q}_1 = Q_0 + \Delta\tilde{Q}$

Then we obtain the new forces $Q_1 = Q_0 + \Delta Q$ by projection of $\tilde{Q}_1$ on to the convex C_0 of the plastically admissible forces, what is written :

$Q_1 = \pi_{C_0}\, \tilde{Q}_1$

We focus on the fact that the scalar product in this projection is not the classical Euclidian product but is defined as an energy product by :

$\langle Q', Q'' \rangle = Q' . \Lambda\, Q''$

We have shown in [1] that if Q represents the forces of a beam finite element, the application of this method to the beam-structures leads to the same results as the well-known plastic hinges method. The advantage of this theoritical approach is to deliver a general frame for developing of the method ; for instance, we mentionned the three dimensional analysis as a possible development. In this paper we are interested in the study of hardening.

2.2 Introduction of hardening

As discuss in [3], we know that a direct extrapolation of infinitesimal laws to obtain a finite incremental model leads to plastically not admissible stresses. To prevent this difficulty, a kind of incremental Hencky model has been proposed in the former paragraphe, that can be regarded has an implicit sheme because the normal law is true at the new forces Q_1.

It is also possible to develop an implicit sheme with hardening and to use iterative methods to solve the equations. But with the complexity of convex areas for a beam element [2], it would lead to hight computing costs.

For this reason, we propose the following method that can be regarded as an implicit-explicit sheme, because firstly we compute the response in perfect plasticity with the upper method and after that we use explicit formulas. A single iterative procedure on a scalar unknown is necessary to obtain plastically admissible forces.

2.2.1 The evolution of stresses

For an equilibrium stress field Q_0 and for a given incremental generalized displacements field dq, the incremental stresses field dQ can be written [3] :

$$dQ = d\overline{Q} + \frac{e}{E}\,(d\tilde{Q} - d\overline{Q})$$

where :

$d\tilde{Q} = \wedge\, dq$ is the elastic response,

$d\overline{Q} = \pi_{T(Q_0)}\, d\tilde{Q}$ is the response for a perfectly plastic material ; $\pi_{T(Q_0)}$ is the energetic projection on to the hyperplan tangent to C_0 in Q_0,

E and e are the Young modulus and the instantaneous hardening modulus.

In our computer program, we choose to extrapolate the upper formula in the case of finite increments :

$$\Delta Q = \Delta\overline{Q} + \frac{e}{E}\,(\Delta\tilde{Q} - \Delta\overline{Q})$$

what can be written (Fig. 1) :

$$Q_1 = \overline{Q}_1 + \frac{e}{E}\,(\tilde{Q}_1 - \overline{Q}_1)$$

2.2.2 The evolution of hardening parameters

This evolution depends obviously on the hardening theory we choose, and our purpose is not to discuss these theories here. In order to illustrate our method, we consider the cinematic hardening with which are obtained the results of the picture 5.

The stresses have been computed with the former position K_0 of the center of the elastic convex C. For a classical cinematic hardening, we have to translate C in the direction of the plastic incremental strain. In order to have here an explicit evolution law, we take for this direction the Euclidian normal in Q_0 : $n(Q_0)$ and we set :

$$\Delta K = k\, n(Q_0)$$

k is computed so as to the boundary of the convex in its new position include the point Q_1.

An approximate value of k can be obtain by extrapolation of the infinitesimal formula :

$$k = n(Q_0).\Delta Q$$

2.3 Application to beam elements

As exposed in [1], in our method Q represents the forces of a complete beam element and not, as is usual in computer programs, the forces in a section. Under some assumptions. that we do not detail here, JOUVE [2] has shown that if the normal plasticity is true for the material, it is also true in terms of generalized forces in the extreme sections of a beam. So we have just to determine the admissible convex for a beam element in perfect plasticity. This convex is shown on the picture 2 in the case of plane bending ; N_P [M_P] is the maximum admissible normal force [bending moment] if there is no bending moment [normal force].

3. Experimentation

The description of an experimental device can be found on picture 3.

The structure is a very simple frame made of usual steelwork (normalized : E26). The traction curve of the steel can be found on the picture 4. It seems to be quite a perfectly plastic material but we are going to see that we cannot neglect the hardening.

With the assumption of perfect plasticity, the plastic hinge of the section (IPE 80×46) is reached for the bending moment : M_P = 5 970 N×m.

Several experimentations have been made on this structure but in the present paper, we just consider the cyclic loading, obtained with the aid of two jacks, the sections of which are differents but which are moved by the same pump. So we have a radial loading (Fig. 5) depending on the single parameter λ.

Forces were measured between the jacks and the structure. The displacements were measured in three points as indicated on picture 3.

In the experimentation related on picture 5 we chose to have a given range of the vertical displacement u.

4. Comparison of results and conclusion

The results of experimentation and of computation are sumerized on picture 5.

The dotted curve was obtained with the assumption of a perfectly plastic material. We can see that the displacements are widely overestimated and this result could be expected : as shown in [1], our method for a perfectly plastic material leads to the same results as the plastic hinges method. In the present case, the value of λ we reach is very close to the value λ_M for which we have the collapse of the structure with the theory of plastic hinges. So, for λ close to λ_M we have very large displacements.

If we introduce the linear cinematic hardening in the model, with the hardening modulus read on the traction curve of the material, we obtain satisfying numerical results, despite some differences with the experimental results.

As expected, the cinematic hardening leads to a first cycle which is also the stabilized cycle. Nevertheless, as the experimental stabilized cycle is reached very quickly and has a maximum close to the first load maximum, we do not think it useful to look for a more sophisticated model of material behaviour.

The shape of the experimental force-displacement relationship is more "curved" that the numerical one, what we think to be due to the modelized behaviour of a beam element in plasticity : we obtain this behaviour as a combination of the elastic and of the perfectly plastic behaviours. The perfectly plastic behaviour itself is simplified because a section is either elastic or entierely plastified with no intermediate position ; and also the perfect plasticity does not allows the expanding of the plastic area in the close sections.

Despite these small differences between the experimentation and the computation, it seems that the proposed modelling of hardening, based on the global study of a beam element (not on the separate study of each section) leads to very interesting results in the computation of beam-structures. We focus on the fact that the results are obtained with the same model of the structure, as for an elastic problem. We recall that in the case of perfect plasticity this method has been validated according to the plastic hinges theory.

REFERENCES

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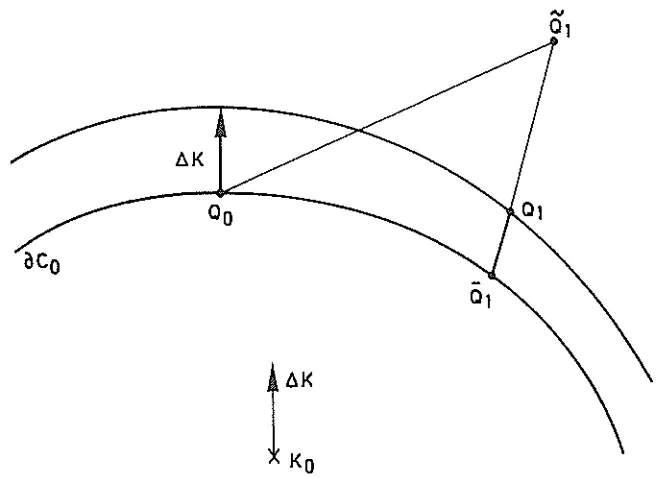
[3] P. LABBE, Etude de structures stratifiées en comportement élasto-plastique par des éléments finis mixtes, Thèse, Rennes, 1980.



Evolution of forces and of hardening parameters

Fig. 1

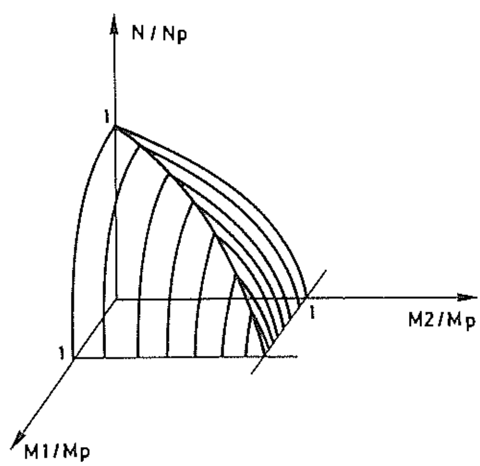
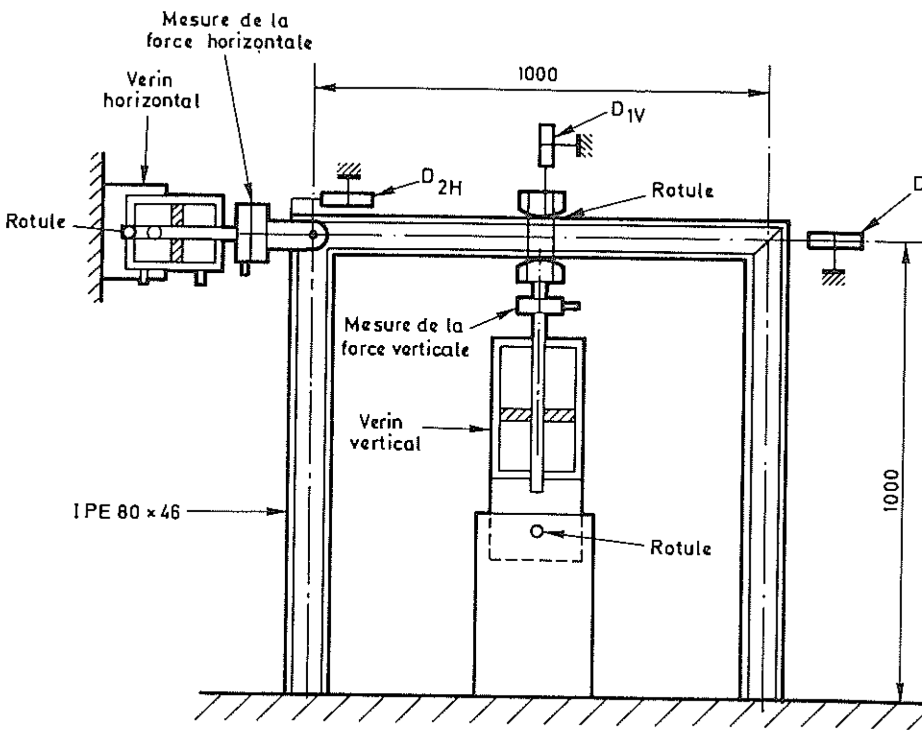


Fig. 2



D_{1V} . D_{2H} . D_{3H} = capteurs de déplacement.

Fig. 3

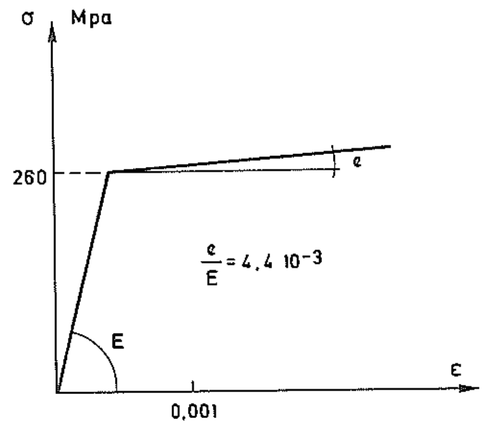

σ Mpa
260
e
$\frac{e}{E} = 4,4 \ 10^{-3}$
E
ε
0,001

Fig. 4

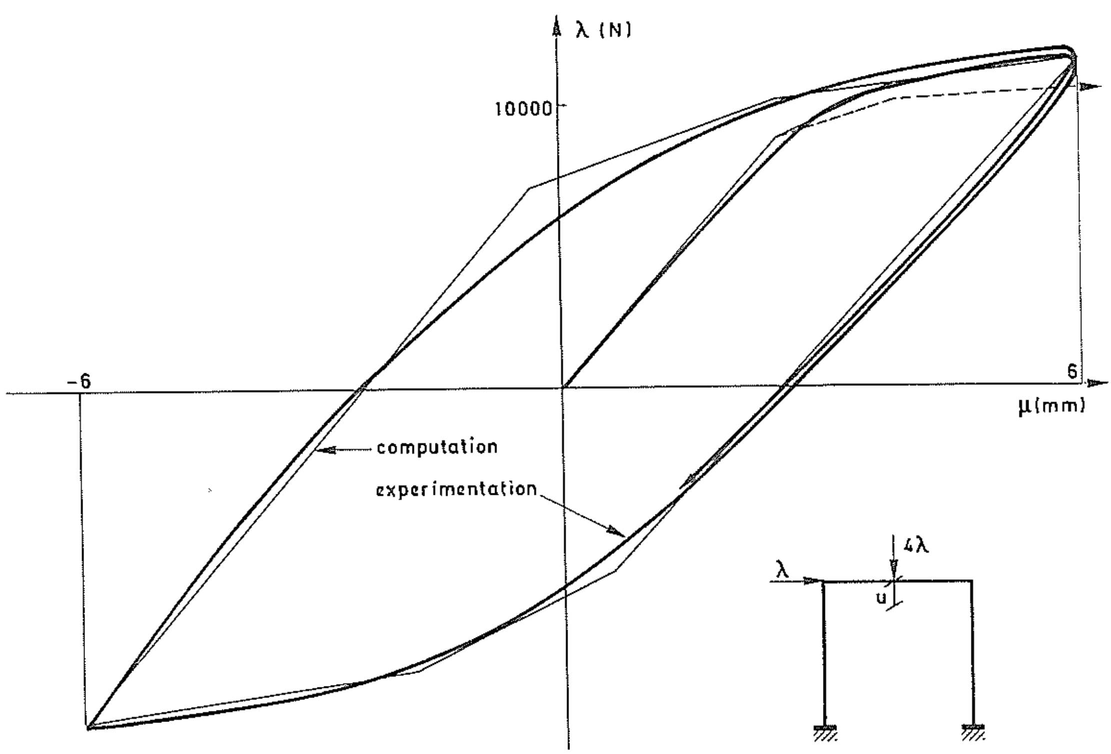

λ (N)
10000
-6
6
μ(mm)
computation
experimentation
4λ
λ
u

Fig. 5