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ON EFFICIENT COMPUTATIONAL PROCEDURES TO ASSESS STRUCTURAL RELIABILITY

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ABSTRACT

In this paper, firstly, the requirements for reliability assessment procedures are stated, i.e. critically reviewed. Secondly, a theoretical concept based on the Response Surface Method (RSM) and advanced simulation procedures is shown to meet these requirements. Thirdly, a software concept to make this procedure feasible for utilization by the engineering practice is briefly described. Finally, a numerical example is shown and conclusions are drawn for future developments.

1. INTRODUCTION

Modern developments in structural mechanics concentrate not only on a realistic mechanical modeling of structures but also on the respective representation of material properties and loading conditions. Extensive investigations showed very clearly that this can only be accomplished by considering, i.e. taking into account the statistical properties of the various parameters involved. As a consequence, traditional analysis and design procedures - which are generally based on very accurate structural analyses (mostly carried out by utilizing discretization procedures such as the Finite Element Method (FEM)), but using a quite simplified, i.e. deterministic load and material description along with mostly empirically developed safety factors - do not suffice to process this additional information on the parameters. In other words, the parameters are not described only by their "maximum" (e.g. load), "minimum" (e.g. strength) or average (e.g. Youngs modulus) values, but also by utilizing the information on their uncertainties in space and time by including information on variance, correlation, etc. which may be modeled either by random variables and/or stochastic processes. Hence the accuracy of the structural modeling is only one - although a very important one - among various issues for a rational structural design and analysis.

As a result of the additional effort which is required when performing the analysis by considering the parameter uncertainties, i.e. a so-called probabilistic analysis, its safety and reliability can be *quantified.* It is frequently overlooked by analysts and designers that this does not apply to traditional procedures, where load and response quantities are checked only pointwise. This concept opens new avenues of analysis and design, where structures and their components can then be designed according to a uniform safety level, i.e. target reliability, which can be specified according to the need - and also resource allocation - of society.

Needless to say, that the processing of the additional information on the uncertainties of the various parameters involved in structural analysis, requires the development of most efficient computational procedures in order to be able to assess structural reliability. This in turn requires the development of the respective software.

It should be noted, that the procedures as described here also apply not only to structures but also to mechanical components (due to their large size) not accessible to series tests.

2. Requirements for Reliability Assessment Procedures

Before addressing some aspects of software development - which in fact led for example to the breakthrough of the Finite Element Method (FEM) for practical application, i.e. its utilization by practicing engineers - a discussion on the requirements of structural reliability procedures, as well as the methodology which meets those requirements, is appropriate:

(1) Mechanical modeling as utilized in deterministic analyses must also apply for probabilistic analyses. In other words no additional simplifications should be introduced. This, of course, holds for structures both under static as well as dynamic loading.
(2) Design criteria for deterministic and probabilistic analysis should be compatible, e.g. such as serviceability, total and partial collapse, fatigue, brittle fracture, etc..
(3) Modern, i.e. stochastic methods have to be efficient and accurate. In other words, computational requirements should remain in acceptable limits and at the same time the reliability estimates should be accurate.

From (1) it becomes rather clear that probabilistic procedures have to be formulated such, that sophisticated modeling methods such as the Finite Elements - as used in deterministic analysis - can be utilized. Only in this case the more sophisticated modeling of load and material parameters within probabilistic concepts represent an important step forward in the overall development, i.e. in comparison with traditional deterministic analysis and design procedures. For being applicable to problems of the engineering practice this quite naturally leads to the development and application of numerical procedures also in probabilistic analysis.

As a consequence of this a complete new generation of structural analysis procedures is now in the stage of development. In the following the concept of a computational procedure, denoted by COSSAN (Computational Stochastic Structural Analysis) - which meets the requirements (1) to (3) - is introduced and discussed in some detail. Early developments of the procedures go back to 1989 [1]. Expansions to total collapse failure estimation of larger MDOF-systems may be found in [2,3]. The consideration of fatigue and brittle collapse failure is shown in e.g. [4,5].

3. A Brief Review of the Theoretical Concept

The design criterion of structures with random systems parameters - to be represented by a random vector $\mathbf{x}$ - can be characterized by the limit state function $g(\mathbf{x})$, which is defined by

$$\begin{array}{ll} g(\mathbf{x},T) > 0 & \text{no failure} \\ g(\mathbf{x},T) \leq 0 & \text{failure} \end{array} \qquad (1)$$

where T is the time range of observation, e.g. design life of a particular structure. The above formulation includes also time effects e.g. due to crack propagation, corrosion, embrittlement, etc.. For the determination of the effect of the various randomly distributed input parameters, such as load, geometric and material parameters, etc. on the structural response, the so-called Response Surface Method (RSM) proved to be most useful, particularly in view of reducing computational effort, as well as generality and versatility of the mechanical model used. Since, by structural analysis, only points on the response surface - indicating the limit state - are calculated, the limit state function, as defined by eq. (1) is to be approximated, by interpolation, by a function $\bar{g}(\mathbf{x})$, which is defined by

$$\bar{g}(\mathbf{x}) = a + \sum_{i=1}^{r} b_i x_i + \sum_{j=1}^{r} c_{ij} x_i x_j \qquad (2)$$

where the coefficients a, b and c are constants to be determined by structural analysis and r represents the number of variables considered in the analysis. When treating realistic problems, the generally large number of variables may be reduced considerably by considering only those variables which are of importance for characterizing the response surface and hence influencing the failure probability. This procedure is based on an eigenvalue analysis (see e.g. [1]). The effect of the remaining variables on the system behavior is taken into account by the pairwise combination as shown in eq. (2).

This procedure fulfills all the requirements as stated in section 2, i.e. the analysis may include all important mechanical effects. In other words, the method allows the utilization of the most modern structural analysis procedures. Within the further analysis the actual limit state function $g(\mathbf{x})$ as defined by eq. (1) is represented by the response surface $\bar{g}(\mathbf{x})$, as defined by eq. (2). The error, which is introduced by this approximation can be quantified (see [1]). Experience shows that in most cases its influence on the estimate of the failure probabilities is small.

The procedure as described here is quite general and maybe applied e.g. to structures to be modeled to MDOF-systems with hysteretic behavior, P-Δ effects and loads as general as evolutionary processes (see e.g. [2,3]). In this case the points at the limit state surface have to be determined iteratively by integrating the nonlinear equations of motion by respective nonlinear FE analyses.

Following the approximation of the limit state function by a response surface, the failure probability can be calculated by evaluation the following multidimensional integral:

$$p_f(t) = \int_{g(\mathbf{x},T) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \qquad (3)$$

where $f_{\mathbf{X}}(\mathbf{x})$ represents the joint probability density of the random (load and resistance) variables $\mathbf{x}$ and $g(\mathbf{x},T)$ describes the time variant failure domain as defined by eq. (1). For evaluation of eq. (3) various procedures are available. For this purpose quite frequently, advanced simulation procedures, which prove to be both efficient and accurate (see e.g. [6] for a review), are given preference. Approximate, first and second order methods (FORM, SORM; see e.g. [7]) may be also applied. However, it should be noted that for the quantification of the errors due to approximation, advanced simulation procedures have to be applied on top of it, which, of course limit the practicability of these methods.

4. SOFTWARE CONCEPT

It has been stated, that the extension of structural analysis and mechanics includes the consideration of the statistical uncertainties of the parameters involved. Its application in practice, however, will only be feasible when it is accompanied by a respective software development. Hence, respective attempts for such developments have already been made. For a brief review it is referred to e.g. [9]. In this context works on a *code* called COSSAN (Computational Stochastic Structural Analysis) date back as early as 1986 (in terms of the ISPUD-code, see [8]) and were then continued [10,1,2]. The basic structure of this development is sketched in Fig. 1. It should be noted that the concept as outlined here - as an option - in its extended form can also treat random material properties, as required when performing stochastic finite elements. The basis of the software is a user friendly command interpreter (denoted by SLANG (Structural Language) with all its well-known features. The program structure is modular and has an object oriented data management [11]. The code also utilizes commercially available software for graphical representation of input and output information [12].

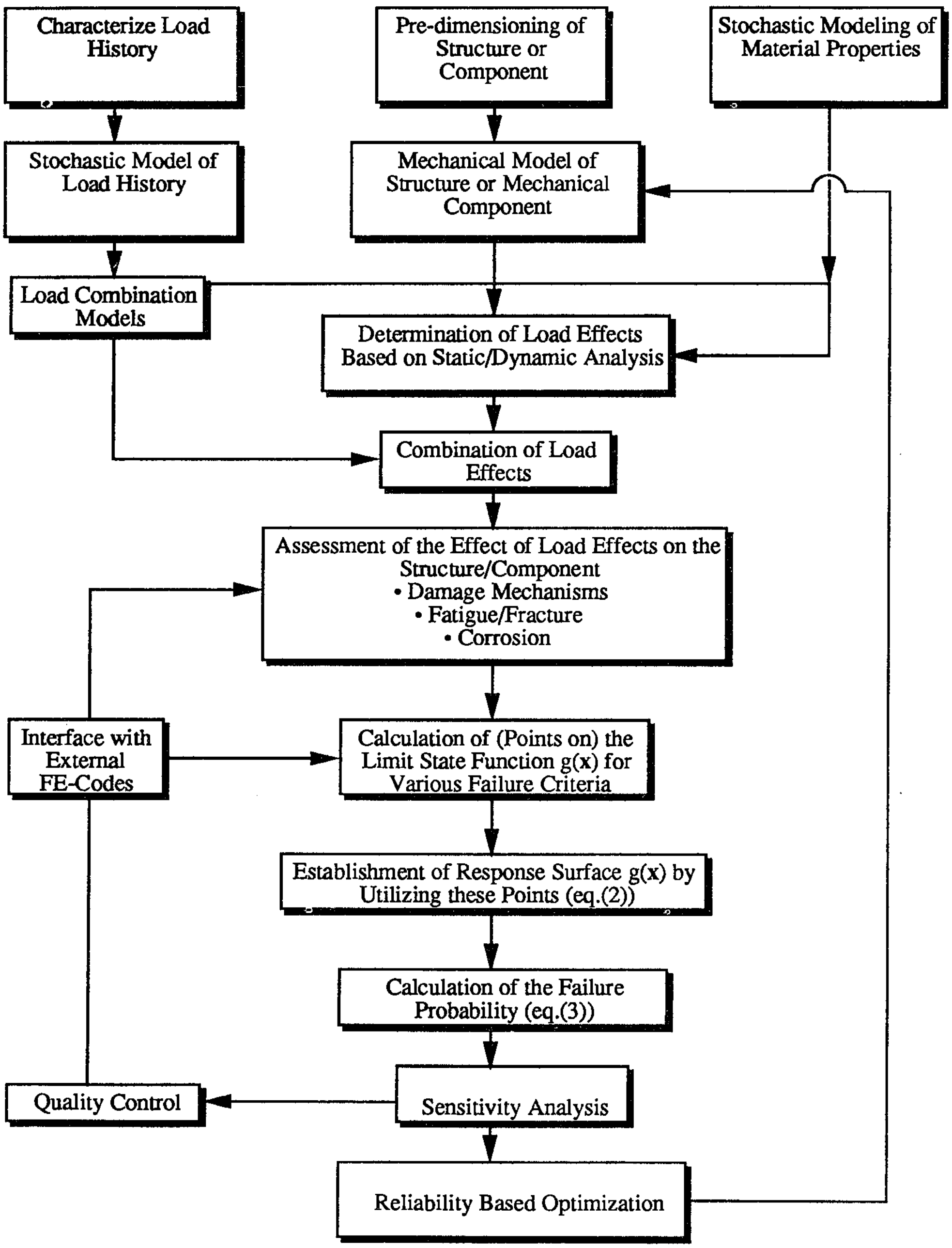

Fig. 1: COSSAN (Computational Stochastic Structural Analysis) - Schematic Flow Chart

5. NUMERICAL EXAMPLE

The procedure as outlined in section 3 is now applied to a problem, where the limit state function cannot be expressed by a smooth function but reveals some noise. In this context it can also be seen that linearization procedures i.e. the first order methods (FORM) for calculating the failure probability, may lead to erroneous results. In other words the requirements of efficiency and accuracy do not apply to all methods available for computing failure probabilities. To exemplify this a two dimensional linear limit state surface with an artificial noise is considered:

$$g(\mathbf{x}) = 9 - 1.5x_1 - 3.0x_2 - \alpha\sum_{i=1}^{2}\sin(100x_i) = 0 \qquad (4)$$

The random variables are assumed to be standard normally distributed. The results for different values of α are summarized in table 1. From this it can be seen that the methods based on linearization (e.g. FORM) provide accurate results only for α values as mall as 0.05. With increasing importance of the noise term of eq. (4) the results of FORM become meaningless. This, despite of the fact that a very efficient optimization procedure to identity the design point is used [13]. For reference, the result for the failure probability when neglecting the noise term in eq. (4) completely, is $0.4 \cdot 10^{-2}$. Both importance (ISPUD) [8] and adaptive (ADSAP) [10,14] sampling procedures provide accurate results.

Tab. 1: Results of the numerical example

α	0.05	0.08	0.1	0.2	0.5	0.8
p_f (FORM) [7]	$0.4 \cdot 10^{-2}$	$0.14 \cdot 10^{-3}$	0.4	0.08	0.17	0.5
p_f (ISPUD) [8]	$0.4 \cdot 10^{-2}$	$0.4 \cdot 10^{-2}$	$0.3 \cdot 10^{-2}$	$0.4 \cdot 10^{-2}$	$0.4 \cdot 10^{-2}$	$0.4 \cdot 10^{-2}$
S_{IE}[%]*	5	22	28	30	50	22
FAC	1.0	1.4	1.4	1.4	1.4	1.4
No. of Simulations	1024	1024	1024	1024	1024	1024
p_f (ADSAP) [10]	$0.4 \cdot 10^{-2}$	$0.4 \cdot 10^{-2}$	$0.4 \cdot 10^{-2}$	$0.4 \cdot 10^{-2}$	$0.4 \cdot 10^{-2}$	$0.5 \cdot 10^{-2}$
S_{IE}[%]*	6	7	5	5	6	6
No. of Simulations	$2 \cdot 512$	$2 \cdot 512$	$2 \cdot 512$	$2 \cdot 512$	$2 \cdot 512$	$3 \cdot 512$

*statistical error of estimate

6. CONCLUSIONS

It is quite clear that a breakthrough in application i.e. utilization of the new concepts in structural mechanics and analysis - i.e. by taking into account the statistical uncertainties of the parameters involved - will only be accomplished, i.e. accepted by the engineering profession on a larger scale when a respective user friendly software is available. (Similar developments were observed with the spread of the Finite Element method within the engineering profession). Engineering practice also requires the utilization of mechanical modeling of the structures and components at the same level sophistication as it is common practice in traditional, i.e. deterministic analysis.

The benefit of the modern, stochastic procedures is the fact that the reliability of the structures can than be *quantified* which is a most important step towards developing rational analysis and design procedures. It is shown that the Response surface Method (RSM) along with advanced simulation procedures are most instrumental to meet these requirements.

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