Modelling the Time-Dependent Behaviour of Concrete

P.E. Roelfstra, F.H. Wittmann
Laboratory for Building Materials, Swiss Federal Institute of Technology,
Chemin de Bellerive 32, CH-1007 Lausanne, Switzerland

SUMMARY

In many concrete structures such as prestressed pressure vessels or secondary safety containments concrete is subjected to thermal and hygral gradients in addition to external loads. A model is developed to describe materials behaviour under this complex time-dependent state of stress.

By means of this model the influence of (a) temperature gradients caused by heat of hydration or external heating, of (b) variation of moisture content, and of (c) further development of degree of hydratation under load on material properties is taken into consideration.

It is shown that crack formation in the tensile zone severely influences time-dependent deformation. Crack formation can be expressed as strain softening.

The model is first checked and verified by comparison with experimental findings obtained by means of test series on plain concrete. Then the materials model-law is used within a comprehensive computer program. With this program it is possible to calculate the time-dependent stress-strain relation of advanced concrete structures. Examples will be described in detail. Theoretical predictions will be compared with measurements on real structures.

In this contribution an advanced materials science approach is used to model actual materials behaviour in such a way that it can be used directly in computerized structural analysis.

1. INTRODUCTION

In many concrete structures such as prestressed pressure vessels or secondary safety containments, in addition to external loads concrete is subjected to thermal and hygral gradients. The present state of numerical methods and the availability of high-speed computers enable us to calculate this complex time-dependent state of stress (Willam /1/). The precision of the results of these calculations depends strongly on the assumptions made to describe the materials properties. In particular the time-dependent behaviour has to be introduced realistically. In most cases these model laws are based on simplifying relations which are determined phenomenologically. These computation rules, however, are only valid for special conditions such as constant stress, constant temperature etc, and they can't be used to describe real material behaviour under complex conditions.

In recent years much progress has been made in materials science in describing the time-dependent behaviour of hardened cement paste with models based on real mechanisms. One of these models by which good results have been obtained is the Munich-model (Wittmann /2/). This model, however, describes the time-dependent behaviour on a micro-level and an immediate application on a macro level is not possible. The aim of a research project, started recently in the Laboratory for Building Materials of the Swiss Federal Institute of Technology - Lausanne, is to bridge the gap between materials research on the one side and the mechanical analysis on the other.

The final result of this project will be the presentation of computation rules for the time-dependent behaviour of concrete which can be used in computerized structural analysis.

In this contribution the concept and preliminary results will be outlined.

2. CONCEPT

To clearify this concept first of all an example is given of the current numerical methods which are available to solve a complex time-dependent state of stress in a real structure. For this purpose the structural analysis of a part a safety containment wall is chosen.

In Fig. 1a the geometry and the finite element idealization is drawn.

Fig. 1b shows by means of isotherms the calculated temperature distribution. This temperature distribution is caused by a different temperature at the in- and outside of the containment wall and a high temperature at one spot (hot-spot) due to e.g. a leakage in a steam-pipe. This temperature distribution is calculated as a steady state process.

Fig. 1c shows by means of iso-hygre the calculated moisture distribution at a given time after demoulding. This moisture distribution is caused by the drying process, which is non linear.

At last the isobars of the calculated resulting axial stresses are drawn in Fig. 1d.

Some of the materials properties involved in these calculations are :

- modulus of elasticity;
- Poisson's ratio;
- thermal diffusion coefficient;
- hygral diffusion coefficient;
- thermal strain;
- hygral strain (unrestrained shrinkage);
- creep

All these materials properties depend on water/cement-ratio, degree of hydration, mix-properties of concrete and water content apart from other influences.

Taking also in consideration that concrete is non-homogeneous the complexity of this analysis will be clear.

In the present approach we can subdivide our activities into two levels. On the micro-level we are dealing with hardened cement paste, being a porous otherwise homogeneous material. On the macro-level we consider dense aggregate embedded in a matrix of hardened cement paste. The actual structure of concrete is generated by means of stochastic methods in the computer.

3. HEAT OF HYDRATION

The general equation governing the temperature distribution in a real solid can be written as :

$$\frac{\partial T}{\partial t} = \frac{1}{\gamma c}\frac{\partial S}{\partial t} + \text{div}\,(D_T \text{ grad } T) \qquad (1)$$

In this equation the symbols have the following meaning :

T = temperature;

$\frac{\partial S}{\partial t}$ = rate of liberation of heat of hydration;

γ = density;

c = specific heat;

D_T = thermal diffusion coefficient;

This problem can be solved numerically if the boudary conditions are given and all the materials parameters are known. In this case the density, the specific heat and the thermal diffusion coefficient can be determined easely by appropriate tests.

The main problem is the determination of the rate of liberation of heat of hydration because the rate of liberation of heat of hydration depends strongly on the actual temperature.

In Fig. 2 the calculated temperature distribution within a specimen is plotted. These results correspond well with experimental findings.

If the behaviour of hardened cement paste made of different types of cement and having different types of cement and having different water-cement ratios is known (micro-level) the temperature gradient in a given structural concrete element can be calculated with sufficient accuracy (macro-level).

4. MOISTURE DIFFUSION AND TIME-DEPENDENT DEFORMATION

4.1. Diffusion process

It has been shown by Pihlajavaara /3/ and Bazant /4/ that the diffusion equation for drying of hardened cement paste can be written as :

$$\frac{\partial H}{\partial t} = \mathrm{div}\,(D_H\ \mathrm{grad}\ H) \qquad (2)$$

In this equation the symbols have the following meaning :

H = pore humidity;
D_H = hygral diffusion cceefficient;

Provided that drying proceeds, the pore system is changed and therefore D_H depends on the pore humidity H. That means that drying process is nonlinear. The diffusion coefficient depends in addition on the temperature, the watercement ratio, the type of cement and the degree of hydration.

As to be able to describe the drying process of hardened cement paste realistically (micro-level) the necessary materials properties are being measured in our laboratory. If the drying process of the homogeneous material is well understood drying of a composite material such as concrete can be calculated by taking the random structure into consideration (macro-level).

4.2. Unrestrained shrinkage

An infinite small particle of hardened cement paste exhibits an immediate hygral volume change if the water content is changed. This volume change is called unrestrained shrinkage or unrestrained swelling.

Klug /5/ has determined the relationship between unrestrained shrinkage and the relative humidity. It turned out that unrestrained shrinkage ε_u can be described satisfactorily as function of relative humidity H by a linear relationship if the extreme regions are excluded :

$$\varepsilon_u = a\,H + b \qquad (3)$$

In this equation a and b are materials parameters and thus depend on water/cement-ratio, degree of hydration and type of cement.

4.3. Drying induced time-dependent deformation

If the moisture distribution is calculated following equation (1) the internal state of stress follows directly. This internal state of stress creates a time-dependent deformation usually called shrinkage. It is important to note that crack formation in the outer drying shell essentially influences the observed deformation (see Wittmann and Roelfstra /6/).

One example is shown in Fig. 3. In this figure calculated shrinkage is compared with measured data.

4.4. Creep

Creep of hardened cement paste without loss or gain of capillary water is comparable to the time-dependent deformation of other materials such as soils, ceramics, polymers and metals. The rate theory provides a solid theoretical basis for a rather general approach to study creep processes in solid matter. Therefore rate theory has also been adopted to describe creep of hardened cement paste.

The rate of creep of loaded hardened cement paste is then given by the following equation :

$$\frac{\partial\varepsilon}{\partial t} = Ce^{-\frac{Q}{RT}} \sinh \frac{V}{RT}\,\sigma \qquad (4)$$

In this equation the symbols have the following meaning :

Q = activation energie;
V = activation volume;
R = gas constant;
T = temperature;
σ = stress;
C = a quantity proportional to the density of creepcenters in a unit volume;

Q and V do not depend on the duration of the applied stress. The time-dependence of the rate of creep is given by quantity C. For a constant stress, humidity and temperature the change of C in time can be written as :

$$C = C_1\, t^{-m} \qquad (5)$$

This assumption leads to the well known creep formulea :

$$\varepsilon = at^n \sinh b\sigma \qquad (6)$$

All parameters in this equation are dependent on the moisture content. Therefore within a drying solid different creep functions describe the actual time-dependent deformation following the moisture gradient. It is impossible to take this in consideration analitically. With modern computer technics, however, this complex situation can be simulated realistically.

5. CONCLUSION

In this contribution the concept and preliminary results of a research project are outlined. The final aim of this research project is the presentation of computation rules for the time-dependent behaviour on a macro-level based on real mechanisms of hardened cement paste on a micro-level. The relationships between materials parameters in these computation rules and the water/cement-ratio, water content, type of cement, degree of hydration etc. will be determined by means of several test series.

The preliminary results are promising and it can be hoped that the final aim, i.e. rigorous treatment of the complex materials behaviour, will be reached too.

6. REFERENCES

/1/ ARGYRIS, J.H., WARNHE, E.P. and WILLAM, K.J., "Berechnungen von Temperatur- und Feuchtefeldern in Massivbauten nach der Methode der finiten Elemente", Deutscher Ausschuss für Stahlbeton, Schriftenreihe, Heft 278 (1977).

/2/ WITTMANN, F.H., "Grundlagen eines Modells zur Beschreibung charakteristicher Eigenschaften des Betons", Deutscher Ausschuss für Stahlbeton, Schriftenreihe, Heft 290 (1977).

/3/ PIHLAJAVAARA, S.E., "On the main features and methods of investigation of drying and related phenomena in concrete", State Inst. for techn. research, Helsinki, Ph.d.Thesis, Publ. No 100 (1965).

/4/ BAZANT, Z.P. and NAJJAR, L.J., "Nonlinear waterdiffusion in nonsaturated concrete", Mat. et Constr. 5, 3-20 (1972).

/5/ KLUG, P., "Kriechen, Relaxation und Schwinden von Zementstein", Dissertation, TU München (1973).

/6/ WITTMANN, F.H. and ROELFSTRA, P.E., "Total deformation of loaded drying concrete", Cement and Concrete Research, Vol. 10, 601-610 (1980).

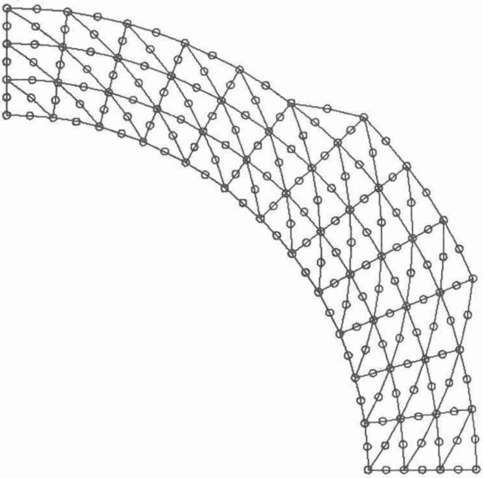

Fig. 1a : Finite element idealization of a containment wall

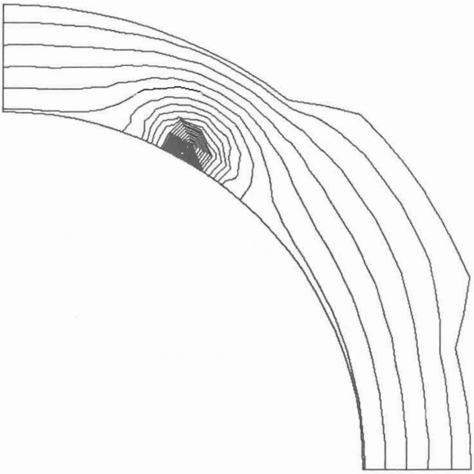

Fig. 1b : Temperature distribution (stendy-state process)

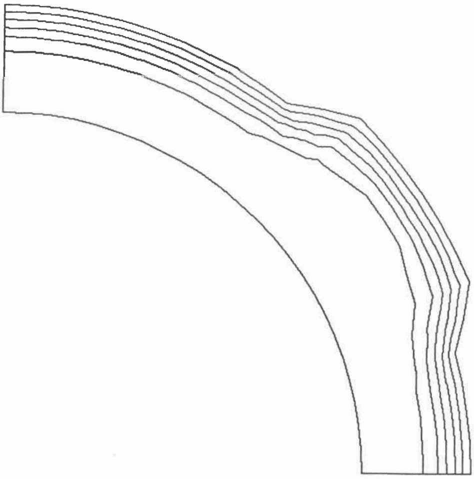

Fig. 1c : Moisture distribution (nonlinear diffusion process)

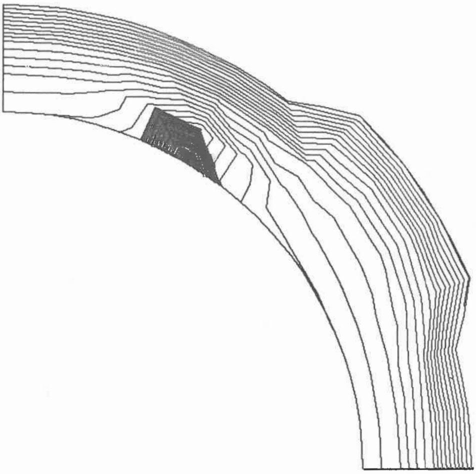

Fig. 1d : Resulting axial stresses

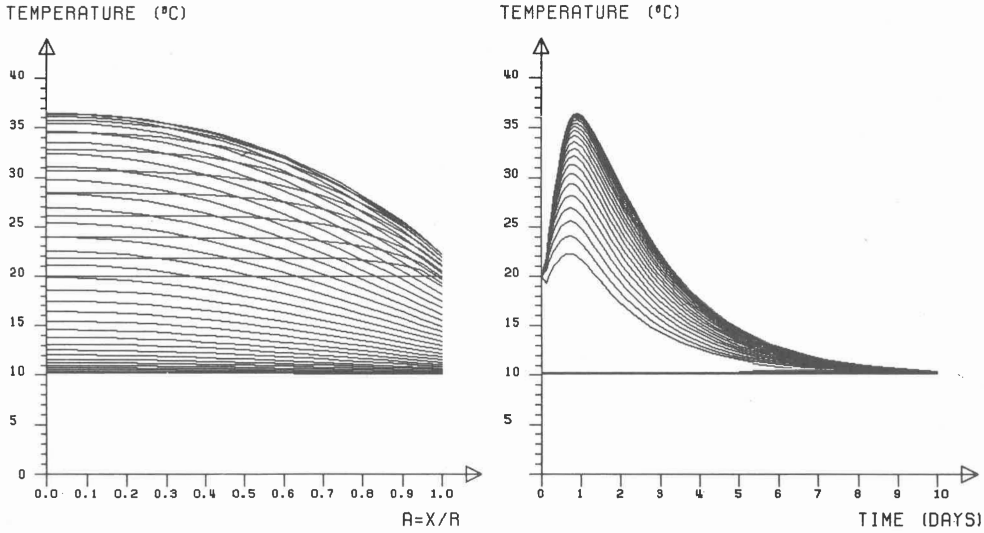

Fig. 2 : Calculated temperature distribution in a specimen due to heat of hydration

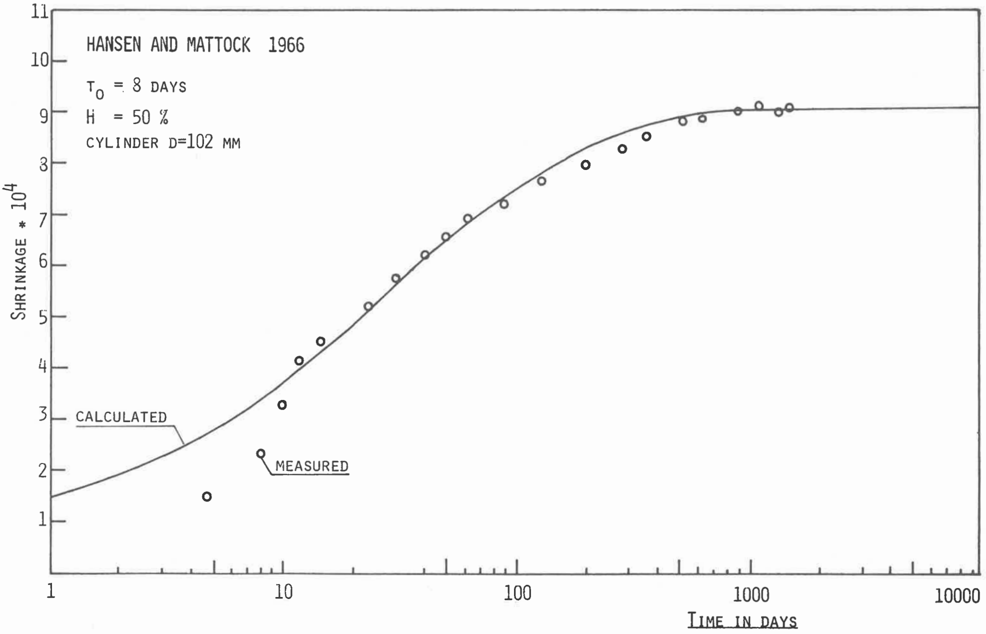

Fig. 3 : Calculated shrinkage compared with measured data