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The influence of plate thickness on damage development in creep conditions

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ABSTRACT: The effect of shear stress in plates, which is known to be pronounced for thick plates, is analysed here for steel plates in high temperature environment. In such circumstances the non-linear creep phenomenon has to be taken into account, followed by a damage growth. The output of numerical calculations is focused on the time to first crack appearance (FCA) t_1 and its relation to the time of final rupture t_2 marked by a through body crack formation. The development of plate cracking for a square and rectangular plates is studied, too.

1 PROBLEM DESCRIPTION

In high temperature environment the creep phenomenon is known to be observed for metallic structural components. As far as safety is concerned the key role is played by the damage growth associated with microstructural changes (void formation, microcracking, etc.). These processes manifest itself on macroscopic level by one (or more) predominant crack formation at time which in the following will be referred to as time to First Crack Appearance (FCA) and denoted by t_1. The consecutive spreading of macrocracks over structure's body lead ultimately to the formation of a crack which spans the characteristic structure's cross -section (the plate thickness, for example). The time when this event happens is referred to as time of Through-body Crack Appearance (TCA), with corresponding notation of t_2. The determination of t_1 allows for lower limit of structure's life-time evaluation, whereas the ratio of t_2/t_1 can be dealt with as safety factor set upon the structure. Both depend on numerous parameters which can be related to material properties (material constants present in the constitutive equations) or to the structure's geometry and loading.

Among these parameters which relate to material properties these will be the interest of present analysis which reflect hypothesis behind damage growth (maximum principal stress or maximum effective stress hypotheses).

Parameters related to structure's geometry will be the thickness-to-width ratio of a plate, and square versus rectangular plate effect. Only clamped plates will be analysed and the load will be evenly distributed over the upper surface of the plates.

The pattern of cracks spread at the time of TCA will be discussed, too.

2 GOVERNING EQUATIONS

Steady-state theory of non-stationary creep (elastic stress is non-negligible) coupled with classical damage theory by Kachanov-Rabotnov is used in the following analysis (cf. Bodnar and Chrzanowski 1987). Under assumption that the total strain tensor ε_{ij} can be decomposed into two components - elastic ε_{ij}^{e} and creep ε_{ij}^{c} ones:

(1) $$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{c} ,$$

the set of constitutive equations is as follows:

(2) $$\varepsilon_{ij}^{e} = D_{ijkl}^{-1}\sigma_{kl} ,$$

(3) $$\frac{\partial \varepsilon_{ij}^{c}}{\partial t} = \gamma\left(\frac{\sigma_e}{1-\omega}\right)^n \frac{\partial \sigma_e}{\partial \sigma_{ij}} ,$$

(4) $$\frac{d\omega}{dt} = A\left(\frac{\sigma_{eq}}{1-\omega}\right)^m ,$$

where σ_{ij} is stress tensor, D_{ijkl} is the elastic constants matrix, γ, n, A, m are creep and damage material constants and t is time.

The equivalent stress σ_{eq} in equation 4 is given by the following formula (Hayhurst 1972):

(5) $$\sigma_{eq} = \alpha\sigma_1 + (1-\alpha)\sigma_e ,$$

where σ_1 and σ_e stand for the maximum principal tensile stress and von Mises effective stress, respectively. A parameter α $(0 \le \alpha \le 1)$ characterises failure mechanism mode; for $\alpha=1$ the brittle (inter-granular) fracture occurs whereas the case of $\alpha=0$ corresponds to the ductile (trans-granular) fracture. The intermediate values of α refer to mixed modes of failure.

With equilibrium equation

(6) $$\frac{\partial \sigma_{ij}}{\partial x_j} = 0$$

and strain-displacement relationship

(7) $$\varepsilon_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$$

equations 1-7 form the set of problem governing equations.

3 NUMERICAL ANALYSIS

As analytical solutions for this problem are not available the Finite Element Method for structure discretisation and Euler's procedure for time integration were used. In the computer code developed by authors, that enables analysis of thin plates as well as those of moderate thickness, the layered isoparametric eight-node Serendipity shell elements with reduced integration were employed. Ten layers and two-point Gaussian quadrature for volume integration were adopted. Details of the algorithm and conditions for numerical stability of solution can be found in the paper by Bodnar et al. (1994). The time t_1 is identified with $\omega = 1$ condition fulfilled in any layer and Gaussian point; when this condition is reached in all ten layers of a Gaussian point the time is referred to as t_2.

Two types of clamped plates have been analysed: square one with the sides length equal to 1.0x1.0 m and rectangular one with the sides length equal to 1.0 x2.0 m. The thickness of both plates were taken as 0.01, 0.05, 0.10 and 0.2 m. The material of the structures was Ti-6Al-2Cr-2Mo alloy which material constants at a temperature of 675 K are that given by Walczak (1981): $E = 0.102 \cdot 10^6 MPa$, $\nu = 0.33$, $\gamma = 1.38 \cdot 10^{-24} (MPa)^{-n} h^{-1}$, $n = 6.8$, $A = 1.08 \cdot 10^{-20} (MPa)^{-m} h^{-1}$, $m = 5.79$. Parameter α was chosen as 0, 0.5 and 1 for analysis, though only the value of α=0.5 fits this material. To allow the comparison of results the loading for analysed plates was chosen in such a way that the maximum of equivalent stress at the time t=0 was the same; this loading is given in Tables 1 and 2 for square and rectangular plates, respectively.

4 RESULTS

The results of analysis for considered plates are summarised in the Tables 1 and 2.

Table 1. Results of analysis for square plates

Thickness [m]	α	Load p [MPa]	Time t_1 [10^5 hr]	Time t_2 [10^5 hr]	Ratio t_2/t_1
0.01	0	0.22648	1.9227	2.3101	1.2016
	0.5	0.21242	1.5783	2.8884	1.8301
	1	0.20000	0.9213	2.0694	2.2462
0.05	0	5.63060	1.8032	2.1236	1.1777
	0.5	5.30060	1.5194	2.7446	1.8064
	1	5.00700	0.8985	2.0033	2.2297
0.10	0	22.24350	1.4723	1.5959	1.0839
	0.5	21.14580	1.4267	2.4925	1.7470
	1	20.15100	0.8826	1.9249	2.1810
0.20	0	83.78500	0.7160	0.7220	1.0083
	0.5	81.99600	1.1945	1.7395	1.4562
	1	80.28200	1.0143	1.8118	1.7863

The spreading of cracks' network as a function of plate thickness is shown for rectangular plates in Tables 3 through 5. The FCA and TCA locations are indicated there by open circles and full triangles, respectively, and cracks' network is shown at time of TCA.

Table 2. Results of analysis for rectangular plates

Thickness [m]	α	Load p [MPa]	Time t_1 [10^5 hr]	Time t_2 [10^5 hr]	Ratio t_2/t_1
0.01	0	0.13583	1.1439	1.6621	1.4531
	0.5	0.12737	0.8555	1.9679	2.3002
	1	0.11990	0.5014	1.3043	2.6018
0.05	0	3.37380	1.1197	1.5843	1.4150
	0.5	3.16190	0.8546	1.9466	2.2776
	1	2.98785	0.5043	1.3063	2.5905
0.10	0	13.24880	1.0566	1.2644	1.1967
	0.5	12.51030	0.8709	1.9720	2.2644
	1	11.84950	0.5220	1.3593	2.6041
0.20	0	49.83800	0.7088	0.7205	1.0166
	0.5	47.94000	0.9166	1.9176	2.0920
	1	46.18150	0.6221	1.6212	2.6060

Table 3. Cracks network for rectangular plates (α=0) at time of TCA

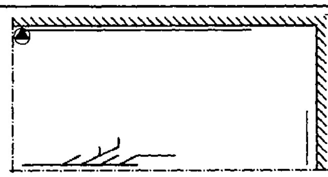
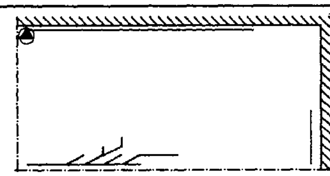
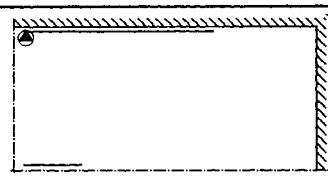
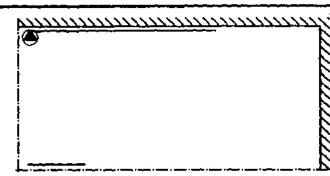
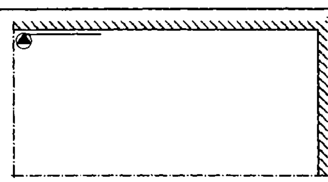
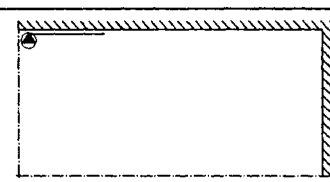
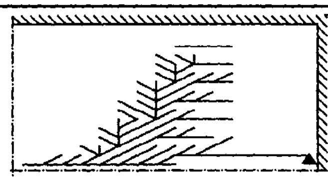
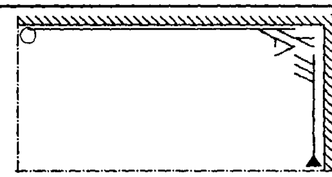
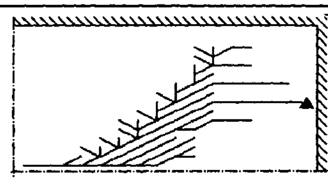
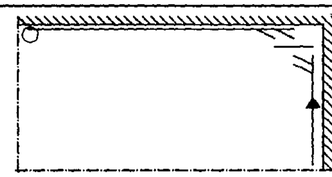
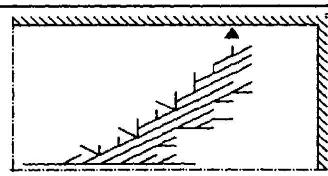
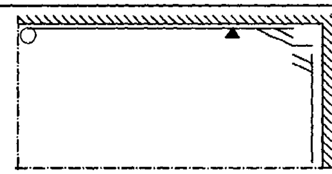
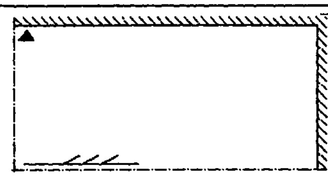
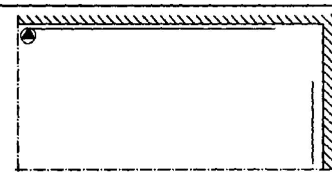
Thickness	View from the bottom of the plate	View from the top of the plate
0.01 m		
0.05 m		
0.10m		
0.20m		

Table 4. Cracks network for rectangular plates (α=0.5) at time of TCA

Thickness	View from the bottom of the plate	View from the top of the plate
0.01 m		
0.05 m		
0.10m		
0.20m		

4 CONCLUSIONS

From the tables 1 and 2 the influence of plate thickness on all: t_1 , t_2 , and t_2/t_1 is clearly seen.

As time to first crack appearance (FCA) - t_1 is considered the following general observations can be made:

- for square plates t_1 decreases as plate thickness increases for all values of α, except for the case of α=1 and plate thickness of 0.2 m,
- for rectangular plates t_1 decreases as plate thickness increases only for $\alpha = 0$. For all other values of α the time t_1 increases.

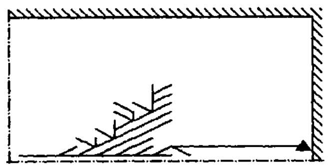
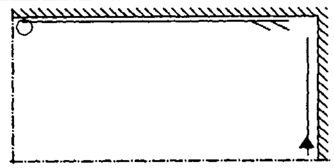
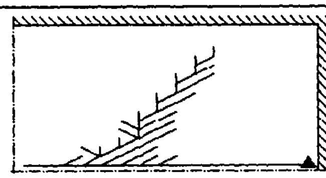
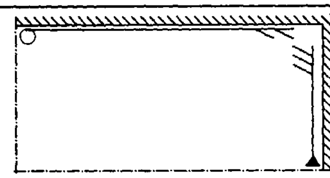
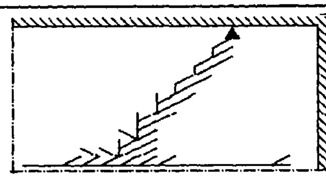
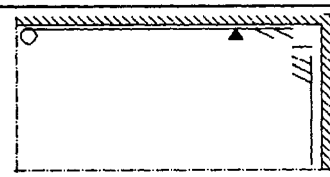
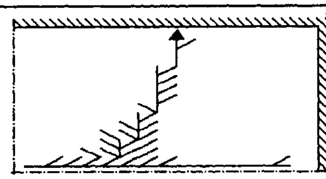
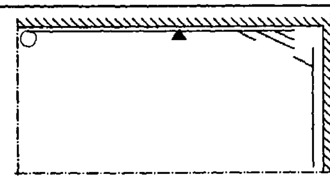
Time to through-body crack appearance (TCA) - t_2 undergoes following observations:

- for square plates t_2 decreases as plate thickness increases for all values of α,
- for rectangular plates t_2 decreases as plate thickness increases only for $\alpha = 0$. For α= 1 it increases, and for $\alpha = 0.5$ is practically constant.

For both square and rectangular plates the ratio of t_2/t_1 decreases always as thickness of a plate increases. The only exception is for a rectangular plate made of material, in which damage growth is controlled by maximum principal stress (α=1).

For the time greater than t_I a system of mackrocraks appears. For all analysed plates the cracks appear first on the upper surface of the plate along its edges: the plates tend to "free" theirself from imposed redundancy, to became ultimately a simply supported plate. Then, cracks develop on the bottom surface, until TCA.

Table 5. Cracks network for rectangular plates (α=1) at time of TCA

Thickness	View from the bottom of the plate	View from the top of the plate
0.01 m		
0.05 m		
0.10m		
0.20m		

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