CREEP ANALYSIS OF BOILER-PODDED PCPV BY THE METHOD OF SLICED SUBSTRUCTURES

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SUMMARY

The authors developed an approximate method of three-dimensional stress analysis for boiler-podded PCPV, "the method of sliced substructures", in which the system stiffness matrix in the finite element form is formed as the combination of the stiffness representing the axi-symmetric component of resistance and the one representing the two-dimensional resistance of horizontally sliced layers of PCPV. The elastic analysis of an actual PCPV revealed that the new method could reduce the computing time to the order of 1/10 of the ordinary full three-dimensional finite element analysis while maintaining enough accuracy. The final degrees of freedom for the system equations are equal to those of the corresponding axi-symmetric problem for a PCPV having the configuration of a solid of revolution.

The present paper applies the same method to the time-dependent creep analysis of the boiler-podded PCPV, in which the need for improvement of computational efficiency is more urgent because numerous steps of elastic analysis have to be repeated.

Adopting a constitutive law based on the rate of flow method for the creep of concrete, a visco-elastic model consisting of a series of a spring, a dashpot and two Kelvin units was used, where the behavior of the model was defined in the axis of pseudo time taken equal to existing unit flow component. Linear dependence of the rate of concrete creep on temperature was assumed. In the formation of three-dimensional constitutive equation, a constant Poisson's ratio was assumed. The perforated zone in the top slab was transformed into an equivalent transversely isotropic continuum and effective creep coefficients were derived.

In a time step of the creep analysis, incremental creep strain under the sustained stress is evaluated, followed by the computation of re-distributing stresses through a cycle of elastic analysis. In the latter part of the time step analysis the method of sliced substructures was incorporated. The system stiffness matrix was decomposed into Cholesky form beforehand of the time step analysis, the latter being greatly facilitated because only the backsubstitution process was needed in solving the system equations in each step.

As a numerical example, an actual PCPV for HTGR was analysed. Stress and deformation histories under sustained prestress and cyclic loading and unloading of inner pressure and thermal gradient taking place in accordance with operation, hot and cold shutdowns to be experienced within 40 years were followed up.

The results of the numerical analysis show the validity and efficiency of the new method, which provides us with a tool of analysis whose computational expense is reasonable for the purpose of the design of PCPV.

1. INTRODUCTION

Three-dimensional stress analysis for the design of prestressed concrete pressure vessels of podded boiler type, which must be carried out for the different phases of elastic reponse, time dependent creep and elasto-plastic behavior, is often heavy burden to designers because of large amount of labor and cost of the computation involved.

A previous paper [1] presented "the method of sliced substructures", an approximate method of three-dimensional stress analysis in the form of finite elements developed for the design of the same type of PCPVs, and verified the accuracy and saving of computation time attainable by the new method with regard to elastic analysis of an actual pressure vessel.

Though the recent advancement of the theory of creep seems to enable us consistently to model the age and temperature dependent creep behavior of concrete and to produce efficient program of computation [2,3], the three-dimensional nature of the stress distribution would still make it difficult to develop a creep analysis program of boiler-podded PCPV efficient enough as a design tool. The object of this paper is to present a method of creep analysis of the PCPV, which was developed by the application of the method of sliced substructures, in stead of the ordinary full three-dimensional method, to creep prediction based on the rate of flow method [4], and also to verify the validity and efficiency of the method by the actual analysis of an example problem of PCPV.

2. Method of Sliced Substructures

Only the brief outline of the method being described herein, the previous paper [1] should be referred for detailed discussion.

The method is based on two fundamental assumptions regarding the stress pattern of the boiler podded PCPVs:

1) the stress disturbance caused by the boiler pod cavities is dominant in the component of σ_r, σ_θ and $\tau_{r\theta}$ which are contained in horizontal layers of the vessel and can be represented by two-dimensional finite element structures of horizontal slices, and

2) the remaining stress components of σ_z and τ_{rz} have, in effect, axisymmetric pattern of distribution and their contribution to the structural stiffness can be evaluated by the axi-symmetric finite element structure into which a representative vertical section of the vessel is divided.

The stiffness matrix contributed by the former components is formed with respect to individual slices as shown in Fig.1 (b) and then condensed by eliminating the degrees of freedom denoted by $\{\bar{d}_{II}\}$ in the figure, only the radial displacements of the nodes resting on the vertical plane of symmetry, $\{\bar{u}_I\}$, being retained. The contribution of the latter conponents to the stiffness matrix is evaluated with respect to the axi-symmetric finite elements of the vertical section as shown in Fig.1 (a).

The two independent groups of the stiffness matrices are combined to form a system stiffness equation by regarding the displacements $\{\bar{u}_I\}$ of the slices to be identical to the corresponding components of the nodal displacements on the vertical section, $\{\bar{u}\}$. Thus, the system equation has the same degrees of freedom as the corresponding axisymmetric problem of PCPV without the boiler pod cavities.

3. Method of Creep Analysis

3.1. Uniaxial Constitutive Model for Concrete

The constitutive relation for concrete under uniaxial stress is formulated based on the rate of flow method [4]. Time dependent total strain of concrete is represented by

$$\varepsilon(t) = \varepsilon_e(t)+\varepsilon_f(t)+\varepsilon_d(t) = \frac{\sigma(t)}{E(t)} + \int_0^t J_f(t-\tau)\frac{d\sigma(\tau)}{d\tau}d\tau + \int_0^t J_d(t-\tau)\frac{d\sigma(\tau)}{d\tau}d\tau \quad (1)$$

where t : age of concrete

τ : time of application load

$\varepsilon(t)$: total strain, $\varepsilon_e(t)$: elastic strain

$\varepsilon_f(t)$: irreversible, flow component of creep strain

$\varepsilon_d(t)$: reversible, delayed elastic component of creep strain

$J_f(t-\tau)$: flow strain by unit stress

$J_d(t-\tau)$: delayed elastic strain by unit stress

According to the rate of flow method, it is assumed that J_f has a prescribed rate of progress as the function of t regardless of time of loading τ, and that J_d, converging to a given limit at $\tau = \infty$, takes a value exclusively depending on the present value of the specific flow J_f.

Further, the actual time axis t is transformed into a pseudo-time axis t', which is scaled in accordance with the progress of the specific flow strain J_f [5], i.e.,

$$J_f(t'-\tau') = t'-\tau' \quad \text{eq. (2)}$$

By the use of this pseudo-time axis, the constitutive relation can be represented by a visco-elastic model which is, in appearance, independent on the age of concrete at which the load is applied. If we assume the specific delayed elastic strain in the form of

$$J_d(t') = J_{d\infty}(1-e^{-\frac{t'-\tau'}{a}}) \quad \text{eq. (3)}$$

the constitutive model in the pseudo-time axis is represented by a Maxwell fluid model plus a Kelvin solid unit connected in series as shown in Fig.2, where the dashpot and the Kelvin unit correspond to the flow and delayed elastic components, respectively.

It is known that the integration for the creep strain of this type of constitutive model can be easily handled by a single step incremental scheme without recourse to the memory of full history of stress and strain [6]. When the integration with respect to the pseudo-time for the whole period of the subject analysis is completed, an inverse transformation of t' into t is carried out to obtain the real history of stress and strain.

The sliced substructures system, a kind of finite element approximation, is incorporated as the means for elastically releasing the unbalanced internal forces caused by the creep strain which has been accumulated during a time step.

3.2. Creep Compliance under Tri-Axial Stress

In this analysis, creep Pisson's ratio is assumed to be constant and equal to its elastic value according to Illston and Jordaan's view [8]. Accordingly, creep compliances, covering both the flow and delayed elasticity, for tri-axial stress state are formulated from the corresponding uniaxial specific creep functions in proportion to the elastic compliance for tri-axial stress.

3.3 Effective Creep Compliance of Perforated Zone in Top Slab

In the perforated zone of the top slab of PCPV, creep strain is accelerated because of the stress concentration around the holes. In order to deal with this portion as a transversely isotropic continuum, an effective creep compliance is formed by the use of the effective compliance in the elastic analysis, which is evaluated by the finite element analysis of unit area of the hole pattern [7].

3.4. Temperature Dependence of Creep of Concrete

It is assumed that the rate of flow component of creep strain expressed in pseudo-time axis is a linear function of temperature T. For uni-axial creep, the following relation is adopted:

$$\varepsilon_f(t', T) = \int_0^{t'} \rho(T)(t'-\tau') \frac{d\sigma(\tau')}{d\tau'} d\tau' \quad \text{eq. (4)}$$

$$\varepsilon_d(t', T) = \int_0^{t'} J_d(\rho(T)(t'-\tau')) \frac{d\sigma(\tau')}{d\tau'} d\tau' \quad \text{eq. (5)}$$

Where $\rho(T)$: linear function of T

4. Computer Program

In the developed computer program, the system stiffness matrix is formed according to the procedure of the elastic analysis [1, 7], which is then decoposed in Cholesky form and kept in the memory to be used repeatedly in the process of stress redistribution in the individual time steps.

In each time step, in-plane creep strains of the sliced layers and axisymmetric creep strains in the vertical section are separately evaluated and the equivalent nodal loads from both the strains are then combined to form the loading terms of the final system equations. For the part of the slices, some modification of the individual nodal loads is necessary before adding to the loads from the vertical section, because some of the nodal displacements of the slices were eliminated by the condensation.

When the system equations for each time step are solved, the axisymmetric components of the redistributed stress are obtained directly from the solution. The nodal displacements and the stresses of the slices are to be obtained via the back-substitution process of the condensed substructures.

5. Analysis of a PCPV

5.1. Object of Analysis

As a numerical example, an actual PCPV as shown in Fig.3 was analyzed and stress and strain histories under 40 years of life were followed up.

Vertical and circumferencial prestressing forces shown in Fig.4, operating inner pressure of $p = 54.6$ kg/cm^2 and prescribed design temperature, whose values of representative points are shown in Fig.5, were assumed to be subjected according to a loading history shown in Fig.6.

5.2. Creep Coefficient for Concrete

Fundamental coefficients for the uniaxial creep of concrete were evaluated and assumed based on Illston's experimental curves [4] and the coefficients of the temperature dependence function were obtained by processing the experimental datas by Browne [9].

Thus the following specific creep function for the uniaxial condition was used:

$$J(t'-\tau') = \frac{1}{E} + \rho(T)(t'-\tau') + Q\{1 - e^{-\lambda\rho(t'-\tau')}\} \quad \text{eq. (6)}$$

where

$E = 3 \times 10^5$ kg/cm^2 $\quad (\nu_c = \nu = 0.17)$

$Q = 1.394 \times 10^{-6}$ cm^2/kg , $\quad \lambda = 2.869 \times 10^6$ kg/cm^2

$t' = (-1.5057 + 0.7738 \log t) \times 10^{-6}$ cm^2/kg $= J_f$

$$P(T) = \frac{1}{50}(T+30)\ ,\ T \text{ in } ^\circ C$$

5.3. Finite Element Subdivision and Time Steps

Fig.7(a) shows the vertical section which was divided into a mesh of axi-symmetric elements and Fig.7(b), a representative horizontal slice which was divided into a mesh of the two-dimensional finite elements.

The period of 40 years to be analyzed was divided into 52 stpes of interval, where the finer division was used for the points nearer to the occurence of variation of loads.

5.4. Computing Time

The final degrees of freedom of the system stiffness equations were 970. The total CPU time required for the analysis was 9,640 sec by the use of IBM 370/158 VS-II machine. CPU time necessary for the analysis of single time step was 175 sec.

5.5. Result of Analysis

Fig.8 and 9 show the stress and strain histories of the representative points of the vessel. Fig.10 shows the deformation and stress distribution just after the beginning of operation at the age of concrete of 2 years, while Fig.11 shows those under operation after 40 years.

6. Conclusion

A method of creep analysis for boiler-podded PCPVs was developed applying the method of sliced substructures to the creep prediction by the rate of flow method. The result of the numerical example worked out on an actual PCPV seems to be valid. The required CPU time indicates that the cost of the analysis will be reasonable for the purpose of design.

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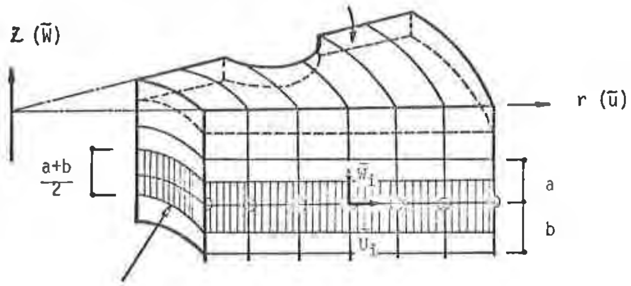
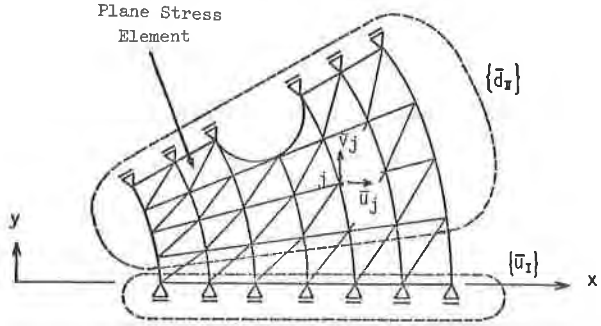


(a)



(b)

Fig.1 Dual System of Finite Elements
(a) Ring Elements and Horizontal Slice
(b) Division of Slice

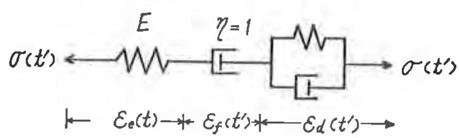


Fig.2 Constitutive Model in Pseudo-Time t'

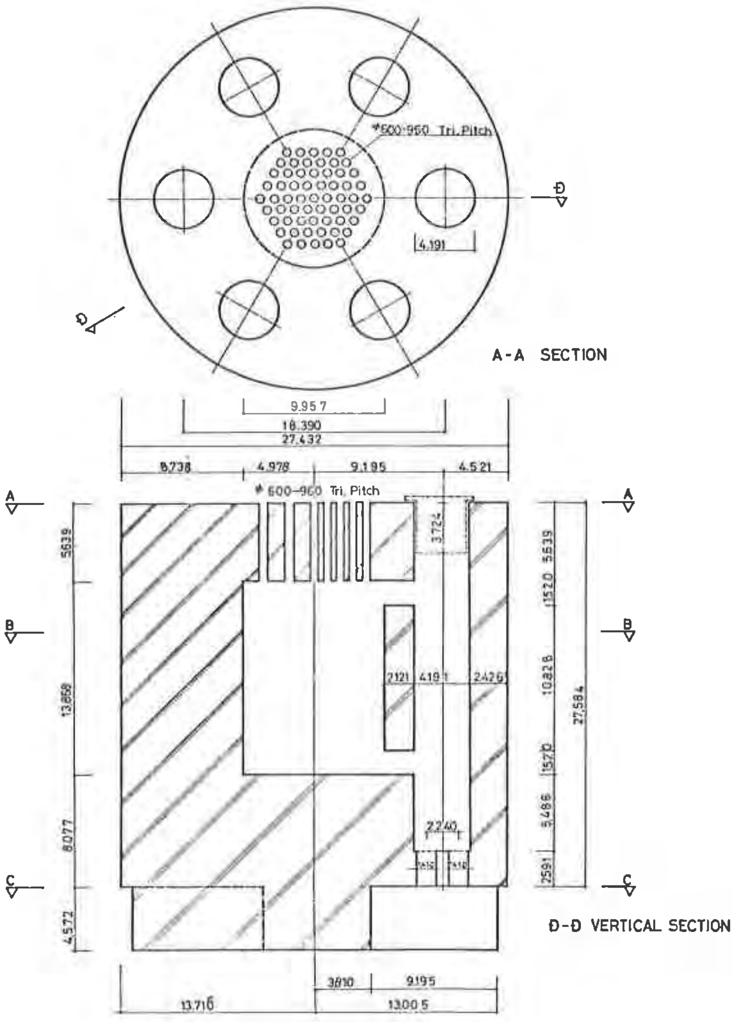


Fig.3 Analyzed PCPV (unit : mm)

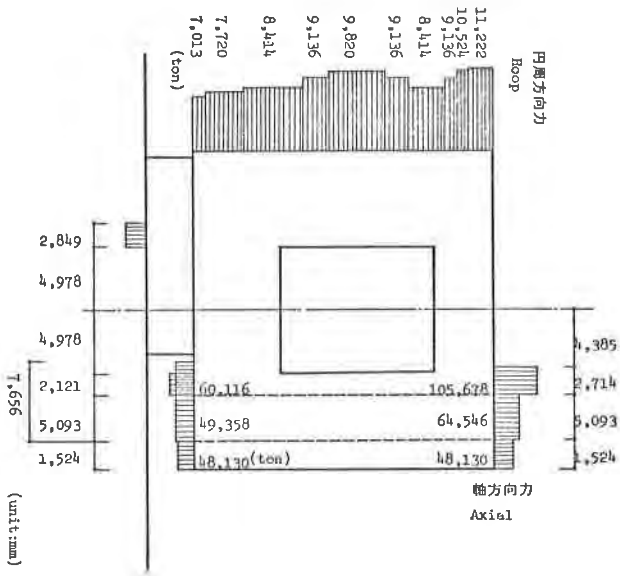


Fig.4 Distribution of Prestressing Forces

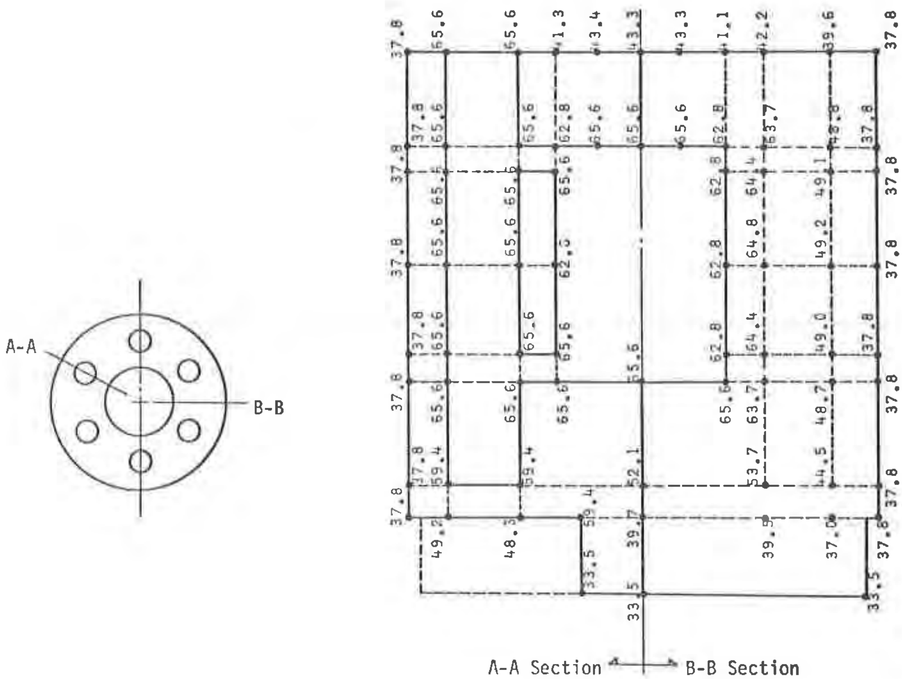


Fig.5 Distribution of Design Temperature (unit : °C)

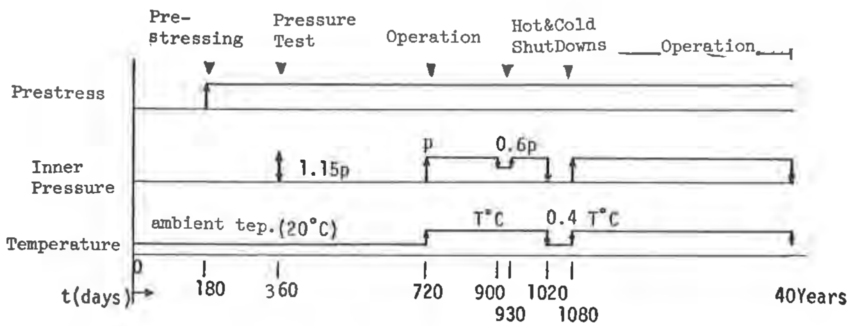
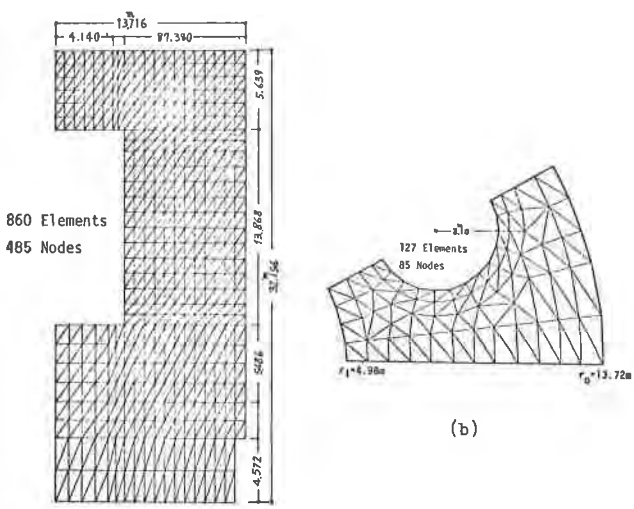


Fig.6 Loading History



(a)

(b)

Fig.7 Finite Element Subdivision
(a) Vertical Section
(b) Horizontal Slice in Mid-Height

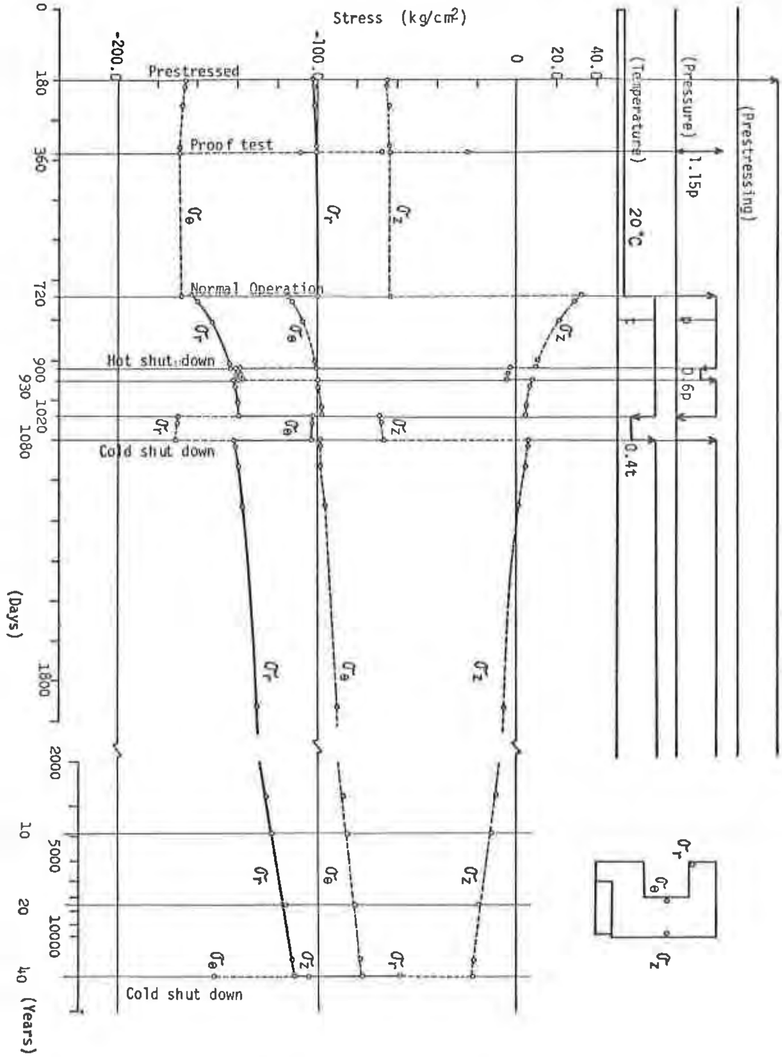


Fig.8 Representative Stress Histories

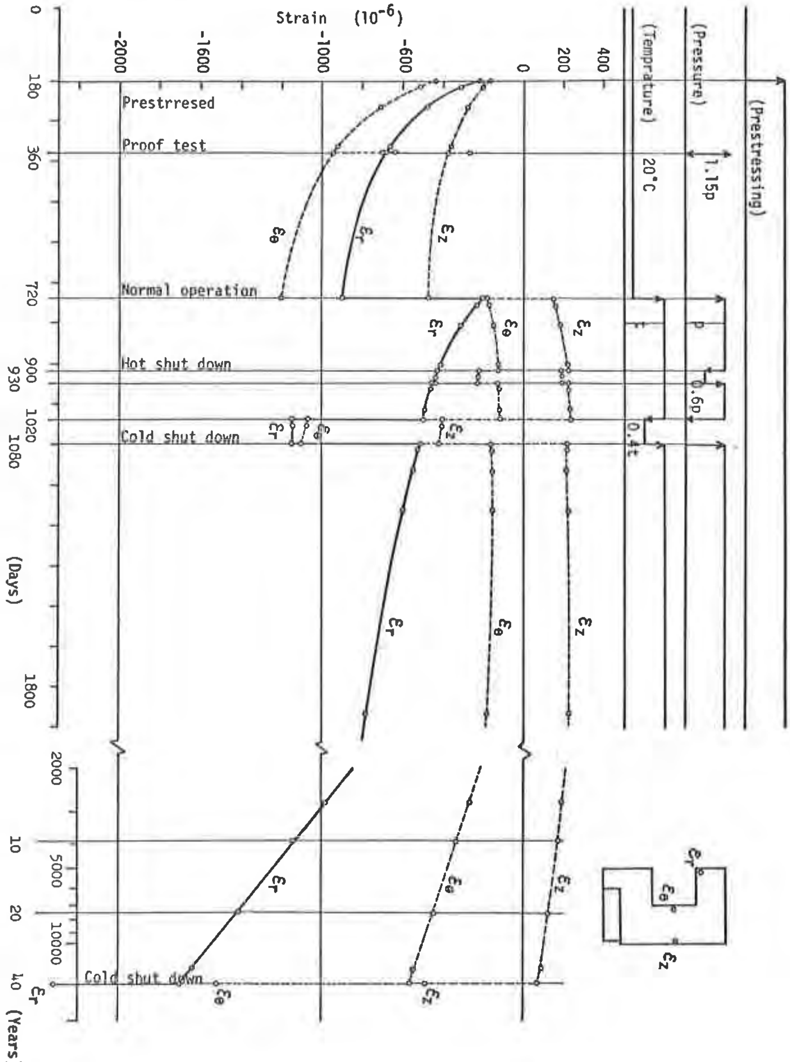


Fig.9 Representative Strain Histories

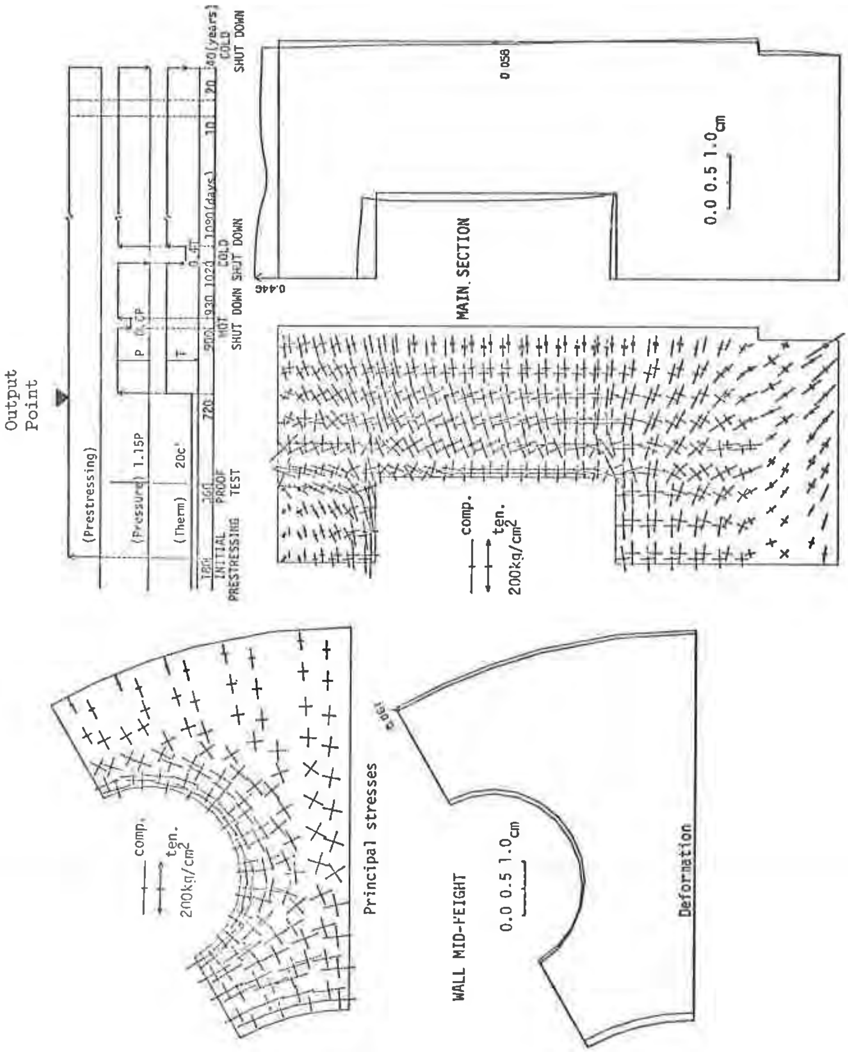


Fig.10 Stress and Deformation-just after Beginning of Normal Operation

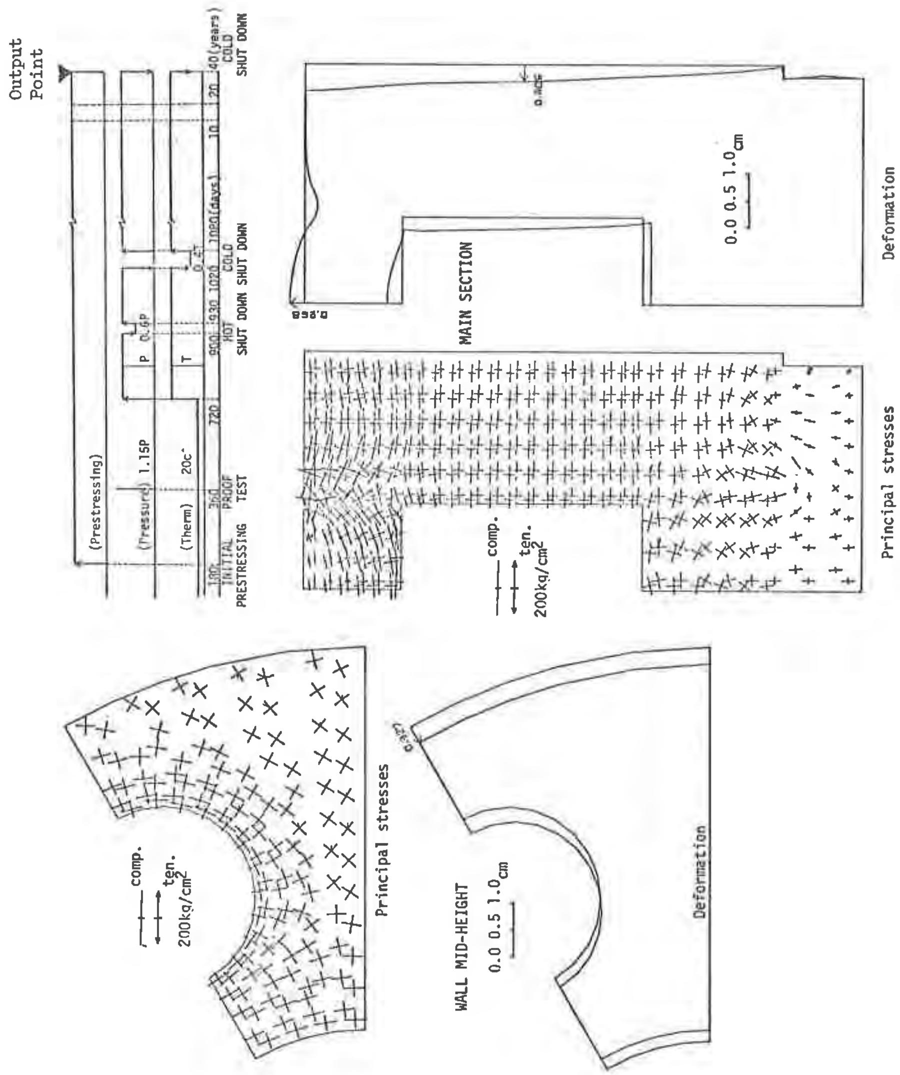


Fig.11 Stress and Deformation-under Normal Operation after 40 years