THE DETERMINATION OF ELASTIC STRESSES NEAR CYLINDER TO CYLINDER INTERSECTIONS

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The present paper gives a survey of the history, the applicability and the limitations as well as the investigation of the accuracy of the solution obtained of the problem of analysis of the stresses in a cylindrical shell with a circular cutout when boundary conditions along the edge of the hole are prescribed. These boundary conditions may be dynamic (given stress resultants and couples), geometric (given displacements and slope) or mixed (e.g. compatibility with a nozzle).

The classical paper on the analysis of the stresses in a cylindrical shell with a circular opening is written by Lur'e in 1946. He based his theory on the so-called shallow shell equations, equivalent to Donnell's equations for the case at issue of cylindrical shells. By means of a power series expansion he obtained a solution which in fact can be used only for small values of a certain geometrical parameter involved, viz. the square of the hole radius divided by the product of shell radius and wall thickness. Other investigators applied his method to somewhat more complicated cases such as reinforced hole boundaries. Between 1950 and 1965 there have been reported several other approaches in literature free from the restriction of Lur'e's analysis. We here mention Withum who used a perturbation method, Van Dyke who used a collocation method in order to enforce the boundary conditions and Myint, Radok and Wolfson who applied a Ritz method. All these methods, although interesting and absolutely correct, suffer from certain drawbacks. They either lead to unwieldy formulae and cumbersome computations or are limited in applicability.

In 1964, the present author reported at the 11th International Congress for Applied Mechanics held at Munich an analysis following roughly Lur'e's method but avoiding the restriction mentioned by using a somewhat different mathematical treatment (in fact by writing an exponential as a Fourier Bessel series rather than expressing it by Krylov functions as Lur'e did). Independently a group in the United States of America (A. Cemal Eringen et al.) and in Japan (Yoshiyuki Yamamoto et al.) treated the problem in similar ways.

At present this analysis has been completed to a certain point. It is possible (and in fact a computer program has been written) to determine stresses and displacements in a cylindrical shell with a circular cutout ensuing from arbitrary boundary conditions along the hole boundary. This for example enables to evaluate influence coefficients referring to a Fourier series expansion. The analysis is based on Donnell's equations and is consequently still of limited applicability. Hence it must be said that in its present form it is probably inaccurate for large radii of the cutout (e.g. larger than a quarter of the shell radius). For these large holes, however, the theory may be modified in order to obtain more reliable results.

The nozzle itself is in many cases of a complicated geometry and not suited for a pure analytical treatment. A method that looks hopeful is to treat the nozzle with a finite element method whereby it is subdivided in ring elements. The unknown displacements are written as Fourier series and determined with the aid of the minimum potential energy principle.

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DISCUSSION

Q J. SPENCE, U. K.

At the Institute of Applied Mechanics, Brno, Czechoslovakia, Ing. I. Lhotakova has been working on this cylinder/cylinder intersection problem. She seems to have some numerical results. Does Professor Lekkerkerker know how this work fits in with the work he has reported ?

A J. G. LEKKERKERKER, The Netherlands

I am acquainted a little with Mrs. Lhotakova's work. However, in a paper she submitted to a Czech Journal she skipped much of the algebra and as a consequence it is not possible to trace the way in which the compatibility between the two structures was arrived at.

Q W. L. QUEENSTREET, U. S. A.

Would you please comment on the different methods used by A. L. Eringen and coworkers ? In one case a collocation scheme was used to enforce boundary conditions at junction, while in the other the b. c. were matched continuously with polar angle φ at non-dimensional radius ρ . Which gives preferable solution in terms of computation ?

A J. G. LEKKERKERKER, The Netherlands

In early papers Eringen applied a kind of perturbation method. Later on he switched to a Fourier series expansion method analogous to the one described in the present paper. I prefer the latter method to collocation although as a matter of fact is should be called "transformed collocation" rather than continuous.
Where Eringen used least square error technique this was not essential but just due.to the fact that the truncations of the summations over $\underline{n}$ and $\underline{l}$ (present notation !) did not correspond, so the number of unknowns was not equal to the number of equations.

Q T. UDOGUCHI, Japan

It is my opinion that there must be a certain limit of the ratio of the diameters of the nozzle and the main shell under which this kind of solution is valid. How do you think about the limiting value of the diameter ratio ?

A J. G. LEKKERKERKER, The Netherlands

To be frank I am a bit pessimistic. From comparison between numerical results obtained using Flügge's equations (after time consuming calculations) and results of the present analysis I tend to fix a 1:5 ratio as the limit for obtaining accurate results, i. e., with erros less than, say, 5%.