Vibration Test of Spherical Shell Structure and Replacing Method into Mathematical Model

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Abstract

To verify the beam-type and oval-type vibratory characteristics of a spherical shell structure, two test specimens were made and vibration tests were carried out. Results of these tests were compared with results of detailed analyses using 3-D FEM and 2-D axisymmetric FEM models. The analytical results of overall vibratory characteristics were in good agreement with the test results, and it was found that the effect of the attached mass should be considered in evaluating local vibration. The replacing method into equivalent beam model was proposed.

1. Introduction

In vibration analyses of spherical shell structures, 3-D FEM or 2-D axisymmetric FEM is usually used. However, these analyses (especially 3-D FEM) spend enormous man-hour and computer fee. Dynamic analysis required for seismic design of spherical shell structures combined with a soil structure interaction model using 3-D FEM or 2-D axi-symmetric FEM analyses are no more practical. The equivalent simplified model of the spherical shell structure is useful in the structural planning stages of initial sizing and parametric studies for seismic response.

Therefore, vibration tests on a spherical shell structure were carried out and one of a modeling technique into simplified beam model based on these test results is proposed in this paper.

2. Vibration Test

2.1 Test Specimen

To verify the beam-type and oval-type vibratory characteristics of the spherical shell structure, two test specimens (model A and B) were made. Tests were carried out to clarify overall beam-type vibration by model A, and to clarify oval-type vibration and local vibration by model B. The upper portion of model B has a larger wall thickness to reduce its natural frequency of beam-type vibration and to investigate the interaction between beam-type vibration and oval-type vibration.

The main specification and the schematic view of the test specimens are shown in Table-1 and Fig.-1, respectively.

2.2 Test Method and Test Results

Three kinds of tests shown in Table-2 were carried out to measure vibratory character-istics. The test facilities used for model A and model B were an electro-magnetic type and an electro-hydraulic type vibration table, respectively.

The natural frequency of the test specimens are shown in Table-2, and examples of the transfer function are shown in Fig.-2 and Fig.-3. The existence of the attached mass did not significantly affect the natural frequency of each beam-type mode or oval-type mode. The consideration of the attached mass is, however, of importance when investigating local vibration of the shell as well as its local dynamic response.

3. Detailed Analysis and Comparison with Test Results

3.1 Overall Vibratory Characteristics

The comparison between analytical and test natural frequencies is shown in Table-3. Beam-type vibration mode calculated by 2-D axi-symmetric FEM (76 nodes, 75 elements) and 3-D FEM (4 nodes shell element; 2233 nodes, 2243 elements) is shown in Fig.-4 and Fig.-5, respectively. The comparison between analytical (2-D axi-symmetric FEM) and test beam-type mode is shown n Fig.-6, and that for oval-type mode is shown in Fig.-7.

With regard to the overall vibratory characteristics, the results of detailed analyses such as 2-D axi-symmetric FEM agree closely with the test results for both beam-type mode and oval-type mode.

3.2 Evaluation of Local Vibration

In the study of model B, a reinforcing plate was installed around the attached mass on inside and outside the spherical shell by spot welding. Because this reinforcing plate was ignored in the preliminary analysis, the analyzed natural frequency of local vibration of the attached mass caused by axi-symmetric FEM was about 20% lower than that obtained in the test. Therefore, to evaluate an equivalent thickness (stiffness) of the reinforcing part, the relation between unit force and deformation of the 3-plate model (inside and outside reinforcing plate, spherical shell) was calculated from the static analysis of 3-D FEM (see Fig.-8 and Fig.-9). Using this reinforcing part thickness into 2-D axi-symmetric FEM, the comparison between analytical and test natural frequency of local vibration is shown in Table-4. This procedure is found to accurately evaluate the local vibration.

4. Replacing Method into Simplified Beam Model

The procedure for constructing a simplified beam model of a spherical shell structure is proposed in this section.

The flow of this procedure is shown in Fig.-10.

The spherical shell structure is deformed as shown in Fig.-11 under seismic loading. Deformation of a certain element i at a height h in a spherical shell is illustrated in Fig.-12. The relation between horizontal force and shear deformation, and the relation between overturning moment and rotational angle are given by Eq.(1) and Eq.(2), respectively, by separating shear deformation and rotational angle.

$$Pi = \frac{G\ Aei}{hi}\ d\delta i \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (1)$$

Pi : horizontal force acting on element i
G : Shear modulus
Aei : equivalent shear area of element i

hi : height of element i
$d\delta i$: increment of shear deformation in element i

$$\pi Ni\ ri = \frac{E\ Iei}{hi}\ d\theta i \quad \text{.......} \quad (2)$$

Ni : axial membrane force in element i
ri : mean radius of element i
E : Young's modulus
Iei : equivalent moment of inertia of element i
$d\theta i$: increment of rotational angle in element i

αAi and αIi are defined as the following equation.

$$\alpha Ai = Aei\ /\ Ai \quad \text{.......} \quad (3)$$

$$\alpha Ii = Iei\ /\ Ii \quad \text{.......} \quad (4)$$

Ai : shear area of element i
Ii : moment of inertia of element i

As shown in Fig.-12, ri, Pi and Ni in Eq.(1) and Eq.(2) are defined as the average value of the top (j+1) and the bottom (j) of the element i. Rotational angle θi, its increment $d\theta i$ and increment of horizontal displacement $d\delta i$ in the element i are obtained as follows.

$$\theta i = u_{j+i}\ /\ r_{j+1} \quad \text{.......} \quad (5)$$

$$d\theta i = \theta_{i} - \theta_{i-1} \quad \text{.......} \quad (6)$$

$$d\delta i = (v_{j+1} - v_{j}) - hi\ \ \theta i \quad \text{.......} \quad (7)$$

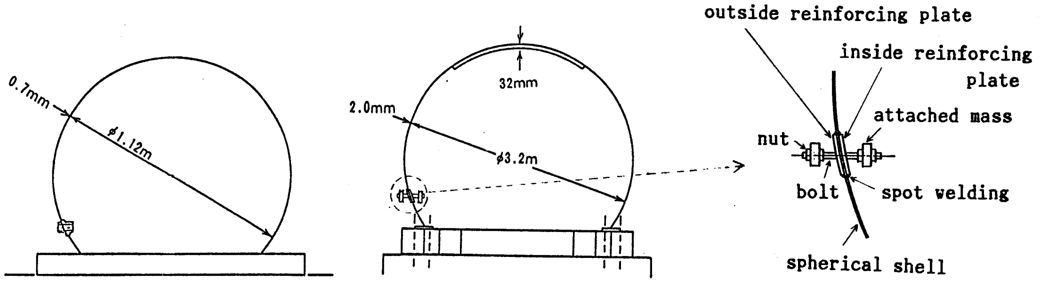
Aei and Iei for element i are given by these representation.

Thus, calculated αA and αI for each of five kinds of models, devided into 3, 5, 10, 20 and 75 beam elements, are shown in Fig.-13. This figure also shows the simplified method's line that approximates the calculated data. The comparison of vibration mode between the detailed analysis and the simplified method (beam model, its stiffness is evaluated by above mentioned line) is shown in Fig.-14. These results agree closely and the validity of the simplified method is thus confirmed.

5. Conclusion

Vibration tests and the detailed analyses were carried out to clarify the vibratory characteristics and to establish the simplified beam model of a spherical shell structure. The following results were obtained.

(1) The effect of the attached mass on overall vibratory characteristics of beam-type and oval-type mode is negligible.
(2) Detailed analysis using 2-D axi-symmetric FEM or 3-D FEM gives results good enough to evaluate the vibratory characteristics of the spherical shell structure.
(3) The structural condition around the attached mass must be considered when evaluating local vibration.
(4) The simple vibration analysis method using a beam model is established, and confirmed by results of both test and detailed analysis.



Model A　　　　Model B

Fig.-1 Schematic View of Test Specimen

Table-1 Main Specification of Test Specimen

		Model A	Model B
Test purpose		Clarification of overall beam type vibration	Clarification of oval type vibration and local vibration
Vibration table		Electro magnetic type	Electro hydraulic type
Frequency range		１００～４００Hz	２０～５０Hz
Dimension	Diameter	１.１２ m	３.２ m
	Wall thickness	０.７ mm	２.０ mm
Manufactaring method		Spinning process	Pressing process

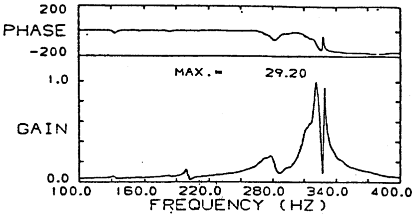


Fig.-2 Transfer Function
(Model A, top of shell
Sinusoidal Sweep Test)

Table-2 Natural Frequency Resulting from Test

(unit : Hz)

Item \ Specimen / Attached Mass	Model A		Model B		Remarks
	without	with	without	with	
Sinusoidal Sweep Test	322	313	39	40	1st beam
			—	47	local
Random Wave Test	323	—	—	40	1st beam
			—	48	local
Free Vibration Test	319	317	42	42	1st beam
			—	48	local
	—	—	111	111	1st oval
			199	198	2nd oval
			219	220	3rd oval
			246	246	4th oval

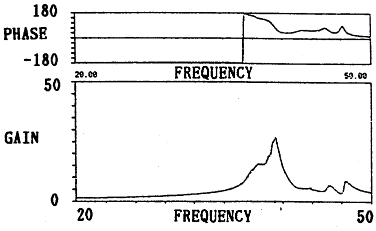


Fig.-3 Transfer Function
(Model B, top of shell
Sinusoidal Sweep Test)

Table-3 Comparison between Analytical and Test Natural Frequency

(unit : Hz)

Mode Type		2-D Axi-symmetric	3－D	Test	Remarks
Beam		322	321	322	Model A
Oval	1st	105	—	111	Model B
	2nd	214	—	199	
	3rd	236	—	219	
	4th	268	—	246	

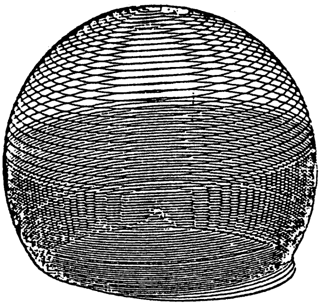


Fig.-4 Vibration Mode
(2-D axi-symmetric FEM)

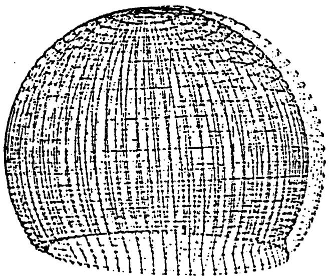


Fig.-5 Vibration Mode (3-D FEM)

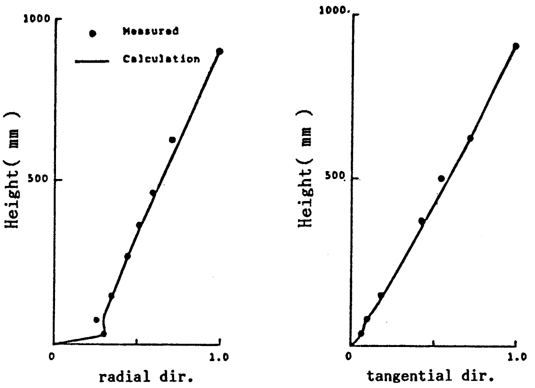
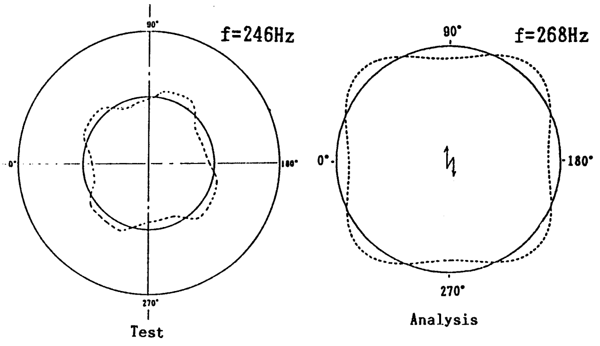


Fig.-6 Comparison of Beam Mode



Test

Analysis

Fig.-7 Comparison of Oval Mode

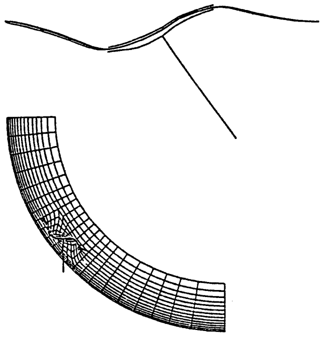


Fig.-8 Deformation of 3-Plate Model

Table-4 Comparison of Natural Frequency for Local Vibration

(unit : H z)

test			analysis	
			reinforcing plate	
sinusoidal sweep	random wave	tapping	ignored	considered*
47.3	48.0	48.0	38.9	47.4

(note) * : equivalent thickness of reinforcing plate

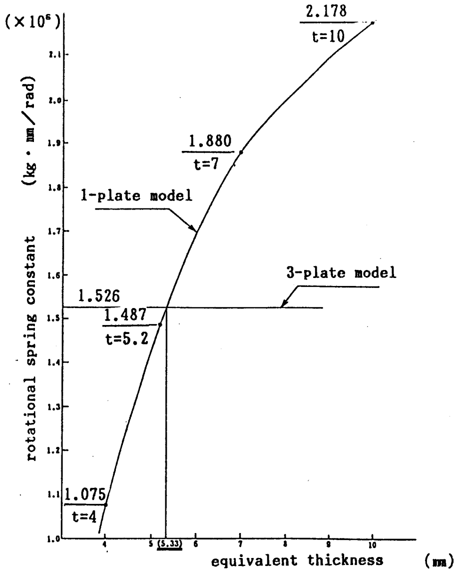


Fig.-9 Estimation of Equivalent Thickness

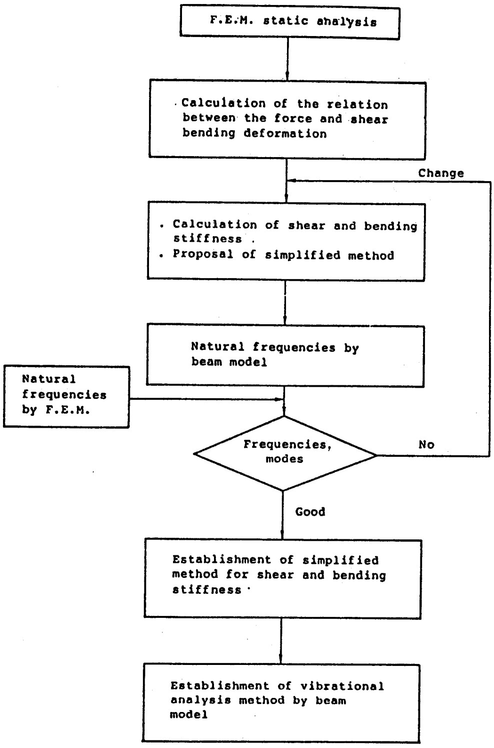


Fig.-10 Flow of The Simplified Method

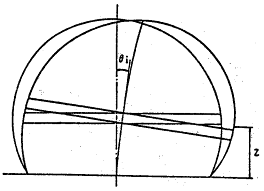


Fig.-11 Deformation of Spherical Shell

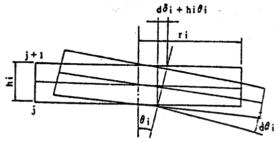


Fig.-12 Deformation of Element

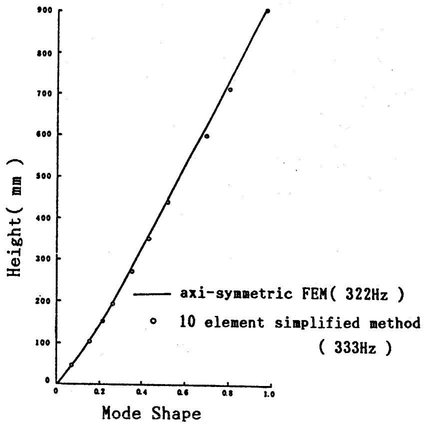


Fig.-14 Comparison of Vibration Mode

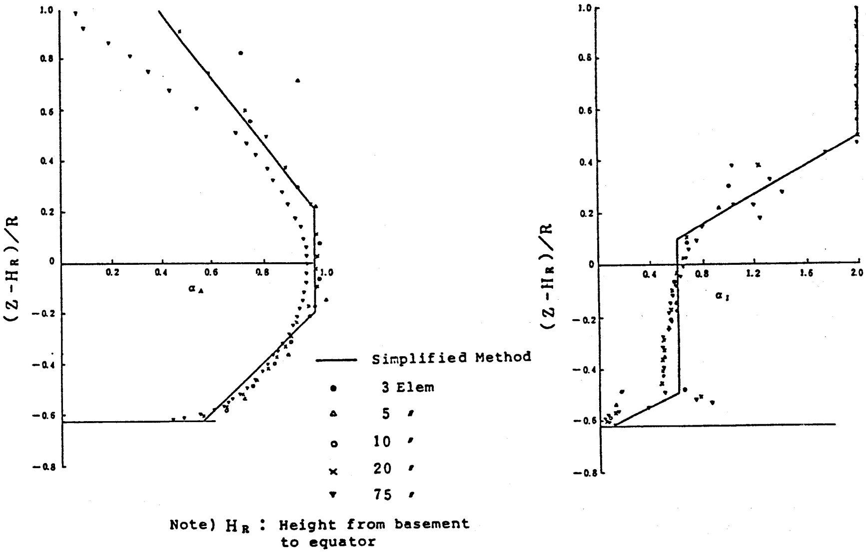


Fig.-13 Simplified Method for α A and α I