Thermal Shock Problems in a Plate

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SUMMARY

This report is concerned with thermal shock problems in a plate with finite thickness. In other words, the problems considered are coupled dynamic thermoelastic analysis in a plate. In a convensional analysis of thermal shock problems in a plate, like Manson's treatment, time enters only as a parameter in the treatment, because the inertia effect and the thermomechanical coupling effect are neglected in the governing equations. As a result, the treatise becomes a quasi-static, and it is not rigorously applicable to thermal shock problems having a steep time-gradient of temperature in thermal and mechanical fields. Besides, in the engineering fields, it is important problem to know that which effect is considerable or neglisible for machine design. In this report, first we try to examine a problem of the coupled dynamic thermal stress problem with small time approximation for the finite region. There have been many papers for the coupled or dynamic thermal stress problems, but most of papers have only treated the region of infinite or semi-space. Therefore they gave the first stress wave propagation. In next step, we treatise both effects individually by pursuing rigorous analysis without small time approximation. Finally we consider thermal shock problems in a plate against different values of heat transfer coefficient(Biot's number) for the time.

In conclusion, for usual materials, the inertia effect may be disappered in the pure thermal problems in contrast to the coupling effect which bring to small lags in the temperature and thermal stress distributions. For the consideration of the maximum thermal stress problems, Manson's uncoupled quasi-static results give enough approximation to the thermal shock problems without significant error from our numerical results.
The analysis is developed by the use of Laplace transforms and several useful graphical illustrations are given.

It should be pointed out that the maximum stress at the surface must be decreased in the first period due to the coupling effect. Against, in the next step of heating, the maximum stress must be increased.

1. INTRODUCTION

When thermal stresses are generated by a sudden change in temperature, this is called a thermal shock problem. Here we consider, as an example, the problem in which a long plate with finite thickness is suddenly heated or cooled by liquid. On this subject, it is well known that Manson has given a conventional treatise for thermal shock problems in a plate under an unsteady-state temperature field, rests on the assumption that the inertia term may be neglected in the governing equation of motion, and that the thermomechanical coupling term may be neglected in the heat conduction equation. This hypothesis, based on the quasi-static process, is known to yield useful results in practical engineering application without significant errors. Strictly speaking, however, in a conventional analysis such as Manson's, time enters only as a parameter in transient thermal and mechanical problems. It is evident that the quality of approximation must depend both upon the size of the relevant intrinsic inertia or coupling parameter and on the nature of time variations inherents in the temperature distribution. In particular, if the temperature field exhibits sufficiently steep time-gradients, dynamic effects disregarded in the traditional treatment of the problem may become significant. When the inertia term is taken into account, the character of the problems is considerably altered. Moreover, if presence of a thermoelastic coupling is taken into account, an exact analysis would require the simultaneous determination stress and temperature distributions.

In order to access to the more real phenomena in thermal shock problem, the present report has considered both effects for the thermal shock in a plate with finite thickness. Analytical steps considered here are as follows;

i, Dynamic coupled thermal stress problems in a plate,

ii, dynamic thermal stress problem in a plate without coupling term,

iii ,coupled thermal stress problem in a plate without inertia term,

From our results, it seems important to consider the coupling effect only, and also that analysis gives us reasonable results for engineering fields.

2. DYNAMIC COUPLED THERMAL STRESS PROBLEM IN A PLATE WITH FINITE THICKNESS

In this section, first we consider both effects, then problems become dynamic coupled thermoelasticity. There have been several papers treated dynamic coupled thermal stress problems. Most papers deal with the problems of infinite or semi-infinite regions. As shown in Fig.1, an infinitely long plate with width $2l$ is suddenly heated by liquid at the high temperature T_o. For the sake of convenience, we introduce the following dimensionless quantitites:

$$X=x/l,\ Y=y/l,\ Z=z/l,\ T_d=T/T_o,\ t_d=\kappa t/l^2,\ H=hl,\ u_d=u\{(1-\nu)/(1+\nu)l\alpha T_d\},$$
$$\sigma_{II}=\{(1-\nu)/E\alpha T_o\}\sigma_{ii},\ \xi=V^{-2}=\{\kappa/v_p l\}^2\ \ v_p=\sqrt{2(1-\nu)\mu/(1-2\nu)\rho}$$

The generarized Hooke's law are given by

$$\sigma_{XX} = \{(1-\nu)/(1-2\nu)\}\{u_{D},_{X}+\nu(C_{1D}+C_{2D})/(1-\nu)-T_{D}\}$$
$$\sigma_{YY} = \{(1-\nu)/(1-2\nu)\}\{\nu(u_{D},_{X}+C_{2D})/(1-\nu)+C_{1D}-T_{D}\} \quad (1)$$
$$\sigma_{ZZ} = \{(1-\nu)/(1-2\nu)\}\{\nu(u_{D},_{X}+C_{1D})/(1-\nu)+C_{2D}-T_{D}\}$$

It follows then that the basic equations bocome

$$u_{D},_{XX} - \xi u_{D},_{tt} = T_{D},_{X} \quad (2)$$

$$T_{D},_{XX} - T_{D},_{t_d} = \delta(u_{D},_{X} + C_{1D} + C_{2D}),_{t_d} \quad (3)$$

Applying Laplace transforms to the above equations, we have

$$u^*_{D},_{XX} - p^2\xi u^*_{D} = T^*_{D},_{X} \quad (4)$$

$$T^*_{D},_{XX} - pT^*_{D} = \delta p(u^*_{D},_{X} + C^*_{1D} + C^*_{2D}) \quad (5)$$

where

$$\delta = \{(3\lambda+2\mu)^2\alpha_t^2 T_A\}/\rho^2 g c_v v_p^2 \quad (6)$$

Eliminating T^*_D in the above equations, it follows that

$$u^*_{D},_{XXXX} - \{\xi p^2 + (1+\delta)p\}u^*_{D},_{XX} + \xi p^3 u^*_{D} = 0 \quad (7)$$

Then the solution eq.(7) is written by

$$u^*_{D} = A_1 sinh\ k_1 X + A_2 sinh\ k_2 X \quad (8)$$

where A_1 and A_2 are arbitrary integral constants to be determined from the loading and heat conditions at the boundaries. The quantities k_1 and k_2 are given by

$$k_i^2 = (p/2)[\xi p + (1+\delta) \pm \{\xi p^2 - 2(1-\delta)\xi p + (1+\delta)^2\}^{1/2}] \quad (9)$$

Substituting eq.(8) into eq,(4), and integrating, we obtain

$$T^*_{D} = (k_1^2 - \xi p^2)(A_1/k_1)coshk_1 X + (k_2^2 - \xi p^2)(A_2/k_2)coshk_2 X - \delta(C^*_{1D} + C^*_{2D}) \quad (10)$$

For the traction free boundary condition, the normal stress must vanish at the boundaries

$$\sigma^*_{XX} = 0 \qquad at\ X = \pm 1 \quad (11)$$

It follows that

$$A_1\xi(p^2/k_1)coshk_1 + A_2\xi(p^2/k_2)coshk_2 + \alpha C^*_{1D} = 0 \quad (12)$$

If we assume $H\to\infty$, it follows from eq .(10) that

$$A_1 k_1 coshk_1 + A_2 k_2 coshk_2 + \beta C^*_{1D} = p^{-1} \quad (13)$$

where α and β in eqs.(12) and (13) are arbitrary constants.

The remaining boundary conditions are given for the following three end conditions:

(1) The displacements are not restrained in either the y- and z-directions

$$\int_{-1}^{1}\sigma^*_{YY}dX = \int_{-1}^{1}\sigma^*_{ZZ}dX = 0 \quad (14)$$

(2) The displacement is restrained in the z-direction (or y) only

$$\int_{-1}^{1}\sigma^{*}_{YY}\,dX = 0, \qquad C^{*}_{2D} = 0 \tag{15}$$

(3) The displacements are restrained in both directions

$$C^{*}_{1D} = C^{*}_{2D} = 0 \tag{16}$$

By using the foregoing conditions of eqs.(12)-(16), the unknown constants are now determined. If we limit the short time problem a the begining of heating, it follows that

$$k_1 = \xi^{1/2}p + \delta\xi^{-1/2}/2 + (4-\delta)\delta\xi^{-3/2}p^{-1}/8 \tag{17}$$

$$k_2 = p^{1/2} - \delta\xi^{-1}p^{-1/2}/2 - (4-3\delta)\delta\xi^{-2}p^{-3/2}/8 \tag{18}$$

Then corresponding subsiderary solutions of displacement and temperature for short-time can be obtained. The different expressions for the three end-conditions can be determined for u^{*}_{D} and T^{*}_{D}.

Thus applying the Laplace inversion formulae with the theorem of residue, the final expression of u_D and T_D can be written as [for case(1) and (2)]

$$u_{D,X} = exp\{-\frac{(1-X)}{2}\delta\xi^{-1/2}\}[S_{11}1(\tau)+\{\frac{2}{\sqrt{\pi}}S_{12}\tau^{1/2}+S_{13}\tau+\frac{4}{3\sqrt{\pi}}S_{14}\tau^{3/2}+\frac{1}{2}S_{15}\tau^{2}\}\eta(\tau)]$$
$$-[S_{21}(4t_D)i^{2}erfc(x')+S_{22}(4t_D)^{3/2}i^{3}erfc(x')+S_{22}(4t_D)^{2}i^{4}erfc(x')+S_{24}(4t_D)^{5/2}$$
$$\times i^{5}erfc(x')+S_{25}(4t_D)^{3}i^{6}\,erfc(x')]\xi^{-1} \tag{19}$$

$$T_D = exp\{-\frac{(1-X)}{2}\delta\xi^{-1/2}\}\{T_{11}\tau+\frac{4}{3\sqrt{\pi}}T_{12}\tau^{3/2}+\frac{1}{2}T_{13}\tau^{2}+\frac{8}{15\sqrt{\pi}}T_{14}\tau^{5/2}+\frac{1}{6}T_{15}\tau^{3}\}$$
$$\times\delta\xi^{-1}\eta(\tau)+\{T_{21}erfc(x')+T_{22}(4t_D)^{1/2}i\,erfc(x')+T_{23}(4t_D)i^{2}erfc(x')+T_{24}(4t_d$$
$$)^{3/2}i^{3}erfc(x')+T_{25}(4t_d)^{2}i^{4}erfc(x')\}-(\phi_k\delta/\alpha_k\gamma_k)\{\frac{2}{\sqrt{\pi}}J_1t_d^{1/2}+J_2t_d+\frac{4}{3\sqrt{\pi}}$$
$$\times J_3t_d^{3/2}+\frac{1}{2}J_4t_d^{2}\} \tag{20}$$

$\tau=\{t_d-(1-X)\xi^{1/2}\}$, $\quad x' = (1-X)/2\sqrt{t_d}$ and $\alpha_k, \beta_k, \gamma_k$ are material constants for the each boundary end conditions.

Therefore, we can easily find all the thermal stress components can be obtained after substitution of (19) and (20) into eq.(1). For the sake of brevity, the expressions of these stress components are omitted here.

Numerical result are shown as Fig.2-5. Main data used here are

ξ = 0.01, 0.04, 0.1, 1.0

δ = 0.0, 0.03, 0.1, 0.3, 0.5

ν = 1/3

Fig.3-5 show the distribution of near the surface of the plate. (For case 1)

TABLE 1 A Glance at Dynamic and Coupling parameters *(l=1cm)*

	Cast iron	Steel	Aluminum	Copper	Lead
ξ	$0.0^{12}113$	$0.0^{13}380$	$0.0^{11}216$	$0.0^{11}595$	$0.0^{13}723$
δ	0.0019	0.0085	0.031	0.018	0.079

3 COUPLED THERMAL STRESS PROBLEMS IN A PLATE

Now we reconsider the basic equations of dynamic coupled thermoelasticity. According to Table 1, we understand that the dynamic term in eq.(4) acts very small effect. Therefore it should be concluded that we only have to take into account of the coupling term in eq.(5) in engineering thermal shock problems. In order to find the dynamic effets, we take unrealistic value of ξ in section 2. Because if we take realistic value of ξ for actual materials, the results show the same as undynamic treatment. In summary, as illustrated in Table, it should be pointed out that the coupling parameter in the fundamental heat-conduction equation eventually appears in value to effect the temperature distribution. In this section we limit consideration to the coupled problem of thermoelasticity. As a result, we can obtain exact solution for the whole the time. Solving the foregoing two equations of (4) and (5) when ξ =0, we obtain

$$\overline{T}_D = A\cosh qX-\{\delta/(1+\delta)\}(\overline{C}_{1D}+\overline{C}_{2D}+D) \tag{21}$$

$$\overline{u}_D = (A\sin qX /q-\delta X(\overline{C}_{1D}+\overline{C}_{2D})/(1+\delta)+DX/(1+\delta) \tag{22}$$

Where A and D are integral constants, and $q=\sqrt{(1+\delta)p}$.
Substituting the results of eqs.(21) and (22) into eq.(1), we have stress solutions of the subsidiary form. Thus we consider the boundary and end conditions, and then unknown constants A, D, $\overline{C}_{1D}$ and $\overline{C}_{2D}$ can be determined. Applying the theorem of residue and inverting the transforms, we obtain

$$T_D= 1-\sum_{n=1}^{\infty} B_n\{\cos(\omega_n X)-(R\sin\omega_n)/\omega_n\}e^{-\omega_n^2 t_D/(1+\delta)} \tag{23}$$

$$u_D= \xi X-\sum_{n=1}^{\infty} B_n\{\sin(\omega_n X)/\omega_n-(P\sin\omega_n)X/\omega_n\}e^{-\omega_n^2 t_D/(1+\delta)} \tag{24}$$

$$\sigma_{XX}= 0$$

$$\sigma_{YY}= -\eta-\sum_{n=1}^{\infty} B_n\{-\cos\omega_n X+(Q\sin\omega_n)/\omega_n\}\, e^{-\omega_n^2 t_D/(1+\delta)} \tag{25}$$

$$\sigma_{ZZ}= -\zeta-\sum_{n=1}^{\infty} B_n\{-\cos\,\omega_n X+(S\sin\omega_n)/\omega_n\}\, e^{-\omega_n^2 t_D/(1+\delta)} \tag{26}$$

where ω_n are the n-th positive roots of

$$H\omega_n\cos\omega_n-(\omega_n^2+HR)\sin\omega_n = 0 \quad , \tag{27}$$

and B_n, ξ, η, and ζ are given by the forms as;

$$B_n= \frac{2\omega_n \sin\,\omega_n}{\omega_n^2+\omega_n\sin\omega_n\cos\omega_n-2R\sin^2\omega_n}$$

and $\xi= (1-\nu)/(1+\nu)$, $\eta=\zeta= 0$, for case (1)

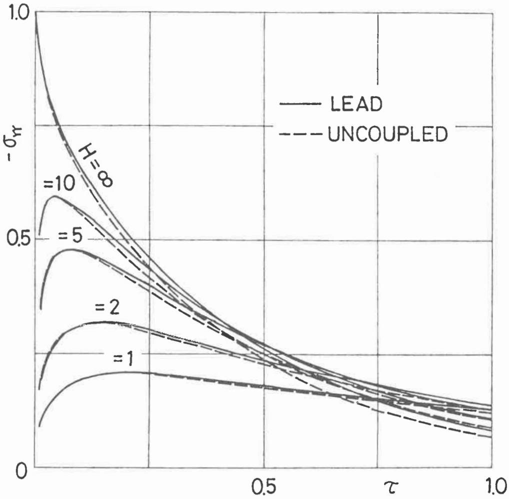
$\xi= 1-\nu$, $\eta = 0$, $\zeta =1-\nu$, for case (2)

$\xi= \eta= \zeta=1$, for case (3)

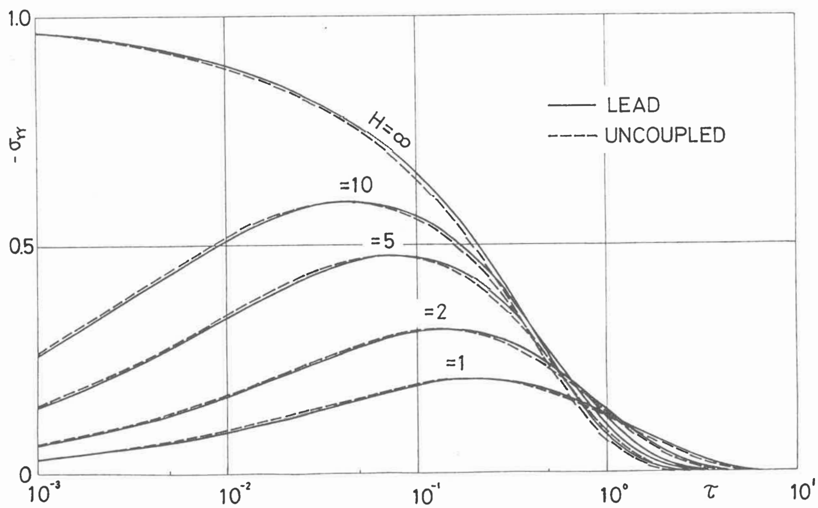
Numerical results are given by Fig.6 and 7 for Case

4 DYNAMIC THERMAL STRESS PROBLEMS IN A PLATE

In this section the treatment has considered both effects separately for the thermal shock problems. We limit consideration to the uncoupled dynamic problem, taking account the inertia term into the basic equation. Thus we can obtain rigorous solution for the whole the time. For the sake of brevity the expressions of solutions are omitted. Several numerical result are shown in Fig.8-10.We take unrealïstic value of ξ to find dynamic effects.



CASE 1



Special Fig.01 and 02 The effects of Biot's number to thermal shock in coupled thermoelastic treatment.

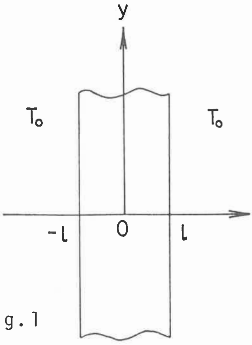


Fig.1

Fig.2

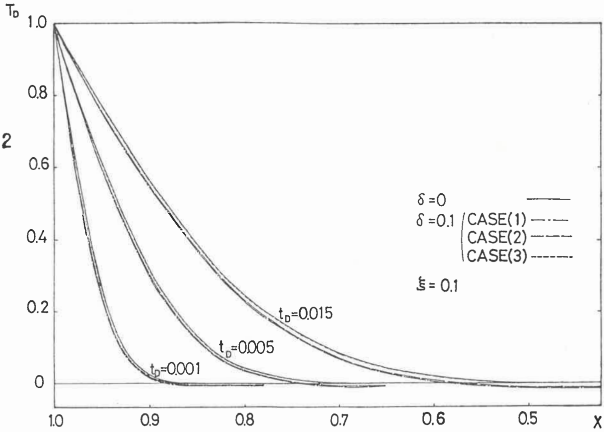


Fig.3

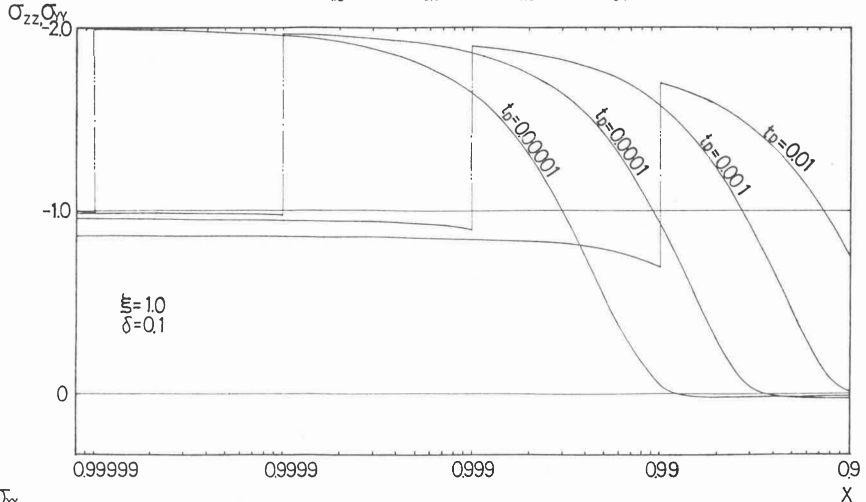
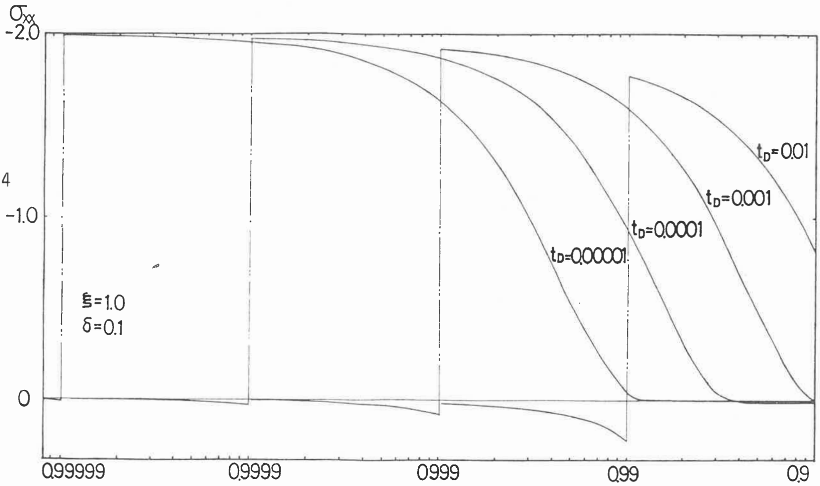
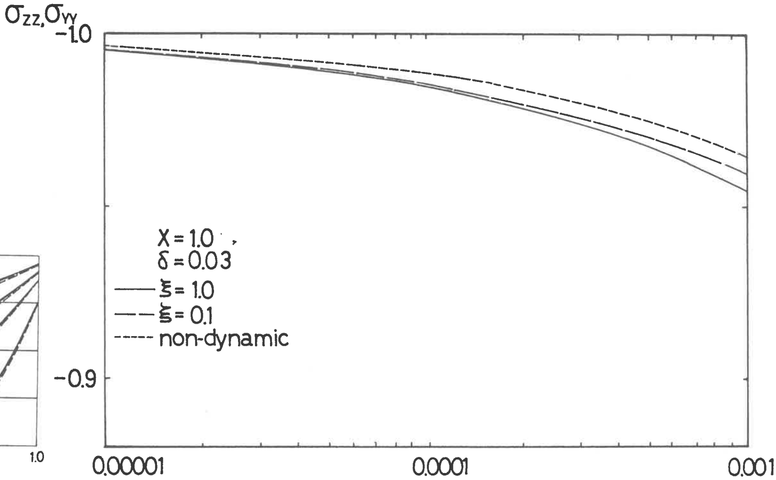


Fig.4





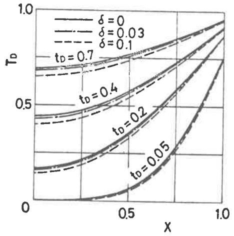


Fig.6(A)Case3

t_D

Fig.5 (CASE 1)

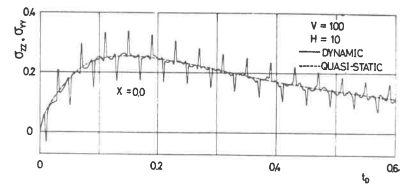


Fig.8

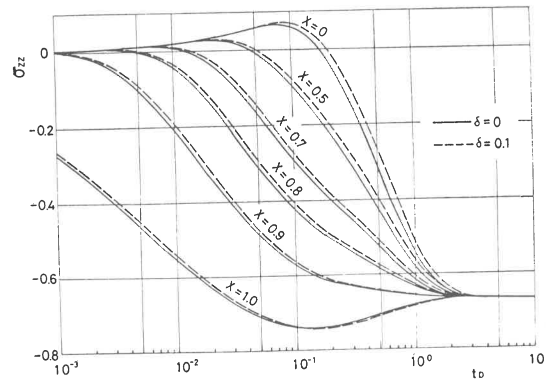


Fig.7 (CASE 2)

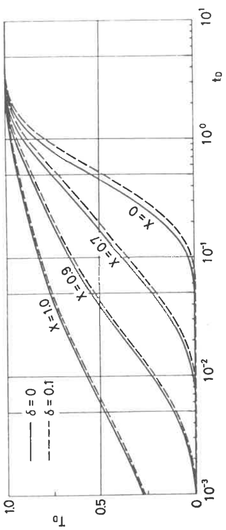


Fig.6(B) CASE 1

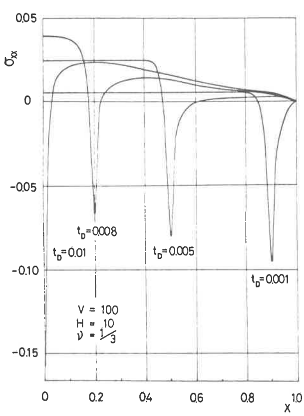


Fig.9

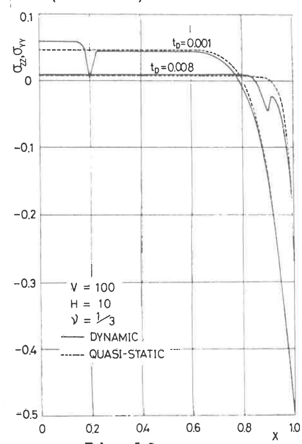


Fig.10