Interrelation between Metallographical Observations and Damage Variables of Continuum Damage Mechanics Applied to High Temperature Structures

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ABSTRACT

Correlation between metallographical parameter "A" and the probability by which cavities on a grain boundary facet is cut by the observation plane is described. By means of an approximate probability analysis, this probability is interrelated to the cavity area fraction and to the cavity number on the cavitating facet. By identifying the average cavity area fraction with the damage variable D of Kachanov–Rabotnov creep damage theory, three different relations between A and D are derived. The resulting equations are compared with each other, as well as with another model, based on the experimental results reported in literatures. Further improvements of the new model and their limitations are elucidated. The application of the proposed relations between A and D are also discussed.

1. INTRODUCTION

Continuum Damage Mechanics (CDM) has originated from the mechanical description of the brittle rupture observed in creep damage, and its promise in the applicability to the engineering practice of design and life prediction of high temperature structures has been advocated for long time. However, in contrast to the success of CDM in the field of geological material and civil engineering, CDM has only limited success in creep, creep–fatigue and fatigue of structural components. The difficulty of CDM as an engineering approach to these problems may consists in the description of damage state in terms of proper mechanical variables as well as in the formulation of the proper evolution equations of the damage variables.

Grain boundary (G.B.) cavities related to creep is one of the most important failure mechanisms occurring in components of high temperature nuclear structures as well as in that of fossil fuel electricity power plants. Among several metallographic parameters proposed for quantitative evaluation of the creep damage due to cavity formation, A–parameter (the number fraction of cavitating grain boundary facets) recommended by CEGB, EPA and EPRI (Cane And Shammas, 1984; Shammas, 1987) has proved itself a potential creep damage parameter. Some theoretical models have been developed to correlate A–parameter with damage variables in continuum damage mechanics, and the corresponding formulas of lifetime prediction have been also derived (Shammas, 1987; Riedel, 1989). The models proposed so far to correlate A–

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parameter and damage variables are all based on the assumption that a cavitating facet behaves as a microcrack of the same geometry as the facet even for small cavity area fraction on that facet. By use of this model the relations of $A \propto D$ or $A \propto \rho$ were developed in literatures (Shammas, 1987; Riedel, 1989), where D is Kachanov damage variable and ρ is a damage variable defined by volume density of microcracks (i.e. cavitating facets). However, as mentioned by Riedel(1989), some uncertainties are associated with these models, i.e. 1) there might be some cavitating facets not counted when measuring A-parameter, if no cavity in these facets is cut by the metallographic section, and 2) it cannot be confirmed whether all the G.B. facets counted are the constrained cavitation facets or not.

In this paper, A-parameter will be correlated to the probability in which cavities on a G.B. facet is cut by the metallographic plane. Firstly, through a simple probability analysis, this probability is interrelated to the cavity area fraction ω on the cavitated G.B. facet and to the cavity number m on the facet. Then, by identifying the average cavity area fraction in the material with the damage variable D, three different relations between A and D are derived, corresponding to different assumptions on the cavity nucleation and growth processes. Some limitations and possible further developments of the models are discussed through the comparisons with experimental results and Riedel(1989)'s model. Finally, the implications of proposed correlation between A-parameter and damage variable D are also discussed with respect to the determination of more proper damage evolution equation.

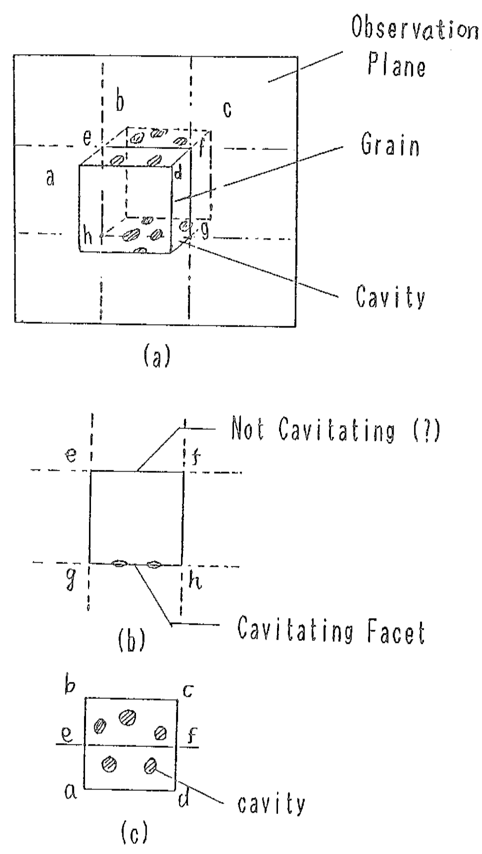

Fig. 1 Grain Boundary Facet Cut by a Observation Plane

2. PROBABILITY ANALYSIS

According to the engineering definition of A-parameter, A is the number fraction of cavitating grain boundary facets[1], where "cavitating grain boundary" implies the grain boundary with cavities observed in the metallographical plane. However, there may be

1 In this paper, the "G.B. facet" or "facet" refer only to those grain boundary facet, which are normal or approximately normal to the direction of applied stress.

some facets which are really cavitated but cannot be counted as "cavitating" facets in the procedure, as the facet a–b–c–d shown in Fig. 1(a) and (c) (where grain is assumed to be cubic for the sake of simplicity). For a facet a–b–c–d, whether a line e–f will intersect the cavities or not is clearly a problem of probability. Now let's denote P as the probability for an arbitrary line (like e–f in Fig. 1) to intersect the cavities on a facet, which is obviously related to the shape and size of facets and cavities etc.. For the sake of simplicity, in following analyses, all the facets will be supposed to have the same shape and size and all the cavitated facet have the same number of cavities with the identical radius r on it, as shown in the Fig. 2(a). Thus, we

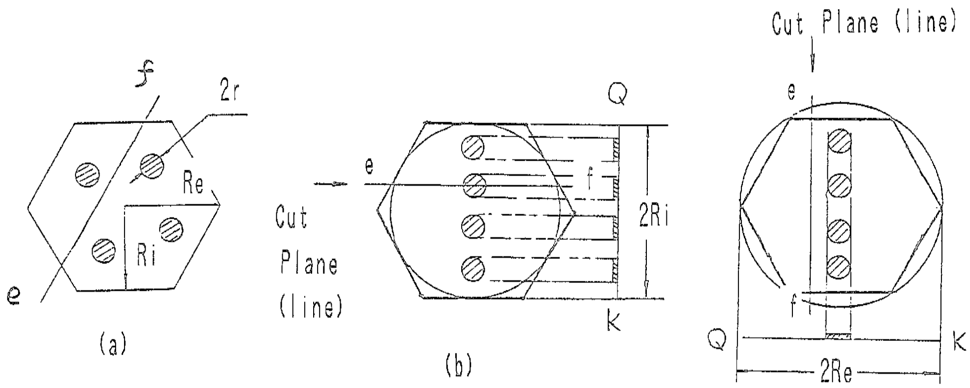

Fig. 2 (a) Grain Boundary Facet with m Cavities on It
(b) Upper Limit for P; $P=m(2r)/(2R_i)$
(c) Lower Limit for P; $P=2r/(2R_e)$

can consider the A–measuring process as a process of finding the probability for an arbitrary line (e–f) to cut the same G.B. facet as in Fig. 2(a), and following relation can be easily obtained,

$$P = \frac{\textit{Numbers of cavity cut events}}{\textit{Numbers of probability test}} = \frac{N_{cut}}{N_c} \quad (1)$$

where N_{cut} is the numbers of facets with cavities observed in the metallographical plane (i.e. the number of "cavitating" G.B facet in A–measuring approach), and N_c is the real numbers of cavitating facets. If we represent the total numbers of G.B. facets counted in the measurement by N_T and the cavitated facet fraction in the material by f, N_c can be determined as

$$N_c = f\,N_T \quad (2)$$

According to the engineering definition of A–parameter and using the above notations, we have the relation

$$A = \frac{N_{cut}}{N_T} \tag{3}$$

Thus, from Eqs. (1) and (2), A is expressed also in following form

$$A = f\,P \tag{4}$$

From this equation, it will be observed that, if P is taken as 1.0 one will get $A = f$ just as in the previous works (Cane and Shammas, 1984). However, according to Fig. 2(a), this situation occurs only when the facet area is almost totally occupied by cavities (i.e., immediately before the final rupture stage). In the present paper, both f and P shall be considered as variables that are related to the cavity nucleation and growth in material.

From Fig. 2(b),(c), the probability P for an arbitrary line e–f to intersect cavities is equal to the ratio of total length of shadow lines to that of line Q–K. An exact analysis of this probability problem is not pursued here, but the upper and lower limit cases are shown in Fig. 2(b),(c). Using the definition of cavity area fraction on a facet

$$\omega = m(\pi r^2)/(facet\ area)$$

together with some simple geometric relations between the facet area and R_i and R_e in the Fig.2, one can easily obtained the relation

$$P = \alpha\sqrt{m\omega} \tag{5}$$

where m is the cavity number on a cavitated facet, ω the cavity area fraction on the facet, and $\alpha = 0.368$ for upper limit and $\alpha = 0.318/m$ for lower limit (Sugita, Liu and Murakami, 1991). Generally speaking, we can expect that the cavities are well separated and randomly distributed on a G.B. facet before ω increase sufficiently large, so the upper limit case in the Fig. 2(b) is supposed more reasonable than the lower limit case in Fig. 2(c). In the following discussions, the upper limit is exclusively used and α in Eq. (5) will be taken as a constant.

3. A–Parameter and Kachanov–Rabotnov Damage Variable

Following the idea of effective area (or effective stress) in the creep damage theory of Kachanov(1958) and Rabotnov(1969) (K–R), the damage variable D can be related to the cavity area fraction ω. If we represent the cavity area fraction on a G.B. facet by ω, we can have the relation

$$D = f\,\omega \tag{6}$$

where ω and D can be interpreted as the local and the average damage variable, respectively. From this equation and the definition of ω, we have

$$D = f\,\omega = m\,f\left(\frac{r}{R}\right)^2 \tag{7a}$$

where r is the (average) cavity radius and R is the average G.B. facet size. Eqs. (4) and (5), on the other hand, yield the relation

$$A = \alpha f \sqrt{m\omega} = \alpha\ m f \frac{r}{R} \tag{8a}$$

Noting $m \cdot f$ in above equations is the average cavity number per facet in the material, we represent it by a notation

$$M = m f$$

then Eqs. (7a) and (8a) can be rewritten as follows

$$D = M \left(\frac{r}{R}\right)^2 \tag{7b}$$

$$A = \alpha\ M \frac{r}{R} \tag{8b}$$

Clearly M and r/R are related to cavity nucleation and growth respectively. Using different assumptions for the nucleation and growth processes, these equations will lead to different relations between A and D.

If M is supposed as a constant and the damage development is only due to the increase of r/R, elimination of r/R from Eqs. (7b) and (8b) will yield

$$A = c_1 \sqrt{D} \tag{9}$$

where $c_1 = \alpha\sqrt{M}$ is a new constant; i.e., A is proportional to $\sqrt{D}$ which has been derived by present authors in previous paper (Sugita, Liu and Murakami, 1991). In this case, creep damage is controlled by cavity growth alone, and cavity nucleation is supposed to be completed at the instant of loading.

If r/R is supposed as a constant and damage is only due to the increase of M or, increase of m and f, one obtains from Eqs. (7b) and (8b)

$$A = c_2\ D \tag{10}$$

where $c_2 = \alpha \cdot r/R$ is another constant. Namely A is proportional to D which has been applied in previous researches (Cane and Shammas, 1987; Sakurai et al, 1990). In this case creep damage is controlled by the cavity nucleation alone, which implies that the cavities will grow no longer after they nucleate with an initial size r_0.

For most engineering alloys employed at high temperature, cavities has been observed to nucleate continuously, as well as to grow. Thus, more reasonable relation between A and D is required. Since the theories of cavity nucleation have not been entirely successful in describing the observed behaviors, the empirical equation proposed by Dyson(1983) has been widely applied (Riedel, 1985; Wilkinson, 1987). Dyson's equation is based on extensive experimental data, and suggests, for most engineering alloys, the nucleation rates of cavities is proportional to macroscopic creep strain. If the initial density of cavities is neglected, Dyson's equation can be written as

$$n = \beta\ \varepsilon^c \tag{11}$$

where n and ε^c are the average cavity number per unit area of G.B facets and the macroscopic creep strain, and β is the material constant. By use of this equation, we obtain the average cavity number per facet in material, M as follow

$$M = n(\pi R^2) = \beta \varepsilon^c (\pi R^2) \tag{12}$$

where πR^2 is the average area of facets. Substituting Eq. (12) into Eqs. (7b) and (8b), and eliminating r/R from the resulting equation, we have

$$A = c_3 \sqrt{\varepsilon^c D} \tag{13}$$

where $c_3 = \alpha R\sqrt{(\beta\pi)}$ is a constant. In view of Eq. (11), concerning the influences of continuous nucleation, this equation show that A-parameter will depend on both damage variable D and macroscopic strain ε^c.

By using different assumptions on cavity nucleation and growth, we have derived three different relations, Eqs. (9), (10) and (13) between A-parameter and damage variable D. In order to compare these equations with the experimental data reported in the form of A-parameter versus lifetime fraction t/t_f, theoretical relations between ε^c and t/t_f and that between D and t/t_f are required further.

By neglecting the primary creep, Kachanov and Rabotnov theory provides the uniaxial relations of creep-damage coupling

$$\dot{\varepsilon}^c = B \left(\frac{\sigma}{1-D}\right)^N \tag{14}$$

$$\dot{D} = \frac{H}{q+1} \frac{\sigma^p}{(1-D)^q} \tag{15}$$

where B, N, H, p and q are material constants. After integrating these equations, the closed form solutions for damage and creep strain can be obtained as follows

$$D = 1 - (1 - t/t_f)^{\frac{\lambda-1}{n\lambda}} \tag{16}$$

$$\varepsilon^c = \varepsilon_f \left[1 - (1 - t/t_f)^{1/\lambda}\right] \tag{17}$$

where $t_f = (H\sigma^p)^{-1}$ is failure time, and $\lambda = \varepsilon_f / C_m$ the ratio of failure creep strain ε_f to the secondary Monkman-Grant strain $C_m = B\sigma^n \cdot t_f$.

Substituting Eqs. (16) and (17) into Eqs. (9), (10), (13), and noting that A will take its critical value A_{cr} when t tends to t_f, we have the following three equations for the situations where damage is controlled by cavity growth alone, by cavity nucleation alone, and by both respectively:

$$\left(\frac{A}{A_{cr}}\right)^2 = 1 - (1 - t/t_f)^{\frac{\lambda-1}{n\lambda}} \tag{18}$$

$$\frac{A}{A_{cr}} = 1 - (1 - t/t_f)^{\frac{\lambda-1}{n\lambda}} \tag{19}$$

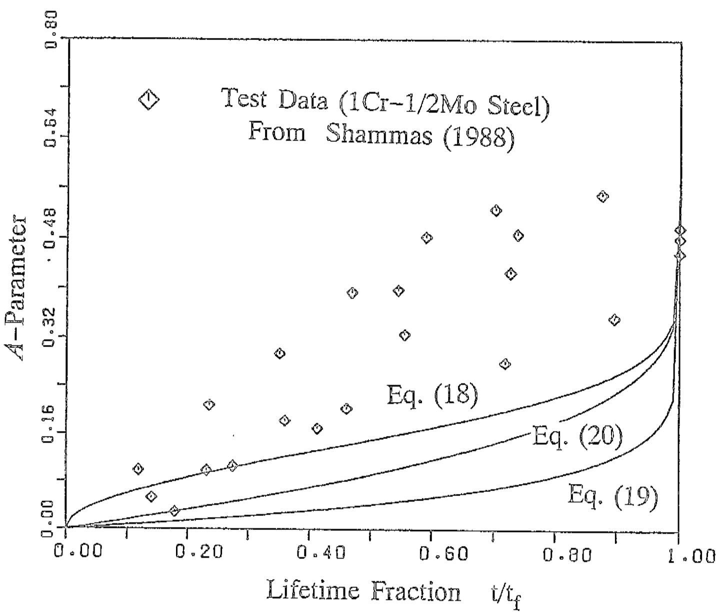

Fig. 3 A-Parameter vs Life Fraction t/t_f, Comparisons among Eq. (18), (19), (20) and the Test Results of 1Cr-1/2Mo HAZ Material from Shammas (1988)

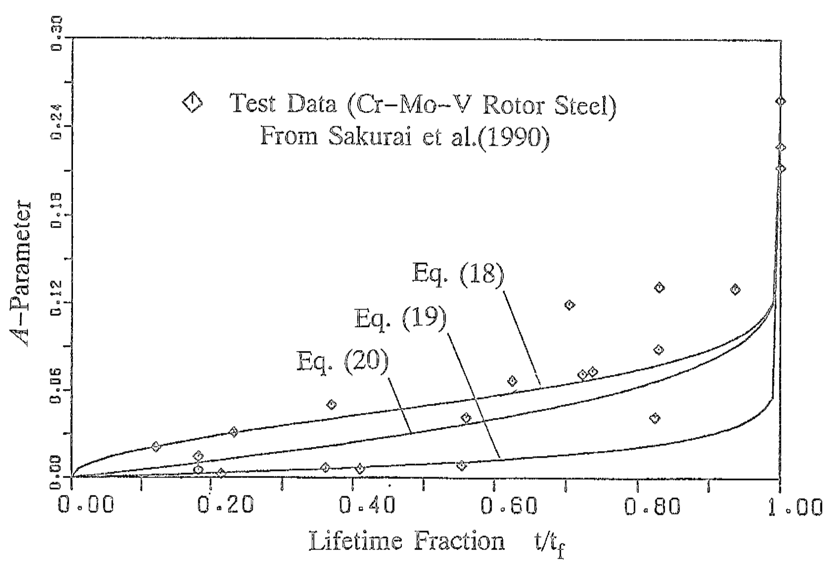

Fig. 4 A-Parameter vs Life Fraction t/t_f, Comparisons among Eq. (18), (19), (20) and the Test Results of Cr-Mo-V Steel from Sakurai et al.(1990)

$$(\frac{A}{A_{cr}})^2 = [1-(1-t/t_f)^{\frac{1}{\lambda}}][1-(1-t/t_f)^{\frac{\lambda-1}{n\lambda}}] \tag{20}$$

In Figs. 3 and 4, these theoretical curves are compared with the test results of two engineering alloys reported in the literatures. Fig. 3 shows the comparisons of Eqs. (18), (19) and (20) with the test results of 1Cr−1/2Mo steel given by Shammas(1988), where λ = 1.5 and n = 3.0 are also taken from the same literature. The critical value A_{cr} is taken as the maximum value of measured A−parameter, i.e. A_{cr}=0.55. Fig. 4 show the comparisons of the above relations with the test results of Cr−Mo−V rotor steel (Sakurai et al, 1990) where λ = 1.28 and n = 4.0 has been reported by the same authors. For this material, A_{max} = 0.26 is used as critical A−parameter A_{cr}. From Figs. 3 and 4 it will be observed that

1) In the case of cavity growth control, the theoretical curve of Eq. (18) gives the largest values of A−parameter among others and is also closest to the test data, especially at the later stage of the lifetime, for both materials. However, in the incipient period of lifetime, the tendency of the curve of Eq. (18) doesn't agree with the test results.

2) In the case of nucleation control, on the other hand, Eq. (19) gives the lowest values of the predicted results. The curve departs from the test data as lifetime increase and the discrepancy becomes considerable in the later period for both materials. A possible explanation for this discrepancy is that, if the creep damage is really controlled only by cavity nucleation (i.e., by the increase of cavity numbers), D versus t/t_f relation of Eq. (16) will be no more appropriate for this situation. It is obvious that the stress−controlled damage equation of K−R theory is not proper for describing the strain−controlled cavity nucleation. A more reasonable equation for strain−controlled damage has been developed by Riedel(1988), and the derived results between A and t/t_f using this equation will be discussed later.

3) Finally, the equation (20), which is similar to Eq. (18) since A is proportional to $\sqrt{D}$ instead of D, and gives the more reasonable correlation with the test results than both Eqs. (18) and (19). In the initial period of lifetime, Eq. (20) predicts correct tendency of test data because the influences of continuous nucleation has been taken into consideration. In the later stage of lifetime, it is very close to the curve of Eq. (18) so is closer to test data points than Eq.(19).

Based on the micromechanics model of constrained cavity growth (Dyson, 1976; Rice, 1981; Hutchinson, 1982), Riedel(1985) has shown that the constrained cavitating facets behave mechanically like microcracks since the local stresses applied on the facets will be greatly relaxed as the cavity area fraction ω on the facet increases. The volume density ρ of cavitated facets (or microcracks) has been suggested as an appropriate damage variable by Riedel(1989). He showed also that this variable is proportional to metallographic parameter A, although there were some uncertainties related to the cavitating facets with no cavities on them being cut by the observed plane, as discussed before. Riedel has suggested a creep−strain−controlled evolution equation for damage variable ρ. Combined with a constitutive model proposed by Hutchinson(1982) for a creeping material containing a dilute concentration of penny−shaped microcracks, Riedel(1989) derived an equation relating A and ρ. Using present donations, this relation can be expressed as follows

$$\frac{t}{t_f} = \lambda(\frac{A}{A_{cr}})^{\lambda-1} - (\lambda-1)(\frac{A}{A_{cr}})^{\lambda} \tag{21}$$

Fig. 5 and Fig. 6 show the comparisons of the above equation with present model of Eq.(20), which has included the influences of continuous nucleation. The test results of Shammas(1988) and Sakurai et al.(1990) are also included in the figures. As shown in the figures, in the initial period of lifetime, Riedel's model gives lower values of predicted A-parameter than present model, but the larger values in the later period.

As regards Eq. (19) which has been employed by some researches (Shammas, 1988; Sakurai et al 1990), although some improvements have been achieved in correlating with the test results in Figs. 5 and 6 by use of the two models considered here, there still exits some discrepancy between theoretical curves and tests data. For Riedel's model, this may be due to the uncertainties mentioned above and the supposition of cavitated facets behaving like microcracks through the whole lifetime, that is clearly unreasonable when ω is very small.

For present model, further efforts may be needed to improve the correlation. Some possible developments will be discussed here briefly. Firstly, non-cavitated facets existing in material during the real damage process will produce constraints to cavity growth, as shown by Dyson(1976) and Rice(1981). This means that the evolution equation of local damage ω will be influenced by the constraints. However, this effects can not be taken into account in the K-R damage theory which has been applied in present research. An engineering damage model with such influences has been developed recently by the present authors, by which the derived A versus t/t_f curve has better correlation with the test data, the results shall be reported elsewhere (Liu and Murakami, 1991). In addition, continuous nucleation of cavities introduced in present model will lead to inhomogeneous distribution of cavities. Generally speaking, the different facets will contain different number of cavities and the cavities on the same facet will have different sizes. Thus the r and m in Eqs. (7) and (8) etc. must be supposed as the average values over total cavitated facets. To account modeling of any of these nonuniform distribution will be complicated and demands further investigation.

Finally, let's discuss the application of the proposed relations between A and D. If the A-parameter in a position of components could be measured by some available techniques (e. q., the replica technique), the remaining lifetime fraction of the components can be estimated by using one of the theoretical curves in Figs. 3 to 6. In fact, this estimation can be made more precisely by use of the empirical relation directly fitted from the test data; One of such relation has been given by Shammas(1988) for 1Cr-1/2Mo steel. However, such technigue is valid only for the condition of constant stress and constant temperature. For more general condition of variable temperature and non-steady states of stress, we need to employ some analytical or numerical techniques based on the evolution equation of damage variable D and related constitutive equations. The correlation between A and D here clearly provides a physical foundation to identify the relevant damage state D from the observed value of A. In previous analyses of this paper, constant q in damage equation (15) is determined indirectly by the damage-induced acceleration of creep strain in the tertiary stage. One of this kind of approaches (Riedel, 1987) gives

$$q+1 = \frac{n\lambda}{\lambda-1} \tag{22}$$

as used in Eqs. (16) and (17). Obviously, more reasonable approach is to determine q by fitting the damage evolution equation

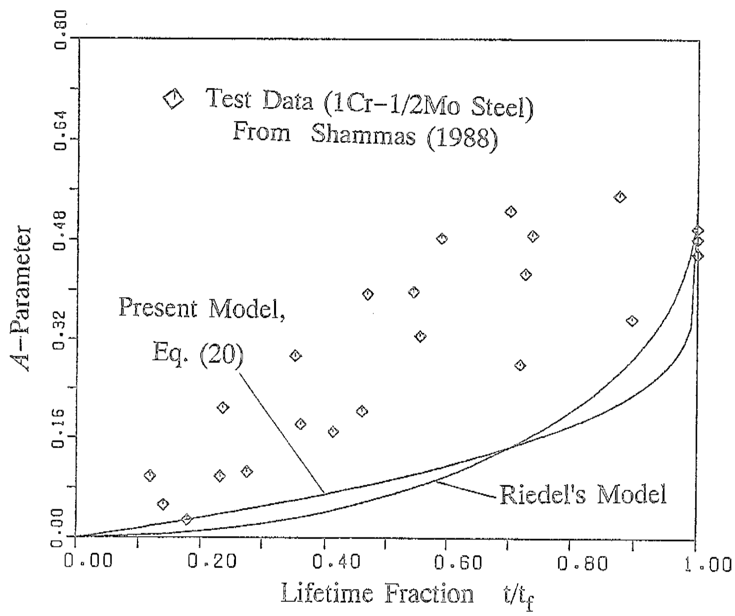

Fig. 5 A-Parameter vs Life Fraction t/t_f, Comparison between Eq. (20) and Riedel's Model. The Test Results of 1Cr-1/2Mo HAZ material from Shammas (1988)

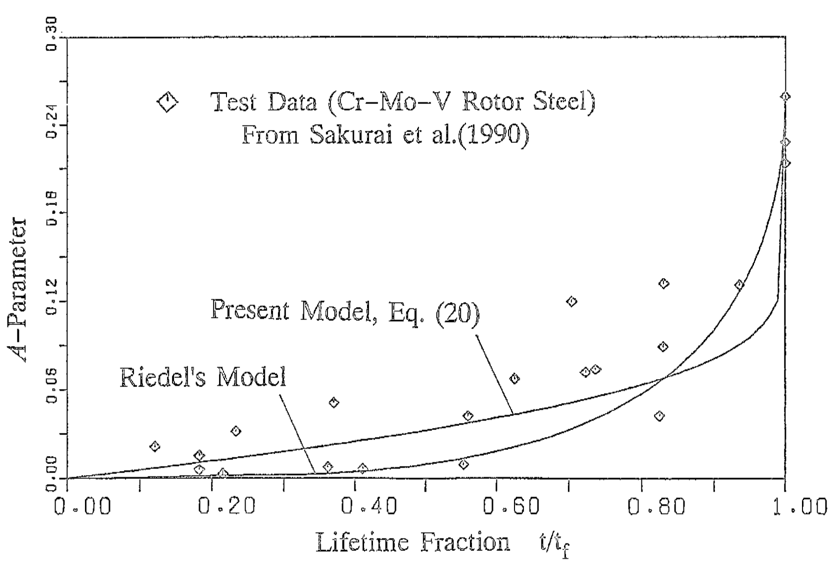

Fig. 6 A-Parameter vs Life Fraction t/t_f, Comparison between Eq. (20) and Riedel's Model. The Test Results of Cr-Mo-V Steel from Sakurai et al.(1990)

$$D = 1-(1-t/t_f)^{\frac{1}{q+1}} \tag{23}$$

directly with the measured evolution of A-parameter, by applying the proposed relations between A and D. These relations are recapitulated from Eqs. (9), (10) and (13) as follows

$$A = c_1\sqrt{D} \tag{24}$$

$$A = c_2\ D \tag{25}$$

$$A = c_3\sqrt{\varepsilon^c D} \tag{26}$$

where $c_1 = c_2 = A_{cr}$ and $c_3 = A_{cr}/\sqrt{\varepsilon_f}$ can be easily obtained. More discussions on this problem can be found elsewhere (Liu and Murakami, 1991).

4. SUMMARY

In present paper, by calculating the cavity–cut probability P, the metallurgical parameter A was correlated to the damage variable D in continuum damage mechanics. The probability P is dependent on the cavity area fraction ω on a G.B. facet which were averaged to give the damage variable D in CDM.

By considering the influences of continuous nucleation of cavities, previously proposed relation of $A \propto D$ (Cane and Shammas, 1984) and $A \propto \sqrt{D}$ (Sugita, Liu and Murakami, 1991) were shown to be the extreme cases of damage evolution by cavity nucleation alone, and cavity growth alone, respectively. A general correlation between A and D has been derived approximately, in which the influences of macroscopic creep strain were included. Based on the comparisons with experimental results reported in literatures, the general equation for correlating A and D has been shown more reasonable.

Finally, the present results suggests that, metallurgical parameter A can provides a proper physical foundation for damage variable and its evolution equation used in CDM. However, some weaknesses of Kachanov–Rabotnov creep damage theory have also been revealed through the correlation, one of which is that damage evolution equation in K–R theory has been developed for the average damage variable D, and thus the influences due to the inhomogeneity of local damage variable ω (cavity area fraction) can not be taken into accounts.

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