Uniformly Probable Design Response Spectra

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ABSTRACT

In the seismic analysis of nuclear power plant structures a common practice is to prescribe a site dependent design response spectrum as the basic formulation of the ground motion to be applied to structural system analyses. A complementary requirement on the amount and distribution of the seismic power, on the main interest frequency range of the problem, has been proposed through a target power spectral density.

One methodology based on non-exceeding probability computation of the design response spectrum ordinates obtained directly from the power spectral density functions is proposed to evaluate the existing compromise between these two functions and to develop uniformly probable design response spectra.

Applications are made on two pairs of function values and conclusions are oriented on the direction of the convenience to use the procedure to improve the consistency on the choice of target power spectral density functions and to generate uniformly probable design response spectra.

INTRODUCTION

In the seismic analysis of nuclear power plant (NPP) structures a common practice is to prescribe a site dependent design response spectrum (DRS) as the basic formulation of the ground motion to be applied to the structural system analyses. As far as response spectrum analyses are used this definition should be enough.

However, if one wishes to develop a time history or a probabilistic analysis, this definition remains incomplete. In the former case, this happens because the generation of an accelerogram to match a response design spectrum to a given tolerance is not an unique solution problem and the resultant time histories may vary significantly; for the latter situation, because no information is given to reasonably characterize the site ground motion random process.

To overcome these difficulties the prescription of a target power spectrum density function (PSD_T) has been adopted [2] given information about the total power of the ground motion process and its distribution along the frequency axis main interest region.

In a previous work [4], one proposes a methodology based on a random structural analysis technique to identify and to quantify in probabilistic terms the relation between these two functions, DRS and its PSD_T. Sequentially, one shows also how to use this procedure to choose among a set of accelerograms, generated to match to a given tolerance a DRS, the one which most conveniently will envelop the DRS.

Now, one works on the idea of a uniformly probable response spectrum, i.e., its ordinates represent a barrier with equal probability not to be exceeded by the response of a single degree of freedom system (SDOF) excited at its base by ground motions belonging to a stationary random process defined by a PSD_T function. In sequence, the idea is applied to criticize the fitness of DRS and PSD_T function pairs.

The methodology is based on the systematic resolution of a first passage problem [3] in the back forward direction, in which the PSD of the one degree of freedom system response functions is computed as the multiplication of the similar ground motion function by the system frequency response squared modulus.

This computation formulation is implemented and applied, in the design context of a Brazilian NPP, to criticize two pairs of DRS and PSD_T functions. The concepts of minimum (0.8 PSD_T), target and 1.3 PSD_T functions [2] are analyzed under the same point of view.

METHODOLOGY

The first passage problem formulation suggested by Vanmarcke [3], equations (1) and (3), may be advantageously applied to compute the probability function of a SDOF response function not to exceed, for instance, a specified DRS level. This formulation is based entirely on the PSD response function moments, and is written as:

$$F(r)=\left(1-e^{\frac{-r^2}{2}}\right)\cdot\exp\left(\frac{-\frac{1}{\pi}\cdot\sqrt{\frac{\lambda_2}{\lambda_0}}\cdot\exp\left(\frac{-r^2}{2}\right)\cdot\left(1-\exp\left(-\sqrt{\frac{\pi}{2}}\cdot r\cdot\left(\sqrt{1-\frac{\lambda_1^2}{\lambda_2\cdot\lambda_0}}\right)^{1.2}\right)\right)}{\left(1-e^{\frac{-r^2}{2}}\right)}\cdot t\right) \tag{1}$$

In the above:

r - the reduced response level,
t - ground motion duration;
λ_i - the i-th order spectral moment;

$$r=\frac{a}{\sqrt{\lambda_0}} \tag{2}$$

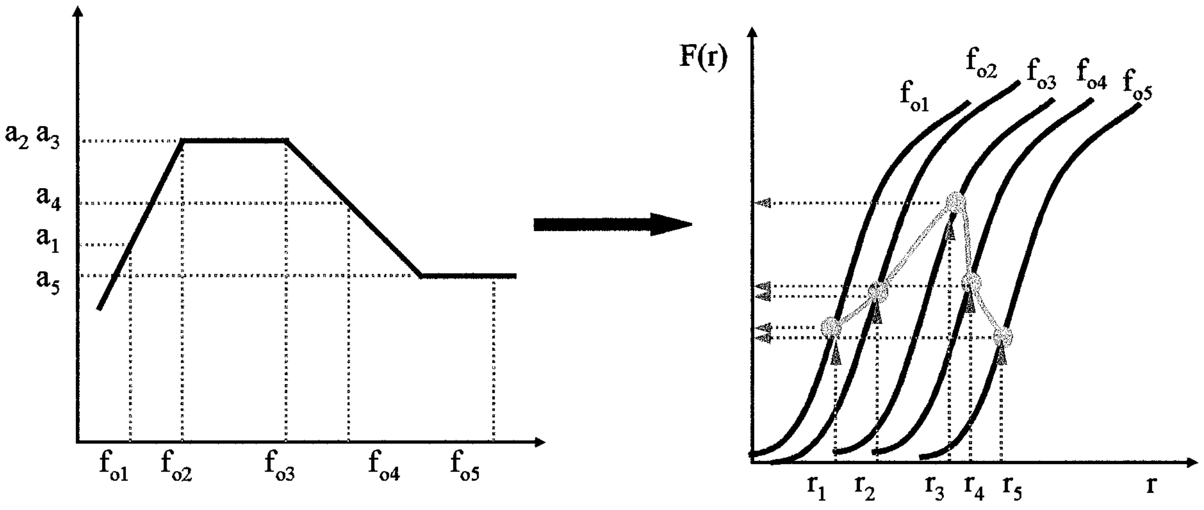
$$\lambda_i=\int\omega^i\Phi_{RR}(\omega)d\omega \tag{3}$$

where a is the acceleration response level and ω is the circular frequency. In this way, one may write:

$$\Phi_{RR}(\omega)=\left|H(\omega)\right|^2\cdot\Phi_{GG}(\omega) \tag{4}$$

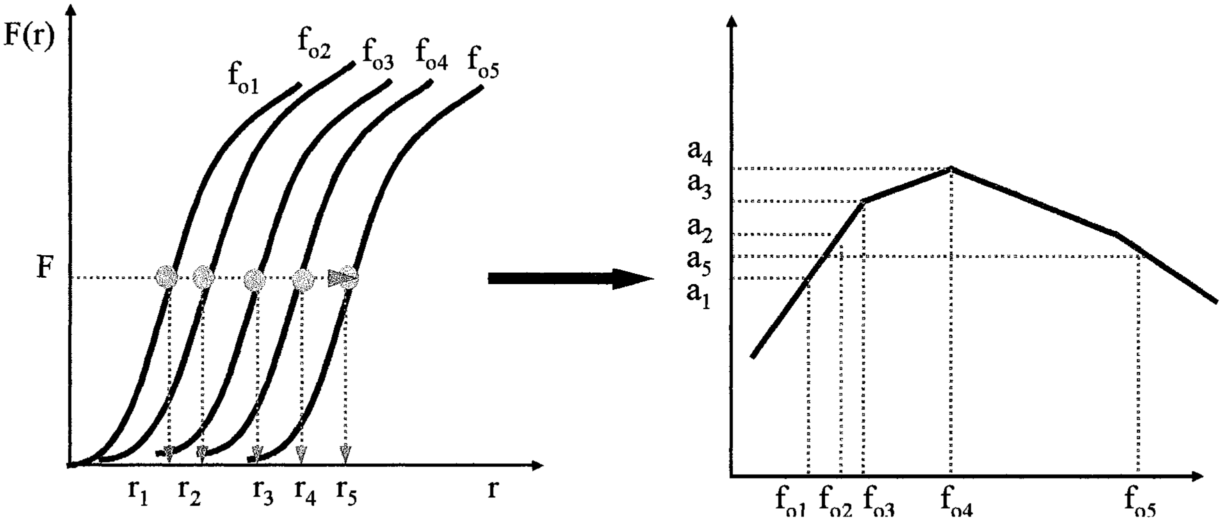
in which, $\Phi_{RR}(\omega)$ is the response power spectral density, H(w) is the frequency response function of a base excited SDOF system and $\Phi_{GG}(\omega)$ the ground motion acceleration PSD function.

Naturally, the problem can be solved on the back forward direction, when one search for a reduced barrier level, r, starting from a given distribution probability, F(r). So working, one may to determine a uniformly probable response spectra, i.e., its ordinates represent a barrier with equal probability of not to be exceeded by the response of a SDOF system excited at its base by ground motions proceeding from a stationary random process defined by a PSD_T function. Figures (1) and (2) show computational procedure schematic views of these two situations.



(a) Design response spectrum (b) Probability distribution functions

Figure 1 Computational procedure schematic view. Probability function of a SDOF system response function not to exceed a specified level



(a) Probability distribution function

(b) Uniformly probable response spectrum

Figure 2 Computational procedure schematic view. Uniformly probable response spectrum

APPLICATION

In the context of a Brazilian NPP design, the just described methodology is applied to a pair of sets, DRS-PSD$_T$ functions, to evaluated the existing probabilistic association in the set and to analyze them under the uniform probability point of view. The two examples are formed as follows:

- Example I: The 2% damped design response spectrum [1] and its target power spectrum density function [2], scaled to 1.0g, both recommended by USNRC.
- Example II: An alternative approach to a site dependent design response spectrum.

Tables 1 and 2 and Figures 3 and 4 show a comparative view of the four functions, two by two.

Table 1. Design response spectrum functions – Example I (USNRC) and II

f_o (Hz)	Example I Acceleration (m/s^2)	Example II Acceleration (m/s^2)
2.5	42.5	35.0
5.0	38.0	35.0
9.0	35.4	33.0
20.0	16.0	14.0
33.0	10.0	10.0

Table 2. Power spectrum density functions - Example I (USNRC) and II

f_o (Hz)	Example I (m^2/s^3)	Example II (m^2/s^3)
2.5	4.17×10^{-1}	2.98×10^{-1}
5.0	1.20×10^{-1}	1.09×10^{-1}
9.0	4.17×10^{-2}	3.93×10^{-2}
20.0	1.25×10^{-3}	1.99×10^{-4}
33.0	0.00	0.00

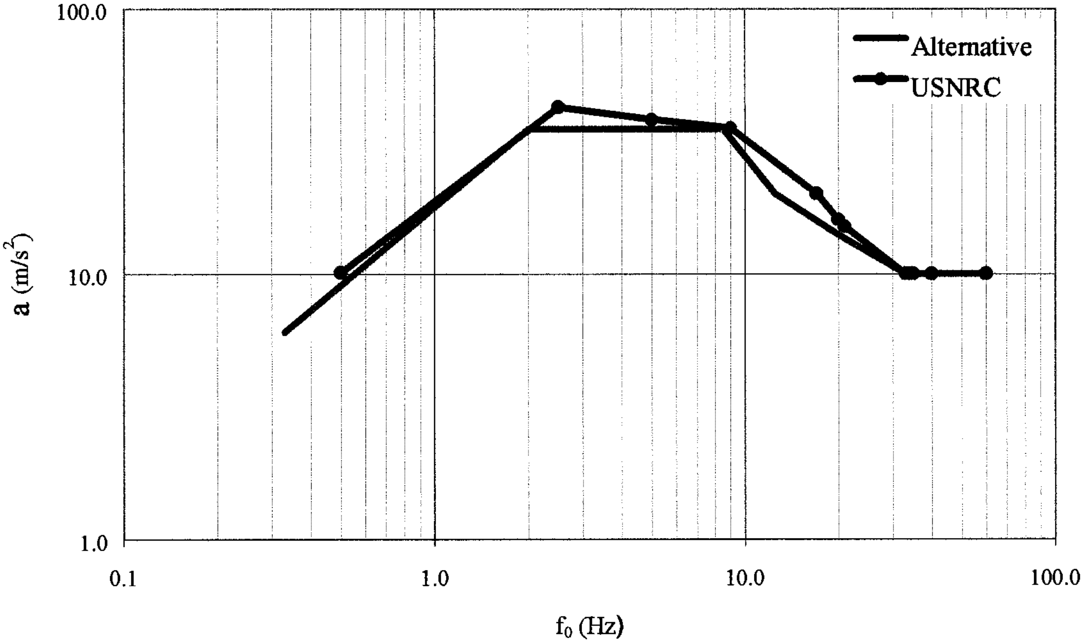


Figure 3 Design response spectrum functions – Example I (USNRC) and II

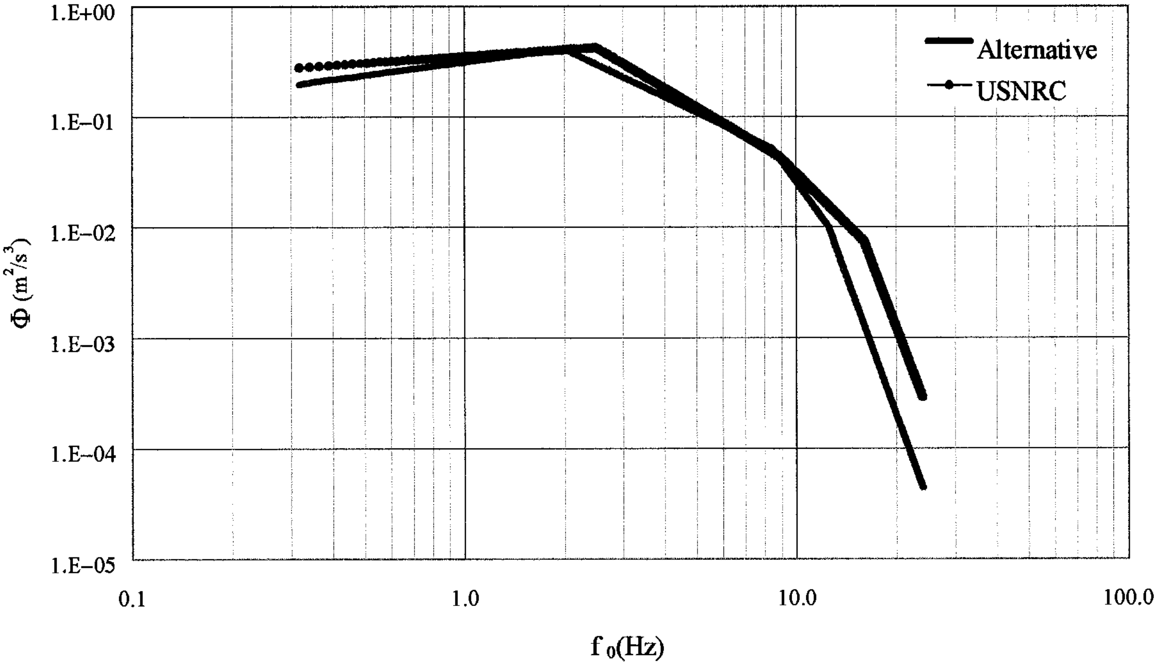


Figure 4 Power spectrum density functions - Example I (USNRC) and II

RESULTS AND DISCUSSION

Initially, one computes probabilities not to exceed the 2% damped DRS acceleration values by the response spectrum ordinates produced by a ground motion belonging to the trial-space defined by the PSD_T function; similar probability values are obtained for the 0.8 PSD_T and 1.3 PSD_T cases, Tables 3.

Table 3. Probabilities not to exceed the 2% damped DRS acceleration values by the response of a SDOF system to a ground motion belonging to the trial-space defined by three PSD_T levels

Example I (USNRC)

f_0 (Hz)	0.8xPSD_T	PSD_T	1.3xPSD_T
0.5	0.52	0.39	0.26
2.5	0.82	0.64	0.38
5.0	0.83	0.57	0.23
9.0	0.93	0.73	0.31
20.0	0.90	0.60	0.11
33.0	0.45	0.10	0.0027

Example II (Alternative)

f_0 (Hz)	0.8xPSD_T	PSD_T	1.3xPSD_T
0.5	0.34	0.23	0.14
2.5	0.743	0.53	0.28
5.0	0.78	0.51	0.18
9.0	0.88	0.61	0.19
20.0	0.99	0.94	0.65
33.0	0.77	0.41	0.06

This makes it possible to analyze the compromise between the DRS and their associated PSD_T. Tables 3 show that the probabilities of the DRS acceleration levels not to be exceeded by the response spectrum ordinates, produced by ground accelerograms belonging to the trial-space defined by the corresponding PSD_T functions, vary along the main interest frequency band; from 0.57 to 0.73, in Example I, and from 0.51 to 0.94, in Example II. In all cases, the accelerogram duration is made equal to 15 seconds.

One also verifies, as it could be expected, that the power reduction from 1.3 to 0.8 PSD_T produces a non-exceeding probability value increasing all over the frequency axes. The very low non-exceeding probability values at the (33.0 Hz – 1.3 PSD_T) table corners confirm that on the very-high frequency region of the spectra the system acceleration values go to 1.0g with probability 1. Similar behavior occurs on the very-low frequency region where the system acceleration values tend to zero.

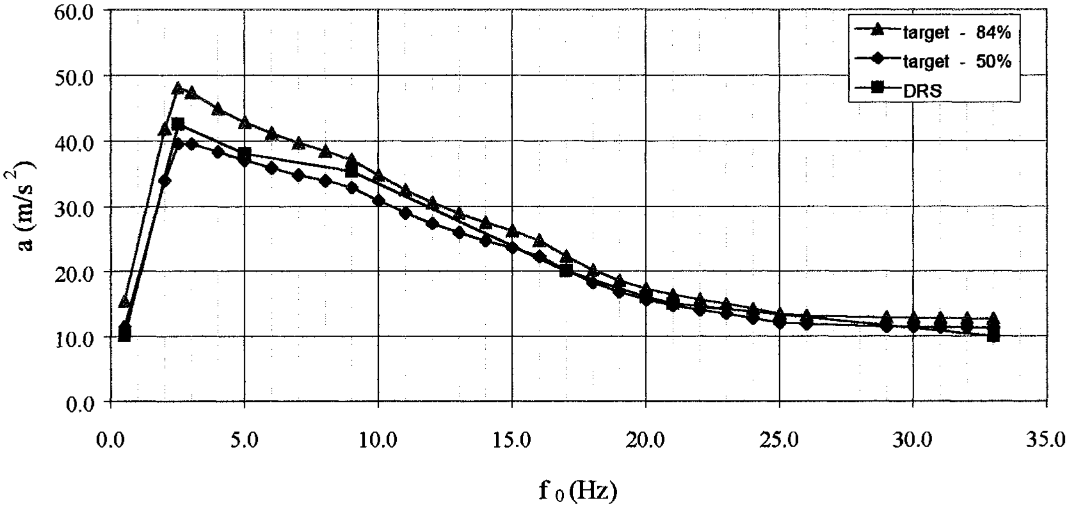


Figure 5 Uniformly, 50% and 84%, probable response spectra and design response spectrum PSD_T – Example I (USNRC)

Now, if one imagines a uniformly probable response spectrum, the question is: Which non-exceeding probability value should be desirable?

Figure 5 shows that the two uniformly probable response spectra follow the same general trends of the DRS. On the very-low frequency region all the three curves go to zero, as it should be expected; however, on the very-high frequency region the uniformly probable spectra are asymptotic to abscissa values greater than 1.0g indicating, in general terms, that the PSD_T has some power excess. The uniformly 50% probable spectrum is enveloped by the DRS and this one by its turn, is enveloped by the 84% spectrum. In this way, one can say that the DRS approaches a uniformly probable spectrum with non exceeding probability between 50 and 84%.

So, the answer to the question is a subject of the plant acceptability criterion. Naturally, this situation changes importance depending on the usage which is made of the PSD_T function, if just to choose the more convenient accelerogram to employ in a time domain analysis or if one intends to develop directly a probabilistic analysis.

Finally, after that, uniformly 50% probable response spectra are computed for the same three PSD_T levels, and compared with the prescribed DRS, Figures 6 and 7.

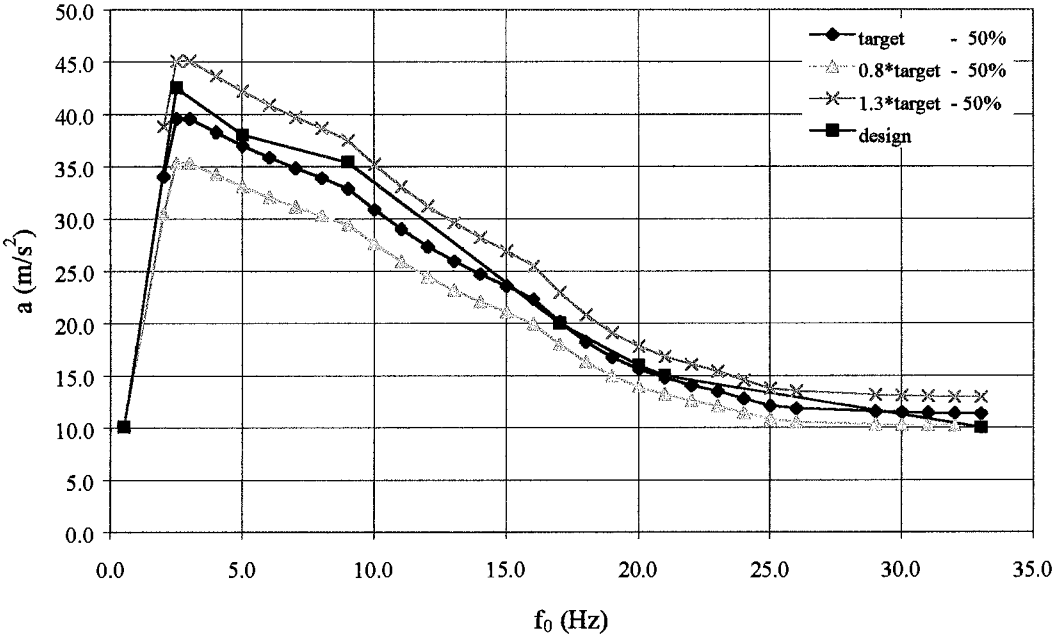


Figure 6 Uniformly 50% probable response spectra and prescribed DRS – Example I (USNRC)

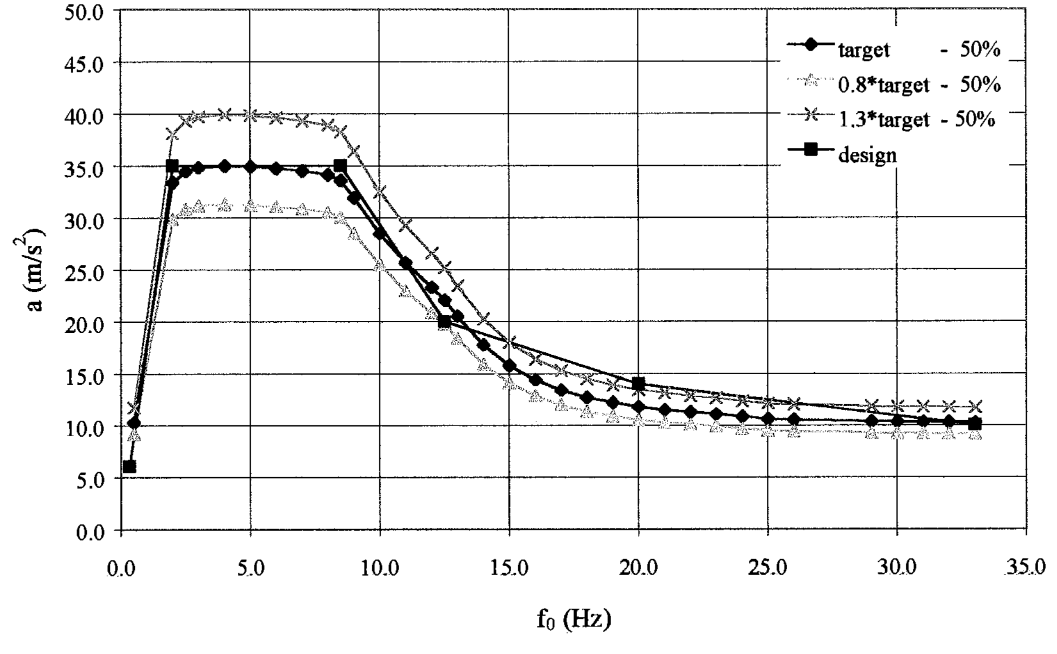


Figure 7 Uniformly 50% probable response spectra and prescribed DRS – Example II (Alternative)

From Figures 6 and 7 one can say, for both examples, that the median uniformly probable response spectra, i.e., response spectra with 50% non-exceeding probability, follow very closely the shapes of the respective DRS.

Figure 6 also shows that, in general, the median uniformly probable response spectrum related to the 0.8 PSD_T is enveloped by the DRS and the one made from a 1.3 PSD_T envelops the DRS ordinates. For the full PSD_T function, the median uniformly probable response spectra follows very closely the USNRC/DRS, except along the 2.0-15.0 Hz frequency band, where some more power would be necessary to make them even.

Similarly in Example II, Figure 7 shows up that exists some lack of power on the frequency range from 14.0 to 33.0 Hz. To reduce this lack of power one may introduce a scale factor on the PSD_T in such a way to get an optimized matching of the two curves.

CONCLUSION

The methodology using the design response spectrum ordinates as a barrier in the first passage problem solution can be used with advantage in the following situations regarding the compromise between a design response spectrum and its associated spectral density functions:

- to choose among a set of ground motion time-histories, generated to match to a given tolerance a design response spectrum, the one which more conveniently approaches the target power spectral density function;
- to criticize the compromise between a design response spectrum and its target power spectral density function and to recommend a way to adjust them;
- to generate a uniformly probable design response spectrum associated to a given target power spectral density function of the ground motion and, in this way, to contribute in establishing the acceptability criteria in a structural probabilistic seismic analysis.

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