Development Work on Vibration Acceptance Criteria of Piping Systems

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ABSTRACT

This paper presents a development work, both analytical and numerical, which has been done on beam mode vibration of piping systems. Using equations of motion, bending stress and bending moment, it has been shown that the level of vibration is directly proportional to bending stress and is a function of size and frequency, rather than layout.

The outcome of the work can be used to establish acceptable levels of vibration for piping systems. This could be utilized for screening purposes, or for estimating the level of vibration stresses, simply by measuring velocity of vibration. The fatigue stress can be further used for assessment of piping systems in a plant life extension program.

INTRODUCTION

The piping systems represent a large portion of the mechanical equipment used in the power generating and many other industries. The piping systems are subjected to fluid flow. The fluid flow represents a source of energy, that can induce and sustain vibration of piping systems.

The flow-induced vibration is a complicated phenomenon. The mathematical modelling and exact characterization of the fluid-structure interaction are still incomplete.

The problem of fluid-structure interaction contains the two following parts; fluid flow and structural response. This paper is concerned with the part of structural response only, which can be addressed separately.

The objective of the paper is to provide information for acceptance criteria, which can be used for assessing vibration levels of piping systems. The objective can be presented in the following two parts:

(a) Formulation of a mathematical relationship between alternating stresses and displacement or velocity due to beam mode vibration.

(b) An extensive computational work performed to confirm and define the relationship.

MATHEMATICAL FORMULATION

The equation of motion, neglecting fluid velocity in the pipe, is:

$$EI \frac{\partial^4 y}{\partial x^4} + m \frac{\partial^2 y}{\partial t^2} = 0 \qquad (1)$$

Where, E is a modulus of elasticity, I is a section modulus, m is the mass per unit length of the pipe and fluid, y is a lateral deformation due to vibration, x is a longitudinal coordinate along a pipe axis and t is a time.

Let the second antiderivative of $\frac{\partial^4 y}{\partial x^4}$ be $\frac{\partial^2 y}{\partial x^2}$ we get;

$$EI \frac{\partial^2 y}{\partial x^2} = - m \int_o^L \int_o^x \frac{\partial^2 y}{\partial t^2} \partial x \, \partial x^1 \qquad (2)$$

Alternating bending stress at the midsurface of the pipe wall is:

$$\sigma = \pm i \frac{MR}{I} \qquad (3)$$

where, i is a stress intensification factor for pipe discontinuities, R is a midsurface radius of the pipe and M is a bending moment expressed as:

$$M = EI \frac{\partial^2 y}{\partial x^2} \qquad (4)$$

Make use of equations 2, 3 and 4 to obtain absolute value of the stress:

$$\sigma = i M \frac{R}{I} \int_o^L \int_o^x \frac{\partial^2 y}{\partial t^2} \partial x \, \partial x^1 \qquad (5)$$

Allow a solution of the equation 1 to be:

$$y = y_o \eta \tau \qquad (6)$$

where,

$\eta = f (\lambda \frac{x}{L})$ is a nondimensionalized function of a deformed

piping shape

$\tau = f (\omega t)$ is a nondimensionalized function of time

A circular frequency ω is:

$$\omega = 2 \pi f_N = (\frac{\lambda}{L})^2 \quad \frac{EI}{m} \qquad (7)$$

where, λ is a mode shape factor, L is a pipe length and f_N is a structural frequency.

Second partial derivative of the equation 6 is:

$$\frac{\partial^2 y}{\partial t^2} = y_o \omega^2 \eta \tau^1 \quad (8)$$

Now from the equation 5, we can write:

$$\int_o^L \int_o^x \frac{\partial^2 y}{\partial t^2} \partial x \partial x^1 = y_o \omega^2 \tau^1 \int_o^L \int_o^x \eta \partial x \partial x^1$$

Let the second antiderivative of $\eta \partial x \partial x^1$ be $\zeta (\frac{L}{\lambda})^2$ and the equation 5 become,

$$\sigma = i m y_o \omega^2 \tau^1 \zeta (\frac{L}{\lambda})^2 \frac{R}{I} \quad (9)$$

Make use of equations 6, 7 and 9 to obtain displacement due to vibration:

$$y = \frac{\eta \tau}{\zeta \tau^1} \frac{1}{i} \frac{1}{\omega} \frac{\sqrt{I}}{R} \frac{1}{\sqrt{m}} \frac{1}{\sqrt{E}} \sigma \quad (10)$$

where,

$\frac{\eta \tau}{\zeta \tau^1} = \beta$ is a nondimensionalized factor

$I = \pi R^3 h$, h is a pipe wall thickness

$$m = 2 \pi R h \rho (1 + \frac{R_p^2}{2 Rh} \frac{\rho_{fl}}{\rho} + \frac{R_{in}}{R} \frac{h_{in}}{h} \frac{\rho_{in}}{\rho})$$

or $m = 2 \pi R h \rho b$

ρ is a density of pipe material, ρ_{fl} is a density of a fluid inside the pipe, ρ_{in} is a density of the material of pipe insulation, R_p is a pipe inside radius, R_{in} is a midsurface radius of the insulation and h_{in} is a thickness of insulation.

Finally, we get:

$$y_p = \beta \frac{1}{i} \frac{1}{2 \pi \sqrt{2}} \frac{1}{f_N} \frac{1}{\sqrt{E}} \frac{1}{\sqrt{\rho}} \frac{1}{\sqrt{b}} \sigma_{max} \quad (11)$$

where,

$$b = 1 + \frac{R_p^2}{2 Rh} \frac{\rho_{fl}}{\rho} \frac{R_{in}}{R} \frac{h_{in}}{h} \frac{\rho_{in}}{\rho} \quad (11a)$$

The equation 11 shows that zero to peak displacement is proportional to maximum alternating stress and is a function of pipe size, fluid inside the pipe, thermal insulation, material of the pipe and the frequency of vibration.

The factor ß is a function of mode shape factor or frequency, size and configuration of pipe as well as piping end conditions. The factor cannot be mathematically determined.

ESTABLISHING THE FACTOR ß

In order to establish the factor ß, which would numerically define the equation 11, computational work is required. The factor ß, as a ratio of peak displacement to maximum alternating stress, can be calculated by using modal analysis of a desired piping system. The analyses of various piping configurations, combined with different pipe sizes and end conditions, have been orderly performed.

The configurations used in the analyses were from a simple one straight run (no elbows) to a complex piping system having five straight runs. Every chosen configuration has been analyzed for a minimum of the following two sizes, 50 mm and 750 mm. In addition, several piping systems of a nuclear generating station, having pipe sizes of 250 mm, 350 mm, 450 mm and 600 mm with branches and reducers, have been included in the analyses. The selected end conditions were anchor-anchor and anchor-guide (restrained in lateral directions only). The analyses were performed using the stress intensification factor i=1. Each piping configuration was analyzed in the frequency range of 0 to 100 Hz. A total of more than 100 different piping systems was analyzed. These resulted in 3000 mode shapes or frequencies each with the related factor ß. Volume of the work done is equivalent to two man-years.

From the results obtained in the analyses, the following can be summarized:

(a) For one straight line (like a header) the factor ß is a constant value for all pipe sizes, end conditions and frequencies as shown in Figure 1.

(b) For complex piping systems having at least one elbow, the factor is not a function of pipe size, piping end condition and piping configuration, but is a function of frequency as shown in Figure 2.

To keep a conservative approach in setting up acceptable levels of vibration, a minimum value of factor ß should be used.

Finally, the following two ß factors are:

(a) ß = 0.92 for a straight pipe line;

(b) In chart form shown in Figure 3 for all piping systems.

ACCEPTANCE CRITERIA

Assuming that vibration of a piping system is a combination of sinusoidal oscillations, we can write the following as a peak vibration velocity:

$$V_p = 2 \pi f_N y_p \qquad (12)$$

make use of equations 11 and 12 to obtain:

$$V_p = \beta \frac{1}{i} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{E}} \frac{1}{\sqrt{\rho}} \frac{1}{\sqrt{b}} \sigma_{max} \quad [\frac{m}{s}] \qquad (13)$$

This equation can be expressed as an acceptable level of vibration if stress is limited to the allowable stress from a fatigue curve for the material of the pipe. Furthermore, the equation 13 shows that the relationship between peak vibration velocity and maximum alternating stress is a function of the factor ß. Since there are two different types of factor ß the following two acceptable levels of vibration are:

(a) For a straight pipe line

$$V_{Pacc} = \frac{650}{i} \frac{1}{\sqrt{E}} \frac{1}{\sqrt{\rho}} \frac{1}{\sqrt{b}} \sigma_{all} \quad [\frac{mm}{s}] \qquad (14)$$

(b) For all piping systems:

$$V_{Pacc} = \beta \frac{707}{i} \frac{1}{\sqrt{E}} \frac{1}{\sqrt{\rho}} \frac{1}{\sqrt{b}} \sigma_{all} \quad [\frac{mm}{s}] \qquad (15)$$

where, for both equations, b is given by the equation 11a, i for nuclear class piping is equal to C_2K_2 (stress indexes) or can be used as a safety factor and the units used are Pa, $\frac{kg}{m^3}$ and mm.

A piping system vibrates in more than one mode. Actually a peak vibration amplitude (displacement, velocity or acceleration) is a combination - "resultant" of all vibrating modes. An actual vibration peak amplitude is obtained from time history data. At the same time, a frequency content of the peak value is obtained from spectrum data. The peak value vibrates in a "resultant frequency" which is not determinative. Since the piping systems are linear and homogeneous, the "resultant frequency" of a peak value cannot be less than the frequency of the smallest excited structural mode. This smallest frequency can be found from the spectrum data.

In the equation 15, the factor ß should be used from Figure 3 for the smallest frequency taken from the spectrum data.

For example, carbon steel nuclear Class 1 piping system has a diameter of 275 mm, wall thickness of 10 mm, contains water at 25°C, no insulation, $C_2K_2 = 1.95$, smallest frequency of vibration is 5 Hz (from recorded spectrum), ß = 0.3 from Figure 3, acceptable zero to peak velocity would be 105 $\frac{mm}{s}$.

CONCLUSIONS

It has been shown that the maximum alternating stress due to beam mode vibration of a piping system is directly proportional to the level of vibration and is a function of pipe size, material and frequency rather than piping configuration.

The equations 14 and 15 present the vibration acceptance levels, for piping systems, in terms of a peak velocity due to beam mode vibration. These equations can be used for screening purposes to establish acceptability of piping vibration in the industry and/or for estimating the level of vibration stresses, simply by measuring the velocity of vibration. This can be further utilized for assessing the piping systems in the life extension program.

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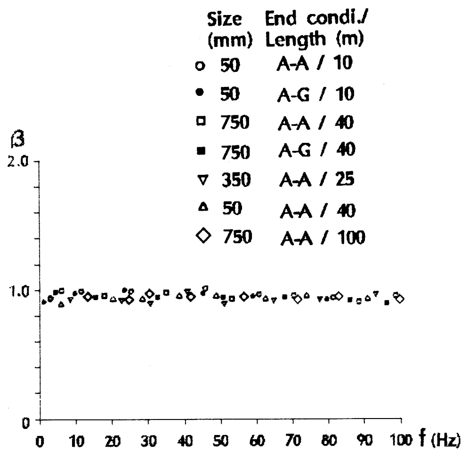

Fig. 1 Factor ß for straight lines with different pipe sizes, lengths and the following end conditions; anchor-anchor; A-A & anchor-guide; A-G

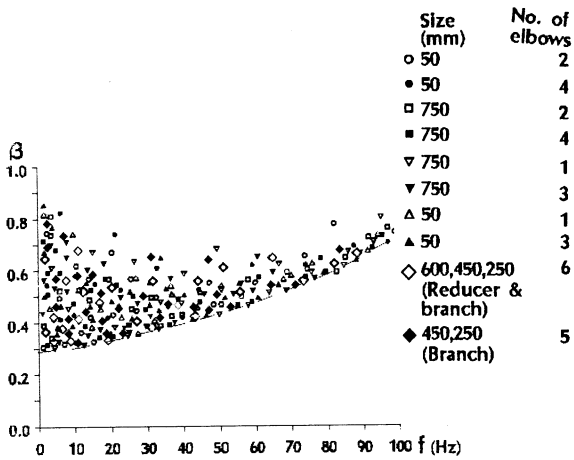

Fig. 2 Factor ß for complex piping systems with different pipe sizes and configurations; No. of elbows

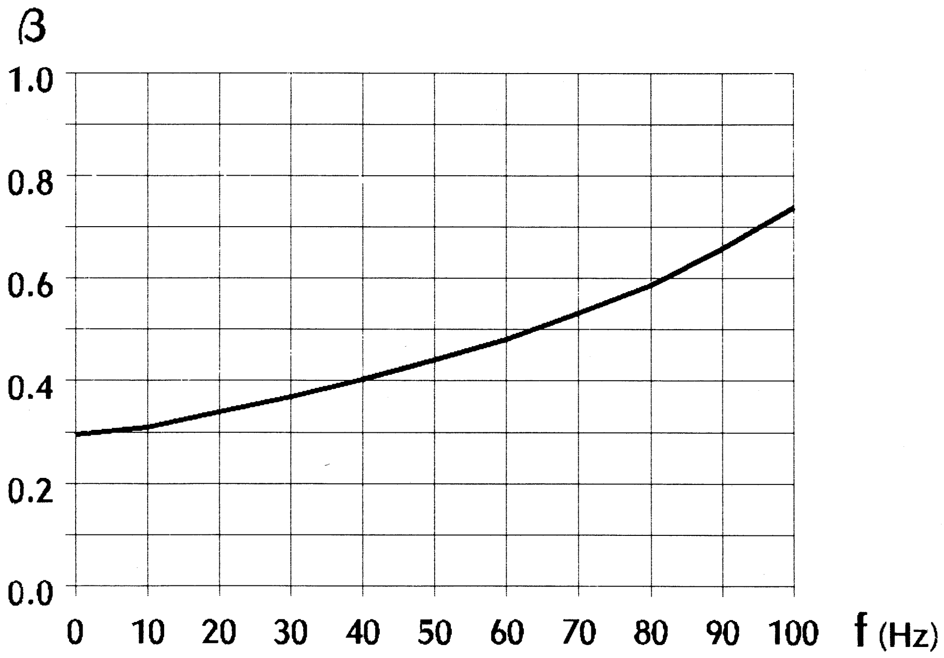

Fig. 3 Factor ß for Piping Systems