CHARACTERIZATIONS OF CONTINUING FRACTURE IN ELASTIC-PLASTIC SOLIDS

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ABSTRACT

The paper examines the essential and the desirable features that constitute a viable approach for treating the onset of crack extension, stable crack extension and fracture instability under consitions of contained and large scale plasticity. Relevant experimental and analytical crack growth studies are summarized; the evidence suggest that a resistance curve approach based on the J-integral and the crack opening displacement (COD) are most appropriate for treating problems of continuing crack extension. From a theoretical basis, the two approaches are similar if certain requirements are satisfied. By exploiting the dominance of the HRR singularity in the crack tip region, it can be shown that there is a direct relationship between the J-integral and the COD. The coefficient relating the two parameters depends on the yield stress, hardening exponent and the ratio of the yield stress to elastic modulus. Similarly, the slope of the J-resistance curve is relatable to an appropriately defined crack opening angle for extending cracks. These relationships are corroborated by experimental and detailed numerical results from a variety of sources.

The relative merits and difficulties associated with the J or COD approaches for treating stable crack growth and fracture instability in structural components are discussed. In applications involving relatively small amounts of crack extension, the J-resistance approach appears to be the easier approach to employ. For large amounts of crack extension, the evidence seems to favor a COD approach.

Requirements for J-controlled crack growth are presented in some detail. Restrictions on the amount of crack extension, the slope of the J-resistance curve and the size requirements are examined in the context of unirradiated and irradiated materials. If these requirements are satisfied, then the J-resistance curve is a material property and fracture stability can be treated by an R curve approach where the crack driving forces are formulated in terms of J.

A simple approach for handling stable crack growth and instability is discussed. The procedure exploits the scaling laws associated with linear elastic and fully plastic deformation. Crack driving forces (in terms of J or COD) determined by an explicit estimation scheme are shown for the compact specimen. The driving force contours together with the material J-resistance curve are employed to predict the deformation behavior of the compact specimen; the predicted behavior and the experimental data are in good agreement.

1. Introduction

Ferritic pressure vessel steels like A533B and A508 have a ductile-to-brittle fracture transition temperature which is close to room temperature. Below the transition temperature the brittle fracture mode is usually cleavage. In this regime, life predictions can be made using linear elastic fracture mechanics and plane strain fracture toughness K_{IC}. Above the transition temperature, the fracture mechanism is usually dimpled rupture, and large plastic strains are required for crack initiation and propagation. Raising the temperature above the transition temperature first increases the toughness which is then followed by a region of relatively constant toughness - the upper shelf. In this paper, we examine the viable parameters for characterizing crack growth under conditions of large scale plasticity and which could be employed for crack growth analyses in pressure vessels operating in the upper shelf temperature range. A discussion of the objectives of the fracture investigations and a summary of the key findings are given by Shih and co-workers (1978)*.

2. Assessment of Parameters

An assessment of the viability of the various parameters for crack growth and instability analyses in pressure vessel steels is reported by Shih and co-workers (1977, 1978). The assessments were performed in two separate phases. In the first phase, the behavior of the potential fracture criteria were observed by "forcing" the crack growth in the finite element model to follow the experimentally measured relationship between the load line displacement (LLD) and crack extension. The potential parameters, e.g., J and dJ/da (near the remote contours), crack opening displacement and angle, etc., are computed at every step in the numerical simulation of crack growth. In the second phase of the study the process is reversed; the acceptable or viable parameters are employed to govern crack extension, for e.g., crack initiation occurs at the critical value of J and crack extension is governed by the J resistance (J-R) curve. A summary of the essential and desirable features that constitute a viable fracture criteria and a comparison of how the parameters rank in terms of these features is given in Table 1.

In terms of the first six requirements, all the parameters are about equally good. However, in terms of the last four requirements, the trend is clear. From the computation and experimental measurement point of view, the viable candidates are the J-integral and the crack opening displacement.

3. Relationship Between J-Integral and the Crack Opening Displacement

The extensive numerical modeling and experimental studies strongly suggest that a resistance curve approach based on J or δ is most appropriate for treating problems of continuing crack extension under contained and large scale plasticity conditions and this is no mere coincidence. By exploiting the HRR singularity Shih (1978) showed the following relationship between the crack tip opening displacement and J, namely,

$$\delta_t = d_n \, J/\sigma_o \qquad (1)$$

where $d_n = (\alpha\, \sigma_o/E)^{1/n}\, (\tilde{u}_x + \tilde{u}_y)^{1/n}\, \tilde{\delta}/I_n$. The dimensionless quantities $\tilde{u}_x$, $\tilde{u}_y$, $\tilde{\delta}$ and I_n are determined from the HRR singularity. Values of d_n for a range of hardening exponents n and σ_o/E are shown in Figs. 1 and 2 for plane strain and plane stress situations.

Under somewhat more restrictive conditions, the slope of the COD-resistance curve or the

*See also paper G5/2 in 1979 SMiRT Proceedings.

crack opening angle $d\delta/da$ is related to the slope of the J-resistance (J-R) curve by

$$d\delta/da = [n/(n + 1)](d_n/\sigma_o)dJ/da \tag{2}$$

Expressions (1) and (2) are obtained on the assumptions that the HRR singularity dominates in the crack tip region and that deformation plasticity theory is applicable to small amounts of crack growth. Experimental data giving the values of J at initiation, the J-resistance curve and COD measurements are provided in investigations by Andrews and Shih (1977), Shih and co-workers (1977), Robinson (1976), Garwood (1977), Garwood and Turner (1977), Griffis (1975), Kass and co-workers (1974). Based on these experimental data, namely J_c and dJ/da, the critical value of δ and $d\delta/da$ are predicted using (1) and (2). The predicted values appear to be consistent with the actual experimental measurements. A summary of the results is given in Table 2. The plane strain value of d_n were used in most of the cases examined; in situations where the specimens dimensions do not appear to meet plane strain conditions, the plane stress values of d_n were employed.

4. Requirements for J-Controlled Crack Growth

Just as the stress intensity factor K plays the role of the crack driving force and the K resistance curve K_R is the material resistance to crack extension (ASTM STP 527, 1973) under small scale yielding conditions, the J-integral could have similar roles under large scale yielding conditions if certain requirements are satisfied. The requirements for J-controlled crack growth have been discussed by Hutchinson and Paris (1977) and by Shih and Dean (1979). The J-integral is the amplitude of the so-called Hutchinson-Rice-Rosengren (HRR) singularity and one condition for J-controlled growth is

$$\Delta a << R \tag{3}$$

i.e., the amount of crack extension Δa be much smaller than the characteristic radius R of the region governed by the HRR singularity.

From considerations that the work done in a fixed material volume due to an increase in J dominate that due to an increase in crack length a, another condition for J-controlled growth is

$$\frac{R}{J}\frac{dJ}{da} >> 0.13 \tag{4}$$

i.e., the slope of the material J-R curve be sufficiently large. In the fully plastic state, R is some fraction of the uncracked ligament c, and condition (4) can be restated as

$$\omega = \frac{c}{J}\frac{dJ}{da} >> \frac{0.13}{g_n} \tag{5}$$

where g_n is a numerical constant. An assessment of the dominance of the HRR singularity in a number of crack configurations (McMeeking and Parks, 1978 and Shih and German, 1978) suggest that $\omega >> 2.5$ for configurations with ligaments subjected primarily to bending, and $\omega >> 20$ for configurations with ligaments subjected primarily to tension. Furthermore a sufficient condition for J-controlled growth which follows from (4) and (5) is

$$\psi = \frac{1}{\sigma_o}\frac{dJ}{da} >> 0.1 \tag{6}$$

An additional condition concerning the size of the uncracked ligament and possibly other relevant dimensions appears to be necessary, namely,

$$c \geq M\frac{J}{\sigma_o} \tag{7}$$

where the multiple M is about 25 for compact specimens and about 200 for center-cracked panels.

5. Applications of J-Controlled Growth

Several consequences ensue under the conditions of J-controlled crack growth. The material J-R curve obtained under large scale yielding including fully plastic conditions will be the same as the J-R curve obtained under small scale yielding conditions as long as plane strain or plane stress conditions prevail in both situations; the J-R curve will also be independent of crack configuration. In other words, the J-R curve is a material property.

Secondly, stable crack extension and crack instability under large scale plasticity can be analyzed by the J-resistance curve approach which is a generalization of Irwin's resistance curve approach for small scale yielding based on the elastic stress intensity factor K (ASTM STP 527, 1973). In fact the R curve approach based on J will be applicable to crack analyses for the complete range of elastic and elastic-plastic deformation.

Lastly, the J-integral crack driving force can be determined from analyses using deformation theory of plasticity. The formulas, obtained by Rice, Paris and Merkle (1973) for deeply cracked specimens and the estimation schemes developed by Shih (1976) and Shih and Hutchinson (1976) for more general situations, could be employed to estimate the J driving force without recourse to detailed numerical calculations. A simple engineering procedure for analyzing crack growth and instability using the J-integral is discussed by Shih (1979). In essence, the procedure exploits the functional forms of the fully plastic solutions and linear elastic solutions to obtain simple approximate formulas for quantities such as J, δ and other pertinent parameters; these formulas interpolate over the range from small scale yielding to large scale yielding.

5.1 Estimation Schemes for J-Integral and Other Crack Parameters

In linear elasticity, crack parameters like the J-integral, crack opening displacement δ, and the load line displacement Δ_c (due to crack) can be expressed in the following forms:

$$\frac{J^e}{\sigma_o \varepsilon_o a} = \left(\frac{\sigma^\infty}{\sigma_o}\right)^2 \hat{J}^e(a/b)$$

$$\frac{\delta^e}{\varepsilon_o a} = \left(\frac{\sigma^\infty}{\sigma_o}\right) \hat{\delta}^e(a/b) \tag{8}$$

$$\frac{\Delta_c^e}{\varepsilon_o a} = \left(\frac{\sigma^\infty}{\sigma_o}\right) \hat{\Delta}_c^e(a/b)$$

where σ^∞ is the remotely applied stress, σ_o and ε_o are some reference stress and strain (the connection $\sigma_o = E\,\varepsilon_o$ can always be made, but is not necessary), $\hat{J}^e$, $\hat{\delta}^e$ and $\hat{\Delta}_c^e$ are dimensionless functions of crack length to width ratio a/b and the superscript e denote elastic quantities.

For a fully plastic material where $\varepsilon \propto \sigma^n$, J, δ and other relevant crack parameters have the following forms (Goldman and Hutchinson, 1975; Shih, 1976)

$$\frac{J^P}{\sigma_o \varepsilon_o a} = \left(\frac{\sigma^\infty}{\sigma_o}\right)^{n+1} \hat{J}^P(a/b,\ n)$$

$$\frac{\delta^P}{\varepsilon_o a} = \left(\frac{\sigma^\infty}{\sigma_o}\right)^{n} \hat{\delta}^P(a/b,\ n) \tag{9}$$

$$\frac{\Delta_c^P}{\varepsilon_o a} = \left(\frac{\sigma^\infty}{\sigma_o}\right)^n \hat{\Delta}_c^P(a/b,\ n)$$

The functional forms (9) are similar to those for linear elasticity (8). Consequently, it is feasible to tabulate the fully plastic solutions $\hat{J}^P$, $\hat{\delta}^P$ and $\hat{\Delta}_c^P$ corresponding to specific values of a/b and n for crack configurations similar to those tabulated in elastic crack handbooks. The fully plastic solutions in plane stress are readily obtained from conventional finite element techniques. In plane strain, the incompressible deformation introduces constraint/constraints on the displacement gradients and special techniques are required to handle fully plastic problems. An efficient technique for solving incompressible nonlinear problems is discussed by Needleman and Shih (1978).

By exploiting the functional forms of the fully plastic solutions (9) and linear elastic solutions (8), simple approximate formulas have been obtained for quantities such as J, δ and Δ_c which interpolates over the range from small scale yielding to fully plastic conditions. In essence, the interpolation formulas are of the form

$$\begin{aligned} J^{eP} &= J^e(a_{eff}) + J^P(a,\ n) \\ \delta^{eP} &= \delta^e(a_{eff}) + \delta^P(a,\ n) \\ \Delta_c^{eP} &= \Delta_c^e(a_{eff}) + \delta^P(a,\ n) \end{aligned} \tag{10}$$

where a_{eff} is the Irwin's effective crack length modified to account for strain hardening. Formulas (10) have been found to be in good agreement with elastic-plastic finite element calculations for high and low hardening materials and for several crack configurations and in stability analyses (Shih and co-workers, 1977, 1978, 1979).

Recently such formulas have been employed in the investigation of the stability of crack growth under J-controlled growth conditions (Hutchinson and Paris, 1977). A refinement of the interpolation formulas and analyses of crack growth stability in simple crack configurations flawed structures is discussed by Shih and co-workers (1979).

5.2 Crack Growth and Stability Analyses

The influence of systems compliance and the type of loading on crack growth stability in 304 stainless steel is examined here. From the least square fit of the measured stress-strain curve, $n = 5.42$, $\alpha = 1.69$, $\sigma_o = 30 \times 10^3$ psi. Using the estimation scheme discussed in Section 5.1 in conjunction with the above material properties, the J-integral crack driving force is determined for the short and long CCP (subjected to tension) and SECP (subjected to bending). The crack driving force contours are shown in Figs. 3 through 4. A J-R curve for 316 stainless steel obtained from Tanaka's paper (1978) is superimposed on the contours of the crack driving forces; the solid lines are for load held fixed and the dash-dot lines are for displacement held fixed. The key features are summarized below:

(a) The crack driving force in specimen 1 (short CCP) under displacement controlled situation remain approximately constant with increase in crack length and crack growth is always stable. However the driving force in specimen 2 (long CCP) increases dramatically with crack length. It appears that instability will develop after stable growth of about 0.2 inch.

(b) The crack driving force in specimen 3 (short SECP) under displacement controlled situation falls rapidly with increase in crack length. This suggests a highly stable situation. In specimen 4 (long SECP), the driving force rises initially but again falls after at larger crack length. The latter configuration is marginally stable and the stability of the specimen will be sensitive to the changes in the slope of the material J-R curve with increasing amount of crack growth.

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7. Acknowledgment

This work is supported by the Electric Power Research Institute (EPRI), Palo Alto, California. The author wishes to acknowledge the encouragement of Drs. Ted Marston and Robin Jones of EPRI.

TABLE 1. A Comparison of Some Fracture Parameters

	J-Integral	Crack Opening Displacement	Local Crack Opening Angle	Work Done in a Process Zone	Generalized Energy Release Rate
Measure of Near-Tip Field	Yes	Yes	Yes	Yes	Yes
Constant During Crack Extension	No*	No*	Yes	Yes	Yes
Independent of Specimen Geometry	Mildly Dependent	Mildly Dependent	Mildly Dependent	Insufficient Data	Insufficient Data
Applicability to Instability Analyses	Yes†	Yes	Yes	Yes	Yes
Generalizable to Mixed Mode Fracture	Probable	Uncertain	Uncertain	Probable	Probable
Applicability to 3D Crack Geometries	Yes	Yes	Yes	Yes	Uncertain
Sensitivity to Mesh, Process Zone and Increment Size	No	Mildly	Moderately	Moderately	Yes
Ease of Computation	Simple	Simple	Moderate	Moderate	Difficult
Direct Local Measurement	No	Yes	Yes (Difficult)	No	No
Direct Global Measurements	Yes	No	No	No	No

*J and the nominal COD could be employed in an R curve approach. The slopes of the J and COD R curves are practically constant in the earlier stages of crack growth.

†For limited crack extension.

TABLE 2

Comparison of Predicted and Measured Values of COD and COA

Material	$\frac{\sigma_y}{E}$	n	J_c MJ/m^2	d_n Plane Strain	d_n Plane Stress	Predicted COD $\delta_c = \frac{d_n}{\sigma_o} J_c$ Plane Strain	Predicted COD $\delta_c = \frac{d_n}{\sigma_o} J_c$ Plane Stress	Measured COD mm	$\frac{dJ}{da}$ MJ/m^3	Predicted COA $\frac{d\delta}{da} = \frac{d_n}{\sigma_o}(\frac{n}{n+1})\frac{dJ}{da}$	Measured COA	References
A533-B	.002	13	.16-.27	.553	-	.21-.35	-	.22-.34	213-305	.26-.37	.27-.36	Andrews, Shih, 1977
A533-B	.002	13	.19-.25	.553	-	.25-.33	-	.3	180-270	.22-.33	.3	Andrews, Shih, 1977
En24 Low Alloy Steel (Q+T)	.005	12.5	.058	.61	.84	.034	.046	.057	-	-	-	Robinson, 1976
			.09			.053		-				Garwood, 1977
En32 Mild Steel	.00137	5	.16-.18	.29	.41	.17-.19	.24-.27	.27-.32	-	-	-	Robinson, 1976
	.00142	5	.12	.30	.42	.125	.175	.2	160	.14 (plane strain)		Garwood & Turner, 1977
										.195 (plane stress)		
HY80	.003	9	.16	.507	-	.138	-	.140*	209	.163	.205	Griffis, 1975 (*our estimate)
AP15L × 65 Pipeline Steel	.0025	13	.35	.57	.8	.39	.546	.42	650	.67	.64	Garwood, 1978
Zircaloy	.0034	13	.087	.59	-	.156	-	} .147			.246	Kass, et al., 1974
		9		.514	-	.136						

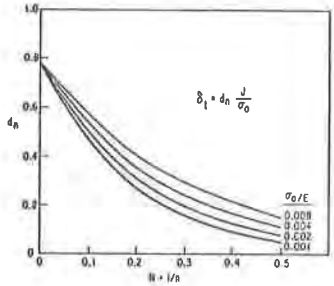


Fig. 1. Variation of d_n with n and σ_o/E for plane strain.

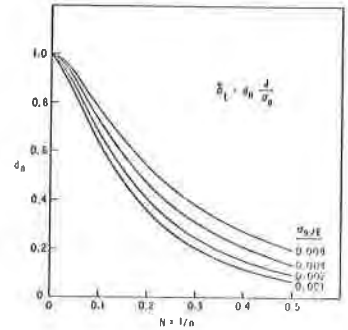
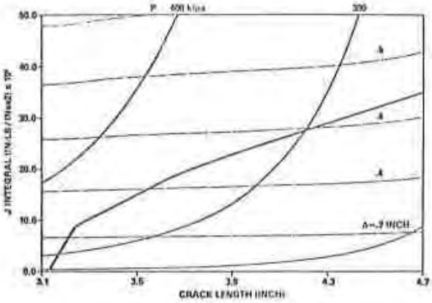
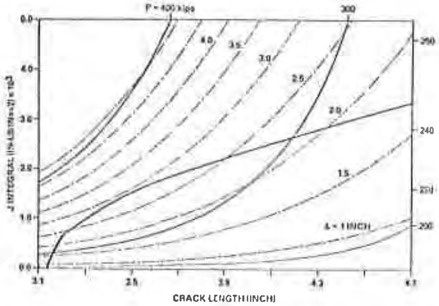


Fig. 2. Variation of d_n with n and σ_o/E for plane stress.

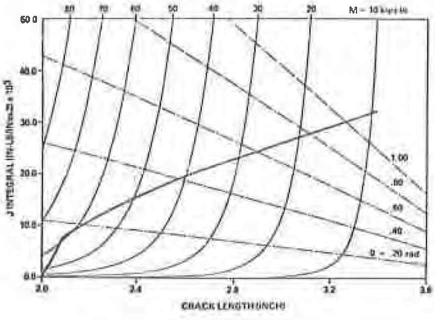


(a)

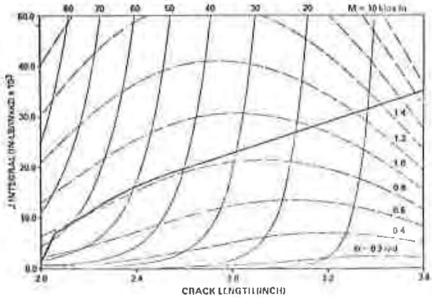


(b)

Fig. 3. J-integral crack driving force for short (h/b = 2) and long (h/b = 100) CCP - A533B steel, b = 6 inches.



(a)



(b)

Fig. 4. J-integral crack driving force for short (h/b = 2) and long (h/b = 100) SECP - A533B steel, b = 4 inches.