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METHODOLOGICAL ISSUES IN THE OVERALL RPV FAILURE PROBABILITY ASSESSMENT

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ABSTRACT

The basic contribution to the failure probability assessment for a component submitted to potentially severe loading conditions is the failure probability during each of these conditions. In the case of a RPV, failures are induced by specific transients (PTS – Pressurized thermal shocks). However, the isolated calculation of each contribution is not sufficient to perform an overall risk assessment. It requires to integrate all these basic terms in a rigorous way.

Some questions arising from this integration are clarified in this paper. Firstly some general probabilistic notions are discussed. They can appear in every structural reliability assessment (not only RPVs). The first issue is the relationship between occurrence frequency and probability of events. No general relationship is available. Particular relationships can result from specific assumptions, which need to be justified like for the use of Poisson processes. The second general issue is the validity and conservatism of the commonly utilized “independence” assumption. It is not valid for global crack failures, but could be examined for failures during one given transient. A partial proof of conservatism is given. Finally, a rigorous evaluation of the overall risk assessment under the independence assumption is proposed. It requires to identify the average number of cracks and the average (or maximum) individual crack failure probability.

Keywords: reactor pressure vessel, structural reliability, probabilistic fracture mechanics, pressurized thermal shock, crack

1. PROBLEM DESCRIPTION

1.1 General physical phenomenon

Reactor Pressure Vessels (RPV) of Pressurized Water Ractors (PWRs) are subjected to cumulative neutron irradiation exposure over their operating life, resulting in increasing embrittlement (i.e. reduced ductility and fracture toughness) of the RPV steel wall zones close to the reactor core, and thus potential cleavage or brittle fracture in these zones. The degree of RPV steel embrittlement is quantified by the shift of the nil-ductility transition temperature RT_{NDT}, that is a function of the chemical composition of the steel, the neutron irradiation exposure and the initial unirradiated transition temperature RT_{NDT} of the ferritic steel constituting the base metal.

In PWRs, transients can occur that result in severe overcooling (thermal shock) of the RPV concurrent with or followed by high repressurization. The pressure and the stress gradients originated from the thermal shock can lead to crack opening for existing flaws located on or near the inner wall surface, all the more since the low temperature during the transient can reach the brittle/ductile transition zone of the steel, where the fracture toughness is significantly reduced.

These transients correspond to particular loading (operating) conditions. These conditions are generally classified as follows (Ardillon, 2003): normal and upset conditions, emergency conditions, faulted conditions.

1.2 Probabilistic analysis

RPV brittle fracture (considered as failure) probability assessments have been performed for many years in various countries. Since the mid-1970s, several probabilistic fracture mechanics computer codes have been developed in the United States and used by the U.S. Nuclear Regulatory Commission (NRC), as recently the FAVOR code (Dickson, 1995). Electricité de France also has been performing such assessments for more than one decade, as mentioned in Persoz (2000), and Turato (2003). They rely on Probabilistic Fracture Mechanics (PFM), and are one of the major applications of structural reliability methods in the nuclear industry.

The basic evaluation is the conditional failure probability for one given transient, flaw, and RPV age. Failure can be understood as cleavage initiation, or involve other mechanisms like the possibility of crack arrest, depending on the accepted criterion. The age is generally the postulated end of life. This evaluation of course relies on a mathematical model of the limit state of the structure submitted to the irradiation damage, including deterministic and random variables, as shown for example in (Persoz, 2000).

However, these basic evaluations have to be aggregated to calculate the global RPV failure annual probability or frequency. This global annual frequency is the aim of the evaluations, as the RPV failure is one major (and direct) core damage initiating event, and a particularly significant input to the calculation of the core damage frequency (CDF) performed in Level 1 Probabilistic Safety Assessments (PSA). This failure frequency is supposed to be given by the general formula :

$$F_{RPV} = \sum_{i=1}^{N_T} f_{T_i} * P(RPV\ \ failure / T_i) \qquad \text{(per reactor.year)} \qquad \text{(Eq.1)}$$

where N_T denotes the total number of thermal transients - generally supposed to be limited to pressurized thermal shocks (PTS) - contributing to the global F_{RPV}, T_i transient n°i and f_{Ti} its annual occurrence frequency, and P(RPV failure / T_i) the conditional RPV failure probability given transient T_i.

Although this general evaluation frame is commonly utilized (Dickson 01), its practical application can lead to approximations that need to be clearly pointed out. The paper provides some recommendations (currently in progress) to deal correctly with the following issues.

1.3 Issues addressed in the paper

First of all, the following aspects related to the (complex) evaluation of the conditional failure probability P(RPV failure / T_i) are out of discussion: assumptions in the French RPV integrity conventional assessments, affecting the failure mode definition (cleavage fracture directly induced by crack initiation considered as RPV failure, involving or not crack arrest, warm prestressing), thermomechanical RPV model, thermal transient description, statistical modelling of input variables, probabilistic numerical methods.

This paper will concentrate on typically probabilistic issues involved in the overall risk assessment and its general formulation (Eq. 1):

- General probabilistic notions affecting the overall risk assessment: relationships between occurrence probability and occurrence frequency, validity and conservatism of the assumption of events independence;
- Treatment of the whole number of flaws located in a component: for a given flaw distribution in the component (location and dimensions), how to calculate the whole probability P(component failure) or P(component failure / T_i)? Note that this section will apply to every kind of flawed component, not only to vessel shells.

2. OVERALL RISK ASSESSMENT: GENERAL PROBABILISTIC NOTIONS

The evaluation of the overall failure risk raises various questions related to probabilistic fundamental notions appearing in different parts of the calculation. It is therefore better to establish general properties. Moreover, they could apply to structural reliability assessments performed for other components than RPVs.

2.1 Distinction and relationship between occurrence probability and frequency of an event

The basic evaluation of (Eq. 1) involves both the occurrence frequency of a transient or of the vessel and the (conditional) probability of failure; the two notions of occurrence frequency and probability are present.

Note that in other reliability evaluations these notions are sometimes supposed to be equivalent; this point has to be discussed.

Moreover, structural reliability assessments generally give the failure probability, and not the failure frequency. So it is important to show the consistency of the RPV evaluation with the other ones.

2.1.1 Definitions

Let us denote N_T the number of occurrences of the event Ev during the period T. N_T is a random variable. The probability of occurrence P_{occ} of event Ev during the period T is then defined as follows:

$$P_{occ}(Ev) = P(N_T \geq 1) \qquad \text{(Eq.2)}$$

Or equivalently:

$$P_{occ}(Ev) = \sum_{i \geq 1} P(N_T = i) = 1 - P(N_T = 0) \qquad \text{(Eq.3)}$$

On the other side, the occurrence frequency f_{occ} of event Ev during the period T is defined as the mean number of occurrences of Ev during the period T, where E denotes the expectation of a random variable:

$$f_{occ}(Ev) = E(N_T) \qquad \text{(Eq.4)}$$

Or equivalently:

$$f_{occ}(Ev) = \sum_{i \geq 0} i.P(N_T = i) = \sum_{i \geq 1} i.P(N_T = i) \qquad \text{(Eq.5)}$$

Sometimes a normalized frequency is defined as

$$f_{occ}(Ev) = \frac{E(N_T)}{T} \qquad \text{(Eq.6)}$$

Both definitions (Eq. 5 and 6) are equivalent for unit periods, which will be the case in this paper. We will then consider the definition given by (Eq. 4).

2.1.2 General case

A first general and immediate result arising from equations 2 and 5 is that $f_{occ}(Ev)$ is always superior to $P_{occ}(Ev)$, and strictly superior as long as $P(N_T \geq 2) > 0$.

2.1.3 Particular events: unique occurrence

In the particular context of RPV failure assessments the number of annual occurrences of certain events is limited to one: one RPV failure would result at least in a complete stop of the reactor considered for a long period, and the occurrence of one transient arising from emergency or faulted conditions would lead to stop the reactor operation till the end of the current operating cycle.

However, it should be noted that the principle that "a RPV cannot fail twice" (NUREG, 2004) is not completely correct and should be precised: for a vessel containing several cracks it cannot be excluded that a given severe PTS results in various consecutive crack failures during its occurrence. But only one PTS can lead to failure.

In the case of unique occurrence events, obviously:

$$P_{occ}(Ev) = f_{occ}(Ev) \qquad \text{(Eq.7)}$$

2.1.4 Rare events (low probability)

It is sometimes assumed that in this case the two quantities are identical:

$$P_{occ}(Ev) \approx f_{occ}(Ev) \approx P(N_T = 1) \qquad \text{(Eq.8)}$$

In fact, no conclusion can be drawn without further assumptions. If the occurrences of the event considered are totally correlated, this equality may be completely wrong. Correlated occurrences can happen for example with common mode failures. For a non technical example think of administrative forms sent to the employees of the same big firm (10^5 employees); a mistake due to human error may occur with relatively low probability (let us say 10^{-2} by month) and impact 10^5 forms at the same time, thus leading to a high frequency of failed documents (10^3).

And it will be shown hereafter that even for independent occurrences, some temporal schemes may contradict this approximation.

2.1.5 Particular case : independence of multiple occurrences

This is an important case, since the independence assumption is commonly used in probabilistic analyses, even if sometimes the required justifications are not provided. This assumption is of course not relevant for single occurrence events (previous case). It will concern basically transients corresponding to normal and upset conditions.

First of all it has to be noticed that for multiple occurrence events a temporal scheme is implicitly considered, opening the way to the use of random processes to modelize the occurrence of such events. The key issue would be to account for the use of one particular stochastic process.

2.1.5.1 1st scheme

In this example no explicit use of stochastic processes is made. The basic idea is that it is possible to divide the total period T into small time intervals for which the occurrence number of event Ev cannot be superior to 1. As aforementioned in this case, probability and frequency are identical on each such interval. Let us simply assume that it is possible to define an elementary time period T_e such that:

- $T = N_{Te} . T_e$, with $N_{Te} >> 1$;
- On each time interval $I_i = [i.T_e; (i+1).T_e]$:
    - the occurrence number of event Ev cannot be superior to 1
    - occurrences on each interval are mutually independent
    - the occurrence probability on each interval is constant (kind of stationnarity)

Thus:

$$P_{occ}(I_i) = f_{occ}(I_i) = f_{occ}(T)/N_{Te} \qquad \text{(Eq.9)}$$

And finally with the independence assumption:

$$P_{occ}(T) = 1 - [1 - \frac{f_{occ}(T)}{N_{Te}}]^{N_{Te}} \qquad \text{(Eq.10)}$$

If $f_{occ}(T)/N_{Te} << 1$ (which can be accepted for most RPV transients having limited annual frequencies), the approximated following result is valid

$$\mathbf{P_{occ}(T) \approx 1 -} e^{-f_{occ}(T)} \qquad \text{(Eq.11)}$$

Note that in this case

- certain events are excluded, since there is a remaining probability that event Ev does not occur during time T;
- for low frequencies or probabilities, probability and frequency are almost identical.

It has to be noted that in (Eq. 10) the occurrence probability is a function of N_{Te} ; so for a “same” event Ev different probabilities can be found, and this can seem paradoxical. However there is no real contradiction since for different N_{Te} values the base events considered are not identical, and the representation of randomness is different. But the dependency to N_{Te} is in fact very low, and (Eq.11) can be supposed to give “the” value of the probability.

The most questionable point of this scheme is the hypothesis of independence of occurrences from one interval I_i to the next one: for little intervals there should be a correlation. However, one can observe that (Eq.11) does not need such little intervals to be valid, only the condition “$f_{occ}(T)/N_{Te} << 1$”.

2.1.5.2 2nd scheme: Poisson process

Now explicit use of random processes is performed. The Poisson process (Bouleau 1988) is one of the most commonly used random processes (queuing theory, road traffic modeling …) due to its interesting properties. It is adapted to phenomena without ageing, like failures of particular components (electronics).

In addition to the assumption of mutually independent occurrences, some assumptions have to be adopted:

- the occurrences take place during the time interval [0 ; T];
- no simultaneous occurrences;
- the distribution of the duration d between two consecutive occurrences does not depend on the instant of the first occurrence (one definition of “no ageing”);
- duration d between two consecutive occurrences: mutual independence;
- C: average number of occurrences during a unit period.

This is the frame of the Poisson process, and classically the following results can be shown (Bouleau 1988):

- the distribution of the duration d between two consecutive occurrences is an exponential distribution;

- the occurrence frequency of the underlying event Ev is equal to C.T;
- the distribution of the number of occurrences N_T is given by the following expression:

$$P(N_T = n) = e^{-C.T} . \frac{(C.T)^n}{n!} \quad \text{(Eq.12)}$$

As a consequence the probability of occurrence of event Ev can be expressed as follows:

$$P_{occ}(Ev) = P(N_T \geq 1) = 1 - e^{-C.T} \quad \text{(Eq.13)}$$

And thus:

$$P_{occ}(Ev) = 1 - e^{-F_{occ}(Ev)} \quad \text{(Eq.14)}$$

It can be seen that the Poisson scheme gives the same relationship as the first scheme considered above (Eq.11).

As in the first case, for low occurrence frequencies or probabilities, (Eq. 14) leads to the following approximation:

$$P_{occ}(Ev) \approx F_{occ}(Ev) \quad \text{(Eq.15)}$$

Regarding the Poisson scheme validity, some remarks can be done. Firstly, as a consequence of the "no ageing" assumption, the duration between two consecutive occurrences has to be unlimited (Bouleau 1988). And this is not the case for nuclear facilities: this duration is necessarily limited by the plant lifetime. For rare events (on the plant lifetime), the restriction is particularly meaningful, since the Poisson scheme would lead to the conclusion that event Ev will happen mostly after the plant lifetime T, when its occurrence is impossible! Therefore, one could conclude that the validity of Poisson process has to be restricted to normal and upset conditions, to initiating events for which the frequency is high enough for the exponential distribution tail to be neglected; however, the consideration of Poisson non homogeneous processes makes it possible to use a process corresponding to Poisson homogeneous process before the end of life, and after to a process with C=0. It can be shown that the previous formulas (Eq. 14) are still valid before the end of life.

Anyway, a better modelling for rare events would imply an appropriate evaluation of the distribution of the duration between two consecutive occurrences, which has to be bounded (upper bound inferior to the anticipated plant lifetime). In this case, it is impossible to exhibit a general relationship between occurrence frequency and probability; if the durations between consecutive occurrences are not identically distributed, it is even possible to have significant difference between the two quantities.

2.1.6 Partial conclusion

There is no general relationship between occurrence probability and frequency of events, except from the inequality mentioned at §2.1.2. Particular relationships can result from specific assumptions. Firstly for unique occurrence events (like transients corresponding to emergency or faulted conditions), the two quantities are identical. Secondly under the assumption of occurrence independence, a precise relationship can be derived for two particular schemes, including the Poisson process; for rare events, the two quantities are approximately identical in this case. No general relationship can be derived under the assumption of occurrence independence; it would require a precise modeling of the duration between two consecutive occurrences.

2.2 Conservatism of independence assumption

In many cases, the failure of a component (like a RPV) can result from various phenomena, and therefore be considered as the union of various basic events. For instance, the failure of a RPV containing various cracks can be considered as the union of the individual crack failures. For practical reasons, the basic events will be often assumed to be independent; however, in some cases this independence assumption may be irrelevant and correlations between the basic events can be exhibited. This section is concerned with the particular case of positive correlation of events, for which an appropriate definition is provided, and gives a partial proof of the conservatism of the independence assumption used instead of the positive correlation of the basic events: the failure probability derived from the independence assumption is higher than the failure probability under positive correlation of the basic events.

2.2.1 Definition of positive correlation – case of two events

It can be easily imagined that if two events A and B are "positively correlated", the probability of their union, P(AUB), is lower than if they were independent: if A happens, B will happen more often if it is "positively correlated" with A, and therefore the simultaneous occurrence of A and B will be more frequent:

$$P(A \cap B)_{\text{positive correlation}} \geq P(A \cap B)_{\text{independence}} = P(A).P(B) \quad \text{(Eq.16)}$$

Or equivalently:

$$P(AUB)_{\text{positive correlation}} \leq P(AUB)_{\text{independence}} \quad \text{(Eq.17)}$$

The equivalence of the last two definitions is due to the well-known formula:

$$P(AUB) = P(A) + P(B) - P(A \cap B) \quad \text{(Eq.18)}$$

It is therefore possible to define the notion of positive correlation of two events by (Eq.16) or (Eq. 17).

Moreover, in the case of two events, the conservatism of the independence assumption is a direct consequence of these definitions.

Note also that the previous definitions are equivalent to the following definition:

$$P(A/B)_{\text{positive correlation}} \geq P(A/B)_{\text{independence}} = P(A) \quad \text{(Eq.19)}$$

However, the common definition of correlation applies to random variables, not to the events. But this point can be easily solved by considering the indicator functions of the events, denoted as 1_A for event A. This function is a random variable, equal to 1 if A happens and to 0 otherwise.

Now let us prove that definitions given by (Eq. 16) or (Eq. 17) are equivalent to the usual definition of positive correlation of variables 1_A and 1_B: the Pearson linear correlation coefficient is superior to 0. Pearson correlation coefficient is superior to 0 is equivalent to the following condition:

$$E[(1_A - E(1_A)).(1_B - E(1_B))] \geq 0 \quad \text{(Eq.20)}$$

And (Eq. 20) is equivalent to:

$$E[1_A . 1_B] \geq E(1_A).E(1_B) = P(A).P(B) \quad \text{(Eq.21)}$$

That is to say:

$$P(A \cap B) \geq P(A).P(B) \quad \text{(Eq.22)}$$

Therefore definition given by (Eq. 16) and the usual definition involving the Pearson correlation coefficient are equivalent.

2.2.2 Extension to an arbitrary number of events

The question is to prove the following result:

$$P(\bigcap_{i=1}^{n} A_i)_{\text{positive correlation}} \geq \prod_{i=1}^{n} P(A_i) \quad \text{(Eq.23)}$$

This extension of the previous result for an arbitrary number of events is not immediate. It is the topic of ongoing research and cannot be developed in this paper. However it is likely that this extension is valid. The main issue is the extension to three events. The extension to an arbitrary number of events would follow automatically by iteration. An issue is to determine if it sufficient to consider only the positive correlation of couple of events, or if other hypotheses are necessary like in the case of independence, for which the independence of all the couples of events does not automatically imply the mutual independence of the events.

3. TREATMENT OF MULTIPLE FLAWS IN THE PROBABILISTIC ASSESSMENT

This section is generic and the proposed treatment could apply to other components than the vessel.

The basic RPV failure probability (or frequency) assessment has been historically carried out for one single flaw. Then, plant operators have experienced the possibility for one given vessel to contain various defects. Currently the probabilistic approach developed for US RPVs also takes account of the possibility to have a random number of flaws on a vessel (Dickson, 2003). Two cases will be considered in this paper: the first case of a population of given defects, and the second case of a random population of defects.

The starting point of the treatment performed in this section is the “independence” assumption of flaws, as in other frames like for example the FAVOR code methodology for the RPV, also accepted in NUREG documents, which provides a well-known formula for the 1st case of a given population of cracks.

Firstly the “independence” assumption of flaws is discussed in the RPV case and shown to be inadequate but conservative. Then the probabilistic treatment under this assumption is extended to the case of a random population.

3.1 Independence of cracks: what meaning?

Let us firstly mention that the relevant issue is the probabilistic independence, not the mechanical independence. Mechanical independence expresses the fact that the two cracks will not interact, referring to minimal distance between them.

To deal with probabilistic independence, it has to be reminded that in the probability theory the independence only applies to events or random variables (which is equivalent, since an event can be represented by its characteristic function), not to things. In many references this point is neglected (Fang, 2003). So the question is: to what kind of events related to the flaws can be subjected to independence in the probability failure assessment? Some of them are listed below:

- flaw occurrence during the manufacturing process: note that this event has not to be considered when the population of cracks is supposed to be known (given cracks or postulated distribution of cracks); it is relevant only for estimating the distribution of RPV cracks due to the action of physical mechanisms leading to crack occurrence; in this case, the action (local or not) of these mechanisms should be questioned; it is well known that many parameters have an impact on the quality of the welding process, but they may be hard to retrieve;
- failure: this is in fact the relevant question, however it may be linked with other parameters; let us firstly remind that crack failures are PTS-induced, so they cannot be supposed to be uncorrelated, even if PTS don't impact the whole vessel shell the same way; more precisely, the failure of each flaw will be strongly dependent on the occurrence of the most critical transient. It is possible to give a formal simplified proof of this statement: let us assume that only 1 PTS denoted as T can cause failure; then it can be stated that

$$P(fail(a_1)) = P_f(a_1/T).P_{occ}(T) \quad \text{(Eq.24)}$$

$$P(fail(a_2)) = P_f(a_2/T).P_{occ}(T) \quad \text{(Eq.25)}$$

Where $P(fail(a_i))$ denotes the failure probability of crack n°i, $P_f(a_i/T)$ denotes the failure probability of crack n°i during transient T, and $P_{occ}(T)$ the occurrence probability of transient T. Note that this formula is valid only under certain assumptions that are not detailed in this paper.

In the same way:

$$P(fail(a_1)\cap fail(a_2)) = P(fail(a_1)\cap fail(a_2)/T).P_{occ}(T) \quad \text{(Eq.26)}$$

So the independence assumption would involve that:

$$P(fail(a_1)\cap fail(a_2)/T) = P_f(a_1/T).\ P_f(a_2/T).P_{occ}(T) \quad \text{(Eq.27)}$$

This result leads to the fact that global independence of failure of flaws is clearly not compatible with conditional independence of flaws during one transient, which would yield:

$$P(fail(a_1)\cap fail(a_2)/T) = P_f(a_1/T).\ P_f(a_2/T) \quad \text{(Eq.28)}$$

Moreover, (Eq.27) would imply that common failure is very unlikely, especially for rare transients, and this is not acceptable, especially for flaws having close characteristics (close location implying close properties like fracture toughness, irradiation…).

So the assumption of global failure independence is not acceptable.

Can conditional failure independence assumption be accepted? For one given transient, may the behavior of two cracks regarding failure during the transient be considered as independent? The answer to this point is probably no, but is not clear up to now. It would involve metallurgical considerations such as the possibility to correlate the fracture toughness from one point to an other on the same vessel shell, and the same for irradiation and all random properties. Will "bad" shells tend to have a poor fracture toughness everywhere? The answer is clearly not simple and automatic, due to the possibility of specific local microstructural behaviors (e.g. presence of ghost lines, segregation). Anyway, this issue cannot be answered only by probabilistic analysts.

- Flaw characteristics (dimensions): this point also cannot automatically be answered to, it requires physical considerations.

Note also that non failures could be considered to be almost independent, due to the fact that failure probabilities are very low. With previous notations:

$$P(non\text{-}fail(a_1)) \approx P(non\text{-}fail(a_2)) \approx 1 \quad \text{(Eq.29)}$$

And therefore

$$P(non\text{-}fail(a_1)\cap non\text{-}fail(a_2)) \approx 1 \quad \text{(Eq.30)}$$

Leading to

$$P(non\text{-}fail(a_1)\cap non\text{-}fail(a_2)) \approx P(non\text{-}fail(a_1))\ .\ P(non\text{-}fail(a_2)) \quad \text{(Eq.31)}$$

That characterizes independence.

Non failure being the "normal" state, its occurrence provides poor information; in a more formal way, think for example of information theory, stating that the occurrence of event Ev will provide a quantity of information equal to –Log(P(Ev)). Therefore other events may be considered as independent of it. However, one could object that for such almost certain events the notion of independence itself is not really meaningful: the event C (certain), defined for example as [failure or non-failure], can be considered as independent of itself, and can also be considered as totally correlated with itself! Anyway, the independence of non-failures cannot help to provide a good estimation of failure probabilities of two cracks.

3.2 General failure probability bounds

Let us mention that without any specific assumption like independence, it is possible to find bounds for the failure probability of a vessel containing N cracks. Let us denote a_i flaw n°i, $P(a_i)$ the failure probability of flaw n°i. So:

$$\mathrm{Max}(P(a_i)) \le P_f(\text{vessel}) = P\Big(\bigcup_{i=1}^{N} fail(a_i)\Big) \le \min\Big[\sum_{i=1}^{N} P(a_i),\ 1\Big] \qquad \text{(Eq.32)}$$

This formula can be improved by Ditlevsen bounds (Ditlevsen, 1979); however these bounds are generally not easy to estimate.

3.3 Case of a given population of flaws

This case is particularly easy to deal with under the global failure independence assumption. Thus, with similar notations as above:

$$P_f(\text{vessel}) = 1 - \prod_{i=1}^{N}\big(1 - P\ (a_i)\big) \qquad \text{(Eq.33)}$$

In this case it is easy to retrieve the upper bound of (Eq.32).

However, sometimes wrong formulas are provided even in this simple case (Fang, 2003).

3.4 First attempt for a random population

In this part cracks are supposed to have the same individual failure probability P(a). If N is the (random) number of cracks, the following expression is valid (under global independence assumption):

$$P_f(\text{vessel}) = 1 - \sum_{n=0}^{+\infty} P(N=n).\big(1 - P(a)\ \big)^n \qquad \text{(Eq.34)}$$

It would be seducing to assume that generally, n.P(a) << 1, and therefore

$$P_f(\text{vessel}) \approx E(N)\ .\ P(a) \qquad \text{(Eq.35)}$$

Where E(N) denotes the mean number of flaws.

However, the assumption “n.P(a) << 1” is not acceptable for all n values, unless the number of flaws is bounded by N_{max} and N_{max} . P(a) << 1. Moreover, the assumption of an identical individual failure probability is generally wrong. Therefore other methods have to be looked for.

3.5 Random population: proposed treatment

The following method relies on the following (not restrictive) assumptions:

- The number of flaws is bounded, limited to N_{max}; however this number can be as high as requested;
- It is possible to make a mesh description of the component (here a vessel shell), each element of the mesh containing one crack at most; the number of mesh cells is equal to N_V (so N_V has to be superior to N_{max});
- Each crack is conventionally affected to one unique element of the mesh (e.g. the element containing the left peak of the crack, or any other procedure);
- The cracks are uniformly distributed in the component (vessel shell); the probability of each mesh cell containing a crack is equal to x; however, the size of the cracks can follow any type of distribution.

With these assumptions, let us denote:

$p_f(a,i)$: failure probability of a crack of size a located in cell n°i

$p_i(a).da$: conditional probability for cell n°i containing a crack that the crack size is equal to a.

Note that the failure probability is not supposed to be constant in the component (vessel shell), even for a given crack size.

Now it can be shown easily that the probability $P_f(i)$ of cell n°i to lead to failure is equal to:

$$P_f(i) = x\ .\ \int_{a_{min}}^{a_{max}} p_f(a,i)\,.\,p_i(a).da \qquad \text{(Eq.36)}$$

Moreover, the failure probability of the component (vessel shell) is equal to the probability that at least one mesh cell leads to failure. Note that the events denoted as “failure of cell n°i” are mutually independent, since the crack failures are supposed to be independent and the number of cracks per cell is limited to 1. Therefore:

$$P_f(\text{vessel}) = 1 - \prod_{i=1}^{N_V}\big(1 - P_f\,(i)\big) \qquad \text{(Eq.37)}$$

It has to be noted that with the assumptions made in this section, and the notations of section 3.4,

$$x = \frac{E(N)}{N_V} \qquad \text{(Eq.38)}$$

Thus, combining the three last equations:

$$P_f(\text{vessel}) = 1 - \prod_{i=1}^{N_V}\left(1 - \frac{E(N)}{N_V}.\int_{a_{\min}}^{a_{\max}} p_f(a,i).p_i(a).da\right) \qquad \text{(Eq.39)}$$

As N_V can be extremely high, the vessel failure probability can then be approximated by:

$$P_f(\text{vessel}) = 1 - e^{-\frac{E(N)}{N_V}.(\sum_{i=1}^{N_V}\int_{a_{\min}}^{a_{\max}} p_f(a,i).p_i(a).da)} \qquad \text{(Eq.40)}$$

Note that the probability depends on the initial mesh selection, and this can seem paradoxical. However, as explained in paragraph 2.1.5, there is no real contradiction since for different N_V values the base events considered are not identical, and the representation of randomness is different. But the dependency to N_V is in fact very low, and (Eq.40) can be supposed to give "the" value of the probability.

It can be seen that in fact the term "$\frac{1}{N_V}.(\sum_{i=1}^{N_V}\int_{a_{\min}}^{a_{\max}} p_f(a,i).p_i(a).da)$ " expresses the mean failure probability over the mesh, denoted as $\overline{P}_a$, this mean is calculated by taking account of the variability due to flaw size and other factors submitted to spatial variability, having an impact on the failure probability. Thus:

$$P_f(\text{vessel}) = 1 - e^{-E(N).\overline{P}_a} \qquad \text{(Eq.41)}$$

If the failure probability is constant (does not depend on flaw size or location in the component), $\overline{P}_a$ is equal to this constant value.
If a maximum value of the probability, P_{max}, can be identified, then:

$$P_f(\text{vessel}) \le 1 - e^{-E(N).P_{\max}} \qquad \text{(Eq.42)}$$

4. CONCLUSIONS

The basic contribution to the failure probability assessment for a component submitted to potentially severe loading conditions is the failure probability during each of these conditions. In the case of a RPV, failures are induced by specific transients (PTS – Pressurized thermal shocks). However, the isolated calculation of each contribution is not sufficient to perform an overall risk assessment. It requires to integrate all these basic terms in a rigorous way.

Some questions arising from this integration have been clarified in this paper. Firstly some general probabilistic notions have been discussed. They are not specific to the RPV failure probability assessment, but can appear in every structural reliability assessment.

The first issue is the relationship between occurrence frequency and probability. However, there is no general relationship between occurrence probability and frequency of events. Particular relationships can result from specific assumptions. Firstly for unique occurrence events (like transients corresponding to emergency or faulted conditions), the two quantities are identical. Secondly under the assumption of occurrence independence, a precise relationship can be derived for two particular schemes, including the Poisson process; for rare events, the two quantities are approximately identical in this case. No general relationship can be derived under the assumption of occurrence independence; it would require a precise modeling of the duration between two consecutive occurrences.

The second general issue is the validity and conservatism of the "independence" assumption, commonly employed for practical reasons. The validity for cracks (in fact crack failures) is discussed: the independence assumption is irrelevant for crack failures themselves, but could be examined for crack failures during a given transient. Anyway, even in this case, its justification would require physical, microstructural considerations specific for each component considered, that cannot be addressed in this paper. As for the conservatism, a partial proof of conservatism is given for the case of events that are positively correlated. This should apply to crack failures.

Finally, the paper proposes a complete evaluation of the overall risk assessment under the (convenient) independence assumption. The rigorous evaluation requires to identify the mean number of cracks (that is perfectly known if all cracks have been identified), and the average (or maximum) individual crack failure probability.

To perform a completely rigorous appraisal of the current overall risk assessment, other issues should be clarified, such as the assumptions justifying the acceptance of the general formula, used for example in the FAVOR code. This issue is currently examined at EDF, jointly with the possibility to derive an alternative evaluation.

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