Elasto/Visco-Plastic Dynamic Response of Multi-Layered Shells of Revolution

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1. INTRODUCTION

Many investigations of the elasto/visco-plastic dynamic response of shells have been conducted (Nagarajan and Popov, 1975; Takezono, Tao, 1977, 1980, 1982b, 1986, 1988, 1989; Atkatsh et al., 1983; Murase and Nishimura, 1979). These investigations, however, have been mostly concerned with the case of single-layered shells, and few studies on multi-layered shells have been reported in spite of their importance in engineering.

In this paper, the authors study the elasto/visco-plastic dynamic response of the multi-layered shells of revolution subjected to impulsive loads. The equations of motion and the relations between the strains and displacements are derived by extending Sanders' theory for elastic thin shells (1959). As the constitutive relation, Hooke's law is used in the linear elastic range, and the elasto/visco-plastic equations by Perzyna (1966) are employed in the plastic range. The criterion for yielding used in the analysis is the von Mises yield theory. In the numerical analysis of the fundamental equations for incremental values an usual finite difference form is employed for the spatial derivatives and the inertia terms are treated with the backward difference formula proposed by Houbolt (1950). The solutions are obtained by summation of the incremental values.

As a numerical example, the elasto/visco-plastic dynamic response of a fixed supported two-layered cylindrical shell composed of mild steel and titanium subjected to impulsive load is analyzed. Numerical computations are carried out for three cases of the ratio of the thickness of the titanium layer to the shell thickness.

2. ANALYTICAL FORMULATIONS

2.1 Fundamental equations

If the middle surface of axisymmetrical shells is given by $r=r(\xi)$, where r is the distance from the axis and ξ is the non-dimensional meridional distance measured from a boundary along the middle surface, as shown in Fig.1, the relations among the non-dimensional curvatures $\omega_\xi(=a/R_s)$, $\omega_\theta(=a/R_\theta)$ and the non-dimensional radius $\rho(=r/a)$ become:

$$\omega_\xi=-(\gamma'+\gamma^2)\omega_\theta,\ \omega_\theta=\sqrt{1-(\rho')^2}/\rho,\quad \omega_\theta'=\gamma(\omega_\xi-\omega_\theta),\ \rho''/\rho=-\omega_\xi\omega_\theta,\quad \gamma=\rho'/\rho,\ \xi=s/a,\ (\ )'=d(\ )/d\xi \qquad \cdots\cdots(1)$$

where a is the reference length. An arbitrary point in the shell can be expressed by the orthogonal coordinate system (ξ, θ, ζ).

Adding the inertia terms to the equilibrium equations in Sanders' theory (1959) for thin shells and eliminating the transverse shear forces Q_ξ and Q_θ from these, where the rotatory inertia terms are omitted, the following equations of motion are obtained:

$$
\left.
\begin{aligned}
&a\left[\frac{\partial}{\partial\xi}(\rho\Delta N_\xi)+\frac{\partial}{\partial\theta}(\Delta\bar{N}_{\xi\theta})-\rho'\Delta N_\theta\right]+\omega_\xi\left[\frac{\partial}{\partial\xi}(\rho\Delta M_\xi)+\frac{\partial}{\partial\theta}(\Delta\bar{M}_{\xi\theta})-\rho'\Delta M_\theta\right]\\
&\quad+\frac{1}{2}(\omega_\xi-\omega_\theta)\frac{\partial}{\partial\theta}(\Delta\bar{M}_{\xi\theta})+\rho a^2\left[\Delta P_\xi-\rho_0 h\frac{\partial^2}{\partial t^2}(\Delta U_\xi)\right]=0\\
&a\left[\frac{\partial}{\partial\theta}(\Delta N_\theta)+\frac{\partial}{\partial\xi}(\rho\Delta\bar{N}_{\xi\theta})+\rho'\Delta\bar{N}_{\xi\theta}\right]+\omega_\theta\left[\frac{\partial}{\partial\theta}(\Delta M_\theta)+\frac{\partial}{\partial\xi}(\rho\Delta\bar{M}_{\xi\theta})+\rho'\Delta\bar{M}_{\xi\theta}\right]\\
&\quad+\frac{1}{2}\rho\frac{\partial}{\partial\xi}[(\omega_\theta-\omega_\xi)\Delta\bar{M}_{\xi\theta}]+\rho a^2\left[\Delta P_\theta-\rho_0 h\frac{\partial^2}{\partial t^2}(\Delta U_\theta)\right]=0\\
&\frac{\partial}{\partial\xi}\left[\frac{\partial}{\partial\xi}(\rho\Delta M_\xi)+\frac{\partial}{\partial\theta}(\Delta\bar{M}_{\xi\theta})-\rho'\Delta M_\theta\right]+\frac{1}{\rho}\frac{\partial}{\partial\theta}\left[\frac{\partial}{\partial\theta}(\Delta M_\theta)+\frac{\partial}{\partial\xi}(\rho\Delta\bar{M}_{\xi\theta})\right.\\
&\quad\left.+\rho'\Delta\bar{M}_{\xi\theta}\right]-a\rho(\omega_\xi\Delta N_\xi+\omega_\theta\Delta N_\theta)+\rho a^2\left[\Delta P_\zeta-\rho_0 h\frac{\partial^2}{\partial t^2}(\Delta W)\right]=0
\end{aligned}
\right\}\cdots(2)
$$

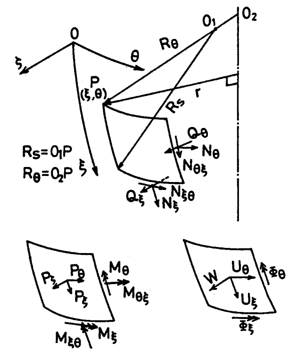

Fig.1 Coordinates and notations

where Δ refers to the incremental value, the notations h , t and ρ_0 in the inertia terms are thickness of the shell, time and mean mass density through thickness, respectively, and $\bar{N}_{\xi\theta}$ and $\bar{M}_{\xi\theta}$ are modified stress resultant and modified stress couple as follows:

$$\Delta\bar{N}_{\xi\theta}=(\Delta N_{\xi\theta}+\Delta N_{\theta\xi})/2+[(1/R_\theta)-(1/R_s)](\Delta M_{\xi\theta}-\Delta M_{\theta\xi})/4\ ,\qquad \Delta\bar{M}_{\xi\theta}=(\Delta M_{\xi\theta}+\Delta M_{\theta\xi})/2 \qquad\cdots\cdots\cdots\cdots(3)$$

The others are shown in Fig.1.

On the boundary, effective membrane force $\hat{N}_{\xi\theta}$ and effective transverse shear force $\hat{Q}_\xi$ per unit length are defined as follows:

$$\Delta\hat{N}_{\xi\theta}=\Delta\bar{N}_{\xi\theta}+\frac{1}{2}\left(\frac{3}{R_\theta}-\frac{1}{R_s}\right)\Delta\bar{M}_{\xi\theta}\ ,\qquad \Delta\hat{Q}_\xi=\frac{1}{a\rho}\left[\frac{\partial}{\partial\xi}(\rho\Delta M_\xi)+2\frac{\partial}{\partial\theta}(\Delta\bar{M}_{\xi\theta})-\rho'\Delta M_\theta\right] \qquad\cdots\cdots\cdots\cdots(4)$$

The strains of the middle surface are given by:

$$\Delta\varepsilon_{\xi m}=\frac{1}{a}\left[\frac{\partial}{\partial\xi}(\Delta U_\xi)+\omega_\xi\Delta W\right],\quad \Delta\varepsilon_{\theta m}=\frac{1}{a}\left[\frac{1}{\rho}\frac{\partial}{\partial\theta}(\Delta U_\theta)+\gamma\Delta U_\xi+\omega_\theta\Delta W\right],\quad \Delta\varepsilon_{\xi\theta m}=\frac{1}{2a}\Big[\frac{1}{\rho}\frac{\partial}{\partial\theta}(\Delta U_\xi)+\frac{\partial}{\partial\xi}(\Delta U_\theta)-\gamma\Delta U_\theta \quad\cdots\cdots(5)$$

where $\varepsilon_{\xi\theta m}$ is half the usual engineering shear strain.

The relations between the bending distortions χ_ξ , χ_θ , $\chi_{\xi\theta}$ and the displacements are:

$$
\left.
\begin{aligned}
&\Delta\chi_\xi=\frac{1}{a}\frac{\partial}{\partial\xi}(\Delta\Phi_\xi),\qquad \Delta\chi_\theta=\frac{1}{a}\left[\frac{1}{\rho}\frac{\partial}{\partial\theta}(\Delta\Phi_\theta)+\gamma(\Delta\Phi_\xi)\right]\\
&\Delta\chi_{\xi\theta}=\frac{1}{2a}\left[\frac{1}{\rho}\frac{\partial}{\partial\theta}(\Delta\Phi_\xi)+\frac{\partial}{\partial\xi}(\Delta\Phi_\theta)-\gamma(\Delta\Phi_\theta)+\frac{1}{2a}(\omega_\xi-\omega_\theta)\left\{\frac{1}{\rho}\frac{\partial}{\partial\theta}(\Delta U_\xi)-\frac{\partial}{\partial\xi}(\Delta U_\theta)-\gamma\Delta U_\theta\right\}\right]
\end{aligned}
\right\}\cdots\cdots\cdots\cdots(6)
$$

where

$$\Delta\Phi_\xi=\frac{1}{a}\left[-\frac{\partial}{\partial\xi}(\Delta W)+\omega_\xi\Delta U_\xi\right],\qquad \Delta\Phi_\theta=\frac{1}{a}\left[-\frac{1}{\rho}\frac{\partial}{\partial\theta}(\Delta W)+\omega_\theta\Delta U_\theta\right] \qquad\cdots\cdots\cdots(7)$$

Under the Kirchhoff-Love hypothesis and the neglect of terms of oder ζ/R_s and ζ/R_θ relative to unity, the strains at the distance ζ from the middle surface are:

$$\{\Delta\varepsilon\}=\{\Delta\varepsilon_m\}+\zeta\{\Delta\chi\} \qquad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(8)$$

where

$$\{\Delta\varepsilon\}^T=\{\Delta\varepsilon_\xi,\Delta\varepsilon_\theta,\Delta\varepsilon_{\xi\theta}\}\ ,\quad \{\Delta\varepsilon_m\}^T=\{\Delta\varepsilon_{\xi m},\Delta\varepsilon_{\theta m},\Delta\varepsilon_{\xi\theta m}\},\quad \{\Delta\chi\}^T=\{\Delta\chi_\xi,\Delta\chi_\theta,\Delta\chi_{\xi\theta}\} \qquad\cdots\cdots\cdots\cdots\cdots(9)$$

and $\{\ \}^T$ represents the transposed matrix.

Now, we shall use the elasto/visco-plastic equations by Perzyna for the constitutive relations

$$\dot{\varepsilon}_{ij}=\frac{1+\nu}{E}\dot{S}_{ij}+\frac{1-2\nu}{E}\dot{S}\delta_{ij}+\gamma_0\langle\Phi(F)\rangle S_{ij}J_2^{-1/2} \qquad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(10)$$

where the dot denotes partial differentiation with respect to time; ε_{ij} , S , S_{ij} and J_2 are strain, mean stress, deviatoric stress and the second invariant, respectively; and E , ν and γ_0 are Young's modulus, Poisson's ratio and the viscosity constant of the material. The symbol $<\Psi(F)>$ is defined as follows:

$$\langle\Phi(F)\rangle=0\ :\ F\leq 0\ ,\qquad \langle\Phi(F)\rangle=\Phi(F)\ :\ F>0 \qquad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(11)$$

where function F is:

$$F=(\bar{\sigma}-\sigma^*)/\sigma^* \qquad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(12)$$

and $F=0$ denotes the von Mises yield surface, $\bar{\sigma}$ is the equivalent stress $(=\sqrt{3J_2}\ )$

and σ^* is the statical stress determined from the elasto-plastic stress-strain relation in a usual tension test, and becomes function of the equivalent plastic strains $\bar{\varepsilon}^{vp}$ in general.

In the plane stress state, as usually assumed in ordinary shell theories, the constitutive relation (10) may be expressed in incremental form as follows:

$$\{\Delta\varepsilon\}=[D]^{-1}\{\Delta\sigma\}+\{\Delta\varepsilon^{vp}\} \qquad \cdots\cdots(13)$$

where

$$\{\Delta\sigma\}=\{\Delta\sigma_\xi,\ \Delta\sigma_\theta,\ \Delta\sigma_{\xi\theta}\}^T \qquad [D]=\frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & 1-\nu\end{bmatrix}$$
$$\{\Delta\varepsilon^{vp}\}=\{\Delta\varepsilon_\xi^{vp},\ \Delta\varepsilon_\theta^{vp},\ \Delta\varepsilon_{\xi\theta}^{vp}\}^T \qquad \cdots\cdots(14)$$

$$\{\Delta\varepsilon^{vp}\}=\{\dot{\varepsilon}^{vp}\}\Delta t=\gamma_1<\Phi\left(\frac{\bar{\sigma}-\sigma^*}{\sigma^*}\right)>\frac{1}{\bar{\sigma}}\begin{bmatrix}1 & -1/2 & 0\\ -1/2 & 1 & 0\\ 0 & 0 & 3/2\end{bmatrix}\{\sigma\}\Delta t, \qquad \gamma_1=(2/\sqrt{3})\gamma_0 \qquad \cdots\cdots(15)$$

Substituting Eqs.(8) into Eqs.(13) and solving them for stress increments, the stress increments are given:

(16) $\{\Delta\sigma\}=[D](\{\Delta\varepsilon_m\}+\zeta\{\Delta\chi\})-\{\Delta\sigma^{vp}\}$ $\cdots\cdots(16)$

where

(17) $\{\Delta\sigma^{vp}\}=[D]\{\Delta\varepsilon^{vp}\}$ $\cdots\cdots(17)$

From Eqs.(16), the increments of the resultant forces and the resultant moments for the multi-layered shell (Fig.2) may be expressed by the following:

$$\begin{Bmatrix}\Delta N\\ \Delta M\end{Bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix}\Delta\sigma\\ \Delta\sigma\zeta\end{Bmatrix}d\zeta=\sum_{i=1}^{n}\int_{\zeta_{i-1}}^{\zeta_i}\begin{Bmatrix}\Delta\sigma\\ \Delta\sigma\zeta\end{Bmatrix}d\zeta=\begin{bmatrix}\bar{A} & \bar{B}\\ \bar{B} & \bar{C}\end{bmatrix}\begin{Bmatrix}\Delta\varepsilon_m\\ \Delta\chi\end{Bmatrix}-\begin{Bmatrix}\Delta N^{vp}\\ \Delta M^{vp}\end{Bmatrix} \cdots(18)$$

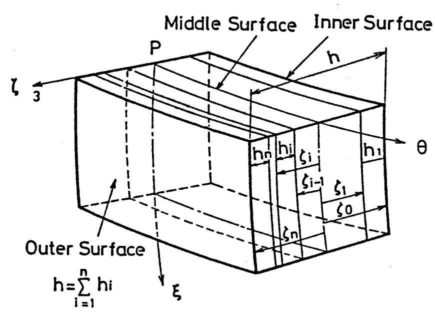

Fig.2 Multi-layered shell element

where

$$\{\Delta N\}=\{\Delta N_\xi,\ \Delta N_\theta,\ \Delta\bar{N}_{\xi\theta}\}^T \qquad \{\Delta M\}=\{\Delta M_\xi,\ \Delta M_\theta,\ \Delta\bar{M}_{\xi\theta}\}^T$$

$$\{\Delta N^{vp}\}=\{\Delta N_\xi^{vp},\ \Delta N_\theta^{vp},\ \Delta\bar{N}_{\xi\theta}^{vp}\}^T=\int_{-h/2}^{h/2}\{\Delta\sigma^{vp}\}d\zeta=\sum_{i=1}^{n}[D_i]\int_{\zeta_{i-1}}^{\zeta_i}\{\Delta\varepsilon^{vp}\}\cdot d\zeta$$

$$\{\Delta M^{vp}\}=\{\Delta M_\xi^{vp},\ \Delta M_\theta^{vp},\ \Delta\bar{M}_{\xi\theta}^{vp}\}^T=\int_{-h/2}^{h/2}\{\Delta\sigma^{vp}\}\zeta d\zeta=\sum_{i=1}^{n}[D_i]\int_{\zeta_{i-1}}^{\zeta_i}\{\Delta\varepsilon^{vp}\}\zeta d\zeta$$

$$[D_i]=\frac{E_i}{1-\nu_i^2}\begin{bmatrix}1 & \nu_i & 0\\ \nu_i & 1 & 0\\ 0 & 0 & 1-\nu_i\end{bmatrix} \qquad \cdots\cdots(19)$$

and

$$\bar{A}=\int_{-h/2}^{h/2}[D]d\zeta=\sum_{i=1}^{n}[D_i]\int_{\zeta_{i-1}}^{\zeta_i}d\zeta=\sum_{i=1}^{n}[D_i](\zeta_i-\zeta_{i-1})$$
$$\bar{B}=\int_{-h/2}^{h/2}[D]\zeta d\zeta=\sum_{i=1}^{n}[D_i]\int_{\zeta_{i-1}}^{\zeta_i}\zeta d\zeta=\frac{1}{2}\sum_{i=1}^{n}[D_i](\zeta_i^2-\zeta_{i-1}^2)$$
$$\bar{C}=\int_{-h/2}^{h/2}[D]\zeta^2 d\zeta=\sum_{i=1}^{n}[D_i]\int_{\zeta_{i-1}}^{\zeta_i}\zeta^2 d\zeta=\frac{1}{3}\sum_{i=1}^{n}[D_i](\zeta_i^3-\zeta_{i-1}^3) \qquad \cdots\cdots(20)$$

In Eqs.(18)-(20), the subscript i refers to the i th layer.

A complete set of field equations for 32 independent variables: $\Delta N_\xi, \Delta N_\theta, \Delta\bar{N}_{\xi\theta}, \Delta M_\xi, \Delta M_\theta, \Delta\bar{M}_{\xi\theta}, \Delta N_\xi^{vp}, \Delta N_\theta^{vp}, \Delta\bar{N}_{\xi\theta}^{vp}, \Delta M_\xi^{vp}, \Delta M_\theta^{vp}, \Delta\bar{M}_{\xi\theta}^{vp}, \Delta\sigma_\xi, \Delta\sigma_\theta, \Delta\sigma_{\xi\theta}, \Delta\sigma_\xi^{vp}, \Delta\sigma_\theta^{vp}, \Delta\sigma_{\xi\theta}^{vp}, \Delta\varepsilon_{\xi m}, \Delta\varepsilon_{\theta m}, \Delta\varepsilon_{\xi\theta m}, \Delta\varepsilon_\xi^{vp}, \Delta\varepsilon_\theta^{vp}, \Delta\varepsilon_{\xi\theta}^{vp}, \Delta\chi_\xi, \Delta\chi_\theta, \Delta\chi_{\xi\theta}, \Delta\Phi_\xi, \Delta\Phi_\theta, \Delta U_\xi, \Delta U_\theta, \Delta W$, is now given by 32 equations: (2), (5)-(7), (15)-(19).

2.2 Non-dimensional equations

In order to analyze the problem of the shells under arbitrary unsymmetrical loads, the distributed loads and the 29 independent variables, except $\{\Delta\varepsilon^{vp}\}$, are expanded into Fourier series. Substituting these into the above 32 equations and appropriately eliminating the variables, the simultaneous differential equations for $\Delta u_\xi^{(n)}, \Delta u_\theta^{(n)}, \Delta w^{(n)}$ and $\Delta m_\xi^{(n)}$ can be obtained:

$$\begin{bmatrix}a_1 & 0 & a_6 & 0\\ 0 & a_{13} & a_{16} & 0\\ a_{20} & a_{23} & a_{26} & a_{29}\\ 0 & 0 & a_{35} & 0\end{bmatrix}Z''+\begin{bmatrix}a_2 & a_4 & a_7 & a_9\\ a_{11} & a_{14} & a_{17} & 0\\ a_{21} & a_{24} & a_{27} & a_{30}\\ a_{32} & 0 & a_{36} & 0\end{bmatrix}Z'+\begin{bmatrix}a_3 & a_5 & a_8 & a_{10}\\ a_{12} & a_{15} & a_{18} & a_{19}\\ a_{22} & a_{25} & a_{28} & a_{31}\\ a_{33} & a_{34} & a_{37} & a_{38}\end{bmatrix}Z=\begin{Bmatrix}d_1\\ d_2\\ d_3\\ d_4\end{Bmatrix}+\begin{Bmatrix}\partial\Delta U_\xi/\partial\tau^2\\ \partial\Delta U_\theta/\partial\tau^2\\ \partial\Delta w/\partial\tau^2\\ 0\end{Bmatrix} \qquad \cdots\cdots(21)$$

where $Z=\{\Delta u_\xi, \Delta u_\theta, \Delta w, \Delta m_\xi\}^T$, and τ is non-dimensional time:

$$\tau=\sqrt{E/\rho_0}\,(t/a),\ \rho_0=\sum_{i=1}^{n}\rho_i h_i/h \qquad \cdots\cdots(22)$$

a_1-a_{38} are constants which depend on shell geometries and materials, and d_1-d_4 are determined from the loads, and the membrane forces and moments due to viscoplasticity.

If $i=1,\ \zeta_1 = h/2,\ \zeta_0 = -h/2$ are assumed in Eqs.(20), then $\bar{B}=0$, and Eqs.(21) coincide with the equations for single-layered shells previously derived by Takezono et al.(1989).

Once the solutions for Δu_ξ, Δu_θ, Δw and Δm_ξ have been calculated, the internal forces at any point in the shells can be found.

The stress increments (in the i th layer) can be expressed with the solution z of Eqs.(21) as follows:

$$\{\Delta s_i\}=\{F_i\}\,z'+\{H_i\}\,z+\{\Delta s_i^{vp}\} \qquad \cdots\cdots\cdots\cdots(23)$$

where

$$\{\Delta s_i\}=\{\Delta s_{\xi i},\ \Delta s_{\theta i},\ \Delta s_{\xi\theta i}\}^T \qquad \{\Delta s_i^{vp}\}=\begin{Bmatrix} \dfrac{E_i}{E_0}\dfrac{\zeta}{a}\dfrac{1}{1-\nu_i^2}\dfrac{1}{C_1}\Delta m_\xi^{vp}-\Delta s_{\xi i}^{vp} \\ \dfrac{E_i}{E_0}\dfrac{\zeta}{a}\dfrac{1}{1-\nu_i^2}\dfrac{\nu}{C_1}\Delta m_\xi^{vp}-\Delta s_{\theta i}^{vp} \\ -\Delta s_{\xi\theta i}^{vp} \end{Bmatrix}$$

$$\zeta_{i-1}\leqq\zeta\leqq\zeta_i\,, \qquad \cdots\cdots\cdots\cdots(24)$$

F_i and H_i are determined from the shell geometries, the materials and the thickness of each layer.

By the use of Eqs.(19), the increments of internal forces and stresses related to visco-plasticity are given as follows:

$$\left.\begin{aligned} &\sigma_0 h_0\sum_{n=0}^{\infty}\{\Delta n_\xi^{vp(n)},\ \Delta n_\theta^{vp(n)},\ \Delta n_{\xi\theta}^{vp(n)}\}[A_n]=\sum_{i=1}^{n}\int_{\zeta_{i-1}}^{\zeta_i}\{\Delta\varepsilon_\xi^{vp},\ \Delta\varepsilon_\theta^{vp},\ \Delta\varepsilon_{\xi\theta}^{vp}\}[D_i]d\zeta \\ &(\sigma_0 h_0^3/a)\sum_{n=0}^{\infty}\{\Delta m_\xi^{vp(n)},\ \Delta m_\theta^{vp(n)},\ \Delta m_{\xi\theta}^{vp(n)}\}[A_n]=\sum_{i=1}^{n}\int_{\zeta_{i-1}}^{\zeta_i}\{\Delta\varepsilon_\xi^{vp},\ \Delta\varepsilon_\theta^{vp},\ \Delta\varepsilon_{\xi\theta}^{vp}\}[D_i]\zeta d\zeta \\ &\sigma_0\sum_{n=0}^{\infty}\{\Delta s_\xi^{vp(n)},\ \Delta s_\theta^{vp(n)},\ \Delta s_{\xi\theta}^{vp(n)}\}[A_n]=\{\Delta\varepsilon_\xi^{vp},\ \Delta\varepsilon_\theta^{vp},\ \Delta\varepsilon_{\xi\theta}^{vp}\}[D_i] \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots(25)$$

where

$[A_n]=[\cos n\theta, \cos n\theta, \sin n\theta]$ (diagonal matrix)

The visco-plastic strain rates on the right-hand sides can be related to the stresses by Eq.(15).

3. NUMERICAL METHOD

A finite difference method is employed for solution. The usual central difference formulas are used for every mesh point except the discontinuity points and the boundary points of the shell. For the discontinuity points and the boundary points forward and backward difference equations are employed. The second derivative with respect to time in the inertia terms in eq.(21) is treated with the backward difference formula proposed by Houbolt (1950).

Applying these difference formulas for the fundamental equations (21), the boundary conditions and the continuity equations, the simultaneous equations with respect to $z_{i,j}$ (z at any point, at any time) can be obtained. The solutions at any time are obtained by summation of the incremental values at each calculating stage.

4. NUMERICAL EXAMPLE

As a numerical example of elasto/visco-plastic dynamic response of multi-layered shells of revolution, a fixed supported two-layered cylindrical shell composed of mild steel and titanium subjected to impulsive load is treated (Fig.3).

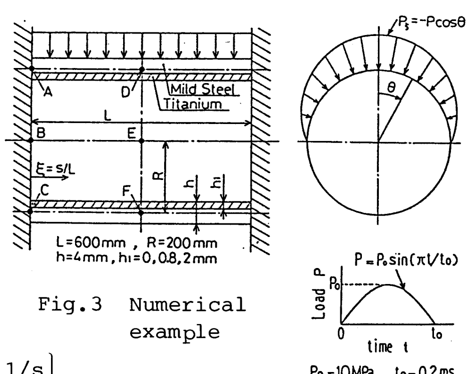

Fig.3 Numerical example

The material constants of mild steel and titanium employed in the calculations are as follows (Perzyna 1966; Takezono 1982a),

Titanium:

$$\left.\begin{aligned} &E=91\ \text{GPa},\ \nu=0.33,\ \rho_1=4.51\ \text{g/cm}^3,\ \gamma_1=800\ 1/\text{s} \\ &\sigma^*=216.0\ \text{MPa},\quad \Phi(F)=\{(\bar{\sigma}-\sigma^*)/\sigma^*\}^{7.4} \end{aligned}\right\} \qquad \cdots\cdots\cdots(26)$$

Mild steel:

$$\left.\begin{array}{l} E=200\ \text{GPa},\ \nu=0.29,\ \rho_2=7.86\ \text{g/cm}^3,\ \gamma_1=40.4\ 1/\text{s} \\ \sigma^*=261.7\ \text{MPa},\quad \Phi(F)=\{(\bar{\sigma}-\sigma^*)/\sigma^*\}^{5.0} \end{array}\right\} \quad \cdots\cdots(27)$$

Some of the essential features of the solutions are shown in Figs.4-12. Numerical computations are carried out for three cases of the ratio of the thickness of the titanium layer to the shell thickness. It is found from the computations that the stress distributions and the deformations vary significantly depending on the thickness ratio.

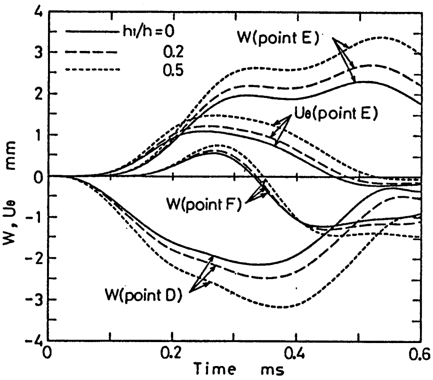

Fig.4 W , U_θ

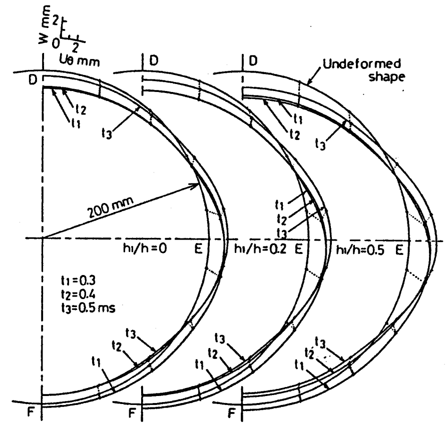

Fig.5 Deformations (ξ =0.5)

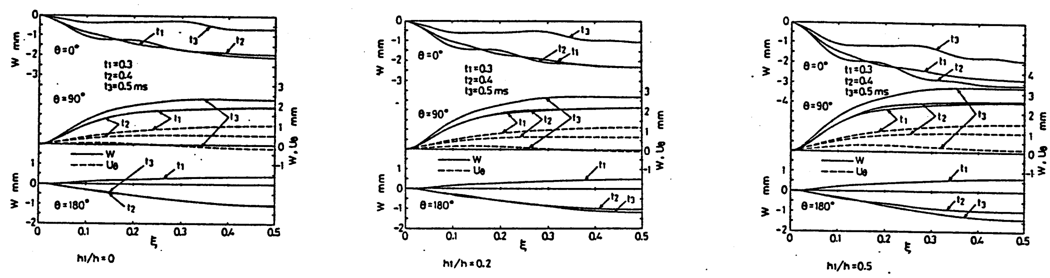

Fig.6 Axial distributions of W , U_θ

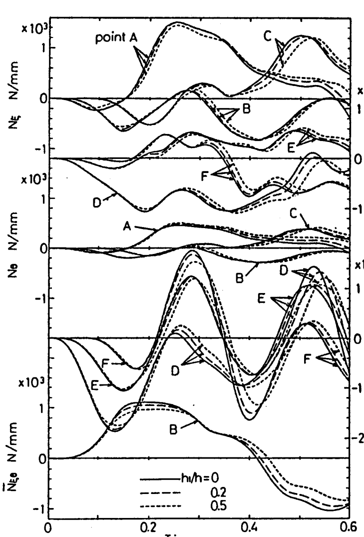

Fig.7 Resultant forces

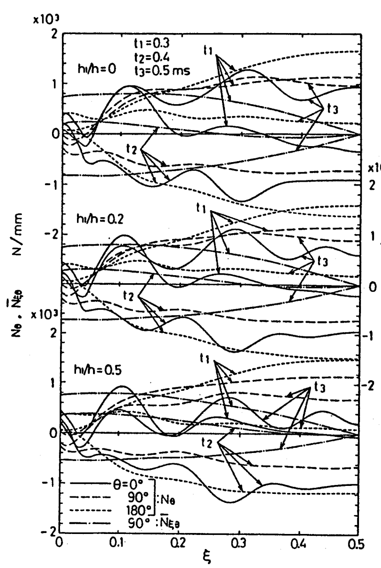

Fig.8 N_θ , $\bar{N}_{\xi\theta}$

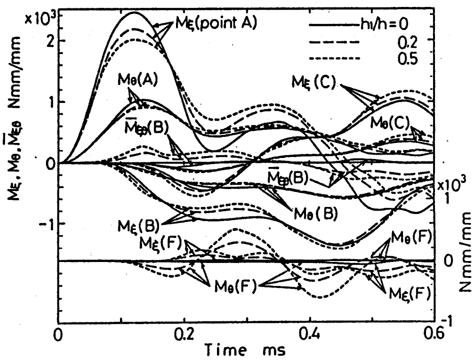

Fig.9 Resultant moments

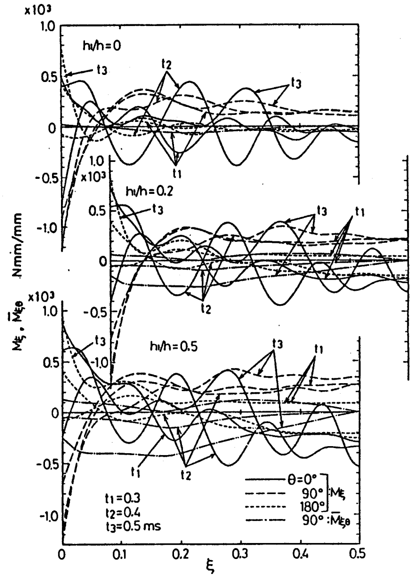

Fig.10 M_ξ , $\bar{M}_{\xi\theta}$

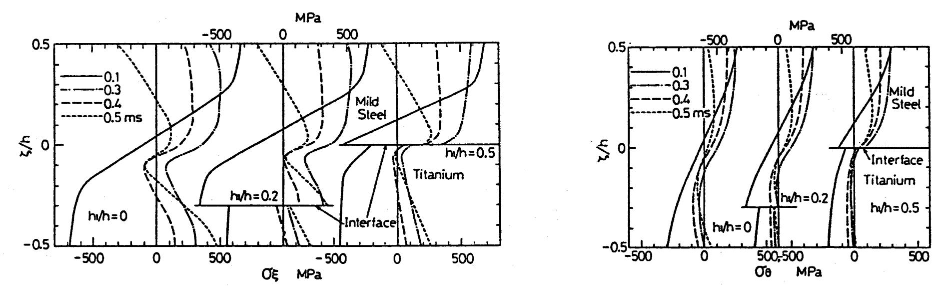

Fig.12 Stress distributions at point A

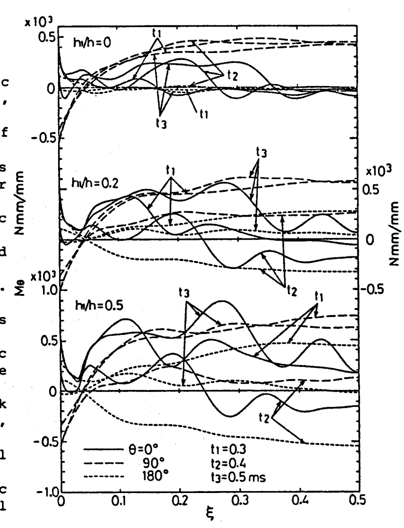

Fig.11 M_θ

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