NONLINEAR DYNAMIC ANALYSIS OF PIPING SYSTEMS USING THE PSEUDO FORCE METHOD

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Abstract

Simple piping systems are composed of linear elastic elements and can be analyzed using conventional linear methods. The introduction of constraint springs separated from the pipe with clearance gaps to such systems to cope with the pipe whip or other extreme excitation conditions introduces nonlinearities to the system, the nonlinearities being associated with the gaps. Since these spring-damper constraints are usually limited in number, discretely located, and produce only weak nonlinearities, the analysis of linear systems including these nonlinearities can be carried out by using modified linear methods. In particular, the application of pseudo force methods wherein the nonlinearities are treated as displacement dependent forcing functions acting on the linear system were investigated.

The nonlinearities induced by the constraints are taken into account as generalized pseudo forces on the right-hand side of the governing dynamic equilibrium equations. Then an existing linear elastic finite element piping code, EPIPE, was modified to permit application of the procedure. This option was inserted such that the analyses could be performed using either the direct integration method or via a modal superposition method, the Newmark-Beta integration procedure being employed in both methods. The modified code was proof tested against several problems taken from the literature or developed with the nonlinear dynamics code OSCIL. The problems included a simple pipe loop, cantilever beam, and lumped mass system subjected to pulsed and periodic forcing functions. The problems were selected to gage the overall accuracy of the method and to insure that it properly predicted the jump phenomena associated with nonlinear systems.

Implementation of the method was found to be straightforward with the simplest iteration procedure for the psuedo force vector sufficing. The results predicted with the method agreed in all important aspects with existing solutions as well as those generated with other methods. As with linear analyses, the modal superposition solution mode was found to be the most efficient, however, exhibiting slightly greater inaccuracies.

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1. Introduction

Nuclear piping systems normally incorporate pipe whip constraint devices. These devices are usually limited in number and exhibit nonlinear stiffness characteristics associated with clearance gaps and energy absorbing elements. These devices are provided to minimize the consequences of postulated pipe break events.

The dynamic analysis of these systems for normal conditions of loading can be undertaken with general purpose nonlinear dynamics methods. This approach, however, is exceedingly costly and unwarranted. Instead, since these systems can be assumed to respond elastically to normal loads, a more efficient approach is to perform the normal loading evaluations using modified linear methods. The advantages are that the linear methods are readily available and well developed and the cost of analysis can be expected to be greatly reduced.

This paper describes an evaluation of one such technique, the psuedo force method. In this method the nonlinearities are treated as displacement dependent forcing functions acting on the otherwise linear system. The governing dynamic equilibrium equation is then

$$[M] \ [\ddot{W}] + [C] \ [\dot{W}[+ [K] \ [W] = [F] + [P] \tag{1}$$

where

[M], [C], and [K] are the mass, damping, and stiffness matrices;

[W], $[\dot{W}]$, and $[\ddot{W}]$ are the displacement, velocity, and acceleration vectors;

[F] is the external load vector; and

[P] is the pseudo force vector of nonlinear external forces due to constraints.

The pseudo forces can be calculated as:

$$P_i^n (u_i, \dot{u}_i) = k_i^n (u_i^n - g_i^n) + C_i^n \dot{u}_i^n \qquad \text{if } u_i^n > g_i^n \tag{2}$$

$$= 0 \qquad \text{if } u_i^n \leq g_i^n$$

where the superscript n and subscript i connotate node n in the i^{th} direction. k_i, C_i, and g_i are the spring constants, damping coefficients, and clearance gaps of the constraints.

The dyanmic equilibrium eq. (1) can be solved by either the direct integration procedure [1] or the normal mode theory [2]. Both methods were considered, the Newmark-Beta integration procedure [3,4] being employed in both.

2. The Direct Integration Method

For the direct integration method the velocity and displacement at t+Δt are given by

$$\dot{W}_{t+\Delta t} = \dot{W}_t + [(1-\delta)\ddot{W}_t + \delta\ddot{W}_{t+\Delta t}] \ t \tag{3}$$

$$W_{t+\Delta t} = W_t \ \dot{W}_t\Delta t + [(1/2-\alpha)\ddot{W}_t + \ \ddot{W}_{t+\Delta t}]\Delta t^2 \tag{4}$$

where α and δ, chosen as α=1/4, δ=1/2 for this study, are parameters that can be selected to achieve integration accuracy and stability.

Using eqs. (3) and (4) the equilibrium equation at t+Δt may be expressed as

$$[K^{eff}]\ [W_{t+\Delta t}] = [F^{eff}] \tag{5}$$

where

$$[K^{eff}] = [K] + A_o[M] + A_1[C] \tag{6}$$

and

$$[F^{eff}] = [F_{t+\Delta t}] + [P_{t+\Delta t}] + [g(W_t, \dot{W}_t, \ddot{W}_t)] \tag{7}$$

the coefficients A_o and A_1 and the function g being developed in accordance with the Newmark Beta integration scheme [4]. As indicated $[F^{eff}]$ is dependent on $[P_{t+\Delta t}]$, an unknown vector which is in general a function of $W(t+\Delta t)$ and $\dot{W}(t+\Delta t)$.

The procedure used to estimate $[P_{t+\Delta t}]$ is as follows:

1) estimate $W_{t+\Delta t}$ assuming $[P_{t+\Delta t}] = 0$, i.e., compute $[Y_o]$ where $[K^{eff}]\ [Y_o] = [F_{t+\Delta t}] + [g(W_t, \dot{W}_t, \ddot{W}_t, \Delta t)]$;

2) Compute bumper forces corresponding to Y_o $[P_{t+\Delta t}] = [K_s]\ [Y_o] + [C_s]\ [\dot{Y}_o]$ were $[K_s]$ is the matrix of bumper spring constants and $[C_s]$ is the matrix of bumper damping coefficients;

3) Compute the additional displacement corresponding to these bumper forces $[B] = [K^{eff}]^{-1}\ [P_{t+\Delta t}]$;

4) Estimate $W_{t+\Delta t}$ as

$$[W_{t+\Delta t}] = [Y_o] + [B].$$

This procedure can be repeated until the difference $(W^{N}_{t+\Delta t} - W^{N-1}_{t+\Delta t})$ achieves some tolerance. We have found that iteration does not strongly effect the results and is not warranted. The solutions were obtained by following steps 1 through 4 without iteration.

3. <u>The Normal Mode Method</u>

For this method, the natural frequencies and the associated mode shapes for the system are computed first, then the equilibrium equation, eq. (1) is expressed in terms of the generalized coordinates X(t) as

$$\ddot{X}(t) + \nabla X(t) + \Omega^2 X(t) = \Phi^T F + \Phi^T P \tag{8}$$

where

$$W(t) = \Phi X(t),\ \Phi^T M \Phi = I,\ \Phi^T K \Phi = \Omega^2 \tag{9}$$

Φ being the modal matrix and Ω^2 being the diagonal spectral matrix of eigenvalues and

$$\nabla = \Phi^T C = \text{diag.}\ (2\omega_i \xi_i) \tag{10}$$

where ζ_i is the damping ratio in the i^{th} mode, proportional damping being assumed.

The system of equations in eq. (8) are decoupled, so the solutions of the generalized displacements can be obtained by time integration without the inversion of the effective stiffness.

The time integration used is again the Newmark-Beta method, where the equilibrium equation, eq. (8), is considered at time t+Δt, we have

$$\begin{bmatrix} \ddot{X}_{t+\Delta t} \\ \dot{X}_{t+\Delta t} \\ X_{t+\Delta t} \end{bmatrix} = A \begin{bmatrix} \ddot{X}_{t} \\ \dot{X}_{t} \\ X_{t} \end{bmatrix} + L[f_{t+\Delta t}] + L[p_{t+\Delta t}] \tag{11}$$

where A and L are the appropriate operators for the Newmark-Beta method and f and p are the modal load vector due to external forces and the modal load reactor due to constraint forces respectively.

Again, the constraint force vector at t+Δt is unknown and is determined through the same iteration procedure as described for the direct integration method above. It may be commented that this iteration is not so straightforward for the modal method as a transformation from generalized coordinates to system coordinates must be made at each iteration to determine the constraint forces (i.e., the nonlinear constraint force-displacement relations are defined in terms of system coordinates only).

4. Numerical Examples

For the first example, a three-dimensional nonlinear piping system, shown in Figure 1, consisting of nine straight pipes and two elbows with three gap-bumper supports was considered. The system was subjected to the dynamic loads shown in Figure 2. Solutions to this problem by two techniques are given in Ref. [2]. The results obtained with the modified EPIPE code are shown in Figure 3. This figure depicts the nodal forces at each of the three bumpers as a function of time for both the direct integration and modal superposition solution modes for a time integration step of 0.0000625 seconds. As can be seen, the solutions look identical for both modes. In addition these results look identical to the graphical results presented in Ref. [2]. A comparison of the numercial results to those of the reference show slight differences which are attributed to fact that completely different analytical methods were used.

For the second example, a system consisting of three concentrated masses interconnected and attached to ground with linear springs and separated from the side walls with gap-nonlinear spring bumpers, as shown in Figure 4A, was considered. For the analysis, the interelement springs were modeled with truss elements while the linear base springs were modeled with massless beam elements. The bumper elements had an initial gap of 0.001" and a cubic nonlinear spring constant given by

$$k = 1000\ (D-G) + 100{,}000\ (D-G)^3.$$

The input excitation was the ground motion acceleration time history depicted in Figure 4B. All the pertinent system parameters are shown in Figure 4A.

The response time histories were determined by both the modal superposition method and direct integration using an integration time step of 0.001 second.

Also, to provide a means to corroborate the results, the system response was again determined using the BNL nonlinear dynamics code OSCIL [6]. This code, developed to evaluate HTGR ocre block response, is designed to evaluate the response of multimass systems containing nonlinear spring elements with general characteristics. It incorporates a numerical integration scheme which automatically varies the integration time step size as a function of event severity.

The bumper forces predicted by the three methods are shown in Figure 5. In each of these computer plots the upper curve coincides to the left-side bumper and the lower curve to the right-side bumper. As can be noted, the graphical results again look identical. A comparison of the numerical results showed minimal differences with the direct integration method providing a solution closer to that produced by OSCIL. Additional runs with finer time steps produced further convergence to the OSCIL solution.

As a last test of the method, the three mass problem was again used to investigate whether the psuedo force method would correctly predict the multiple response roots inherent in this nonlinear system. Using the direct integration solution mode and the OSCIL code, the response of the system to a sweeping frequency sinusoidal forcing functions, the frequency being both swept up and swept down, was determined. The excitation was a concentrated force given by $F=20 \cos(\omega t)$ acting, in phase, on each mass point. For the direct integration solution the integration time step was $1x10^{-4}$ seconds while the OSCIL code adapted minimum time steps as small as $1x10^{-5}$ seconds.

Both solution methods predicted the same response curve, shown in Figure 6. This curve is typical for a system with a hardening characteristic [5] and was to be expected for the problem being considered. In the frequency range 4→6 cps two distinct response roots are evident, the root exhibited by the system excited at these frequencies being determined by the prior history transients. Clearly, this result further confirms the adequacy of the psuedo force method as a nonlinear analysis method.

5. Conclusions

The implementation of the psuedo force method into an existing elastic finite element code was found to be straightforward. It required the addition of some simple algorithms and the substitution of the Newmark-Beta integration scheme for the original integration algorithm. These modifications were most readily made for the direct integration solution mode while requiring some additional coding for the modal superposition solution mode (gap effects necessitated a constant return to system coordinates). Lastly, potential problems foreseen for the development of a pseudo force iteration scheme proved unfounded as the simplest, most direct procedure provided acceptable accuracy.

Concerning the problem solutions contained herein, the results predicted with the BNL psuedo force code option agreed in all important aspects the existing solutions. This was true for both the direct integration and the modal superposition solution modes with the latter exhibiting slightly greater, but insignificant, inaccuracys. As with linear system analyses, the modal superposition solution mode was found to be the most rapid exhibiting the reduction in computer running time normally associated with it.

6. References

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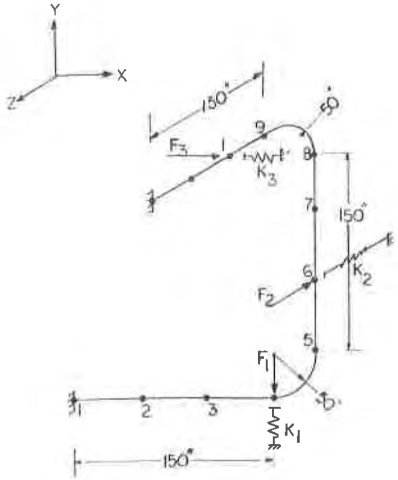


Figure 1 3-D Piping System W/Gap-Bumpers

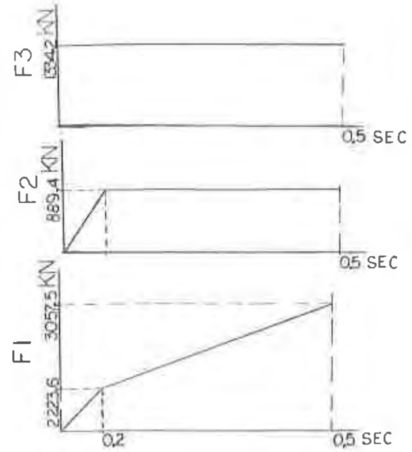


Figure 2 Applied Forcing Functions

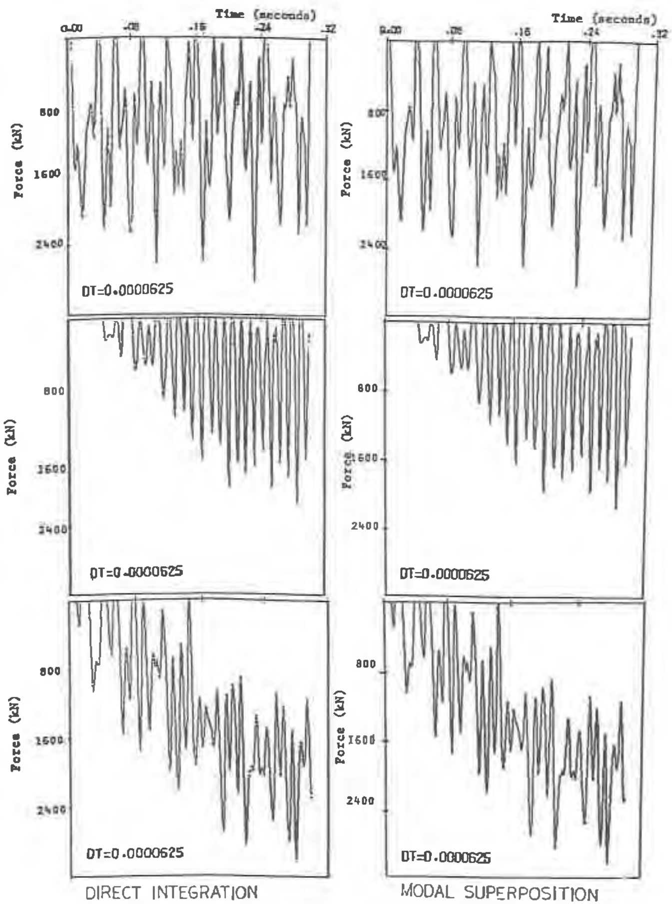
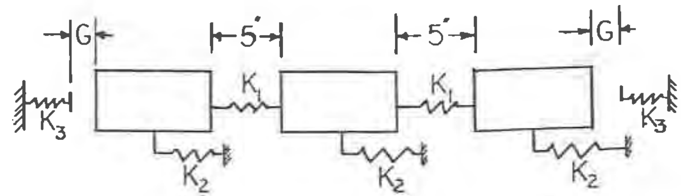


Figure 3 Bumper Forces at Node 4, 6, and 10



K_1 = 10000 Lb/IN

K_2 = 18 Lb/IN

$K_3 = 1000(D-G) + 100000(D-G)^3$

D = NODAL DISPLACEMENT HISTORY

Figure 4A 3-Masses System

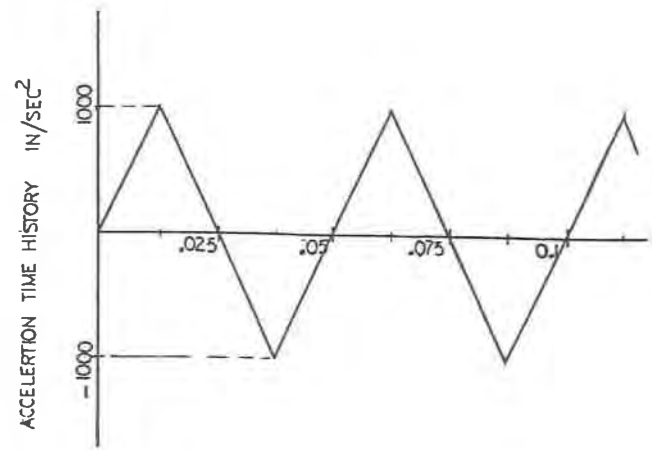


Figure 4B Ground Acceleration

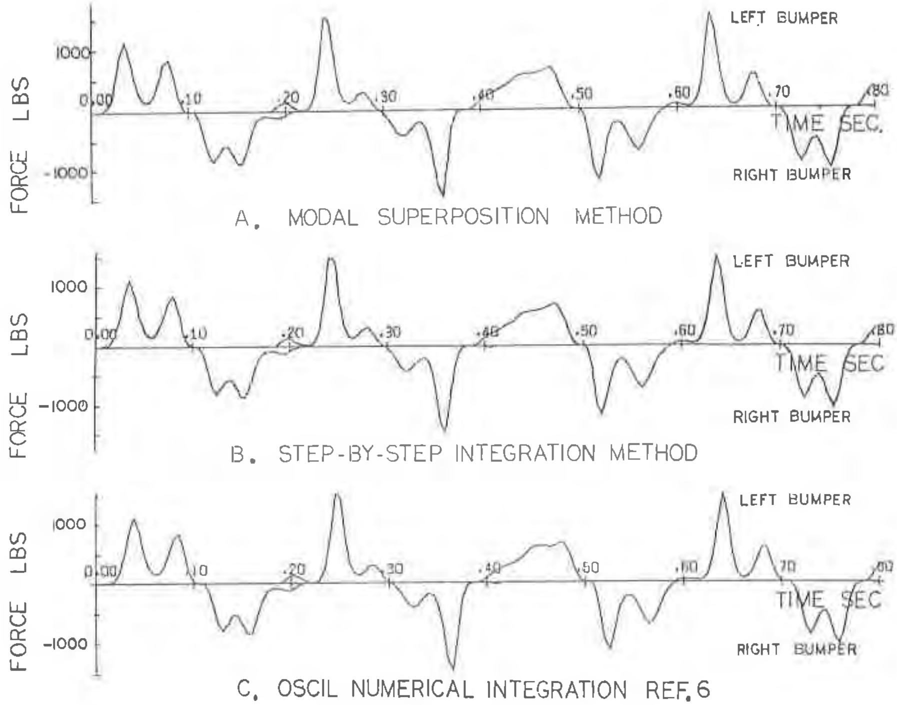


Figure 5 Bumper Forces

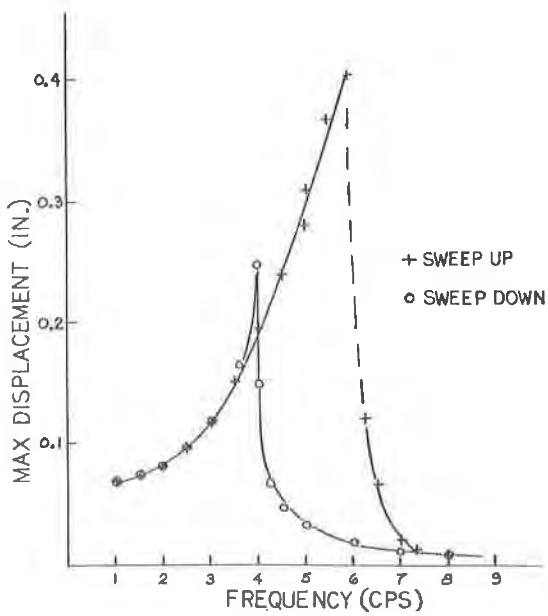


Figure 6 Response 3-Mass System