ABSTRACT

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In this research we have studied the effect of trade on economic growth through comparative advantage without including any of the usual channels that trade could affect growth, such as the scale effect, research and development, technology transfer or even foreign investment. First, we consider a 2 sector endogenous growth model in which two goods are being produced using 2 types of reproducible factors of production with Cobb-Douglas type production functions. In this model there are two countries that could trade in goods but there is no international lending or borrowing. In present of free international trade, our model implied that countries would completely specialize in producing one good on the Balanced Growth Path (BGP) and both countries enjoy higher growth rates with trade. Next we used a more general form of production functions. In this section we showed that the production patterns on the BGP will be of 2 types, depending on the relative price of the countries under autarky to the price after trade. We showed the existence and uniqueness of a price at which both goods are being produced on the BGP. This price would be the price of the country under autarky. However if the price of the country under autarky is different from the price after trade, the country will shut down the production of one sector. On the transition to the steady state also complete specialization could happen. Finally in the last section we considered testing the implication of a simpler model where 2 goods being produced, one good being purely consumption and one being investment, using only one type of capital. In this model

countries will also completely specialize on the BGP. The country that specializes in consumption goods and imports capital could enjoy higher growth rate with trade. The other country specializes in producing capital goods and so imports consumption good and its growth rate will remain unchanged with trade. We tested this implication using a large panel of 92 countries over the period of 1965-2000. We used an instrumental variable approach when using yearly panel data. We also report the GMM estimates of the model using non-overlapping five year averages. We used an estimation method suggested by Arellano and Bond (1990) to get the consistent estimates. Our results imply that specializing in consumption goods affects the growth rates significantly positive. However we did not find any significant effect for specializing in capital goods.

Essays on International Trade and Endogenous Growth

by
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DEDICATION

To

my parents, Nader and Shidokht,

my husband, Omid,

and

my brother, Arasp.

BIOGRAPHY

Maryam Arabshahi has born in December, 24 of 1973 to a loving parents in Tehran, Iran. She received her Bachelor degree in economics in summer of 1996 from Shahid Beheshti University in Tehran, Iran. She then came to the United State of America to pursue her higher education. She started her Master program in economics at Illinois State University in August 1997 and received her degree in December 1999. She worked as RA in Economics as well as Women Studies program while she was attending at Illinois State University.

In winter of 2000 she came to Raleigh, North Carolina to start her Ph.D program in Economics at North Carolina State University. During her program she worked as TA and lecturer for various courses in economics. She also worked as an intern in Research Triangle Institute, Health Economics division for about a year. She is now married and living in Raleigh.

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INTRODUCTION

Since the emergence of endogenous growth models there have been a flow of papers on endogenous growth in the literature. Many of these works have contributed in studying the relationship between endogenous growth and international trade. In these literatures there are many ways that relates trade to economic growth. For instance trade can affect growth through increases in the global stock of knowledge which increases the efficiency of research and development in countries that engage in trade. Also trade can be the channel through which countries can learn about new and advanced technologies developed in their trading partner countries. Trade also can increase the rate of economic growth through increases in size of markets or through demographic changes in countries. These are all important channels however we could ask whether trade could affect growth through the mechanism of comparative advantage in a dynamic model similar to the way that it raises income level in static models. Although in some of the studies, such as Grossman and Helpman (1991, ch7) and Acemuglu and Ventura (2002) the role of comparative advantage has been considered in determining the pattern of specialization but trade affects growth through the channels other than comparative advantage. Seater (2007) however explores this matter by considering a model in which there is no research and development, technology transfer, scale effect or even international investment, in order to focus on the effect of trade through the mechanism of comparative advantage only. He extends the two sector endogenous growth model described in Barro and Sala-i-Martin (1995, ch5). In his model he considers two countries each producing two types of goods using two reproducible factors of production. When engage in trade, there will be a stable steady state in which each country will completely specialize in production of one sector according to their comparative advantage pattern. The growth rates will be higher with trade relative to that under autarky for each country. However the existence of the balanced growth path and the effect of trade on growth depend on the absolute advantage of the countries engage in trade. When each country has absolute advantage in producing of one of the goods then balance growth path exists and international trade raises the growth rates on the balanced growth path and countries grow at the same rate. In addition, he shows that the resulting growth rates in this case are those that would result from technology transfer, even though no technology transfer occurs.

In his model, Seater (2007) assumes that technologies are different across sectors and between countries. However these differences of technologies are reflected through the factor productivity parameters alone since he considers equality between factor shares across sectors and between countries in the production functions of Cobb-Douglas type. The first objective of this research is to relax this simplified assumption and consider the more general form of production functions in which factor shares across sectors of production as well as between countries are different. Then using these general forms of production function, we also show that a balanced growth path in which all variables that are growing grow at the same rate, exists and it is stable. Also the production patterns are similar to what Seater (2007) has obtained. On the balanced growth path each country completely specializes in production of one sector and the growth rates rise with trade and countries grow at the same rate. However these results depend on the value of the equilibrium world price. If the value of the world price stays between the autarky prices then the balanced growth path exists and the growth rates of the countries are higher with trade and are of those that would result from technology transfers. When using the general form of production functions, these results do not reflect the requirements of absolute advantage. In the other world the condition of absolute advantage that guarantees the existence of balanced growth path as obtained in Seater (2007), does not hold when we consider the general model. We could further show that the absolute advantage requirement is the result of considering the simple version of production functions as Seater (2007) did.

The second objective of this research is to generalize the form of production functions even more. By doing so we could study the interaction between international trade and the capital accumulation process. Following Bond and Trask (1997), we utilized a dynamic general equilibrium model in which there are two traded goods produced in two different sectors and using two types of reproducible factors of production. Output is similarly assumed to be produced under conditions of constant returns to scale and perfect competition. Therefore the model exhibits endogenous growth because it has constant returns to scale in the reproducible factors. The model that we considered in this section, which is similar to what we considered in the previous sector, is related to the two by two Heckscher-Ohlin model of international trade theory, since it has two primary factors and two traded goods. However notice that in this model both factors are reproducible. Therefore it is different from dynamic Heckscher-Ohlin models since in the latter types of models one factor (physical capital) is determined endogenously while the other factor (labor force) is exogenously given. We show that when both factors are production of reproducible, the model produces the Ricardian type of results. We first establish the existence of a balance growth path and show the relationship between the world price and the pattern of production on the balanced growth path. When the world price is the same as the price under autarky then the country is incompletely specialized. For any price other than this price the country will specialize in production of one sector. We then show that there will be a unique factor ratio on the balanced growth path for the equilibria with specialization in one of the traded goods and we establish the saddle path stability of these equilibria.

In the final section we considered a two sector model in which two goods, a pure consumption and a pure investment good is being produced using one type of capital. Similarly when the equilibrium world price is between the autarky prices, on the balanced growth path each country specializes in production of one sector. However the effect of trade on growth rates depend on the type of good a county is importing. When a country specializes in producing consumption good and an import capital good, its growth rate rises with trade. However for the country that specialized in capital good and import consumption good, the growth rate remain unchanged. Most of the studies that examine the relationship between growth rate and international trade usually do not consider the type of trading good and instead they focus on the effect of trade in general. The purpose of this section is to test whether the specialization matters when it comes to the effect of trade on growth rates. We extend an specification developed by Bond, Leblebicioglu and Schiantarelli (2006) to allow the long run growth rate in each country depend on the variable that our model predict to affect the long run growth rate. We estimate this specification using a panel of 92 countries over the period of 1965-2000. To capture the effect of the nature of trading goods we use three measures, import share, export share (both as a percentage of GDP) and an specialization index known as Laffay index. We also report the effect of trade specialization on growth using both yearly panel and non-overlapping five-year averages. When considering yearly panel we estimate the model using instrumental variable approach using the different lagged values of endogenous variables as well a few additional instruments.

Also report the estimates of the model for the five-year average data using an approach developed by Arellano and Bond (1991) to obtain the consistent estimates. Our results show that specializing in consumption goods increase the long-run growth rate statistically significant while we could not find any evidence of specializing in capital goods.

CHAPTER 1

TRADE, GROWTH AND COMPARATIVE ADVANTAGE

1.1 INTRODUCTION

There are varieties of models which try to link the cross-country differences in long-run rate of growth to international trade through different channels. For instance, studies such as Kremer (1993), Barro and Sala-i-Martin (1997) and Connolly (2000) consider the case where opening to trade increases the size of markets for producers leading to more specialization and higher growth. In the models where research and development is the source of growth, trade in specialized inputs raises the level and the rate of growth of real output. Grossman and Helpman’s (1991) works are examples of this type.

Trade can also affect long-run growth rate through exchange of technology and knowledge. For instance, Rivera-Batiz and Romer (1991) consider two models with different specifications of the research and development sector. In their model trade opens countries to each other’s knowledge and therefore integration could increase the worldwide long-run rate of growth through the growth in stock of knowledge in each country.

Although all these studies consider important channels through which trade could affect growth but most of them do not provide a strong link between comparative advantage and the patterns of trade between countries. Seater (2007) however, studies this importance by providing a mechanism where trade could raise the rate of growth through comparative advantage. He considers the two sector AK model introduced in Barro and Sala-i-Martin (1995, ch5), where two goods are being produced using two types of reproducible factors of production and Cobb-Douglas production functions. One sector produces a unified good that could be used as consumption or investment good and the other sector produces another type of capital good and both goods are tradable. He extends this closed economy model to examine the effect of free trade on the growth performances of countries engaged in trade. He assumes that the technology for producing each type of good is different within sectors of production in each country as well as across countries. He also assumes that the differences in technologies arise only from the differences in total factor productivity parameters. His model therefore is a simple two country – two good model, which has been simplified to focus on the growth effects of trade through comparative advantage without recourse to scale effects, technology transfer, research and development, or even international investment. His results show that if each country has an absolute advantage in the production of one of the traded goods then trade could raise the growth rate of both countries and balance growth path exists for individual countries as well as the world. In addition although there is no technology transfer however by trading in goods that are factors of production the growth rates imply the results that would emerge if countries exchanged technology, Comparative advantage however determines the pattern of trade within the countries.

The purpose of this section is to extend Seater's (2007) work to consider the more general case where the source of technology differences within the sectors and across the countries is not just reflected in total factor parameters. We could show that even in the general model similar to the simple model that Seater (2007) considered, free trade could raise the growth rated of countries through the channel of comparative advantage. In what fallows, we first review some of the existing models that study the relationship between trade and growth. Then we explain the model Seater's (2007) have studied and present some of his results. Then describe the model with more general form of the production function and examine the effect of free trade on the growth performances.

1.2 REVIEW OF SOME LITERATURES

Although most of the early researches on the trade and growth have adopted different versions of the neoclassical model to examine wide range of issues in the field of international trade, since Romer (1986) suggested a model that endogenizes the growth rate of economies, variety of this idea has been examined to study the effects of international trade on growth rate of output and income. For examples Grossman and Helpman (1991, chapter 7, 8 and 9), Rivera-Batiz and Romer (1991) and Acemoglu and Ventura (2002) have extended endogenous growth models of technological progress to analyze the effect of international trade on growth. Similarly learning by doing models or the models with human capital accumulation has been used to study this relationship. Lucas (1988, 1993), Young (1991), Stokey (1991), Bond and Trask (1997) and Seater (2007) are examples of such studies. Backus, Kehoe, and Kehoe (1992), Kremer (1993) relate growth and international trade through channels of scale effect. Barro and Sala-I-Martin (1997) and Connolly (2000) investigate the interaction between trade and growth through innovation and imitation of products. In what follows we briefly summarize some of these researches.

In a model where research and development is the engine of growth, Grossman and Helpman (1991, ch7) investigate the determinants of endogenous comparative advantage when innovators develop new varieties of horizontally differentiated products. Their results imply that countries in which human capital is relatively abundant will specialize in research on the steady state. Therefore this county acquires the know-how to produce a relatively wider range of innovation goods. In addition in the world with free trade each country have access to the wider range of innovative products which makes the growth rates to be higher with trade than that under autarky for both countries.

Grossman and Helpman (1991, ch9) also provided an analysis linking long-run rate of growth to trade and international condition in an endogenous growth model in which knowledge is the engine of growth due to the presence of scale economies. They consider a model with two countries and 3 production activities; production of final good, production of a continuum of varieties of differentiated middle products and R&D sector that produces designs for new products using primary resources and previously accumulated knowledge. They consider two countries that have similar production functions but have different endowments. They showed free trade in goods and inputs bring international spillovers of knowledge, then trade leads to an increase in the growth rates of both countries.

Rivera-Batiz and Romer (1991) consider two types of trade between two economies that are similar in their endowments and technologies and are initially separated and are at their balanced growth path. They consider two different ways of trade between the economies: free trade in goods but not ideas, and free trade in ideas but not goods. Following Romer (1990) their model as well includes two types of manufacturing activities; production of consumption goods and production of the physical units of types of capital that have already been invented. Research and development is the third activity, which creates designs for new types of capital goods. They consider two specifications of the technology for R&D. One specification assumes that human capital and knowledge are the only inputs that influence the output of designs (knowledge driven specification), while another assumes that technology for R&D uses the same inputs, in the same proportions as the manufacturing technology (lab equipment model). These specifications permit a sharp distinction between flows of goods and flows of ideas. To study the effect of trade, first they allow for the full integration so that economies are completely integrated into a single economy. After full integration takes place, regardless of the specification of the technology for R&D the rate of growth will increase. In the next step they only allow for trade in goods but restrict the flow of ideas, under this assumption they show that trade in goods has no effect on the long-run rate of growth under the knowledge driven specification for R&D technology. However when they consider the effects of trade in goods under the lab equipment specification opening trade in goods alone causes the same permanent increase in the rate of growth as complete integration. Then they include the effect of opening communications networks along with trade in goods in the knowledge driven specification for R&D, under this assumption their results show that allowing for flow of ideas creates a permanently higher growth rates.

Acemoglu and Ventura (2002) also show that international trade –based on specialization- could lead to a stable world income distribution even if there is no diminishing return in production and technological spillovers. They model the world as a collection of economies a la Rebelo (1991) with growth resulting from accumulation of capital. Two final good (consumption and investment) and a stream of intermediate goods are being produced. Capital and intermediates are being used in production of both goods while the technology of producing intermediates requires one unit of capital to produce one unit of intermediate. The ability of producing higher number of intermediates imply how advanced the technology of a country is. In this world, countries could trade in intermediate goods. On the steady state countries specialize in a set of intermediate goods. Countries with greater production for each variety of intermediates experience a decline in their terms of trade which translates to slower capital accumulation and reverse happens for the other countries. As the result, on the steady state countries will grow at the same rate and it is stable.

Models of diffusion of technology are another type of models that have been considered in studying the relationship between trade and growth. Barro and Sala-I-Martin (1997) try to construct a model linking the long-run growth implications of the endogenous growth theories with the convergence implications of the neoclassical growth model. They consider two countries that are engaged in producing final consumable good and production of intermediates and R&D production that aimed at learning about new varieties of intermediates. An agent can learn by inventing a new type of good or by imitating a product that is known to each country. They assumed that one country is the technological leader and the other is the follower. Therefore the number of types of intermediates is larger in the technological leader than that in the follower country. Imitation is typically cheaper than invention and therefore follower prefer to imitate the intermediate goods known in the leader country rather than inventing them. The lower cost of imitation implies that the follower would grow fast and tends to catch up to the leader. In the long run, all economies would grow at the rate of discovery in the leading country. Therefore in this type of model trade expands markets for intermediate goods and the production of those goods and as the result the growth rates are higher. Connolly (2000) studies the transitional dynamics for a developed and less developed country when North-South trade leads to technological diffusion through reverse engineering of intermediate goods in a quality ladder model of endogenous growth. Domestic technological progress occurs via innovation or imitation, while growth is driven by technological advances in the quality of domestically available inputs, regardless of country of origin. For reasonable parameter values, the rates of innovation and imitation are both falling in transition to steady-state and yet remain above that under autarky. Increased interaction between the North and the South, through increased openness to imports of Northern intermediate goods, leads to higher world growth.

Kremer (1993) in the other hand constructs a model in which population growth could affect the long-run rate of growth. He first shows that the growth rate of population is proportional to the level of population. In his model, each person's chance to be smart enough to invent something is independent of population. However even if each person's productivity is independent of population, total research output will increase with population and so he argues that trade would increase the population size and therefore could raise the rate of technology improvements. Similarly Backus, Kehoe, and Kehoe (1992) studied the scale effects of trade on growth by deriving the theoretical relations between scale and growth in 3 different models where growth is driven from learning by doing, investment in human capital and from development of specialized inputs to production. They further examine the scale effects predicted by those theories of trade and growth. Their evidence however showed mixed support for scale effects of trade on growth.

Lucas (1988) extends his one-sector model of accidental learning by doing to a two-good model and examines the roles of human capital accumulation in international trade. He considers two sectors producing two consumptions good using only labor as input. However workers can accumulate experience or human capital by working in a firm. The technologies are of Ricardian type in which the output of a good is equal to the efficiency units of labor input. He considers one sector to be the "high-technology" sector in which $a_1 > a_2$, where a_i

>0 is a measure of the efficiency of learning. If consumption goods are poor substitutes then the steady state tends to be stable with diversification in production of two goods, when economy is closed. Lucas (1993) then extends this model to trade, assuming a continuum of small countries facing exogenously given world prices under free trade. The comparative advantage of a country depends on the country's autarkic price ratio and the world's price ratio. Countries tend to be completely specialized, but each country will accumulate only the type of human capital that is specific to the good produced. Therefore when different countries produce different goods under free trade and so they will have different growth rates. Other models of trade with learning by doing also have been suggested. Young (1991) considers accidental learning and allows for spillovers across goods. He however shows that when Less Developed Countries (LDCs) open to trade, these countries will specialize in traditional goods where learning has been exhausted. Therefore they experience lower growth rates under free trade relative to the autarky.

All these models provide important insights in relation between trade and rate of economic growth however in most of these studies comparative advantage plays no role. Comparative advantage might define the pattern of trade in some of these studies such as Grossman and Helpman (1991) or Acemoglu and Ventura (2001), however the effect of trade on growth comes from channels other than the effect of comparative advantage mechanism similar to the one we consider to raises the income level in static models without growth. Seater (2007) however proposes an approach in which trade, in and of itself, raises growth through the comparative advantage by considering a model in which there is no scale effects, no research and development or no technology transfer and no international investment. He considers a two sector endogenous growth model in which two goods are being produced using two type of reproducible input. His results imply that comparative advantage guarantees trade, however the effect of trade on growth depends on the absolute advantages of the trading countries. When each country has an absolute advantage in producing something, trade could raise both countries' growth rates and replicates the result that would emerge if countries exchanged technologies, even though no technology transfer actually takes place. In the following section we describe his model and results with more detail.

1.3 THE MODEL

1.3.1 THE MODEL IN THE ABSENCE OF TRADE

In this section without getting into the details, we first summarize the model for the closed economy, which has been studied by Barro and Sala-I-Martin (1995-ch5). We then continue by reviewing Seater's (2007) in which he has modified the model to study the effects of trade on growth.

Consider an economy that produces two goods, good *Y* that can be used as consumption good or as gross investment in physical capital *K*, and the good *Z*. Good *Z* could be used as only a type of capital (*H*), for instance human capital[1]. These two goods are assumed to be produced in different sectors and by different technologies. Also both reproducible factors are essential to production of both goods. Each sector's technology is assumed to be Cobb-Douglas. Therefore the production technologies could be written as,

$$Y_t = C_t + I_t \quad = A(v_t K_t)^{\alpha} (u_t H_t)^{1-\alpha} , \qquad (1)$$

[1] It does not need necessarily be human capital. It could be any other types of capital which is not K. It could also be thought of any capital that could augment labor but is not human capital. Example of such capital could be computers or computer software.

$$Z_i = \dot{H}_t + \delta H_t = B[(1-\nu_t)K_t]^\eta[(1-u_t)H_t]^{1-\eta}, \quad (2)$$

where A, $B > 0$ are technological parameters, α and η are the shares of K-type capital in the outputs of each sector, v and u are the fractions of K-type and H-type capital, respectively, used in the production of good Y [2]. Both type of capital are assume to depreciate at the same rate, δ.

The gross domestic product for the economy, Q, can be defined as

$$Q = Y + p(\dot{H} + \delta H), \quad (3)$$

where p is the price of H in terms of Y.

The utility function is assumed to be CRRA, therefore

$$U = \int_0^\infty \frac{C_t^{1-\theta} - 1}{1-\theta} e^{\rho t} dt, \quad (4)$$

is the lifetime utility.

The representative agent maximizes (4) subject to (1) and (2). This problem is a standard model of household optimization and therefore we can write the Hamiltonian as,

$$V_t = \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \phi_t\left[A \cdot (\nu_t K_t)^\alpha \cdot (u_t H_t)^{1-\alpha} - \delta K_t - C_t\right]$$
$$+ \psi_t\left[B \cdot [(1-\nu_t) \cdot K_t]^\eta \cdot [(1-u_t)H_t)]^{1-\eta} - \delta H_t\right]. \quad (5)$$

Using the first-order conditions, the growth rate of consumption could be obtained as[3]

$$\gamma_C = \frac{1}{\theta}\left[\alpha A\left(\frac{\nu K}{uH}\right)^{-(1-\alpha)} - \delta - \rho\right]. \quad (6)$$

[2] Note that here it is assumed that labor is a constant normalized to be one.
[3] Since the details for deriving the solution has been provided in Seater (2007) we only present the final solutions.

On the balanced growth path the growth rates of *K*, *H*, *Y* and *Q* will all be equal to the growth rate of *C* (see Seater 2007) for details, and the common growth rate can be calculated as

$$\gamma = \frac{1}{\theta}\left[A^{\frac{\eta}{1-\alpha+\eta}} B^{\frac{1-\alpha}{1-\alpha+\eta}} \alpha^{\frac{\alpha\eta}{1-\alpha+\eta}} (1-\alpha)^{\frac{(1-\alpha)\eta}{1-\alpha+\eta}} \eta^{\frac{(1-\alpha)\eta}{1-\alpha+\eta}} (1-\eta)^{\frac{(1-\alpha)(1-\eta)}{1-\alpha+\eta}} - \delta - \rho \right], \qquad (7)$$

where the value of p is

$$p = \left(\frac{A}{B}\right)\left(\frac{\alpha}{\eta}\right)^{\eta}\left(\frac{1-\alpha}{1-\eta}\right)^{1-\eta}\left[\left(\frac{A}{B}\right)^{\frac{1}{1-\alpha+\eta}}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\alpha+\eta}}\left(\frac{\alpha}{\eta}\right)^{\frac{\eta}{1-\alpha+\eta}}\left(\frac{1-\alpha}{1-\eta}\right)^{\frac{1-\eta}{1-\alpha+\eta}}\right]^{\alpha-\eta}. \qquad (8)$$

As mentioned this is the model where the county is assumed to be closed. Next we discuss how Seater (2007) extends this model to include trade.

1.3.2 INTRODUCING TRADE, THE SIMPLE MODEL

To introduce foreign trade into the model Seater (2007) begins with a simple case. He considers a Ricardian type model of trade with two countries having different technologies and no fixed factors. In addition he assumes that the factor share for *K* (*H*) to be the same within two sectors of production as well as between the two countries. In the other word he assumes,

$$\alpha = \eta \Rightarrow 1-\alpha = 1-\eta$$

This restriction implies that the cross-country and cross-industry technology differences will be captured only by the total factor productivity parameters *A* and *B*. In the general case where $\alpha \neq \eta$, it could be shown that the following relation between v, u, α, and η holds,

$$\left(\frac{\eta}{1-\eta}\right)\left(\frac{\nu}{1-\nu}\right) = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{u}{1-u}\right) \tag{9}$$

Therefore assuming $\alpha = \eta$ implies $v = u$. This indeed simplifies the production functions for *Y* and *H* as,

$$Y_i = A_i \cdot v_i \cdot K_i^{\alpha} \cdot H_i^{1-\alpha},$$

$$Z_i = \dot{H}_i + \delta H_i = B_i \cdot (1-\nu_i) \cdot K_i^{\alpha} \cdot H_i^{1-\alpha},$$

where *i* indicates the country.

Now the optimization problem is to maximize the lifetime utility function described in equation (4) subject to these modified production functions. Similarly by using the first order and necessary conditions of such optimization problem the autarkic growth rate for each country[4] will be obtain as,

$$\gamma_{i,T} = \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{(1-\alpha)} A_i \left(\frac{A_i}{B_i}\right)^{\alpha-1} - \delta - \rho\right] \tag{10}$$

To study the effect of trade, Seater (2007) considers two relatively large countries such that their population is fixed and of equal size and each country consists of a large number of competitive firms with identical production functions. When countries are open they could trade in both goods freely. There are however no international lending and borrowing. Consider *X* to be the exports of good *Y* and if *p* still represents the relative price of *H* in terms of *Y*, then the constraints for country *i* would be modified as follow,

$$Y_i = C_i + \dot{K}_i + \delta K_i + X_i$$

$$= A_i v_i K_i^{\alpha} H_i^{1-\alpha}, \tag{11}$$

[4] For details please refer to Seater (2007).

$$Z_i = \dot{H}_i + \delta H_i = B_i(1-\nu_i)K_i^{\eta} H_i^{1-\eta} + \frac{1}{p} X_i \,. \tag{12}$$

Now each county should choose X_i along with everything else. Notice that neither of countries is small and they are related to each other through trade, therefore we need to determine the growth rate for countries simultaneously. To do this, Seater (2007) solves the problem in two steps. First he solves each country's problem taking p as given and then imposes the trade balance condition $X_1 = -X_2$ to find the equilibrium value for p.

In each country the representative agent maximizes (4) subject to (11) and (12), and therefore the Hamiltonian is,

$$V_i = \frac{C_i^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \phi_i \left[A_i \cdot \nu_i \cdot K_i^{\alpha} \cdot H_i^{1-\alpha} - C_i - \delta K_i - X_i \right]$$

$$+ \psi_i \left[B_i \cdot (1-\nu_i) \cdot K_i^{\alpha} \cdot H_i^{1-\alpha} - \delta H_i + \frac{1}{p} X_i \right]. \tag{13}$$

First order and necessary conditions could be obtained as the usual forms. However there is one condition that needs a little more attention. Consider the first order condition with respect to X_i ,

$$\frac{\partial V_i}{\partial X_i} = -\phi_i + \frac{1}{p} \psi_i = 0 \,. \tag{14}$$

As it can be seen this equation does not depend on any control variable. This is the standard case of bang-bang control for X_i, which in turn implies that equation (14) might not be always satisfied. Notice that when countries are engage in trade with each other p represents the international price of H in terms of Y. Now we know that the costate variables φ_i and ψ_i are, respectively, country i's marginal value of Y-good and H-good (Z) from internal production. Therefore their ratio is the marginal value of *H* in terms of *Y* which is the country *i*'s internal price of *H* in terms of *Y*. Therefore in general these two might not be equal to each other. Consequently we can consider 3 cases as follow,

i. $\frac{\partial V_i}{\partial X_i} = -\phi_i + \frac{1}{p}\psi_i > 0 \Rightarrow \frac{\psi_i}{\phi_i} > p$

ii. $\frac{\partial V_i}{\partial X_i} = -\phi_i + \frac{1}{p}\psi_i = 0 \Rightarrow \frac{\psi_i}{\phi} = p$

iii. $\frac{\partial V_i}{\partial X_i} = -\phi_i + \frac{1}{p}\psi_i < 0 \Rightarrow \frac{\psi_i}{\phi} < p$

The first case where $\frac{\psi_i}{\phi_i} > p$ implies that the marginal value of *X* is positive regardless of the value of *X* therefore country *i* would choose *X* as high as possible. In other words it will choose to produce only *Y* and export it to obtain *H*. However if $\frac{\psi_i}{\phi_i} < p$, then the reverse will happen, in this case the marginal value of *X* will always be negative and therefore country *i* will choose to set *X* as low as possible and therefore produce only *H* and trade it to obtain *Y*. Finally if $\frac{\psi_i}{\phi_i} = p$ the marginal value of *X* will always be zero. Therefore the country would be indifferent to the value of *X*. When $\alpha = \eta$, it could be shown that $\frac{\psi_i}{\phi_i} = \frac{A_i}{B_i}$ [5]. Considering this equality the bang-bang condition could be rewritten as, 1)

[5] It could be easily obtained by putting $\alpha = \eta$ in equation (14). In addition Seater (2007) has obtained this equality by using the first order and necessary conditions of the optimization problem for this case.

country i specializes in producing Y when $\frac{A_i}{B_i} > p$, 2) when it $\frac{A_i}{B_i} = p$ then the country continue to operate in both sections and 3) it will specialize in producing H when $\frac{A_i}{B_i} < p$.

In order for countries to be interested in trade with each other, the equilibrium world price should fall between $\frac{A_1}{B_1}$ and $\frac{A_2}{B_2}$, internal prices of country 1 and country 2 respectively. Without loss of generality, it could be assumed that $\frac{A_1}{B_1} > \frac{A_2}{B_2}$[6] and so the necessary condition for trade is, $\frac{A_2}{B_2} \leq p \leq \frac{A_1}{B_1}$. According to what we explained above, the latter condition implies that country one specializes in producing good Y and country 2 specializes in producing good H. Now having this information the optimization problem for each country can be rewritten and solved for each country. Here we only present the results of such problems[7].

Country 1 specializes in producing Y and trade this to obtain H, which implies country 1 sets v=u=1. The Hamiltonian for this country can be written as

$$V_1 = \frac{C_1^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \phi_1 \left[A_1 \cdot K_1^{\alpha} \cdot H_1^{1-\alpha} - C_1 - \delta K_1 - X_1 \right] + \psi_1 \left[-\delta H_1 + \frac{1}{p} X_1 \right] \qquad (15)$$

and country 2 does the opposite therefore it sets v=u=1. The Hamiltonian for this country is then,

[6] The reverse could be assumed and the only difference would be that our results will be reverse for the countries.
[7] More detail could be found in Seater (2007).

$$V_2 = \frac{C_2^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \phi_2 \left[-C_2 - \delta K_2 - X_2 \right] + \psi_2 \left[B_2 \cdot K_2{}^{\alpha} \cdot H_2{}^{1-\alpha} - \delta H_2 + \frac{1}{p} X_2 \right] \qquad (16)$$

When countries are open to trade with each other, they should do so at the world price *p*.

Therefore for each country now $\frac{\psi_i}{\phi_i}$ is equal to *p*.

Now, using the necessary and first-order conditions the growth rate for each country will be obtained as,

$$\gamma_{1,T} = \frac{1}{\theta} \left[\alpha^{\alpha} (1-\alpha)^{(1-\alpha)} A_1 \left(\frac{1}{p} \right)^{1-\alpha} - \delta - \rho \right] \qquad (17)$$

$$\gamma_{2,T} = \frac{1}{\theta} \left[\alpha^{\alpha} (1-\alpha)^{(1-\alpha)} B_2 P^{\alpha} - \delta - \rho \right] \qquad (18)$$

For any $p \in \left[\frac{A_2}{B_2} \ , \ \frac{A_1}{B_1} \right]$, it could be seen that these growth rates are greater than the growth rates under autarky. Consider country 1 for which $p < \frac{A_1}{B_1}$, then,

$$\gamma_{1,T} = \frac{1}{\theta} \left[\alpha^{\alpha} (1-\alpha)^{(1-\alpha)} A_1 p^{\alpha-1} - \delta - \rho \right] > \frac{1}{\theta} \left[\alpha^{\alpha} (1-\alpha)^{(1-\alpha)} A_1 \left(\frac{A_1}{B_1} \right)^{\alpha-1} - \delta - \rho \right] = \gamma_{1,Au} \,,$$

For country 2, we have $p > \frac{A_2}{B_2}$, therefore,

$$\gamma_{2,T} = \frac{1}{\theta} \left[\alpha^{\alpha} (1-\alpha)^{(1-\alpha)} B_2 p^{\alpha} - \delta - \rho \right] > \frac{1}{\theta} \left[\alpha^{\alpha} (1-\alpha)^{(1-\alpha)} B_2 \left(\frac{A_2}{B_2} \right)^{\alpha} - \delta - \rho \right] = \gamma_{2,Au}$$

These results imply that trade could raise the growth rates of both countries for any $p \in \left[\frac{A_2}{B_2} \ , \ \frac{A_1}{B_1} \right]$. However notice that the growth rates are not equal for any arbitrary *p*.

To get equal growth rates it is necessary and sufficient for p to be equal to $\frac{A_1}{B_2}$. If this condition holds then the common growth rate would be,

$$\gamma \equiv \gamma_{1,T} = \gamma_{2,T} = \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha} A_1^{\alpha} B_2^{1-\alpha} - \delta - \rho\right] \tag{19}$$

Now from the constraints for country 1 and country 2 we have,

$$\dot{K}_1 = A_1 \cdot K_1^{\alpha} \cdot H_1^{1-\alpha} - C_1 - \delta K_1 - X_1 \Rightarrow \frac{\dot{K}_1}{K_1} = A_1\left(\frac{K_1}{H_1}\right)^{\alpha-1} - \frac{C_1}{K_1} - \frac{X_1}{K_1} - \delta \tag{20.1}$$

$$\dot{K}_2 = -C_2 - X_2 - \delta \Rightarrow \frac{\dot{K}_2}{K_2} = -\frac{C_2}{K_2} - \frac{X_2}{K_2} - \delta \tag{20.2}$$

In addition, the trade balance implies $X_1 = -X_2$. A balanced growth path for each individual country requires that the level of consumption and the stock of K and H capital grow at the same rate, i.e. $\gamma_C = \gamma_K = \gamma_H$. Therefore, all the terms on the right sides of equations (19) and (20) should be constant, which in turn requires that K_i, H_i, X_i, C_i and Y_i all grow at the same rate in each country. However notice that $X_1 = -X_2$, therefore for each country to have balanced growth individually, it is necessary that the two countries have the same growth rate on the balanced growth path. As discussed earlier the equality of growth rates could be achieved only when $p = \frac{A_1}{B_2}$.

It is important to note that this argument holds only when p falls in the interval of $\left[\frac{A_2}{B_2}, \frac{A_1}{B_1}\right]$,. However, there is no guarantee that $\frac{A_1}{B_2}$ fall between $\frac{A_1}{B_1}$ and $\frac{A_2}{B_2}$, and if

$\frac{A_1}{B_2}$ falls outside this interval, then *p* can not be equal to it, and balance growth path for the world as well as for individual counties could not exist.

To obtain the condition that guarantees the existence of a balance growth path, consider the case where $\frac{A_1}{B_2}$ falls inside the interval, i.e. $\frac{A_2}{B_2} < p = \frac{A_1}{B_2} < \frac{A_1}{B_1}$. This inequality implies that $A_1 > A_2$ and $B_2 > B_1$. As argued before the source of difference between technologies across countries or within the sections of production are from the differences between their total factor productivities *A* and *B*. $A_1 > A_2$ and $B_2 > B_1$ then means that country 1 has absolute advantage in producing *Y* and country 2 has absolute advantage in producing *H*.

Therefore the requirement for the existence of balance growth path for each country or for the world as a whole is that each country to have absolute advantage in producing something. In the model considered here, this condition implies that country 1 to have absolute advantage in producing good *Y* and country 2 have absolute advantage in producing good *H.* If this condition meet then growth rate of each country will be higher with free trade, balanced growth is possible and is asymptotically stable[8] and the common growth rate would the one described in (19). As it can be seen this common growth rate depends on the factor productivity parameter of efficient technology in each country, i.e. production of good *Y* in country 1 and production of good *H* in country 2. This in turn implies that each country abandon its inferior technology and substitute it by its trade partner's efficient technology.

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[8] See Seater (2007) for details.

Now consider where $\frac{A_1}{B_2}$ falls outside the interval, then p can not be equal to $\frac{A_1}{B_2}$ but it will be equal to the value of the boundaries whichever it is closer to. For instance if $\frac{A_2}{B_2} < \frac{A_1}{B_1} < \frac{A_1}{B_2}$ then p will be equal to $\frac{A_1}{B_1}$. In this case the growth rate of one country will stay the same as it was under autarky while the growth rate of the other country will rise, and growth rates of countries will not be equal.

For instance, suppose $\frac{A_1}{B_2} > \frac{A_1}{B_1}$. In this case p can not be equal to $\frac{A_1}{B_2}$, and so $p = \frac{A_1}{B_1}$. Now the growth rates for each country after trade would be obtained as

$$\gamma_{1,T} = \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{(1-\alpha)} A_1 p^{\alpha-1} - \delta - \rho\right] = \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{(1-\alpha)} A_1\left(\frac{A_1}{B_1}\right)^{\alpha-1} - \delta - \rho\right] = \gamma_{1,Au}\,, \quad (21)$$

and,

$$\gamma_{2,T} = \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{(1-\alpha)} B_2 p^{\alpha} - \delta - \rho\right] > \frac{1}{\theta}\left[\alpha^{\alpha}(1-\alpha)^{(1-\alpha)} B_2\left(\frac{A_2}{B_2}\right)^{\alpha} - \delta - \rho\right] = \gamma_{2,Au}\,. \quad (22)$$

Notice that we have assumed $\frac{A_1}{B_1} > \frac{A_2}{B_2}$, therefore $\gamma_{1,T} > \gamma_{2,T}$.

Now $\frac{A_1}{B_2} > \frac{A_1}{B_1} > \frac{A_2}{B_2}$ implies $A_1 > A_2$, and $B_1 > B_2$. This means that country 1 has an absolute advantage in both goods while country 2 has only a comparative advantage in producing *H*. In this case country 1 continues to produce both goods and its growth rate depends only on its own technology and it will stay the same as it was under autarky. However country 2 specializes in H, thus producing only H and exports it to obtain Y and so its growth rate rises but stays below that for country 1.

Case of Small Country: So far it has been assumed that countries are not small relative to each other. However Seater (2007) also studies the case where one of the countries is small relative to the rest of the world. As we know in the case of small country the world price would be exogenous to the small country and therefore unaffected by its choice. If the small country's TFP ratio ($\frac{A_s}{B_s}$) is greater than p, the country produces only Y and if it is smaller than p it will produce only H and the rest of the world will be unaffected by trade. In this case if small country has absolute advantage in producing one good and the world has absolute advantage in the other good then trade will raise small country's growth rate to the world's growth rate. However if small country does not have absolute advantage in any of the goods its growth rate will increase by trade but it remains below the world's growth rate. Finally if the small country has an absolute advantage in both goods then its growth rate will be higher than that for the world and it will be unaffected by trade.

1.3.3 TRADE AND GROWTH, THE GENERAL MODEL

In the previous section we reviewed a simple case of two good two country model that Seater (2007) has considered in studying the effect of trade on growth. As we described earlier, in his model Seater (2007) has assumed equality of factor shares of capitals between two sections of production as well as across countries. In this section, we will relax this assumption and examine the relationship between trade and economic performance in a more general case where the factor shares are not only different between two production sections but also across countries[9].

When countries are not open to trade with each other, the optimization problem for each country would be exactly the same as the one described in section 1.2. Assuming different factor shares between countries and production sections, the equations 1, 2, 7 and 8 represent the production function for each sector, growth rate and the price level on the balanced growth path in autarky. Again these equations are,

$$Y_i = C_i + I_i$$

$$= A_i (v_i K_i)^{\alpha_i} (u_i H_i)^{1-\alpha_i} \tag{23}$$

$$\dot{H}_i + \delta H_i = B_i [(1-v_i)K_i]^{\eta_i} [(1-u_i)H_i]^{1-\eta_i} \tag{24}$$

$$\gamma_i = \frac{1}{\theta}\left[A_i^{\frac{\eta_i}{1-\alpha_i+\eta_i}} B_i^{\frac{1-\alpha_i}{1-\alpha_i+\eta_i}} \alpha_i^{\frac{\alpha_i \eta_i}{1-\alpha_i+\eta_i}} (1-\alpha_i)^{\frac{(1-\alpha_i)\eta_i}{1-\alpha_i+\eta_i}} \eta_i^{\frac{(1-\alpha_i)\eta_i}{1-\alpha_i+\eta_i}} (1-\eta_i)^{\frac{(1-\alpha_i)(1-\eta_i)}{1-\alpha_i+\eta_i}} - \delta - \rho \right] \tag{25}$$

$$p_i = \left(\frac{A_i}{B_i}\right)\left(\frac{\alpha_i}{\eta_i}\right)^{\eta_i}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{1-\eta_i}\left[\left(\frac{A_i}{B_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{\eta_i}\right)^{\frac{\eta_i}{1-\alpha_i+\eta_i}}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{\frac{1-\eta_i}{1-\alpha_i+\eta_i}}\right]^{\alpha_i-\eta_i} \tag{26}$$

Notice here if we impose $\alpha = \eta$, we get the case that Seater (2007) has considered. As discussed in section 3.1.2 after countries open to free trade they specialize completely in the sector that has comparative advantage in and by doing so each country' s growth rate will rise and will become equal on the balanced growth path. These results however are the interior solution that will be guaranteed if each country has an absolute advantage in producing something. The corner solution would be somewhat different. For a country at the

[9] Other case where the factor share is only different between sectors or only across the countries has also been studied. Some of the results are being presented in appendix F.

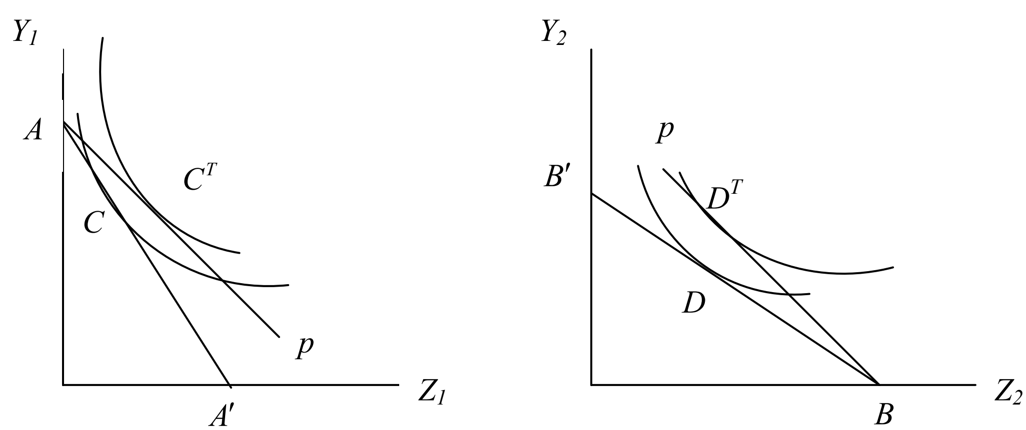
corner, that is the country whose autarkic price will be the same as the world price, trade would not raise its growth rate and it will continue to operate in both sections of production. The growth rate of the other country also will rise but it will stay below that of its trading partner if the country does not have absolute advantage in any production sector.

In this section then we are trying to examine if we could obtain these results after relaxing the assumption of equality of factor shares. As discussed earlier, in the case of interior solution both countries are going to completely specialize meaning that they shut down production of one of the sectors. Each country produces only one factor of production domestically and imports the other factor. This result is what we obtain in the classical Ricardian international trade model where there is only one factor of production and the productivity of labor in production of goods is the source of technology differences and therefore comparative advantage. In appendix 1.A we have shown that when $\alpha = \eta$ the production possibility frontier for each country become linear with a slope of $\frac{A_i}{B_i}$. Recall that A and B are the total factor productivity parameters in production of Y and Z respectively. It has be shown (Seater (2007)) that in this case the ratio of total factor productivity parameters represents the domestic price of good Z in units of good Y.

When $\alpha = \eta$, the production possibility frontier for each country has been illustrated in figure 1-1 (a) and (b) by line AA' for country 1 and BB' and country 2. When there is no trade, country 1 produces and consumes at point C and country 2 produces and consumes at point D. Notice that the slope of the production possibility frontier is the domestic relative price when countries are closed, i.e. $\frac{A_i}{B_i}$.

After countries open to trade with each other, the price level at which they trade will be different from that under autarky. For country 1 the world price is lower than the autarkic price ($p < \frac{A_1}{B_1}$) while it will be higher than the autarkic price for country 2 ($p > \frac{A_2}{B_2}$).

Country 1 then will set *X* (its export of *Y* good) as high as possible and produces at point A. At this point it produces only *Y* and exports it to obtain *Z.* Also its consumption point will move to point C^T as the result of trade. Instead country 2 will shut down the production of good *Y* and produces only good *Z* since it will set *X* as low as possible. The production point for this country moves to point B and its consumption point moves to D^T. Therefore results that have been obtained here are similar to what Ricardian model is predicted in a static setting.



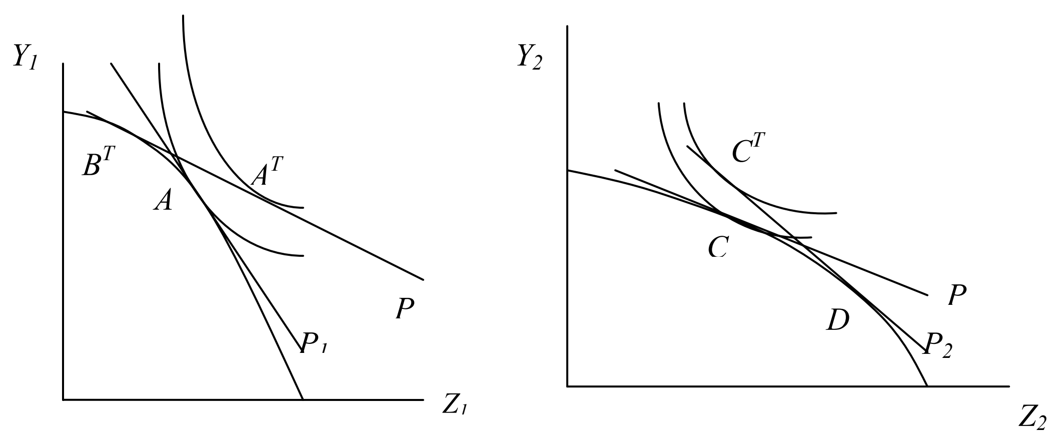
(a): Country 1 (b): Country 2

$$p < \frac{A_1}{B_1} \Rightarrow -\phi_1 + \frac{1}{p}\psi_1 < 0 \qquad\qquad p > \frac{A_2}{B_2} \Rightarrow -\phi_2 + \frac{1}{p}\psi_2 < 0$$

Figure 1-1: Production possibility frontiers for the case where $\alpha = \eta$

Now consider the case where $\alpha \neq \eta$. It could be shown that for this case the production possibility frontiers are no longer linear[10]. Figure 1-2 (a) and (b) illustrate the production possibilities frontiers for each country when $\alpha \neq \eta$.

Without loss of generality, we again assume that $p_1 > p_2$ where p_1 and p_2 are the autarkic prices for country1 and country 2 respectively. From the principles of (static) international trade when countries are not open to trade with each other, then each country will produce and consume where the slope of production possibility frontier is equal to its domestic price of the country under autarky. This is shown by point A for country 1 and by point C for country 2. When countries engage in international trade then the world price would be p, such that $p_2 < p < p_1$.



(a): Country 1

(b): Country 2

Figure 1-2: Production possibility frontiers for the case where $\alpha \neq \eta$

[10] See Appendix 1.B for more detail.

After trade then the price level is going to change for each country. Each country's production point will now move to the point at which the price line p is tangent to the production possibility frontier. Country 1's production point will move to point B (figure 2-2 (a))and country 2's production point will move to point D(figure 2-2 (b)). In the other words country 1 will produce more of good Y and export it in order to import good H (incomplete specialization in good Y). Country 2 will do the opposite, produce more of good H and export it to obtain good Y (incomplete specialization in good H).

By solving the dynamic optimization problem for the case where $\alpha = \eta$ the production pattern on the balanced growth path was similar to what static Ricardian model of international trade predicts. Next we examine how these results might change when we relax the assumption of equality of factor shares.

Although we will no longer assume the equality of factor shares between the sectors and countries but we continue to assume that countries are not small relative to each other, they have fixed population of equal size and each country consists of a large number of competitive firms with identical production functions.

Similarly let X denotes exports of Y in each country. The accumulation constraints then are,

$$Y_i = C_i + \dot{K}_i + \delta K_i + X_i ,$$

$$= A_i (v_i K_i)^{\alpha_i} (u_i H_i)^{1-\alpha_i} , \qquad (27)$$

$$Z_i = \dot{H}_i + \delta H_i = B_i [(1 - v_i) K_i]^{\eta_i} [(1 - u_i) H_i]^{1-\eta_i} + \frac{1}{p} X_i . \qquad (28)$$

In each country the representative agent will maximize its lifetime utility function as described in (4) subject to (27) and (28). Similarly each country must choose *X* along with everything else and the equilibrium value of *p* will depend on the decision of both countries. To solve this dynamic problem we will follow Seater (2007) and will find the world equilibrium in two steps. First, we try to solve each country's optimization problem assuming *p* is given. Then in the second step we will impose the trade balance condition $X_1 = - X_2$ to find the value for equilibrium *p.*

The Hamiltonian and the necessary and first order conditions are as follow,

$$V_i = \frac{C_i^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \phi_i \left[A_i \cdot (v_i K_i)^{\alpha_i} \cdot (u_i H_i)^{1-\alpha_i} - C_i - \delta K_i - X_i \right]$$

$$+ \psi_i \left[B_i \left((1-v_i) K_i\right)^{\eta_i} \left((1-u_i) H_i\right)^{1-\eta_i} - \delta H_i + \frac{1}{p} X_i \right], \qquad (29.1)$$

$$\dot{K}_i = \frac{\partial V_i}{\partial \phi_i} = A_i \cdot (v_i K_i)^{\alpha_i} \cdot (u_i H_i)^{1-\alpha_i} - C_i - \delta K_i - X_i, \qquad (29.2)$$

$$\dot{H}_i = \frac{\partial V_i}{\partial \psi_i} = B_i \cdot \left((1-v_i) K_i\right)^{\eta_i} \cdot \left((1-u_i) H_i\right)^{1-\eta_i} - \delta H_i + \frac{1}{p} X_i, \qquad (29.3)$$

$$\dot{\phi}_i = -\frac{\partial V_i}{\partial K_i} = -\phi_i \left[\alpha_i \cdot v_i \cdot A_i \cdot (v_i K_i)^{\alpha_i - 1} \cdot (u_i H_i)^{1-\alpha_i} - \delta \right] - \psi_i \left[\eta_i \cdot (1-v_i) \cdot B_i \cdot \right.$$

$$\left. \left((1-v_i) K_i\right)^{\eta_i - 1} \cdot \left((1-u_i) H_i\right)^{1-\eta_i} \right], \qquad (29.4)$$

$$\dot{\psi}_i = -\frac{\partial V_i}{\partial H_i} = -\phi_i \left[(1-\alpha_i) \cdot u_i \cdot A_i \cdot (v_i K_i)^{\alpha_i} \cdot (u_i H_i)^{-\alpha_i} \right] - \psi_i \left[(1-\eta_i) \cdot (1-u_i) \cdot B_i \cdot \right.$$

$$\left. \left((1-v_i) K_i\right)^{\eta_i} \cdot \left((1-u_i) H_i\right)^{-\eta_i} - \delta \right], \qquad (29.5)$$

$$\frac{\partial V_i}{\partial C_i} = 0 \Rightarrow C_i^{-\theta} e^{-\rho t} - \phi_i = 0, \qquad (29.6)$$

$$\frac{\partial V_i}{\partial v_i} = 0 \Rightarrow \phi_i \left[\alpha_i \cdot K_i \cdot A_i \cdot (v_i K_i)^{\alpha_i - 1} \cdot (u_i H_i)^{1-\alpha_i}\right]$$

$$-\psi_i \left[\eta_i \cdot K_i \cdot B_i \cdot \left((1-v_i)K_i\right)^{\eta_i - 1} \cdot \left((1-u_i)H_i\right)^{1-\eta_i}\right] = 0, \tag{29.7}$$

$$\frac{\partial V_i}{\partial u_i} = 0 \Rightarrow \phi_i \left[(1-\alpha_i) \cdot H_i \cdot A_i \cdot (v_i K_i)^{\alpha_i} \cdot (u_i H_i)^{-\alpha_i}\right]$$

$$-\psi_i \left[(1-\eta_i) \cdot H_i \cdot B_i \cdot \left((1-v_i)K_i\right)^{\eta_i} \cdot \left((1-u_i)H_i\right)^{-\eta_i}\right] = 0, \tag{29.8}$$

$$\frac{\partial V_i}{\partial X_i} = 0 \Rightarrow -\phi_i - \frac{1}{p}\psi_i = 0 \,. \tag{29.9}$$

Notice that equation (29.9) does not depend on any control variable, indicating bang-bang control for X_i. Again the costate ϕ_i is the marginal value of good Y and ψ_i is the marginal value of good H from internal production. Therefore $\frac{\psi_i}{\phi_i}$ represents the marginal value of good Z in terms of good Y that is the internal price of good Z in units of good Y.

Having the bang-bang control for X_i, we can consider 3 cases 1) $\frac{\partial V_i}{\partial X_i} > 0 \Rightarrow -\phi_i - \frac{1}{p}\psi_i > 0 \Rightarrow p < \frac{\psi_i}{\phi_i}$, where country i will try to set X_i as high as possible and if $\frac{\partial V_i}{\partial X_i} < 0 \Rightarrow -\phi_i - \frac{1}{p}\psi_i < 0 \Rightarrow p > \frac{\psi_i}{\phi_i}$, country i will try to set X_i as low as possible, and finally if $\frac{\partial V_i}{\partial X_i} = 0 \Rightarrow -\phi_i - \frac{1}{p}\psi_i = 0 \Rightarrow p = \frac{\psi_i}{\phi_i}$ country i is indifferent with respect to the value of X_i. Similarly we assume $p_1 > p_2$ and since we are interested in the case where trade occurs, then $p_2 < p < p_1$. Under this assumption then country 1 will try to produce more of good Y and export it to obtain Z and country 2 will try to produce more of good Z and export it to obtain Y (Figure 2-2(a) and (b)). Note again that after countries open to trade they do so at the world prices, therefore with trade $\frac{\psi_i}{\phi_i} = p$.

From equation (29.6) we have:

$$\frac{\partial V_i}{\partial C_i} = 0 \Rightarrow C_i^{-\theta} e^{-\rho t} - \phi_i = 0 \Rightarrow \frac{\dot{C}_i}{C_i} = \gamma_i = -\frac{1}{\theta}\left[\frac{\dot{\phi}_i}{\phi_i} + \rho\right]. \tag{30}$$

Using equation (29.4) and (29.7) we can show that[11],

$$\frac{\dot{\phi}_i}{\phi_i} = -\alpha_i \cdot A_i \cdot \left(\frac{v_i K_i}{u_i H_i}\right)^{\alpha_i - 1} + \delta , \tag{31}$$

Substituting for $\frac{\dot{\phi}_i}{\phi_i}$ from (31) into equation (30) the growth rate of consumption for country 1 could be written as,

$$\gamma_{C_1} = \frac{1}{\theta}\left[\alpha_1 \cdot A_1 \cdot \left(\frac{v_1 K_1}{u_1 H_1}\right)^{\alpha_1 - 1} - \delta - \rho\right]. \tag{32}$$

It also can be shown that the growth rates of K, H, Y and Q all are equal to the growth rate of C[12]. As described above with international trade, $p = \frac{\psi_i}{\phi_i}$, therefore $\gamma_p = \gamma_\phi - \gamma_\psi$. Note that trade balance requires $p \in [p_2 , p_1]$ (assuming $p_1 > p_2$), where the autarkic price level for each country is,

$$p_i = \left(\frac{A_i}{B_i}\right)\left(\frac{\alpha_i}{\eta_i}\right)^{\eta_i}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{1-\eta_i}\left[\left(\frac{A_i}{B_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{\eta_i}\right)^{\frac{\eta_i}{1-\alpha_i+\eta_i}}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{\frac{1-\eta_i}{1-\alpha_i+\eta_i}}\right]^{\alpha_i-\eta_i} .$$

[11] See appendix 1.C-I for details.
[12] See appendix 1.D.

Therefore the equilibrium world level of prices *p* must fall in a constant interval. Also a balanced growth path requires everything that grows should do so at a constant rate. Therefore *p* as well must grow at a constant rate along the balanced growth path. These two conditions together imply that the only rate that *p* could grow at and stay in a constant interval is zero. Now if $\gamma_p = 0$ then $\gamma_p = \gamma_\phi - \gamma_\psi = 0$.

Using (29.4), (29.5), (29.7) and (29.8) and zero growth rate of price level on the balanced growth path, the growth rates of costate variables can be obtain as[13],

$$\frac{\dot{\phi}_i}{\phi_i} = -\alpha_i \cdot A_i \cdot \left(\frac{\nu_i K_i}{u_i H_i}\right)^{\alpha_i - 1} + \delta\,, \tag{33}$$

and,

$$\frac{\dot{\psi}_i}{\psi_i} = -\frac{1}{p} \cdot (1-\alpha_i) \cdot A_i \cdot \left(\frac{\nu_i K_i}{u_i H_i}\right)^{\alpha_i} + \delta\,. \tag{34}$$

As explained these growth rates are equal on the balanced growth path. Therefore by equating them for country 1 we can obtain,

$$\frac{\nu_1 K_1}{u_1 H_1} = p \cdot \frac{\alpha_1}{1-\alpha_1}\,, \tag{35}$$

Substituting for the capital ratios from equation (35) into (32), we get the growth rate for country 1 in the presence of trade as,

$$\gamma_{1,T} = \frac{1}{\theta}\left[\alpha_1 \cdot A_1 \cdot \left(p \cdot \frac{\alpha_1}{1-\alpha_1}\right)^{\alpha_1 - 1} - \delta - \rho\right],$$

or,

[13] Details are presented in appendix 1.C.

$$\gamma_{1,T} = \frac{1}{\theta}\left[\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{(1-\alpha_1)} \cdot A_1 . p^{\alpha_1 - 1} - \delta - \rho\right]. \tag{36}$$

Similarly, in Appendix 1.C we have shown the growth rates of φ and ψ could also be obtained as,

$$\frac{\dot{\phi}_i}{\phi_i} = -p \cdot \eta_i \cdot B_i \cdot \left(\frac{(1-v_i)K_i}{(1-u_i)H_i}\right)^{\eta_i - 1} + \delta, \tag{37}$$

and,

$$\frac{\dot{\psi}_i}{\psi_i} = -\left[(1-\eta_i) \cdot B_i \cdot \left(\frac{(1-v_i)K_i}{(1-u_i)H_i}\right)^{\eta_i} - \delta\right]. \tag{38}$$

Equating equations (37) and (38) we get,

$$\frac{(1-v_i)K_i}{(1-u_i)H_i} = p \cdot \frac{\eta_i}{1-\eta_i}. \tag{39}$$

The growth rate for consumption as shown in (30) is,

$$\frac{\dot{C}_i}{C_i} = \gamma_i = -\frac{1}{\theta}\left[\frac{\dot{\phi}_i}{\phi_i} + \rho\right]. \tag{40}$$

Then substitute (39) into (37) for country 2 we get,

$$\frac{\dot{\phi}_2}{\phi_2} = -p \cdot \eta_2 \cdot B_2 \cdot \left(p\frac{\eta_2}{1-\eta_2}\right)^{\eta_2 - 1} + \delta, \tag{41}$$

and then substituting (41) into (40) the growth rate of country 2 will become,

$$\gamma_{C_2} = \frac{1}{\theta}\left[p \cdot \eta_2 \cdot B_2\left(p\frac{\eta_2}{1-\eta_2}\right)^{\eta_2 - 1} - \delta - \rho\right], \tag{42}$$

or alternatively,

$$\gamma_{2,T} = \frac{1}{\theta}\left[\eta_2^{\eta_2} . (1-\eta_2)^{(1-\eta_2)} \cdot B_2 \cdot p^{\eta_2} - \delta - \rho\right]. \tag{43}$$

The growth rates for both countries are greater than the growth rates under autarky for any p in the interval (p_2 , p_1). To see this, remember the growth rate and the price level under autarky are,

$$\gamma_i = \frac{1}{\theta}\left[A_i^{\frac{\eta_i}{1-\alpha_i+\eta_i}} B_i^{\frac{1-\alpha_i}{1-\alpha_i+\eta_i}} \alpha_i^{\frac{\alpha_i\eta_i}{1-\alpha_i+\eta_i}} (1-\alpha_i)^{\frac{(1-\alpha_i)\eta_i}{1-\alpha_i+\eta_i}} \eta_i^{\frac{(1-\alpha_i)\eta_i}{1-\alpha_i+\eta_i}} (1-\eta_i)^{\frac{(1-\alpha_i)(1-\eta_i)}{1-\alpha_i+\eta_i}} - \delta - \rho \right],$$

$$p_i = \left(\frac{A_i}{B_i}\right)\left(\frac{\alpha_i}{\eta_i}\right)^{\eta_i}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{1-\eta_i}\left[\left(\frac{A_i}{B_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{\eta_i}\right)^{\frac{\eta_i}{1-\alpha_i+\eta_i}}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{\frac{1-\eta_i}{1-\alpha_i+\eta_i}}\right]^{\alpha_i-\eta_i}.$$

Appendix 1.E.I and 1.E.II show that we can simplify the autarkic growth rate for each country as,

$$\gamma_{1,Au} = \frac{1}{\theta}\left[\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{1-\alpha_1} \cdot A_1 \cdot p^{\alpha_1-1} - \delta - \rho\right],$$

$$\gamma_{2,Au} = \frac{1}{\theta}\left[\eta_2^{\eta_2} \cdot (1-\eta_2)^{1-\eta_2} \cdot B_2 \cdot p^{\eta_2} - \delta - \rho\right].$$

Now $p < p_1$, then

$$\gamma_{1,T} = \frac{1}{\theta}\left[\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{(1-\alpha_1)} \cdot A_1 . p^{\alpha_1-1} - \delta - \rho\right],$$

$$= \frac{1}{\theta}\left[\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{(1-\alpha_1)} \cdot A_1 . \left(\frac{1}{p}\right)^{1-\alpha_1} - \delta - \rho\right],$$

$$> \frac{1}{\theta}\left[\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{(1-\alpha_1)} \cdot A_1 . \left(\frac{1}{p_1}\right)^{1-\alpha_1} - \delta - \rho\right] = \gamma_{1,Au}.$$

also, $p_2 < p$, then

$$\gamma_{2,T} = \frac{1}{\theta}\left[\eta_2^{\eta_2} . (1-\eta_2)^{(1-\eta_2)} \cdot B_2 \cdot p^{\eta_2} - \delta - \rho\right],$$

$$> \frac{1}{\theta}\left[\eta_2{}^{\eta_2} .(1-\eta_2)^{(1-\eta_2)} \cdot B_2 \cdot p_2{}^{\eta_2} - \delta - \rho\right] = \gamma_{2,Au} .$$

These inequalities imply that trade will raise the growth rates of both countries for any $p \in (p_2 , p_1)$. However we should notice that the growth rates in (36) and (43) are not equal for any arbitrary *p*. The price level at which the growth rates in two countries will be equal can be obtained by equating the growth rates in (36) and (43), and that price level will then be,

$$\gamma_{1,T} = \gamma_{2,T} \Rightarrow p = p^* = \left(\frac{\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{(1-\alpha_1)}}{\eta_2^{\eta_2} \cdot (1-\eta_2)^{(1-\eta_2)}} \cdot \frac{A_1}{B_2} \right)^{\frac{1}{1-\alpha_1+\eta_2}} . \tag{44}$$

By substituting this value into equations (36) and (43) we can get the equilibrium growth rate as,

$$\gamma \equiv \gamma_{1,T} = \gamma_{2,T} = \frac{1}{\theta}\left[\alpha_1^{\frac{\alpha_1\eta_2}{1-\alpha_1+\eta_2}} \cdot (1-\alpha_1)^{\frac{(1-\alpha_1)\eta_2}{1-\alpha_1+\eta_2}} \cdot \eta_2^{\frac{(1-\alpha_1)\eta_2}{1-\alpha_1+\eta_2}} \cdot (1-\eta_2)^{\frac{(1-\alpha_1)(1-\eta_2)}{1-\alpha_1+\eta_2}} \cdot A_1^{\frac{\eta_2}{1-\alpha_1+\eta_2}} \cdot B_2^{\frac{(1-\alpha_1)}{1-\alpha_1+\eta_2}} - \delta - \rho \right]. \tag{45}$$

Now consider the growth rates for K_1 and K_2,

$$\dot{K}_1 = A_1 \cdot (v_1K_1)^{\alpha_1} \cdot (u_1H_1)^{1-\alpha_1} - C_1 - \delta K_1 - X_1 \rightarrow \frac{\dot{K}_1}{K_1} = A_1 \cdot \left(\frac{v_1K_1}{u_1H_1}\right)^{\alpha_1-1} - \frac{C_1}{K_1} - \delta - \frac{X_1}{K_1}, \tag{46}$$

$$\dot{K}_2 = A_2 \cdot (v_2K_2)^{\alpha_2} \cdot (u_2H_2)^{1-\alpha_2} - C_2 - \delta K_2 - X_2 \rightarrow \frac{\dot{K}_2}{K_2} = A_2 \cdot \left(\frac{v_2K_2}{u_2H_2}\right)^{\alpha_2-1} - \frac{C_2}{K_2} - \delta - \frac{X_2}{K_2}. \tag{47}$$

If we impose the trade balance condition, i.e. $X_1 = -X_2$, then we can rewrite the equation (47) as,

$$\frac{\dot{K}_2}{K_2} = A_2 \cdot \left(\frac{v_2K_2}{u_2H_2}\right)^{\alpha_2-1} - \frac{C_2}{K_2} - \delta + \frac{X_1}{K_2}. \tag{48}$$

We know that on the balanced growth path everything that grows should do so at a constant rate, therefore the growth rate of K_1 and K_2 also should be constant along this path. Using (46) and (48) it could be shown that 1) $\gamma_{C_1} = \gamma_{K_1} = \gamma_{H_1} = \gamma_{X_1}$, and 2) $\gamma_{C_2} = \gamma_{K_2} = \gamma_{H_2} = \gamma_{X_1}$ [14]. These two conditions therefore require that the growth rates of *K*, *H*, *C*, *Y* and *X* should be the same in two countries. In the other world it is necessary that the two countries have the same growth rate in order for each country to have balanced growth individually. Therefore for the countries and world to have a balanced growth *p* should be at the level stated in (44).

The balanced growth in this case is globally asymptotically stable. To see that take derivative from (36) and (43)

$$\frac{d\gamma_{1,T}}{dP} = -\frac{1}{\theta}\left[\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{(1-\alpha_1)} \cdot A_1 \cdot (1-\alpha_1) \cdot p^{\alpha_1 - 2}\right] < 0$$

$$\frac{d\gamma_{2,T}}{dp} = \frac{1}{\theta}\left[\eta_2^{\eta_2} \cdot (1-\eta_2)^{(1-\eta_2)} \cdot B_2 \cdot \eta_2 \cdot p^{\eta_2 - 1}\right] > 0$$

Now, when $p > \left(\frac{\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{(1-\alpha_1)}}{\eta_2^{\eta_2} \cdot (1-\eta_2)^{(1-\eta_2)}} \cdot \frac{A_1}{B_2}\right)^{\frac{1}{1-\alpha_1+\eta_2}}$, then $\gamma_{1,T}$ will be lower and $\gamma_{2,T}$ will be higher compared to their balanced growth rate. In the other word, *K* will grow slower than *H* and therefore *p* will start to fall, and if $p < \left(\frac{\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{(1-\alpha_1)}}{\eta_2^{\eta_2} \cdot (1-\eta_2)^{(1-\eta_2)}} \cdot \frac{A_1}{B_2}\right)^{\frac{1}{1-\alpha_1+\eta_2}}$ the opposite will happen, $\gamma_{1,T}$ rises and $\gamma_{2,T}$ falls, which makes *K* grows faster than *H* and therefore *p* starts to rise.

[14] See Appendix 1.F for more details.

We already have shown that the production possibility frontier would be concave if $\alpha \neq \eta$. In the static framework usually the result is incomplete specialization in production of goods. For instance in the Heckscher-Ohlin model where there are two goods, two countries and two factors of production predicts that each country would produce more of the good that uses the abundant factor intensively and export this good to obtain the other good. Now consider the equilibrium growth rate stated in equation (45). As we can see this growth rate depends only on the technology parameters of country 1's *Y* sector (α_1 and A_1) and the technology parameters of *H* sector of country 2 (η_2 and B_2) . This result implies that country one which has comparative advantage in production of good *Y*, has abandoned its inferior technology, i.e. production of *H* and replaced it with the superior technology of its trading partner by importing good *H*. Similarly, country 2 which has comparative advantage in production of good *H*, has stopped the production of its disadvantage production, i.e. sector *Y* and replaced it by the more efficient technology of its trading partner through importing ***Y***.

These results could also be obtained by taking into account the complete specialization pattern on the balanced growth path when solving the optimization problem. Under our assumption $p_2 < p < p_1$, therefore the bang-bang control solution for *X* implies that country 2 is going to set its export of *Y* as low as possible and country 1 will set its export of *Y* as high as possible. Therefore we can set *v*=*u*=1 for this country and solve the optimization problem. The Hamiltonian for country 1 will be,

$$V_1 = \frac{C_1^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \phi_1 \left[A_1 \cdot K_1^{\alpha_1} \cdot H_1^{1-\alpha_1} - C_1 - \delta K_1 - X_1 \right] + \psi_1 \left[-\delta H_1 + \frac{1}{p} X_1 \right]. \qquad (49)$$

Also country 2 will shut down the production of section Y and so for this country v=u=0 and therefore its Hamiltonian will reduce to

$$V_2 = \frac{C_2^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \phi_2 \left[-C_2 - \delta K_2 - X_2 \right] + \psi_2 \left[B_2 K_2^{\eta_2} H_2^{1-\eta_2} - \delta H_2 + \frac{1}{p} X_2 \right]. \qquad (50)$$

It is shown in Appendix 1.G that by using the first order and necessary condition for each country's Hamiltonian stated in equations (49) and (50) the growth rate of each country will be obtained as those in equations (36) and (43).

Therefore we can see although the production possibility frontier is not linear in the general case but we still could get the complete specialization results. This is due to the fact that countries could trade in the goods that are being used as factors of production. Therefore in the long-run it might not be profitable for countries to produce both goods on the balanced growth path when they can instead import the good that uses the scarce factor intensively.

Again notice that for countries to be interested in trade the equilibrium world price p^* as stated in equation (44) needs to be between the autarky prices of country 1 and 2, i.e. $p_2 < p^* < p_1$. However there is no guarantee that p^* falls in this interval. If p^* falls outside of this interval then the equilibrium world price will not be equal to this value and the balanced growth will not exist for the world and individual countries. However if the price could take the values of the boundaries then when the equilibrium value (stated in equation (44)) falls outside this interval then the world price could not be equal to this value and instead it will be equal to the value at the boundary i.e. p_1 and p_2 whichever is closer to p^*. In this case one of the countries would be at the corner. The growth rate of the country whose autarkic price will be equal to the world price will not change with trade and so it will continue to operate in both sectors while the other country chooses to complete specialize in production of one sector and its growth rate will rise with trade. Note that the price of the country at the corner in this case remain unchanged and therefore for this country the necessary conditions of its optimization problem will not change and therefore its production pattern will be the same. However the other country's price rises with trade and so it will affect the necessary condition of the optimization problem as well as its production decision.

Remember again that the domestic prices for each country without trade (p_1 and p_2) is,

$$p_i = \left(\frac{A_i}{B_i}\right)\left(\frac{\alpha_i}{\eta_i}\right)^{\eta_i}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{1-\eta_i}\left[\left(\frac{A_i}{B_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{\eta_i}\right)^{\frac{\eta_i}{1-\alpha_i+\eta_i}}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{\frac{1-\eta_i}{1-\alpha_i+\eta_i}}\right]^{\alpha_i-\eta_i}.$$

It is easy to show that this price can be simplified as follows[15]

$$p_i = \left(\frac{\alpha_i^{\alpha_i}(1-\alpha_i)^{(1-\alpha_i)}}{\eta_i^{\eta_i}(1-\eta_i)^{1-\eta_i}}\cdot\frac{A_i}{B_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}$$

Therefore the condition to guarantee the existence of world balanced growth path could be written as $p_2 < p^* < p_1$, or

$$\left(\frac{\alpha_2^{\alpha_2}(1-\alpha_2)^{(1-\alpha_2)}}{\eta_2^{\eta_2}(1-\eta_2)^{1-\eta_2}}\cdot\frac{A_2}{B_2}\right)^{\frac{1}{1-\alpha_2+\eta_2}} < \left(\frac{\alpha_1^{\alpha_1}\cdot(1-\alpha_1)^{(1-\alpha_1)}}{\eta_2^{\eta_2}\cdot(1-\eta_2)^{(1-\eta_2)}}\cdot\frac{A_1}{B_2}\right)^{\frac{1}{1-\alpha_1+\eta_2}} < \left(\frac{\alpha_1^{\alpha_1}\cdot(1-\alpha_1)^{(1-\alpha_1)}}{\eta_1^{\eta_1}\cdot(1-\eta_1)^{(1-\eta_1)}}\cdot\frac{A_1}{B_1}\right)^{\frac{1}{1-\alpha_1+\eta_1}} \quad (50)$$

Notice when $\alpha = \eta$ this condition simplifies to $\frac{A_2}{B_2} < \frac{A_1}{B_2} < \frac{A_1}{B_1}$. We argued in previous section that if country one has absolute advantage in production of *Y*, i.e. $A_1 > A_2$ and

[15] See Appendix 1.H.

country 2 has absolute advantage in production of H, i.e. $B_1 < B_2$, then this inequality will hold. If anything other than this case prevails then this inequality will not hold. For instance, if country one has absolute advantage in both goods and therefore $A_1 > A_2$ and $B_1 > B_2$ while country two has only comparative advantage in good H, i.e. $\frac{A_2}{B_2} < \frac{A_1}{B_1}$ then $p^* = \frac{A_1}{B_2}$ can not be the equilibrium price.

In the case where the factor shares are different between the production sectors as well as across the countries, it is however very hard to find a condition for the parameters to guarantee the inequality in equation (50). Moreover the condition for absolute advantage is not as straight forward as the case where factor shares are equal. A country has an absolute advantage in production of a certain good if it could produce more of it by using the same level of input as the other country does. For instance consider the production of good Y in each country, the production functions are

$$Y_1 = A_1 K_1^{\alpha_1} H_1^{1-\alpha_1},$$

$$Y_2 = A_2 K_2^{\alpha_2} H_2^{1-\alpha_2},$$

or alternatively,

$$Y_1 = A_1 k_1^{\alpha_1} H_1,$$

$$Y_2 = A_2 k_2^{\alpha_2} H_2,$$

where K_i and H_i are the capital used in production of good Y in each country and $k=K/H$. Given that $H_1 = H_2$ and $K_1 = K_2$ then $\frac{Y_1}{Y_2} = \frac{A_1}{A_2} k^{\alpha_1 - \alpha_2}$. If $\alpha_1 = \alpha_2$ then the absolute advantage depends only on the relative value of A_1 and A_2. However when the factor shares are not equal across the countries then the relative production of good *Y* between the two countries depends on the relative values of α_1 and α_2 and the magnitudes of *k*. This implies that for some values of *k* country 1 might have absolute advantage in producing *Y* while for another range of *k* country 2 might have absolute advantage. Therefore in the general case the results from the simple model might not hold.

However we could consider 2 cases,

1) If $B_1 = B_2$ and $\eta_1 = \eta_2$ then $p_2 < p^* = p_1$. In this case country 2 will shut down the production of *Y* and produce only *H* and export it. However country 1, whose domestic price without trade is equal to the world price, will continue to produce both goods.

2) If $A_1 = A_2$ and $\alpha_1 = \alpha_2$ then $p_2 = p^* < p_1$. In this case country 1 will shut down the production of *H* and produce only *Y* and export *Y* in exchange for *H*. Country 2 however will continue to produce both goods.

In the first case where country 1 continues to produce both goods, its growth rate will remain unchanged with trade.

$$\gamma_{1,T} = \frac{1}{\theta}\left[\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{(1-\alpha_1)} \cdot A_1 . p^{*\alpha_1 - 1} - \delta - \rho\right] = \frac{1}{\theta}\left[\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{(1-\alpha_1)} \cdot A_1 . p_1^{\alpha_1 - 1} - \delta - \rho\right] = \gamma_{1,Au},$$

while the growth rate for country 2 will be higher under trade than that under autarky.

$$\gamma_{2,T} = \frac{1}{\theta}\left[\eta_2^{\eta_2} .(1-\eta_2)^{(1-\eta_2)} \cdot B_2 \cdot p^{*\eta_2} - \delta - \rho\right] > \frac{1}{\theta}\left[\eta_2^{\eta_2} .(1-\eta_2)^{(1-\eta_2)} \cdot B_2 \cdot p_2^{\eta_2} - \delta - \rho\right] = \gamma_{2,Au}.$$

Similarly in the second case country 2' growth rate will remain unchanged while the growth rate of country 1 increases under trade.

1.4 Other Cases

So far we have considered two most extreme cases,1) where the factor shares are the same between countries and production sections (Seater, 2007) and 2) the case where the factor shares are different everywhere. We also can think about two other cases where factor shares are the same between production sections while they are different across countries and the case where factor shares are different between sectors but the same across countries. Appendix 1.F considers these cases and shows that different assumption about factor shares does not change the fact that complete specialization happens and growth rate of countries will be higher with trade.

1.5 CONCLUSION

In this paper we generalized the model that Seater (2007) has considered in studying the effects of trade on growth rate of countries that engage in free trade with each other. He considered a very simple case of a 2 sector endogenous growth model in which there are 2 countries producing 2 goods with different technologies using two types of capital. However he made the model simple by assuming that the factor shares in production sectors are the same. Therefore the differences between technologies across countries and between sections are reflected on their total factor productivity parameters. In this framework he showed that when countries open to trade with each other, on the balanced growth path they will completely specialize in production of the sector in which they have comparative advantage. He also discussed that the growth rate of both countries could be higher under trade than that under autarky. However these results could only arise if each country has absolute advantage in producing something.

In this section we consider the case where the factor shares are different in production technologies between sections as well as across countries. Unlike what we expected the results show that after countries engages in free trade, each country shut downs the disadvantage production sector and produces only one good, then trade it to obtain the other good. Also the growth rate of both countries will be higher under trade than that under autarky as long as the world price falls between the autarky prices of countries. If that is the case then it was shown that the balanced growth path could exist for individual countries as well as for the world. However if the value of the prices that equate the growth rates falls outside the interval of autarky price the world price could not be equal to it. In this case one of the countries will be at the corner and its price will not change with trade. For this country the growth rate stays the same and it will continue to produce in both sections. However the growth path will no longer exist for the individual or for the world. The condition of absolute advantage that would guarantee the growth path where factor shares assumed to be equal could not be obtain for the general case.

APPENDIX 1.A

DERIVING THE PPF FOR THE CASE WHERE $\alpha = \eta$

In general the production functions for each country would be:

$$Y_t = A \cdot (v_t K_t)^{\alpha} \cdot (u_t H_t)^{1-\alpha}, \tag{A1-1}$$

$$\dot{H}_t + \delta H_t = B \cdot [(1-v_t)K_t]^{\eta} \cdot [(1-u_t)H_t]^{1-\eta}. \tag{A1-2}$$

If we assume $\alpha = \mu \Rightarrow v = u$, then we get the simplified version of the production functions as following

$$Y_t = A \cdot v_t \cdot K_t^{\alpha} \cdot H_t^{1-\alpha}, \tag{A2-1}$$

$$\dot{H}_t + \delta H_t = B \cdot (1-v_t) \cdot K_t^{\alpha} \cdot H_t^{1-\alpha}. \tag{A2-2}$$

To get the production possibility frontier:

$$\textbf{Max } Y_t = A \cdot v \cdot K^{\alpha} \cdot H^{1-\alpha} = A \cdot (vK)^{\alpha} \cdot (vH)^{1-\alpha}$$

$$\textbf{s.t. } \dot{H} + \delta H = B \cdot [(1-v)K]^{\alpha} . [(1-v)H]^{1-\alpha}$$

Solving this maximization we get:

$$Y = A \cdot \left[K - \frac{Z}{B}\left(\frac{K}{H}\right)^{1-\alpha} \right]^{\alpha} \left[H - \frac{Z}{B}\left(\frac{K}{H}\right)^{\alpha} \right]^{1-\alpha}, \tag{A3}$$

or,

$$Y = A \cdot K^{\alpha} \cdot H^{1-\alpha} - \frac{A}{B} Z,$$

therefore,

$$\frac{dY}{dZ} = -\frac{A}{B}. \tag{A4}$$

APPENDIX 1.B

DERIVING THE SLOPE OF PPF FOR THE CASE WHERE $\alpha \neq \eta$

To get the production possibility frontier for the general case we could use the same approach as we used in Appendix 1.A, however in this case that is a bit complicated so we restrict ourselves to just finding the slope of the production possibility frontier.

Again in general the production functions are:

$$Y_t = A \cdot (vK)^{\alpha} \cdot (uH)^{1-\alpha}, \tag{B1.1}$$

$$\dot{H}_t + \delta H_t = B \cdot [(1-v)K]^{\eta} \cdot [(1-u)H]^{1-\eta}, \tag{B1.2}$$

To get the slope for production possibility frontier, first take the total differential from (B1-1) we obtain,

$$dY = \frac{\partial Y}{\partial (vK)} d(vK) + \frac{\partial Y}{\partial (uH)} d(uH). \tag{B2}$$

Notice that along the production possibility frontier, stock of capital (or any input) will remain constant, therefore,

$$d(vK) = Kdv + vdK = Kdv, \tag{B3.1}$$

$$d(uH) = Hdu + udH = Hdu, \tag{B3.2}$$

Using (B3-1) and (B3-2) into (B2) we obtain,

$$dY = \frac{\partial Y}{\partial (vK)} Kdv + \frac{\partial Y}{\partial (uH)} Hdu,$$

then using (B1-1),

$$dY = \alpha A(vK)^{\alpha-1} Kdv + (1-\alpha)A(uH)^{-\alpha} Hdu,$$

after some simplifications it will become,

$$dY = A\left(\frac{vK}{uH}\right)^{\alpha}\left[\alpha K\left(\frac{vK}{uH}\right)^{-1} dv + (1-\alpha)Hdu\right], \tag{B4}$$

Similarly, taking total differential from (B1-2),

$$d(\dot{H} + \delta H) = \frac{\partial(\dot{H} + \partial H)}{\partial[(1-v)K]} d[(1-v)K] + \frac{\partial(\dot{H} + \delta H)}{\partial[(1-u)H]} d[(1-u)H], \tag{B5}$$

again,

$$d[(1-v)K] = -Kdv + (1-v)dK = -Kdv, \tag{B6.1}$$

$$d[(1-u)H] = -Hdu + (1-u)dH = -Hdu, \tag{B6.2}$$

Taking the same steps by using (B6-1) and (B6-2) in (B-5) we get,

$$d(\dot{H} + \delta H) = -B\left(\frac{(1-v)K}{(1-u)H}\right)^{\eta}\left[\eta K\left(\frac{(1-v)K}{(1-u)H}\right)^{-1} dv + (1-\eta)Hdu\right], \tag{B7}$$

So the PPF slope is:

$$\frac{dy}{d(\dot{H} + \delta H)} = -\frac{A\left(\frac{vK}{uH}\right)^{\alpha}\left[\alpha K\left(\frac{vK}{uH}\right)^{-1} dv + (1-\alpha)Hdu\right]}{B\left(\frac{(1-v)K}{(1-u)H}\right)^{\eta}\left[\eta K\left(\frac{(1-v)K}{(1-u)H}\right)^{-1} dv + (1-\eta)Hdu\right]}, \tag{B8}$$

$$\frac{dy}{d(\dot{H} + \delta H)} = -\frac{A}{B}\left(\frac{vK}{uH}\right)^{\alpha}\left(\frac{(1-v)K}{(1-u)H}\right)^{-\eta} \frac{\left[\alpha \frac{K}{H}\left(\frac{uH}{vK}\right)\frac{dv}{du} + (1-\alpha)\right]}{\left[\eta \frac{K}{H}\left(\frac{(1-u)H}{(1-v)K}\right)\frac{dv}{du} + (1-\eta)\right]}, \tag{B9}$$

$$\frac{dy}{d(\dot{H}+\delta H)} = -\frac{A}{B}\left(\frac{vK}{uH}\right)^{\alpha}\left(\frac{(1-v)K}{(1-u)H}\right)^{-\eta} \frac{\left[\alpha\frac{u}{v}\frac{dv}{du}+(1-\alpha)\right]}{\left[\eta\left(\frac{1-u}{1-v}\right)\frac{dv}{du}+(1-\eta)\right]}. \quad \text{(B10)}$$

From equation (9) in the text,

$$\left(\frac{1-u}{1-v}\right) = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1-\eta}{\eta}\right)\left(\frac{u}{v}\right). \quad \text{(B11)}$$

Substitute for $\left(\frac{1-u}{1-v}\right)$ from (B11) into (B10), and after some simplifications we get:

$$\frac{dy}{dz} = -\frac{A}{B}\alpha^{\eta}(1-\alpha)^{1-\eta}\eta^{-\eta}(1-\eta)^{\eta-1}\left(\frac{vK}{uH}\right)^{\alpha-\eta}, \quad \text{(B12)}$$

which, clearly shows the slope of production possibility frontier is not constant but rather it is changing as we move along a particular PPF.

APPENDIX 1.C

DERIVING THE K/H RATIO IN EACH SECTOR AT STEADY STATE

I. From equation (10) in the text,

$$\dot{\phi}_i = \phi_i\left[\alpha_i \cdot v_i \cdot A_i \cdot (v_i K_i)^{\alpha_i - 1} \cdot (u_i H_i)^{1-\alpha_i} - \delta\right] - \psi_i\left[(1-\eta_i)\cdot(1-u_i)\cdot B_i\right.$$

$$\left.\left((1-v_i)K_i\right)^{\eta_i = 1}\left((1-u_i)H_i\right)^{1-\eta_i}\right]$$

therefore,

$$\frac{\dot{\phi}_i}{\phi_i} = -\alpha_i \cdot \nu_i \cdot A_i \cdot \left(\frac{\nu_i K_i}{u_i H_i}\right)^{\alpha_i - 1} - \delta - p \cdot \eta_i \cdot (1-\nu_i) \cdot B_i \cdot \left(\frac{(1-\nu_i)K_i}{(1-u_i)H_i}\right)^{\eta_i - 1}, \qquad \text{(C1)}$$

From (29.7),

$$\frac{\partial V_i}{\partial v_i} = 0 \Rightarrow \phi_i\left[\alpha_i \cdot K_i \cdot A_i \cdot (v_i K_i)^{\alpha_i - 1} \cdot (u_i H_i)^{1-\alpha_i}\right]$$

$$-\psi_i\left[\eta_i \cdot K_i \cdot B_i \cdot \left((1-v_i)K_i\right)^{\eta_i - 1} \cdot \left((1-u_i)H_i\right)^{1-\eta_i}\right] = 0,$$

which implies,

$$\alpha_i \cdot K_i \cdot A_i \cdot \left(\frac{\nu_i K_i}{u_i H_i}\right)^{\alpha_i - 1} = p \cdot \eta_i \cdot K_i \cdot B_i \cdot \left(\frac{(1-\nu_i)K_i}{(1-u_i)H_i}\right)^{\eta_i - 1}, \qquad \text{(C2)}$$

Now from (C2) substitute into (C1),

$$\frac{\dot{\phi}_i}{\phi_i} = -\alpha_i \cdot \nu_i \cdot A_i \cdot \left(\frac{\nu_i K_i}{u_i H_i}\right)^{\alpha_i - 1} - \delta - \alpha_i \cdot (1-\nu_i) \cdot A_i \cdot \left(\frac{\nu_i K_i}{u_i H_i}\right)^{\alpha_i - 1}$$

$$\frac{\dot{\phi}_i}{\phi_i} = -\alpha_i \cdot A_i \cdot \left(\frac{\nu_i K_i}{u_i H_i}\right)^{\alpha_i - 1} + \delta, \qquad \text{(C3)}$$

From (29.5),

$$\dot{\psi}_i = -\frac{\partial V_i}{\partial H_i} = -\phi_i\left[(1-\alpha_i)\cdot u_i \cdot A_i \cdot (v_i K_i)^{\alpha_i} \cdot (u_i H_i)^{-\alpha_i}\right] - \psi_i\left[(1-\eta_i)\cdot(1-u_i)\cdot B_i \cdot\right.$$

$$\left((1-v_i)K_i\right)^{\eta_i} \cdot \left((1-u_i)H_i\right)^{-\eta_i} - \delta\Big],$$

therefore,

$$\frac{\dot{\psi}_i}{\psi_i} = -\frac{1}{p}\left[(1-\alpha_i)\cdot u_i \cdot A_i \cdot \left(\frac{v_i K_i}{u_i H_i}\right)^{\alpha_i}\right] - \left[(1-\eta_i)\cdot(1-u_i)\cdot B_i \cdot \left(\frac{(1-v_i)K_i}{(1-u_i)H_i}\right)^{\eta_i} - \delta\right]. \quad \text{(C4)}$$

From (29.8),

$$\frac{\partial V_i}{\partial u_i} = 0 \Rightarrow \phi_i\left[(1-\alpha_i)\cdot H_i \cdot A_i \cdot (v_i K_i)^{\alpha_i} \cdot (u_i H_i)^{-\alpha_i}\right]$$

$$-\psi_i\left[(1-\eta_i)\cdot H_i \cdot B_i \cdot \left((1-v_i)K_i\right)^{\eta_i} \cdot \left((1-u_i)H_i\right)^{-\eta_i}\right] = 0,$$

which implies,

$$\frac{1}{p}\left[(1-\alpha_i)\cdot H_i \cdot \left(\frac{v_i K_i}{u_i H_i}\right)^{\alpha_i}\right] = \left[(1-\eta_i)\cdot H_i \cdot B_i \cdot \left(\frac{(1-v_i)K_i}{(1-u_i)H_i}\right)^{\eta_i}\right]. \quad \text{(C5)}$$

Now substituting from (C5) into (C4):

$$\frac{\dot{\psi}_i}{\psi_i} = -\frac{1}{p}\cdot(1-\alpha_i)\cdot A_i \cdot \left(\frac{v_i K_i}{u_i H_i}\right)^{\alpha_i} + \delta\,. \quad \text{(C6)}$$

If $\gamma_\psi = \gamma_\varphi$, then

$$-\frac{1}{p}\cdot(1-\alpha_i)\cdot A_i \cdot \left(\frac{v_i K_i}{u_i H_i}\right)^{\alpha_i} + \delta = -\alpha_i \cdot A_i \cdot \left(\frac{v_i K_i}{u_i H_i}\right)^{\alpha_i - 1} + \delta\,,$$

this implies

$$\frac{v_i K_i}{u_i H_i} = p \cdot \frac{\alpha_i}{1-\alpha_i}\ . \quad \text{(C7)}$$

II. From equation (C1):

$$\frac{\dot{\phi}_i}{\phi_i} = -\alpha_i \cdot \nu_i \cdot A_i \cdot \left(\frac{\nu_i K_i}{u_i H_i}\right)^{\alpha_i - 1} - \delta - p \cdot \eta_i \cdot (1-\nu_i) \cdot B_i \cdot \left(\frac{(1-\nu_i)K_i}{(1-u_i)H_i}\right)^{\eta_i - 1},$$

and from (C2),

$$\alpha_i \cdot K_i \cdot A_i \cdot \left(\frac{\nu_i K_i}{u_i H_i}\right)^{\alpha_i - 1} = p \cdot \eta_i \cdot K_i \cdot B_i \cdot \left(\frac{(1-\nu_i)K_i}{(1-u_i)H_i}\right)^{\eta_i - 1}.$$

Now substitute for the first term in left hand side of (C1) from the left hand side of (C2):

$$\frac{\dot{\phi}_i}{\phi_i} = -p \cdot \eta_i \cdot \nu_i \cdot B_i \cdot \left(\frac{(1-\nu_i)K_i}{(1-u_i)H_i}\right)^{\eta_i - 1} + \delta - p \cdot \eta_i \cdot (1-\nu_i) \cdot B_i \cdot \left(\frac{(1-\nu_i)K_i}{(1-u_i)H_i}\right)^{\eta_i - 1},$$

or,

$$\frac{\dot{\phi}_i}{\phi_i} = -p \cdot \eta_i \cdot B_i \cdot \left(\frac{(1-\nu_i)K_i}{(1-u_i)H_i}\right)^{\eta_i - 1} + \delta. \qquad \text{(C8)}$$

Also from (C4),

$$\frac{\dot{\psi}_i}{\psi_i} = -\frac{1}{p}\left[(1-\alpha_i) \cdot u_i \cdot A_i \cdot \left(\frac{\nu_i K_i}{u_i H_i}\right)^{\alpha_i}\right] - \left[(1-\eta_i) \cdot (1-u_i) \cdot B_i \cdot \left(\frac{(1-\nu_i)K_i}{(1-u_i)H_i}\right)^{\eta_i} - \delta\right],$$

and from (C5),

$$\frac{1}{p}\left[(1-\alpha_i) \cdot H_i \cdot \left(\frac{\nu_i K_i}{u_i H_i}\right)^{\alpha_i}\right] = \left[(1-\eta_i) \cdot H_i \cdot B_i \cdot \left(\frac{(1-\nu_i)K_i}{(1-u_i)H_i}\right)^{\eta_i}\right].$$

Now substituting from (C5) into (C4),

$$\frac{\dot{\psi}_i}{\psi_i} = -\left[(1-\eta_i) \cdot u_i \cdot B_i \cdot \left(\frac{(1-\nu_i)K_i}{(1-u_i)H_i}\right)^{\eta_i}\right] - \left[(1-\eta_i) \cdot (1-u_i) \cdot B_i \cdot \left(\frac{(1-\nu_i)K_i}{(1-u_i)H_i}\right)^{\eta_i} - \delta\right],$$

which would be,

$$\frac{\dot{\psi}_i}{\psi_i} = -\left[(1-\eta_i)\cdot B_i \cdot \left(\frac{(1-v_i)K_i}{(1-u_i)H_i}\right)^{\eta_i} - \delta\right]. \tag{C9}$$

Again if $\gamma_{\psi} = \gamma_{\varphi}$, then:

$$-\left[(1-\eta_i)\cdot B_i \cdot \left(\frac{(1-v_i)K_i}{(1-u_i)H_i}\right)^{\eta_i} - \delta\right] = -p\cdot\eta_i\cdot B_i \cdot \left(\frac{(1-v_i)K_i}{(1-u_i)H_i}\right)^{\eta_i - 1} + \delta,$$

therefore,

$$\frac{(1-v_i)K_i}{(1-u_i)H_i} = p\cdot\frac{\eta_i}{1-\eta_i} \ . \tag{C10}$$

APPENDIX 1.D

GROWTH RATES ON THE BALANCED GROWTH PATH

The following will show that the growth rates of K, H, Y and Q are all equal to the growth rate of C.

From equation (29.2),

$$\dot{K}_i = A_i \cdot (v_i K_i)^{\alpha_i} \cdot (u_i H_i)^{1-\alpha_i} - C_i - \delta K_i - X_i \rightarrow \gamma_K = \frac{\dot{K}_i}{K_i} = \frac{A_i.(v_i K_i)^{\alpha_i} (u_i H_i)^{1-\alpha_i}}{K_i} - \frac{C_i}{K_i} - \delta - \frac{X_i}{K_i},$$

note $Y_i = A_i.(v_i K_i)^{\alpha_i} (u_i H_i)^{1-\alpha_i}$, therefore,

$$\gamma_K = \frac{\dot{K}_i}{K_i} = \frac{Y_i}{K_i} - \frac{C_i}{K_i} - \frac{X_i}{K_i} - \delta , \tag{D.1}$$

$$\dot{H}_i = B_i \cdot \left((1-v_i)K_i\right)^{\eta_i} \cdot \left((1-u_i)H_i\right)^{1-\eta_i} - \delta H_i + \frac{1}{p} X_i,$$

dividing each side by H, we get,

$$\gamma_H = \frac{\dot{H}_i}{H_i} = \frac{B_i.\left((1-v_i)K_i\right)^{\eta_i} \left((1-u_i)H_i\right)^{1-\eta_i}}{H_i} - \delta + \frac{1}{p}\frac{X_i}{H_i},$$

define, $Z \equiv B_i.\left((1-v_i)K_i\right)^{\eta_i} \left((1-u_i)H_i\right)^{1-\eta_i}$, therefore

$$\gamma_H = \frac{\dot{H}_i}{H_i} = \frac{Z_i}{H_i} + \frac{1}{p}\frac{X_i}{H_i} - \delta \cdot \tag{D.2}$$

Also the growth rate of consumption is,

$$\gamma_{C_i} = \frac{1}{\theta}\left[\alpha_i \cdot A_i \cdot \left(\frac{v_i K_i}{u_i H_i}\right)^{\alpha_1 - 1} - \delta - \rho\right].$$

From the production for the Y-sector we can get,

$$\frac{Y_i}{v_i K_i} = A_i \cdot \left(\frac{v_i K_i}{u_i H_i} \right)^{\alpha_1 - 1},$$

therefore,

$$\gamma_{C_i} = \frac{1}{\theta} \left[\alpha_i \frac{Y_i}{v_i K_i} - \delta - \rho \right].$$

We know that everything grows on the balanced growth path at a constant rate and prices remain the same. Now if γ_C is constant then $\frac{Y}{vK}$ would also be constant which in turn it implies,

$$\gamma_{\mathrm{Y}} = \gamma_v + \gamma_K. \qquad \text{(D.3)}$$

As we know on the BGP everything that grows should do so at a constant rate, therefore γ_v should be constant. However $0 \le v \le 1$ and so $\gamma_v = 0$. Therefore (D.3) implies that $\gamma_{\mathrm{Y}} = \gamma_K$. Now if γ_H is constant then from (D.2), then $\frac{Z + \frac{1}{p} X}{H}$ should also be constant, given that *Z*, *X* and *H* are all non-negative and *p* is constant on the BGP, then this implies $\gamma_{\mathrm{Z}} = \gamma_X = \gamma_H$.

Similarly, if γ_K is constant then from (D-1) then $\frac{Y - C - X}{K}$ should also be constant. Since $\gamma_{\mathrm{Y}} = \gamma_K$ therefore $\frac{Y}{K}$ must be constant which in turn implies that $\frac{C + X}{K}$ should be constant. However *C*, *X* and *K* all are non-negative therefore $\gamma_C = \gamma_K = \gamma_X$. These results imply that on the balanced growth path $\gamma_C = \gamma_K = \gamma_X = \gamma_H = \gamma_Y$.

APPENDIX 1.E

DERIVING THE AUTARKY GROWTH RATES AS A FUNCTION OF PRICE LEVEL

I. The growth rate of country 1 under autarky is:

$$\gamma_{1,Au} = \frac{1}{\theta}\left[A_1^{\frac{\eta_1}{1-\alpha_1+\eta_1}} \cdot B_1^{\frac{1-\alpha_1}{1-\alpha_1+\eta_1}} \cdot \alpha_1^{\frac{\alpha_1\eta_1}{1-\alpha_1+\eta_1}} \cdot (1-\alpha_1)^{\frac{(1-\alpha_1)\eta_1}{1-\alpha_1+\eta_1}} \cdot \eta_1^{\frac{(1-\alpha_1)\eta_1}{1-\alpha_1+\eta_1}} \cdot (1-\eta_1)^{\frac{(1-\alpha_1)(1-\eta_1)}{1-\alpha_1+\eta_1}} - \delta - \rho \right]. \quad \text{(E1)}$$

Now

$$A_1^{\frac{\eta_1}{1-\alpha_1+\eta_1}} \cdot B_1^{\frac{(1-\alpha_1)}{1-\alpha_1+\eta_1}} = A_1^{\frac{\eta_1}{1-\alpha_1+\eta_1}-1+1} \cdot B_1^{\frac{(1-\alpha_1)}{1-\alpha_1+\eta_1}} = A_1 \cdot \left(\frac{A_1}{B_1}\right)^{\frac{(\alpha_1-1)}{1-\alpha_1+\eta_1}}, \quad \text{(E2)}$$

$$\alpha_1^{\frac{\alpha_1\eta_1}{1-\alpha_1+\eta_1}} = \alpha_1^{\frac{\alpha_1\eta_1}{1-\alpha_1+\eta_1}-\alpha_1+\alpha_1} = \alpha_1^{\frac{\alpha_1(\alpha_1-1)}{1-\alpha_1+\eta_1}} \cdot \alpha_1^{\alpha_1}, \quad \text{(E3)}$$

$$(1-\alpha_1)^{\frac{(1-\alpha_1)\eta_1}{1-\alpha_1+\eta_1}} = (1-\alpha_1)^{\frac{(1-\alpha_1)\eta_1}{1-\alpha_1+\eta_1}-(1-\alpha_1)+(1-\alpha_1)} = (1-\alpha_1)^{\frac{(1-\alpha_1)(\alpha_1-1)}{1-\alpha_1+\eta_1}} \cdot (1-\alpha_1)^{(1-\alpha_1)}. \quad \text{(E4)}$$

From (E1), (E2), (E3) and (E4) substitute into (E1),

$$\gamma_{1,Au} = \frac{1}{\theta}\left[A_1 \cdot \left(\frac{A_1}{B_1}\right)^{\frac{\alpha_1-1}{1-\alpha_1+\eta_1}} \cdot \alpha_1^{\frac{\alpha_1(\alpha_1-1)}{1-\alpha_1+\eta_1}} \cdot \alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{\frac{(1-\alpha_1)(\alpha_1-1)}{1-\alpha_1+\eta_1}} \cdot (1-\alpha_1)^{(1-\alpha_1)} \cdot \eta_1^{\frac{(1-\alpha_1)\eta_1}{1-\alpha_1+\eta_1}} \cdot (1-\eta_1)^{\frac{(1-\alpha_1)(1-\eta_1)}{1-\alpha_1+\eta_1}} - \delta - \rho \right]$$

$$\gamma_{1,Au} = \frac{1}{\theta}\left[\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{(1-\alpha_1)} \cdot A_1 \cdot \left\{ \left(\frac{A_1}{B_1}\right)^{\frac{\alpha_1-1}{1-\alpha_1+\eta_1}} \cdot \alpha_1^{\frac{\alpha_1(\alpha_1-1)}{1-\alpha_1+\eta_1}} \cdot (1-\alpha_1)^{\frac{(1-\alpha_1)(\alpha_1-1)}{1-\alpha_1+\eta_1}} \cdot \eta_1^{\frac{(1-\alpha_1)\eta_1}{1-\alpha_1+\eta_1}} \cdot (1-\eta_1)^{\frac{(1-\alpha_1)(1-\eta_1)}{1-\alpha_1+\eta_1}} \right\} - \delta - \rho \right]$$

Notice that the terms in the bracket equals to $p_1^{\alpha_1-1}$, therefore the growth rate for country 1 would be,

$$\gamma_{1,Au} = \frac{1}{\theta}\left[\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{(1-\alpha_1)} \cdot A_1 \cdot p_1^{\alpha_1-1} - \delta - \rho \right]. \quad \text{(E5)}$$

II. Now to get the growth rate for country 2 before countries engaged in trade we could similarly start from the growth rate for country 2 under autarky:

$$\gamma_{2,Au} = \frac{1}{\theta}\left[A_2^{\frac{\eta_2}{1-\alpha_2+\eta_2}} \cdot B_2^{\frac{(1-\alpha_2)}{1-\alpha_2+\eta_2}} \cdot \alpha_2^{\frac{\alpha_2\eta_2}{1-\alpha_2+\eta_2}} \cdot (1-\alpha_2)^{\frac{(1-\alpha_2)\eta_2}{1-\alpha_2+\eta_2}} \cdot \eta_2^{\frac{(1-\alpha_2)\eta_2}{1-\alpha_2+\eta_2}} \cdot (1-\eta_2)^{\frac{(1-\alpha_2)(1-\eta_2)}{1-\alpha_2+\eta_2}} - \delta - \rho \right], \tag{E6}$$

Now,

$$A_2^{\frac{\eta_2}{1-\alpha_2+\eta_2}} \cdot B_2^{\frac{\eta_2}{1-\alpha_2+\eta_2}} = A_2^{\frac{\eta_2}{1-\alpha_2+\eta_2}} \cdot B_2^{\frac{(1-\alpha_2)}{1-\alpha_2+\eta_2}+1-1} = \left(\frac{A_2}{B_2}\right)^{\frac{\eta_2}{1-\alpha_2+\eta_2}} \cdot B_2 , \tag{E7}$$

$$\eta_2^{\frac{(1-\alpha_2)\eta_2}{1-\alpha_2+\eta_2}} = \eta_2^{\frac{(1-\alpha_2)\eta_2}{1-\alpha_2+\eta_2}-\eta_2+\eta_2} = \eta_2^{\frac{-\eta_2^2}{1-\alpha_2+\eta_2}} \cdot \eta_2^{\eta_2} , \tag{E8}$$

$$(1-\eta_2)^{\frac{(1-\alpha_2)(1-\eta_2)}{1-\alpha_2+\eta_2}} = (1-\eta_2)^{\frac{(1-\alpha_2)(1-\eta_2)}{1-\alpha_2+\eta_2}-(1-\eta_2)+(1-\eta_2)} = (1-\eta_2)^{\frac{-\eta_2(1-\eta_2)}{1-\alpha_2+\eta_2}} \cdot (1-\eta_2)^{(1-\eta_2)} . \tag{E9}$$

Again substitute from (E7), (E8) and (E9) into (E6),

$$\gamma_{2,Au} = \frac{1}{\theta}\left[\left(\frac{A_2}{B_2}\right)^{\frac{\eta_2}{1-\alpha_2+\eta_2}} \cdot B_2 \cdot \alpha_2^{\frac{\alpha_2\eta_2}{1-\alpha_2+\eta_2}} \cdot (1-\alpha_2)^{\frac{(1-\alpha_2)\eta_2}{1-\alpha_2+\eta_2}} \cdot \eta_2^{\frac{-\eta_2^2}{1-\alpha_2+\eta_2}} \cdot \eta_2^{\eta_2} \cdot (1-\eta_2)^{\frac{-\eta_2(1-\eta_2)}{1-\alpha_2+\eta_2}} \cdot (1-\eta_2)^{(1-\eta_2)} - \delta - \rho \right]$$

$$\gamma_{2,Au} = \frac{1}{\theta}\left[\eta_2^{\eta_2} \cdot (1-\eta_2)^{(1-\eta_2)} \cdot B_2 \cdot \left\{ \left(\frac{A_2}{B_2}\right)^{\frac{\eta_2}{1-\alpha_2+\eta_2}} \cdot \alpha_2^{\frac{\alpha_2\eta_2}{1-\alpha_2+\eta_2}} \cdot (1-\alpha_2)^{\frac{(1-\alpha_2)\eta_2}{1-\alpha_2+\eta_2}} \cdot \eta_2^{\frac{-\eta_2^2}{1-\alpha_2+\eta_2}} \cdot (1-\eta_2)^{\frac{-\eta_2(1-\eta_2)}{1-\alpha_2+\eta_2}} \right\} - \delta - \rho \right]$$

Again notice that the term in the brackets is p^{η_2}, therefore:

$$\gamma_{2,Au} = \frac{1}{\theta}\left[\eta_2^{\eta_2} \cdot (1-\eta_2)^{(1-\eta_2)} \cdot B_2 \cdot p_2^{\eta_2} - \delta - \rho \right] . \tag{E10}$$

APPENDIX 1.F

EQUILIBRIUM PRICE AND GROWTH RATES WITH TRADE, OTHER CASES

1. $\alpha = \eta$ within the production sections but not among the countries.

In this case the fact that we still have $\alpha = \eta$, implies that production possibility frontiers is still linear and complete specialization will occur. In the same manner it could be shown that the growth rates of countries will raise after trade relative to their autarky and the equilibrium price and growth rate would be

$$p = \left[\frac{\alpha_1^{\alpha_1} \cdot (1-\alpha_1)^{(1-\alpha_1)}}{\alpha_2^{\alpha_2} \cdot (1-\alpha_2)^{(1-\alpha_2)}} \cdot \frac{A_1}{B_2} \right]^{\frac{1}{1-\alpha_1-\alpha_2}},$$

$$\gamma \equiv \gamma_{1,T} = \gamma_{2,T} = \frac{1}{\theta}\left[\alpha_1^{\frac{\alpha_1\alpha_2}{1-\alpha_1+\alpha_2}} \cdot (1-\alpha_1)^{\frac{(1-\alpha_1)\alpha_2}{1-\alpha_1+\alpha_2}} \cdot \alpha_2^{\frac{\alpha_2(1-\alpha_1)}{1-\alpha_1+\alpha_2}} \cdot (1-\alpha_2)^{\frac{(1-\alpha_1)(1-\alpha_2)}{1-\alpha_1+\alpha_2}} \cdot A_1^{\frac{\alpha_2}{1-\alpha_1+\alpha_2}} \cdot B_2^{\frac{(1-\alpha_1)}{1-\alpha_1+\alpha_2}} - \delta - \rho \right].$$

Which could have easily obtained if we impose $\alpha_1 = \eta_1$ and $\alpha_2 = \eta_2$ in equations (44) and (45).

2. $\alpha \neq \eta$ within the production sections but $\alpha_1 = \alpha_2$ and $\eta_1 = \eta_2$.

Since now $\alpha \neq \eta$, the production possibility frontier will no longer be linear. This case will be similar to the one studied in section 4. We can proceed as before and show that the growth rates will again be higher under trade and price level and growth rate on the balanced growth path would be:

$$p = \left[\frac{\alpha^{\alpha} \cdot (1-\alpha)^{(1-\alpha)}}{\eta^{\eta} \cdot (1-\eta)^{(1-\eta)}} \cdot \frac{A_1}{B_2} \right]^{\frac{1}{1-\alpha+\eta}},$$

$$\gamma \equiv \gamma_{1,T} = \gamma_{2,T} = \frac{1}{\theta}\left[\alpha^{\frac{\alpha\eta}{1-\alpha+\eta}} \cdot (1-\alpha)^{\frac{(1-\alpha)\eta}{1-\alpha+\eta}} \cdot \eta^{\frac{\eta(1-\alpha)}{1-\alpha+\eta}} .(1-\eta)^{\frac{(1-\alpha)(1-\eta)}{1-\alpha+\eta}} \cdot A_1^{\frac{\eta}{1-\alpha+\eta}} \cdot B_2^{\frac{1-\alpha}{1-\alpha+\eta}} - \delta - \rho \right].$$

Again these values could have been obtain by imposing $\alpha_1 = \alpha_2$ and $\eta_1 = \eta_2$ in equation (44) and (45).

APPENDIX 1.G

GROWTH RATES WITH COMPLETE SPECIALIZATION

For country 1 v=u=1,

$$V_1 = \frac{C_1^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \phi_1 \left[A_1 \cdot K_1^{\alpha_1} \cdot H_1^{1-\alpha_1} - C_1 - \delta K_1 - X_1 \right] + \psi_i \left[-\delta H_i + \frac{1}{p} X_i \right], \qquad \text{(G1)}$$

$$\dot{K}_1 = \frac{\partial V_1}{\partial \phi_1} = A_1 \cdot K_1^{\alpha_1} \cdot H_1^{1-\alpha_1} - C_1 - \delta K_1 - X_1 , \qquad \text{(G2)}$$

$$\dot{H}_1 = \frac{\partial V_1}{\partial \psi_1} = -\delta H_1 + \frac{1}{p} X_1 , \qquad \text{(G3)}$$

$$\dot{\phi}_1 = -\frac{\partial V_1}{\partial K_1} = -\phi_1 \left[\alpha_1 \cdot A_1 \cdot K_1^{\alpha_1 - 1} \cdot H_1^{1-\alpha_1} - \delta \right], \qquad \text{(G4)}$$

$$\dot{\psi}_1 = -\frac{\partial V_1}{\partial H_1} = -\phi_1 \left[(1-\alpha_1) \cdot A_1 \cdot K_1^{\alpha_1} \cdot H_1^{-\alpha_1} \right] - \psi_i \left[-\delta \right], \qquad \text{(G5)}$$

$$\frac{\partial V_1}{\partial C_1} = 0 \Rightarrow C_1^{-\theta} e^{-\rho t} - \phi_1 = 0 , \qquad \text{(G6)}$$

$$\frac{\partial V_1}{\partial X_1} = 0 \Rightarrow -\phi_1 - \frac{1}{p} \psi_1 = 0 . \qquad \text{(G7)}$$

From (G4),

$$\frac{\dot{\phi}_1}{\phi_1} = -\left[\alpha_1 \cdot A_1 \cdot K_1^{\alpha_1 - 1} \cdot H_1^{1-\alpha_1} - \delta \right] = -\alpha_1 \cdot A_1 \left(\frac{K_1}{H_1} \right)^{\alpha_1 - 1} + \delta ,$$

and from (G5),

$$\frac{\dot{\psi}_1}{\psi_i} = -\frac{1}{p} \left[(1-\alpha_1) \cdot A_1 \cdot K_1^{\alpha_1} \cdot H_1^{-\alpha_1} \right] - \left[-\delta \right] = -\frac{1}{p} (1-\alpha_1) . A_1 \left(\frac{K_1}{H_1} \right)^{\alpha_1} + \delta$$

As we discussed in the paper when countries engage in international trade they will do so at the world level of prices and so $p = \frac{\psi_1}{\varphi_1}$. Besides on the balance growth path $\gamma_p = 0$ therefore $\frac{\dot{\psi}}{\psi} = \frac{\dot{\varphi}}{\varphi}$, equating this growth rate then we can get

$$-\alpha_1 \cdot A_1 \left(\frac{K_1}{H_1} \right)^{\alpha_1 - 1} + \delta = -\frac{1}{p}(1-\alpha_1).A_1 \left(\frac{K_1}{H_1} \right)^{\alpha_1} + \delta \, ,$$

which implies,

$$\frac{K_1}{H_1} = p \frac{\alpha_1}{1-\alpha_1} \, . \tag{G8}$$

Substituting into the growth rate of φ,

$$\frac{\dot{\phi}_1}{\phi_1} = -\alpha_1 \cdot A_1 \left(\frac{K_1}{H_1} \right)^{\alpha_1 - 1} + \delta = -\alpha_1 \cdot A_1 \left(p \frac{\alpha_1}{1-\alpha_1} \right)^{\alpha_1 - 1} + \delta \, .$$

Similarly, we can obtain the growth rate of consumption by using (G6) as,

$$\frac{\dot{C}}{C} = -\frac{1}{\theta} \left[\frac{\dot{\phi}_1}{\phi_1} + \rho \right] \Rightarrow \gamma_1 = \frac{1}{\theta} \left[A_1.\alpha_1^{\alpha_1}.(1-\alpha_1)^{1-\alpha_1}.p^{\alpha_1 - 1} - \delta - \rho \right] . \tag{G9}$$

This is the same growth rate that we describe in (36) without imposing the complete specialization condition.

Now to see the growth rate for country 2, we can set v=u=0 since this country is assumed to specialize in producing H and so it is going to shut down section Y.

$$V_2 = \frac{C_2^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \phi_2 \left[-C_2 - \delta K_2 - X_2 \right] + \psi_2 \left[B_2.K_2^{\eta_2}.H_2^{1-\eta_2} - \delta H_2 + \frac{1}{p} X_2 \right], \tag{G10}$$

$$\dot{K}_2 = \frac{\partial V_2}{\partial \phi_2} = -C_2 - \delta K_2 - X_2 \, , \tag{G11}$$

$$\dot{H}_2 = \frac{\partial V_2}{\partial \psi_2} = B_2 \cdot K_2^{\eta_2} \cdot H_2^{1-\eta_2} - \delta H_2 + \frac{1}{p} X_2 , \tag{G12}$$

$$\dot{\phi}_2 = -\frac{\partial V_2}{\partial K_2} = -\phi_2 [-\delta] - \psi_2 \left[\eta_2 \cdot B_2 \cdot K_2^{\eta_2 - 1} \cdot H_2^{1-\eta_2} \right], \tag{G13}$$

$$\dot{\psi}_2 = -\frac{\partial V_2}{\partial H_2} = -\psi_2 \left[(1-\eta_2) B_2 . K_2^{\eta_2} . H_2^{-\eta_2} - \delta \right], \tag{G14}$$

$$\frac{\partial V_2}{\partial C_2} = 0 \Rightarrow C_2^{-\theta} e^{-\rho t} - \phi_2 = 0 , \tag{G15}$$

$$\frac{\partial V_2}{\partial X_2} = 0 \Rightarrow -\phi_2 - \frac{1}{p} \psi_2 = 0 . \tag{G16}$$

We can repeat all the steps that we took to get the growth rate for country 1. First from (G13) and (G14) obtain the growth rate for φ and ψ,

$$\frac{\dot{\phi}_2}{\phi_2} = -[-\delta] - p\left[\eta_2 \cdot B_2 \cdot K_2^{\eta_2 - 1} \cdot H_2^{1-\eta_2} \right] = \delta - p \eta_2 B_2 \left(\frac{K_2}{H_2} \right)^{\eta_2 - 1} ,$$

$$\frac{\dot{\psi}_2}{\psi_2} = -\left[(1-\eta_2) B_2 . K_2^{\eta_2} . H_2^{-\eta_2} - \delta \right] = -(1-\eta_2) B_2 \left(\frac{K_2}{H_2} \right)^{\eta_2} + \delta .$$

Again on the balance growth path $\gamma_p = 0$ therefore $\frac{\dot{\psi}}{\psi} = \frac{\dot{\varphi}}{\varphi}$,

$$\frac{\dot{\phi}_2}{\phi_2} = \frac{\dot{\psi}_2}{\psi_2} \Rightarrow \delta - p \eta_2 B_2 \left(\frac{K_2}{H_2} \right)^{\eta_2 - 1} = -(1-\eta_2) B_2 \left(\frac{K_2}{H_2} \right)^{\eta_2} + \delta ,$$

implying,

$$\frac{K_2}{H_2} = p \frac{\eta_2}{1-\eta_2} . \tag{G17}$$

Substituting into $\frac{\dot{\phi}_2}{\phi_2} = \delta - p\eta_2 B_2 \left(\frac{K_2}{H_2}\right)^{\eta_2 - 1}$,

$$\frac{\dot{\phi}_2}{\phi_2} = \delta - p\eta_2 B_2 \left(p\frac{\eta_2}{1-\eta_2}\right)^{\eta_2 - 1}. \qquad \text{(G18)}$$

As usual the growth rate of consumption,

$$\frac{\dot{C}_i}{C_i} = -\frac{1}{\theta}\left[\frac{\dot{\phi}_i}{\phi_i} + \rho\right].$$

Substituting from (G18) into the growth rate of consumption we can obtain the growth rate for country 2,

$$\gamma_2 = \frac{1}{\theta}\left[B_2.\eta_2^{\eta_2}.(1-\eta_1)^{1-\alpha_1}.p^{\alpha_1 - 1} - \delta - \rho\right], \qquad \text{(G19)}$$

which is the same as we obtained by solving the model without imposing any condition.

APPENDIX 1.H

SIMPLIFYING THE PRICE LEVEL

$$p_i = \left(\frac{A_i}{B_i}\right)\left(\frac{\alpha_i}{\eta_i}\right)^{\eta_i}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{1-\eta_i}\left[\left(\frac{A_i}{B_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{\eta_i}\right)^{\frac{\eta_i}{1-\alpha_i+\eta_i}}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{\frac{1-\eta_i}{1-\alpha_i+\eta_i}}\right]^{\alpha_i-\eta_i},$$

$$p_i = \left(\frac{A_i}{B_i}\right)\left(\frac{\alpha_i}{\eta_i}\right)^{\eta_i}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{1-\eta_i}\left(\frac{A_i}{B_i}\right)^{\frac{\alpha_i-\eta_i}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\frac{\alpha_i-\eta_i}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{\eta_i}\right)^{\frac{\eta_i(\alpha_i-\eta_i)}{1-\alpha_i+\eta_i}}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{\frac{(1-\eta_i)(\alpha_{\_}-\eta_i)}{1-\alpha_i+\eta_i}},$$

$$p_i = \left(\frac{A_i}{B_i}\right)^{1+\frac{\alpha_i-\eta_i}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\frac{\alpha_i-\eta_i}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{\eta_i}\right)^{\eta_i+\frac{\eta_i(\alpha_i-\eta_i)}{1-\alpha_i+\eta_i}}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{(1-\eta_i)+\frac{(1-\eta_i)(\alpha_{\_}-\eta_i)}{1-\alpha_i+\eta_i}},$$

$$p_i = \left(\frac{\alpha_i^{\alpha_i}(1-\alpha_i)^{(1-\alpha_i)}}{\eta_i^{\eta_i}(1-\eta_i)^{1-\eta_i}}\cdot\frac{A_i}{B_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}.$$

CHAPTER 2

TRADE AND GROWTH WITH ENDOGENOUS CAPITAL ACCUMULATION

2.1 INTRODUCTION

In this section we further generalize the model we considered in the previous section. Consider a two sector endogenous growth model with two types of capital accumulation in a relatively large open economy. Reproducible factors of production are considered to be two types of capital, K and H type which are used to produce two goods, a consumption/K-investment good and an H-type investment good. Output of both sectors is assumed to be produced under conditions of constant returns to scale and perfect competition. Therefore, the model exhibits endogenous growth because of constant returns to scale in the reproducible factors of production. Human capital is usually considered as not-traded. Therefore, here we treat H-type capital good as another type of capital other than physical capital and not necessarily human capital. This assumption then implies that both goods could be considered as tradable.

We modify a model developed by Bond and Trask (1997), in which they consider a small open economy with 3 sectors of production and 2 types of capital, physical and human. Final outputs in this economy are the consumption good, investment good and education. However only consumption and investment good is assumed to be traded, education is assumed to be non-traded. Endogenous growth results from the assumption of constant return to scale in the reproducible factors of production. They establish the existence and uniqueness of a balanced growth path in which all 3 goods (consumption, physical capital and human capital) grow at the same rate and the relative prices are constant. They have identified 3 types of production patterns on the BGP depending on the relative price of traded good on autarky with respect to the world price.

In the case where the autarkic price is not the same as the one in the world market, on the BGP the small open economy will completely specialize in the production of one of the traded goods as the result of trade. However when the price of traded good in international market equals to the autarkic price, small open economy continues to produce both goods. In either case however they could show that the growth rate on the BGP under free trade can never be lower than the growth rate on the autarky BGP.

They also show that there is a unique capital/labor ratio on the BGP where small open country completely specializes in one of the traded goods and that this equilibria exhibit the saddle path stability. However when there is incomplete specialization there is a continuum of capital/labor ratios on the balanced growth path.

In the following section we try to establish similar results in a model where there are only two final goods, and both are tradable. First, we show the existence of a balanced growth path at which the growth rate of consumption, *K* and *H* capital are the same and the prices remain unchanged. Then we show that on the BGP exists a unique world price at which the economy could continue to produce both goods. The price at which the open economy will be incompletely specialized is the autarky price on the BGP. If the world price however is greater or smaller than this price then the open economy will completely specialize in production of one sector. When there is complete specialization it could be shown that the growth rate of an open economy is larger than that under autarky. We then examine the properties of the dynamic transition in a two sector endogenous growth model for an open economy. It will be shown that complete specialization could also be happen on the transition to the steady state and the transition path is saddle path stable. We start by introducing the model and then developing the properties of a BGP.

2.2 THE MODEL

Consider a model of endogenous growth in which there are two sectors of production. Two goods are being produced using two reproducible factors of production, *K* and *H*. The production technologies for each sector are assumed to be different but both exhibit constant return to scale in the reproducible factors of production. Also there is assumed to be perfect competition in final good markets as well as the market for factors of production.

One sector produces a unified consumption and one type of capital good (*K*-Type), and the other sector produces another type of capital good (*H*-Type). We denote the unified consumption/*K*-investment good by *Y*, and the other sector by *Z*.

The production technologies for each sector can then be expressed as

$$Y_i = F_i(v_i K_i, u_i H_i) = u_i H_i \; f_i(k_{yi}), \tag{1.1}$$

$$Z_i = G_i\big((1-v_i)K_i, (1-u_i)H_i\big) = (1-u_i)H_i \; g_i(k_{zi_i}), \tag{1.2}$$

where, $k_{yi} \equiv \dfrac{v_i K_i}{u_i H_i}$ and $k_{zi} \equiv \dfrac{(1-v_i)K_i}{(1-u_i)H_i}$, v and u are the shares of *K* and *H* capital allocated to the *Y* sector, respectively. The output per unit of *H* functions, *f* and *g*, are considered to be strictly increasing and strictly concave.

There also is assumed to be two countries that are relatively large with respect to each other and can trade in final goods. Then, *i=1, 2* indicate country one and two. However since the solution for both countries look the same we will drop this subscription unless it is needed.

There is no international lending and borrowing. Therefore the trade balance constraint requires the value of imports to be equal to the value of all exports. Alternatively, the value of the country's excess supplies (demands) must be equal to zero at each point in time.

The trade balance condition is then,

$$(Y_P - Y_C) + p\,(Z_P - Z_C) = 0 \Rightarrow (Y_P - Y_C) = X = -p\,(Z_P - Z_C)\,, \tag{TB}$$

where Y_P and Y_C (Z_P and Z_C) are the domestic production and domestic consumption/demand of good *Y* (*Z*) respectively. Therefore $(Y_P - Y_C)$ and $(Z_p - Z_C)$ are defined as excess supply of good *Y* and *Z*. Then *X* is the value of export of good *Y*. In equation (TB), *p* represents the international price of good *Z* in units of good *Y*.

Using this trade condition, the evolution of the stock of *K*-capital can be written as

$$\dot{K} = uH\,f(k_y) - \delta K - C - X\,, \tag{2.1}$$

where δ is the rate of depreciation of *K*-type capital

The output of the *Z* sector could also be traded or added to the stock of *H*-type capital, therefore

$$\dot{H} = (1-u)\,Hg(k_z) - \eta H + \frac{1}{p}X\,, \tag{2.2}$$

where η is the rate of depreciation of *H*-type capital

A representative agent's optimization problem is

$$\max_{C,u_i,v_i} \int_0^\infty \frac{\mathrm{C(t)}^{1\text{-}\sigma}}{1\text{-}\sigma} e^{-\rho t}\,,$$

$$\text{s.t.}\quad (2.1)\text{-}(2.2), 1 \geq u_i \geq 0,\ 1 \geq v_i \geq 0,\ i = Y, Z\,,$$

$$H(0) = H_0 > 0, \qquad K(0) = K_0 > 0\,.$$

Therefore the current value of Hamiltonian for this problem can be written as

$$V = \frac{C^{1-\sigma}}{1-\sigma} + \mu\left[uH\, f(k_Y) - \delta K - C - X\right] + \lambda\left[(1-u)H\, g(k_Z) - \eta H + \frac{1}{p}X\right]$$

$$+\left[\beta_Y v + \beta_Z(1-v)\right]K + \left[\alpha_Y u + \alpha_Z(1-u)\right]H\,, \tag{3}$$

μ and λ are the costate variables associated with the state variables K and H, respectively. Also α_j and β_j (j=Y, Z) are the Lagrange multiplier for the constraints on the shares of H-capital and K-capital devoted to each sector to be non-negative. The necessary and F.O.C. are

$$\frac{\partial V}{\partial C} = 0 \rightarrow C^{-\sigma} - \mu = 0\,, \tag{4.1}$$

$$\frac{\partial V}{\partial X} = 0 \rightarrow -\mu + \frac{1}{p}\lambda = 0\,, \qquad (4.2)^{1}$$

$$\frac{\partial V}{\partial v} = 0 \rightarrow \mu\left[Kf'(k_Y)\right] + \lambda\left[-Kg'(k_Z)\right] + \beta_Y K - \beta_Z K = 0$$

$$\rightarrow f'(k_y) + \frac{\beta_Y}{\mu} = pg'(k_Z) + \frac{\beta_Z}{\mu}\,, \tag{4.3}$$

$$\frac{\partial V}{\partial u} = 0 \rightarrow \mu\left[H\left(f - k_Y f'(k_Y)\right)\right] + \lambda\left[-H\left(g - k_Z g'(k_Z)\right)\right] + \alpha_Y H - \alpha_Z H = 0$$

$$\rightarrow \left(f - k_Y f'(k_Y)\right) + \frac{\alpha_Y}{\mu} = p\left(g - k_Z g'(k_Z)\right) + \frac{\alpha_Z}{\mu}\,, \tag{4.4}$$

$$\dot{\mu} = -\frac{\partial V}{\partial K} + \rho\mu = -\mu\left[vf'(k_Y) - \delta\right] - \lambda\left[(1-v)g'(k_Z)\right] - \left(\beta_Y v + \beta_Z(1-v)\right) + \rho\mu\,, \tag{4.5}$$

[1] Notice that equation (4.2) does not depend on any control variable. Therefore, we have a bang-bang control for *X*.

$$\dot{\lambda} = -\frac{\partial V}{\partial H} + \rho\lambda = -\mu\left[uf(k_Y) - uk_Y f'(k_Y)\right] - \lambda\left[(1-u)g(k_Z) - (1-u)k_Z g'(k_Y) - \eta\right]$$

$$-\left(\alpha_Y u + \alpha_Z (1-u)\right) + \rho\lambda \,, \tag{4.6}$$

$$\lim_{t\to\infty} e^{-\rho t}\mu(t)K(t) = 0 \,, \tag{4.7}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)H(t) = 0 \,. \tag{4.8}$$

The costate variables μ and λ are the marginal utilities of *Y*-good and *Z*-good respectively. Therefore, $\frac{\lambda}{\mu}$ represents the relative marginal value of *Z* with respect to *Y* or domestic price of *Z* in units of *Y*. We should note that the *p* is the international price of good *Z* in terms of *Y* which in general might or might not be equal to the domestic price of each country. However after trade occurs open country's price for traded goods will be the same as the one in international market.

As mentioned, α_j and β_j (*j=Y, Z*) are the Lagrange multiplier for non negativity constraints on the shares of *H*-capital and *K*-capital devoted to each sector. Hence, if the constraints on the control variables *v* and *u* are satisfied with strict inequality, i.e. the economy produces both goods, corresponding multiplier would then be zero.

Dividing (3.5) and (3.6) by μ and λ respectively, we obtain the rate of growth for the costate variables as,

$$\frac{\dot{\mu}}{\mu} = (\rho+\delta) - v\left[f'(k_Y) - pg'(k_Z) + \frac{\beta_Y}{\mu} - \frac{\beta_Z}{\mu}\right] - pg'(k_Z) - \frac{\beta_Z}{\mu}\,,$$

$$\frac{\dot{\lambda}}{\lambda} = (\rho+\eta) - \frac{\mu}{\lambda}u\left[\left(f - k_Y f'\right) - p\left(g - k_Z g'\right) + \frac{\alpha_Y}{\mu} - \frac{\alpha_Z}{\mu}\right] - (g - k_Z g') - \frac{\alpha_Z}{\lambda}\,.$$

Using (4.3) and (4.4) these equations will be simplified as follows,

$$\frac{\dot{\mu}}{\mu} = (\rho + \delta) - \left(pg'(k_Z) + \frac{\beta_Z}{\mu} \right),$$

$$\frac{\dot{\lambda}}{\lambda} = (\rho + \eta) - \left(g - k_Z g' \right) - \frac{\alpha_Z}{\lambda}.$$

Let $r \equiv f'(k_Y) + \frac{\beta_Y}{\mu}$, denotes the market rental on *K*-capital and $w \equiv \left(f(k_Y) - k_Y f'(k_Y) \right) + \frac{\alpha_Y}{\mu}$ denotes the market rental on *H*-capital in units of *Y*-good. We can then rewrite the first order and necessary conditions as

$$C^{-\sigma} - \mu = 0, \tag{4.1}$$

$$-\mu + \frac{1}{p}\lambda = 0 \Rightarrow p = \frac{\lambda}{\mu}, \tag{4.2}$$

$$r = f'(k_y) + \frac{\beta_Y}{\mu} = pg'(k_Z) + \frac{\beta_Z}{\mu}, \tag{4.3}$$

$$w = \left(f - k_Y f'(k_Y) \right) + \frac{\alpha_Y}{\mu} = p\left(g - k_Z g'(k_Z) \right) + \frac{\alpha_Z}{\mu}, \tag{4.4}$$

$$\frac{\dot{\mu}}{\mu} = \rho + \delta - r, \tag{4.5}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho + \eta - \frac{w}{p}, \tag{4.6}$$

$$\lim_{t \to \infty} e^{-\rho t} \mu(t) K(t) = 0, \tag{4.7}$$

$$\lim_{t \to \infty} e^{-\rho t} \lambda(t) H(t) = 0. \tag{4.8}$$

The necessary conditions together with equations (2.1) and (2.2) can be used to characterize solution to the representative agent's problem.

2.2.1 AUTARKY CONDITION

When there is no trade, X would then be zero and p will be the internal price of good Z in units of Y. Bond, Wang and Yip (1996) have discussed the model for a closed economy. In this case, $\alpha_Y = \beta_Y = 0$ and $\alpha_Z = \beta_Z = 0$ and therefore (4.3) and (4.4) will become

$$r = f'(k_y) = pg'(k_Z), \tag{4.3$'$}$$

$$w = f(k_Y) - k_Y f'(k_Y) = p[g(k_Z) - k_Z g'(k_Z)]. \tag{4.4$'$}$$

Assuming no factor intensity reversal we can use $(4.3)'$ and $(4.4)'$ to solve for the factor intensities in each sector i.e. $k_Y(p)$ and $k_Z(p)$ and factor prices $r(p)$ and $w(p)$, as functions of the output price alone.

Totally differentiate $(4.3)'$ and $(4.4)'$, and using the definition of factor prices, we get

$$\text{(a)}\ \ k'_Y(p) = \frac{rk_Z + w}{(k_Z - k_Y)pf''}, \qquad \text{(b)}\ \ k'_Z(p) = \frac{rk_Y + w}{(k_Z - k_Y)p^2 g''},$$

$$\text{(c)}\ \ r'(p) = f''\, k'_Y(p), \qquad \text{(d)}\ \ w'(p) = -k_Y f''\, k'_Y(p), \tag{5.1}$$

Also,

$$\frac{w(p)}{p} = g(k_Z(p)) - k_Z(p)\, g'(k_Z(p)) \Rightarrow \left(\frac{w(p)}{p}\right)' = -k_Z(p) g'' k'_Z(p), \tag{5.2}$$

Define $k \equiv K/H$, the aggregate factor proportions. Therefore $k = uk_Y(p) + (1-u)k_Y(p)$. We can rewrite the reduced system as follows

$$u(p,k) = [k - k_Z(p)]/[k_Y(p) - k_Z(p)], \tag{6.1}$$

$$y(p,k) \equiv Y/H = u(p,k)f(k_Y(p)), \tag{6.2}$$

$$z(p,k) \equiv Z/H = [1-u(p,k)]g(k_Z(p)). \tag{6.3}$$

For given values of the costate variable *p* and the state variable *k*, we can determine the allocation of resources by using (5) and (6).

As mentioned the relative price of good *Z* is the ratio of costate variable, i.e. $p = \dfrac{\lambda}{\mu}$.

Therefore equations (4.5) and (4.6) can be used to find the growth rate of *p*,

$$\gamma_p = \left(\frac{\dot{\lambda}}{\lambda}\right) - \left(\frac{\dot{\mu}}{\mu}\right) \rightarrow \gamma_p = r(p) - \frac{w(p)}{p} + \eta - \delta,$$

or alternatively,

$$\gamma_p = [r(p) - \delta] - \left[\frac{w(p)}{p} - \eta\right].$$

Using the necessary and first order conditions, Bond, Wang and Yip (1996) have shown that there will exist a unique growth path on which *K*, *H* and *C* will all grow at a constant rate γ^* and relative price of goods will be constant, i.e. $\gamma_p = 0$, where

$$\gamma^* = \frac{1}{\sigma}\left[r(p) - \delta - \rho\right], \tag{7}$$

and there will be a unique *k* ratio associated with BGP.

2.2.2 BALANCE GROWTH PATH WITH TRADE

Now suppose countries open to trade. After trade resumes between countries, the internal price of each country should be equal to the equilibrium world price, *p*, at which the trade balance is satisfied. Since both countries are large therefore *p* should be determined as part of the general equilibrium problem. Also the optimization problems of two countries are related through X therefore they should be solved simultaneously. To solve this problem we first assume that p exists and it is unique and we solve the problem. Then we examine whether the condition for existence and uniqueness of p holds.

From international trade theory, we know that for both countries to be interested in trade, p should lie between the countries' autarkic prices, i.e. $p \in [p_2, p_1]$, where p_1 and p_2 are the autarkic prices for each country (assuming $p_2 < p_1$).

Therefore, if $p_2 \leq p \leq p_1$, there are three possible cases to consider[2],

$$\begin{aligned} &i)\ p_2 < p = p_1, \\ &ii)\ p_2 = p < p_1, \\ &iii)\ p_2 < p < p_1. \end{aligned} \tag{8}$$

Cases (i) and (ii) imply that for at least one country the price of traded goods after trade will remain the same as that before trade. Case (iii) however indicates that the prices for both countries would be different under trade than that under autarky.

[2] Remember that p is the price at which $X_1 = -X_2$, i.e. trade balance condition for the world should be satisfied. From previous chapter recall that this is the price at which both countries will grow at the same rate. With Cobb- Douglas production function in both sectors, $Y = A(vK)^{\alpha}(uH)^{1-\alpha}$, and $Z = B((1-v)K)^{\eta}((1-u)H)^{1-\eta}$ this price is, $p^* = \left(\frac{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}{\eta_2^{\eta_2}(1-\eta_2)^{1-\eta_2}}\frac{A_1}{B_2}\right)^{\frac{1}{1-\alpha_1+\eta_2}}$. To get the conditions in (8) it could be shown that the autarky prices can be written as $p_i = \left(\frac{\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}}{\eta_i^{\eta_i}(1-\eta_i)^{1-\eta_i}}\frac{A_i}{B_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}$ (see appendix A). Now, it can easily be seen, if $\alpha_1 = \alpha_2$ and $A_1 = A_2$ then $p_2 = p^* < p_1$. Also, if $\eta_1 = \eta_2$ and $B_1 = B_2$ then $p_2 < p^* = p_1$. Finally when $\alpha_1 \neq \alpha_2, \eta_1 \neq \eta_2, A_1 \neq A_2$ and $B_1 \neq B_2$ then $p_2 < p^* < p_1$.

2.2.2.1 EXISTENCE OF A UNIQUE BALANCED GROWTH PATH

In what follows we will examine the existence and uniqueness of a balanced growth path (BGP) for a large open economy given the world price. We will show that there will be two types of balance growth path equilibria. Depending on the value of country's autarkic price relative to the equilibrium world price, countries could either completely specialize in production of one good or continue to produce both goods along the BGP.

To do so, we will follow a similar approach used by Bond and Trask (1997). First, we show that the domestic prices and the pattern of specialization consistent with balanced growth path could uniquely be determined by the world price, *p*. Then we use these prices to solve for the *K/H* ratio, growth rates, and consumption/wealth ratio on the BGP. A BGP for each country requires that the level of consumption and the stock of capitals (*K*-capital and *H*-capital) grow at the same rate, $\gamma_C = \gamma_K = \gamma_H > 0$ and the relative price of traded goods be constant $\gamma_p = 0$ [3].

The internal price of *Z* in terms of *Y* for each country $p_i = \dfrac{\lambda_i}{\mu_i}$ will be equal to external (international) price after trade. Therefore $\gamma_p = 0$ implies $\gamma_\lambda = \gamma_\mu$. Using (4.5) and (4.6) this growth rate could be written as,

[3] Other variables' growth rate on the BGP could be determined as follow. On the BGP *p* remains constant and so as $k_Y(p)$ and $k_Z(p)$. Also $\gamma_K = \gamma_H \rightarrow \gamma_k = 0$ therefore from (6.1) *u* will be constant. From (6.2) and (6.3) y and z are constant on the BGP, so $\gamma_y = \gamma_z = 0 \rightarrow \gamma_Y = \gamma_H = \gamma_Z$. From (2.1) $\gamma_K = u\dfrac{f(k_{Yy})}{k} - \delta - \dfrac{C}{K} - \dfrac{X}{K}$, we have shown that on the balanced growth path the growth rate of *K* is constant so everything on the right hand side should be as well constant, therefore $\gamma_X = \gamma_K$. It could then be seen that on the BGP everything that is growing (i.e. *C*, *K*, *H*, *X*, *Y* and *Z*) will grow at an equal constant rate, while *p* and *u* (as well as *v* since it could be written as a function of *u*) remain constant.

$$\gamma_p = 0 \Rightarrow r - \frac{w}{p} + \eta - \delta = 0 ,$$

or alternatively,

$$(r - \delta) - (\frac{w}{p} - \eta) = 0 . \tag{9}$$

This condition implies that on the BGP for each country, the net return on investment in *K*-capital should be equal to the net return on investment in *H*-capital. Following Bond and Trask (1997), we call this condition *intertemporal arbitrage* (*IA*) *condition*.

The solution for w and r consistent with the balanced growth path given the value of p can be obtained from the intertemporal arbitrage condition and the zero profit condition. Zero profit condition requires that unit costs Φ_j $(j = Y, Z)$ should be no less than the price in each sector (strict equality holds in sectors that are producing).

These conditions are,

$$1 \leq \Phi_Y(w, r) , \tag{10.1}$$

$$p \leq \Phi_Z(w, r) . \tag{10.2}$$

Notice that in this model there are two traded goods and two factors of production. In static models typically these unit cost conditions could be used to solve for w and r given p. The result would usually generate a range of factor endowments consistent with production of all goods[4]. However, in the dynamic setting these prices should also satisfy the intertemporal arbitrage condition. Therefore it is less likely that in the dynamic models a given p be consistent with production of both goods on the BGP (i.e. (10.1) and (10.2) both be satisfied with strict equality).

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[4] The 2×2 Heckscher-Ohlin model of international trade theory.

Following Bond, Wang, and Yip (1996), we impose the following condition on the technologies which assures the existence of a solution.

Condition FP (Factor Price). Let $\Omega_{Y,Z} = \{(w,r,p) \,|\, p_j = \phi_j(w,r) \text{ for } j \in \{Y,Z\}\}$, then

$$\sup_{\Omega_{Y,Z}} \left(r - \frac{w}{p} \right) > \delta - \eta > \inf_{\Omega_{Y,Z}} \left(r - \frac{w}{p} \right).$$

This condition ensures that investment in both types of capital is profitable. Using the FP condition and the no arbitrage condition and zero profit conditions described above we can examine the existence of two types of production pattern on the BGP.

i) If condition FP holds there exist a unique world price, *p and corresponding domestic prices (*w** and *r**) and sectoral factor intensities[5] (k_Y and k_Z), at which both sectors continue to produce and the intertemporal arbitrage condition is satisfied.**

Start by solving for the values of *w* and *r* that are consistent with the balanced growth path in which both goods are being produced, i.e. (10.1) and (10.2) holds with strict equality. Therefore,

$$1 = \Phi_Y(w,r), \tag{11.1}$$

$$p = \Phi_Z(w,r). \tag{11.2}$$

These equations can be inverted to solve for *w*, *p* in terms of *r*. Totally differentiating (11.1) and (11.2) we get:

$$\hat{w} = -\frac{1-\theta_{HY}}{\theta_{HY}} \hat{r}, \tag{12.1}$$

$$\hat{p} = \frac{\theta_{HY} - \theta_{HZ}}{\theta_{HY}} \hat{r}, \tag{12.2}$$

[5] Notice that the factor intensities can be written as a function of the price level, *p*.

where a hat over variables denotes a rate of change[6] and θ_{Hi} represents the share of *H*-capital costs in unit costs of good *i*, *i=Y,Z*.

Using (12), it could be shown that *w(r)/p(r)* is a monotone and decreasing function of *r*. Therefore, if condition FP holds, there will exist at most one *r* at which the intertemporal arbitrage condition in (9) be satisfied. Denoted this value by *r**. As the result *p* = p(r*)* is the unique world price consistent with BGP and incomplete specialization in good *Y* and *Z*.

Bond, Wang, and Yip (1996) have shown that when the economy is closed and FP condition holds, there exist a unique BGP with the prices given by the values *r**, *w** and *p**, in which both goods are being produced and intertemporal arbitrage condition is satisfied[7]. Therefore if condition FP holds then there exists a unique world price at which all of the equation in (9)-(10.2) are satisfied with strict equality and the world price associated with incomplete specialization is also the price of the open economy under autarky.

ii) If *p*>*p, there exist unique domestic prices satisfying (9)-(10.2) in which only good *Z* is produced. In this case *r*>*r** and *w/p* is increasing in *p*.**

In this case the relative price of *Z*-good will be higher with trade than that under autarky. Under autarky there will exist a unique price *p** and its corresponding domestic prices *w** and *r** such that,

$$1 = \Phi_Y(w^*, r^*), \tag{13.1}$$

$$p_i = \Phi_Z(w^*, r^*), \tag{13.2}$$

$$(r^* - \delta) - (\frac{w^*}{p^*} - \eta) = 0. \tag{13.3}$$

---

[6] i.e. $\hat{x} = \frac{dx}{x}$ for any variable.

[7] i.e. The prices at which equation (9) as well as (11.1) and (11.2) are satisfied.

Again start with solving for the values of (w, r and p) consistent with balanced growth path and production of only Y sector. In this case the new set of prices should satisfy the intertemporal arbitrage condition (9) and only condition (10.2) with strict equality, that is,

$$p = \Phi_Z(w,r). \qquad (10.2)$$

We can invert that to obtain $w = w(p,r)$, totally differentiating (10.2) will yield,

$$\hat{w} = \frac{1}{\theta_{HZ}}\hat{p} - \frac{1-\theta_{HZ}}{\theta_{HZ}}\hat{r}. \qquad (14)$$

Using (14) it can be shown that w *(p,r)/p* is a decreasing function of r. Therefore, there can be at most one value of r that satisfies the intertemporal arbitrage condition. Also (14) can be used to show that for a given r, w/p is increasing in p. Recall that (10.2) and (9) are also satisfied at the autarky prices (p^*, w^*, r^*) with strict equality. Therefore if $r=r^*$ then $\frac{w}{p} = \frac{w^*}{p^*}$. However since $p > p^*$ then $r>r^*$ and $\frac{w}{p} > \frac{w^*}{p^*}$. This implies that r and w/p are both increasing in p. The values of w/p and r at which (9) and (10.2) are being satisfied

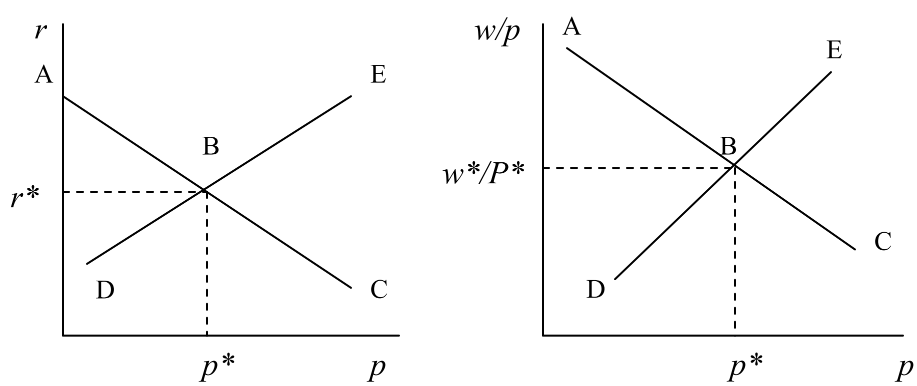

Figure 2-1: Factor prices satisfying intertemporal arbitrage and zero profits

with strict equality could be illustrated by the line DBE in (r, p) space and the ray DBE in (w, p) space in figure 2-1.

Since r and w are increasing in p, then the unit cost of good Y is increasing in p along DBE. Note that the production of Y earns zero profit at p*, (i.e. p=p*, $\Phi_Y(w^*, r^*) = 1$) therefore for $p > p^*$, $\Phi_Y(w, r) > 1$. Hence, the Y sector is not earning positive profit and so it will be shut down. The segment BE in figure 2-1 is consistent with the balanced growth when only good Z is being produced.

iii) If *p*<*p, then there exist unique prices (*w*, *r*) consistent with the balanced growth path at which (9) is satisfied and production of only *Y* sector.**

In this case the autarky price for the economy is higher than that with trade. At the autarky price (p_i=p*) both goods are being produced,

$$1 = \Phi_Y(w^*, r^*)\,, \tag{15.1}$$

$$p_i = \Phi_Z(w^*, r^*)\,, \tag{15.2}$$

and,

$$(r^* - \delta) - (\frac{w^*}{p^*} - \eta) = 0\,. \tag{15.3}$$

Next we solve for the values of *(p, w, r)* consistent with balanced growth when only good Y is being produced. If country two is producing only good Y then (10.1) holds with equality. This condition can be inverted to yield $w = w(r)$. Totally differentiating this condition we obtain,

$$\hat{w} = -\frac{1 - \theta_{HY}}{\theta_{HY}}\hat{r}\,. \tag{16}$$

Using equation (16), it can be shown that for a fixed *p*, $r - \frac{w(r)}{p}$ is an increasing function of *r*, and therefore (9) has at most one solution for a given *p*[8]. By totally differentiating (9) and substituting from (15) we get,

$$\frac{\hat{r}}{\hat{p}} = -\frac{w\theta_{HY}}{r\,p\theta_{HY} + w(1-\theta_{HY})} < 0\,, \tag{17.1}$$

$$\frac{\hat{w}-\hat{p}}{\hat{p}} = -\frac{r\,p\theta_{HY}}{r\,p\theta_{HY} + w(1-\theta_{HY})} < 0\,. \tag{17.2}$$

Using (17.1) and (17.2) it could be shown,

$$\frac{\hat{\phi}_Z}{\hat{p}} = (1-\theta_{HZ})\frac{\hat{r}}{\hat{p}} + \theta_{HZ}\frac{\hat{w}}{\hat{p}}\ ,$$

and then substituting from (16),

$$\hat{\phi}_Z - \hat{p} = -\frac{w(1-\theta_{HZ}) + r\,p\theta_{HY}}{r\,p\theta_{HY} + w(1-\theta_{HY})}\ \hat{p} < 0\,. \tag{18}$$

Equation (18) implies that the cost of producing *Z* is changing more slowly than the price of good *Z*. Since at the autarky prices p^* we have $\phi_Z(w^*, r^*) = p^*$, then for $p < p_i (= p^*), \phi_Z > p$. Therefore sector *Z* is unprofitable in this case and it will be shut down on the BGP. The segment AB in figure 1 would be consistent with producing only *Y* on the balanced growth path.

These results could be summarized as follow,

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[8] Notice that solution exist when $p = p_i = p^*$, for $p < p_i$ solution will exist as long as $p \geq p^{\min}$, i.e. $p^{\min} = \min_{\Omega_Y} w(r+\eta-\delta)^{-1}$, and $\Omega_Y = \{(w,r) \mid p_i = \Phi_i(w,r) \text{ for } i \in \{Y\}\}$

1) $p^*_2 < p = p^*_1$: There is a unique world price at which country 1 continues to produce both good on the balanced growth path, while country 2 shuts down the production of good Y.

2) $p^*_2 = p < p^*_1$: There is a unique world price at which Country 2 continues to produce both goods on the balanced growth path and country 1 shuts down the production of good Z.

3) $p^*_2 < p < p^*_1$: There is a unique world price at which country 1 produces only good *Y* and country 2 produces only good *Z*.

So far we have shown that there exists a unique set of constant domestic prices which satisfy the necessary condition (4.2)-(4.6) given condition FP. To complete the proof of the existence of a unique balanced growth path we need to show that there is a common non-degenerate growth rate and constant values of $c \equiv C/H > 0$, $k \equiv K/H > 0$ and $x \equiv X/H$ that satisfy the remaining necessary conditions and constraints (2.1)-(2.2).

From (4.1) we have,

$$C^{-\sigma} - \mu = 0,$$

consequently,

$$\gamma_C = \frac{1}{\sigma}\left(\frac{\dot{\mu}}{\mu} - \rho\right).$$

Substituting for $\frac{\dot{\mu}}{\mu}$ from (4.5) we get,

$$\gamma_C = \frac{1}{\sigma}\left(r(p) - \delta - \rho\right).$$

On the balanced growth path $\gamma_K = \gamma_H = \gamma_C = \gamma^*$ if $\gamma_p = 0$, where,

$$\gamma^* = \frac{1}{\sigma}\left(r(p) - \delta - \rho\right). \tag{19}$$

For this growth rate to be non-degenerate we need the following condition to hold, $r^*\text{-}\delta - \rho > 0$.

Next we should check if the transversality conditions are satisfied on the balanced growth path. For transversality condition (4.7) to be satisfied, the growth rate of $e^{-\rho t}\mu(t)K(t)$ should be negative, therefore

$$\frac{d(e^{-\rho t}\mu(t)K(t)}{dt} = -\rho + \frac{\dot{\mu}}{\mu} + \frac{\dot{K}}{K} = -\rho - \sigma\gamma + \gamma = -\rho + (1-\sigma)\gamma < 0, \tag{20.1}$$

similarly for (3.8) to be satisfied,

$$\frac{d(e^{-\rho t}\lambda(t)H(t)}{dt} = -\rho + \frac{\dot{\lambda}}{\lambda} + \frac{\dot{H}}{H} = -\rho - \sigma\gamma + \gamma = -\rho + (1-\sigma)\gamma < 0. \tag{20.2}$$

We need an upper bound on the feasible growth rate to guarantee that the transversality condition are satisfied on the balance growth path, i.e. $\rho > (1\text{-}\sigma)\ \gamma\ (p_x{}^{min})$. Therefore, from (17.1) and (17.2),

$$-\rho + (1-\sigma)\gamma < 0,$$

or,

$$\rho > (1-\sigma)\gamma\ . \tag{21}$$

Also it needs to be shown that the growth rate defined in (16) satisfies (2.1) - (2.2).

From (2.2)[9],

$$\dot{H} = (1-u)\,Hg(k_z) - \eta H + \frac{1}{p}X\,,$$

from this equation solve for (1-*u*),

$$(1-u) = \frac{(\dot{H}/H) + \eta - (1/p)(X/H)}{g(k_Z)}\,,$$

since *H* grows at the rate of consumption then,

$$(1-u) = \frac{\gamma^* + \eta - (1/p)(x)}{g(k_Z)}\,. \tag{22}$$

Feasibility then requires $0 < (1-u) < 1$. We use the equation (22) to check this condition. First we check, $0 < (1-u)$ which from equation (22) can be written as,

$$\gamma^* + \eta - \frac{1}{p}x > 0\,.$$

To see if this inequality holds, from (2.2) we have

$$\gamma^* = \frac{\dot{H}}{H} = (1-u)\,g(k_z) - \eta + \frac{1}{p}\left(\frac{X}{H}\right) = y - \eta + \frac{1}{p}x\,,$$

therefore,

$$\gamma^* + \eta - \frac{1}{p}x = y > 0\,.$$

Similarly we can check the left hand inequality, i.e. $(1-u) < 1$ which using equation (22) will be,

$$\frac{\gamma + \eta - (1/p)(x)}{g(k_Z)} < 1,$$

[9] Since on the BGP, $\dot{K}/K = \dot{H}/H$ it is enough if we show the growth rate could satisfy only one of them.

therefore,

$$\gamma + \eta - (1/p)(x) < g\,. \tag{23}$$

Notice that from the competitive profit condition we have,

$$g = \frac{w + rk_Z}{p}\,.$$

Substituting for g from the competitive profit condition in equation (23) we get,

$$\gamma + \eta - (1/p)(x) > \frac{w + rk_Z}{p}\,,$$

or,

$$\gamma - (1/p)(x) - \frac{rk_Z}{p} > \frac{w}{p} - \eta\,.$$

Next, from equation (4.6) and intertemporal arbitrage condition we have,

$$\frac{w}{p} - \eta = \rho - \frac{\dot{\lambda}}{\lambda} = \rho - \frac{\dot{\mu}}{\mu}\,,$$

also we have shown before that, $\gamma = -\frac{1}{\sigma}\left(\frac{\dot{\mu}}{\mu}\right) \Rightarrow \gamma = \frac{1}{\sigma}\left(\frac{w}{p} - \eta - \rho\right)$ and so

$$\frac{w}{p} - \eta = \sigma\gamma + \rho > \gamma > \gamma - (1/p)(x) - \frac{rk_Z}{p}\,.$$

Next we should solve for the values of *k*, *c* and *x* on the balanced growth path. We have shown that if $p = p^*$ both goods are going to be produced and if $p \neq p^*$ only one of the traded goods is going to be produced. Therefore the solution for *k* could be obtained from the full employment condition, those are,

1) if $p = p_i$ then $k(p) = uk_Y(p) + (1-u)k_Z(p)$,

2) if $p > p_i$ then $u = 0$ and therefore $k(p) = k_Z(p)$,

3) if $p < p_i$ then $u = 1$ and therefore $k(p) = k_Y(p)$.

It remains to show that $c \geq 0$. The budget constraint for each country can be written as

$$C + (\dot{K} + \delta K) + p(\dot{H} + \eta H) = wH + rK\,,$$

or,

$$C + (\frac{\dot{K}}{K} + \delta)K + p(\frac{\dot{H}}{H} + \eta)H = wH + rK\,,$$

dividing each side by H,

$$\frac{C}{H} + (\gamma + \delta)\frac{K}{H} + p(\gamma + \eta) = w + r\frac{K}{H}\,,$$

then,

$$c + (\gamma + \delta)k + p(\gamma + \eta)) = w + rk\,,$$

rearranging this equation as follow,

$$c + (k + p)\gamma = p(\frac{w}{p} - \eta) + (r - \delta)k\,. \tag{24}$$

From equation (9) and using equation (19) we get,

$$(r - \delta) = (\frac{w}{p} - \eta) = \sigma\gamma + \rho\,. \tag{25}$$

Substituting from (25) into (24)

$$c + (k + p)\gamma = (p + k)(\sigma\gamma + \rho)\,,$$

dividing each side by $k + p$ and also considering $\rho > (\sigma - 1)\gamma$, we obtain,

$$\frac{c}{k + p} = \rho + (\sigma - 1)\gamma > 0\,,$$

consequently it implies c>0.

The segment BE in figure1 represents the factor prices consistent with the complete specialization in good *Z*. Similarly, segment AB represents the factor prices consistent with complete specialization in good *Y,* and point B represents the factor prices at which both goods are being produced. As it can be seen *r* is the lowest when the relative price equals the autarky prices[10].

Now recall that the growth rates of the economy on the BGP is given by equation (19). The growth rates are positively related to the return on *K*-type and *H*-type capital[11]. It is easy to see that the growth rates then is the smallest at autarky prices. Therefore the growth rates of countries with trade are higher than that under autarky. However, when $p = p_i$, the growth rate of country *i* will remain unchanged with trade while the growth rate of its trade partner would rise with complete specialization in one of the traded goods.

2.2.2.2 EXISTENCE AND UNIQUENESS OF THE INTERNATIONAL PRICE

So far we have shown the existence and uniqueness of a balanced growth path when the international price is given. Now we should show that such unique price level could exist. Suppose $p_2 < p < p_1$[12]. Country 1 is completely specialize in producing *Y* therefore its growth rate is

$$\gamma_{1,T} = \frac{1}{\sigma}\left(r_1(p) - \delta - \rho\right).$$

However country 2 is going to produce only *Z*, then it could be shown

[10] The price level at which both goods are being produced.

[11] $\gamma^* = \frac{1}{\sigma}\left(r(p) - \delta - \rho\right) = \frac{1}{\sigma}\left(\frac{w(p)}{p} - \eta - \rho\right)$ from the intertemporal arbitrage condition.

[12] If p could be equal to the autarky prices then the argument will hold the only difference is that the growth rate of one of the countries does not change.

$$\gamma_{2,T} = \frac{1}{\sigma}\left(\frac{w_2(p)}{p} - \eta - \rho\right).$$

Now $\frac{\partial \gamma_{1,T}}{\partial p} = \frac{1}{\sigma}(r'(p))$ which will be positive if $k_Z > k_Y$ and it will be negative if $k_Z < k_Y$. Also $\frac{\partial \gamma_{2,T}}{\partial p} = \frac{1}{\sigma}\left(\frac{w(p)}{p}\right)'$ and it will negative if $k_Z > k_Y$ and will be positive if $k_Z < k_Y$. Therefore regardless of sectoral factor intensities a unique p will exist and it is stable.

Also notice that on the balance growth path all the variables that are growing should grow at the same rate, i.e. $\gamma^* = \gamma_K = \gamma_H = \gamma_X$. To check if it holds consider (2.1), $\gamma_K = \frac{\dot{K}}{K} = \frac{1}{k}(y - c - x) - \delta$. We have already shown that on the balanced growth path $\gamma^* = \gamma_K = \gamma_H$ therefore for γ_K to be constant x needs to be constant and so $\gamma_H = \gamma_X$.

Now consider the growth rate for country 1,

$$\gamma_1 = \frac{1}{k_1}(y_1 - c_1 - x_1) - \delta = \frac{1}{k_1}\left(\frac{Y_1}{H_1} - \frac{C_1}{H_1} - \frac{X_1}{H_1}\right),$$

from the trade balance condition we have $X_1 = -X_2$, substitute for X_1 in the growth rate of country 1. We can see that on the BGP, $\gamma_1^* = \gamma_{K,1} = \gamma_{H,1} = \gamma_{X,1} = \gamma_{X,2} = \gamma_{H,2} = \gamma_{K,2} = \gamma_2^*$. Therefore the equilibrium level of p should be the level at which the growth rates of countries be equal, i.e. $\gamma_{1,T} = \gamma_{2,T}$ which also implies $r_1(p) - \delta = \frac{w_2(p)}{p} - \eta$. Therefore the trade balance condition requires that at the equilibrium level of p the net return on investment in capitals should be equal across counties. In that case balanced growth could exist for individual countries as well as for the world[13].

However note that the value of price level that solves $r_1(p) - \delta = \frac{w_2(p)}{p} - \eta$ to be the world price should lie between the autarkic prices of the countries. Let's call this value p^*. In the other world if $p^* \in [p_2 \quad p_1]$ then the world price will be equal to this value and the BGP could exist. However if it falls out of the interval then the world price can not be equal to this value, instead it will be equal to one the values at the corner. In the case of corner solution then at least one of the countries incomplete specialize and its growth rate remains unchanged. Appendix 2.B explains describes the condition that guarantees world price to stay in this interval.

2.3 TRANSITIONAL DYNAMICS

In this section we analyze the transitional dynamics of the model described in the previous section in the neighborhood of the balanced growth path by examining the linearized dynamic system around the BGP. Define *k*, *c* and *x* as $k \equiv \frac{K}{H}$, $c \equiv \frac{C}{H}$ and $x \equiv \frac{X}{H}$. The dynamics of the system can be described by the behavior of *c*, *x*, *k* and *p*. From (2.1) and (2.2) we can get,

$$\frac{\dot{K}}{K} = \frac{uf(k_Y) - c - x}{k} - \delta,$$

$$\frac{\dot{H}}{H} = (1-u)g(k_Z) - \eta + \frac{1}{p}x.$$

[13] Condition FP should hold

Therefore the dynamics for *k* and *c* are,

$$\frac{\dot{k}}{k}=\frac{\dot{K}}{K}-\frac{\dot{H}}{H}=\frac{y(k,p)-c-x(k,p)}{k}-\delta-z(k,p)+\eta-\frac{1}{p}x(k,p)\,, \tag{26}$$

$$\frac{\dot{c}}{c}=\frac{\dot{C}}{C}-\frac{\dot{H}}{H}=\gamma_C(p)-z(k,p)+\eta-\frac{1}{p}x(k,p)\,, \tag{27}$$

Also we have shown before,

$$\frac{\dot{p}}{p}=\frac{\dot{\lambda}}{\lambda}-\frac{\dot{\mu}}{\mu}=r(p)-\frac{w(p)}{p}+\eta-\delta\,. \tag{28}$$

The system (26)-(28) describe the dynamics of the system along the transition path for each country.

Linearizing this system around the balanced growth path,

$$\begin{pmatrix}\dot{p}/p\\ \dot{c}/c\\ \dot{k}/k\end{pmatrix}=\begin{pmatrix}a_{11} & 0 & 0\\ a_{21} & 0 & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}\begin{pmatrix}p-p^*\\ c-c^*\\ k-k^*\end{pmatrix}, \tag{29}$$

where,

$$a_{11}=r'(p)-\left(\frac{w'}{p}-\frac{w/p}{p}\right), \tag{30.1}$$

$$a_{21}=v_C'-\frac{\partial z}{\partial p}-\left(\frac{1}{p}\frac{\partial x}{\partial p}-\frac{1}{p^2}x\right), \tag{30.2}$$

$$a_{23}=-\frac{\partial z}{\partial k}-\frac{1}{p}\frac{\partial x}{\partial k}, \tag{30.3}$$

$$a_{31}=\frac{1}{k}\left(\frac{\partial y}{\partial p}-\frac{\partial x}{\partial p}\right)-\frac{\partial z}{\partial p}-\left(\frac{1}{p}\frac{\partial x}{\partial p}-\frac{1}{p^2}x\right), \tag{30.4}$$

$$a_{32}=-\frac{1}{k}, \tag{30.5}$$

$$a_{33} = -\frac{(y-c-x)}{k^2} + \frac{1}{k}\left(\frac{\partial y}{\partial k} - \frac{\partial x}{\partial k}\right) - \frac{\partial z}{\partial k} - \frac{1}{p}\frac{\partial x}{\partial k}. \quad (30.6)$$

Two points should be considered here. First, it is clear that the dynamics of the relative price is independent of *c* and *k*. This is the result of the models in which the prices of factor inputs are determined by output prices alone and are independent of the relative factor supplies. However the signs of $r'(p)$ and $w'(p)$ are dependent on factor intensity of each sector. Therefore a_{11} could be either positive or negative. If $a_{11} > 0$ then the relative price adjustment process is unstable and *p* must jump immediately to the balance growth value. Therefore *p* remains constant along the transition process. Since growth rate of consumption is a function of prices it will also stay constant throughout the transition when $a_{11} > 0$. However when $a_{11} < 0$ then the adjustment of prices along the transition to the balanced growth path will be stable. Therefore *p* or the growth rate of consumption will not be constant in this case.

Also notice that from equation (Q), the Hamiltonian is linear in *X* with the slope equal to $\frac{\partial V}{\partial X} = -\mu + \frac{1}{p}\lambda$. Therefore there are 3 cases to consider,

$$i)\frac{\partial V}{\partial X} > 0 \rightarrow -\mu + \frac{1}{p}\lambda > 0 \rightarrow p < \frac{\lambda}{\mu}, \quad (31.1)$$

$$ii)\frac{\partial V}{\partial X} = 0 \rightarrow -\mu + \frac{1}{p}\lambda = 0 \rightarrow p = \frac{\lambda}{\mu}, \quad (31.2)$$

$$iii)\frac{\partial V}{\partial X} < 0 \rightarrow -\mu + \frac{1}{p}\lambda < 0 \rightarrow p > \frac{\lambda}{\mu}. \quad (31.3)$$

If case (i) prevails then the slope of Hamiltonian with respect to control variable *X* is always positive regardless of the value of *X*. This case is drawn as curve 1 in figure 2 for a specific point in time. The control region is assume to be the close interval [b, a]. Therefore the maximum of Hamiltonian occurs at X=a. Case (iii) in the other hand indicates that the slope of Hamiltonian with respect to X is decreasing, i.e. curve 2. If this case prevails then X=b maximizes the Hamiltonian. Second case is drawn as Curve 3 which indicates that the Hamiltonian slope with respect to X is zero therefore the maximum of V could occur at any value of X.

Recall that μ and λ are the costate variables associated with the state variables K and H, respectively. Therefore in our model, $\frac{\lambda}{\mu}$ represents the domestic price of Z in units of Y. Therefore at any point of time if $\frac{\lambda}{\mu} > p$ ($\frac{\lambda}{\mu} < p$), i.e. case 1 (3), to maximize Hamiltonian we need to set X as high (low) as possible, while when $\frac{\lambda}{\mu} = p$ Hamiltonian can be maximized at any level of X.

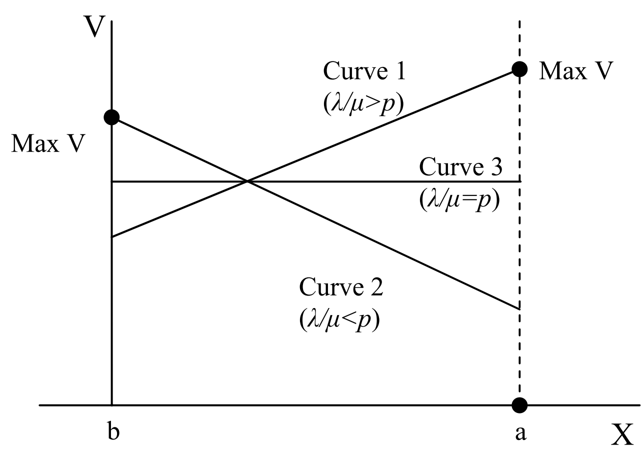

Figure 2-2: The bang-bang condition

Therefore because of the bang-bang control nature of X (control variable) we might have complete specialization on the balanced growth path as well as on the transition. To see this more clearly below we show that there will be a critical value of r such that the economy will specialize in Y for r less than this value and it will specialize in Z for values greater than the critical value along the transition. During the transition to the BGP, the relative price of good Z maybe changing because accumulation rates of the factors are not necessarily constant.

Remember the intertemporal arbitrage condition,

$$\frac{\dot{p}}{p} = \frac{\dot{\lambda}}{\lambda} - \frac{\dot{\mu}}{\mu} = r(p) - \frac{w(p)}{p} + \eta - \delta \,, \tag{32}$$

this equates the net rate of return to capitals on the BGP. On the transition however it might not be the case. If there is capital gain the equalization of returns to capitals require that the difference in net returns between K and H capital equal the rate of capital gain. Therefore factor prices should satisfy (10.1)-(10.2) and (9) along the transition path.

Now consider figure 2-3. In this figure ABC locus is the locus of factor prices consistence with zero profit in the Y sector. Along this locus we have $1 = \Phi_Y(w,r)$. Similarly DBE represents the locus of factor prices consistence with zero profits in Z sector i.e. $p = \Phi_z(w,r)$. Notice that these frontiers are drown for the case where $\theta_{HZ} > \theta_{HY}$. By using equations (12.1) and (12.2) we could show that DBE locus is flatter than ABC.

It is easy to show that the unit cost of Y (i.e. $\Phi_Y(w,r)$) is increasing in r along DBE locus, by using (14)[14].

Therefore for any $r < r_1$, $\Phi_Y(w,r) < 1$, which means that the Y sector earns positive profit for $r<r_1$. This implies, the production specialization should be Y for $r<r_1$. Similarly, using equation (15) we can show that the unit cost of Z is decreasing in r along ABC[15].

Therefore for $r>r_1$ then $\Phi_z(w,r) < 1$, i.e. sector Z is earning positive profit. Accordingly for $r>r_1$ the production specialization should be Z. Assuming $\theta_{HZ} > \theta_{HY}$ there is a critical value of r such that the economy will completely specialize in H-capital intensive good for r greater than this critical value and it will specialize in the other good for the values less than the critical value of r.

---

14 $$\frac{\hat{\Phi}_Y(w,r)}{\hat{r}} = \frac{\frac{d\Phi_Y}{\Phi_Y}}{\hat{r}} = \frac{\frac{\partial \Phi_Y}{\partial w}\cdot\frac{dw}{\Phi_Y} + \frac{\partial \Phi_Y}{\partial r}\cdot\frac{dr}{\Phi_Y}}{\hat{r}} = \frac{\frac{\partial \Phi_Y}{\partial w}\cdot\frac{w}{\Phi_Y}\cdot\frac{dw}{w} + \frac{\partial \Phi_Y}{\partial r}\cdot\frac{r}{\Phi_Y}\cdot\frac{dr}{r}}{\hat{r}}$$

$$\frac{\hat{\Phi}_Y(w,r)}{\hat{r}} = \frac{\theta_{HY}\cdot\hat{w} + (1-\theta_{HY})\hat{r}}{\hat{r}} = \theta_{HY}\left(\frac{\hat{w}}{\hat{r}}\right) + (1-\theta_{HY}) \qquad \text{(i)}$$

Now from equation (14), $\frac{\hat{w}}{\hat{r}} = -\frac{1-\theta_{HZ}}{\theta_{HZ}}$ substituting in (i) above we get,

$$\frac{\hat{\Phi}_Y(w,r)}{\hat{r}} = \theta_{HY}\left(-\frac{1-\theta_{HZ}}{\theta_{HZ}}\right) + (1-\theta_{HY}) = -\frac{\theta_{HY}}{\theta_{HZ}} + 1 > 0, \text{ since } \theta_{HY} < \theta_{HZ}.$$

15 $$\frac{\hat{\Phi}_z(w,r)}{\hat{r}} = \frac{\frac{d\Phi_Z}{\Phi_Z}}{\hat{r}} = \frac{\frac{\partial \Phi_Z}{\partial w}\cdot\frac{dw}{\Phi_Z} + \frac{\partial \Phi_Z}{\partial r}\cdot\frac{dr}{\Phi_Z}}{\hat{r}} = \frac{\frac{\partial \Phi_Z}{\partial w}\cdot\frac{w}{\Phi_Z}\cdot\frac{dw}{w} + \frac{\partial \Phi_Z}{\partial r}\cdot\frac{r}{\Phi_Z}\cdot\frac{dr}{r}}{\hat{r}}$$

$$\frac{\hat{\Phi}_Z(w,r)}{\hat{r}} = \frac{\theta_{HZ}\cdot\hat{w} + (1-\theta_{HZ})\hat{r}}{\hat{r}} = \theta_{HZ}\left(\frac{\hat{w}}{\hat{r}}\right) + (1-\theta_{HZ}) \qquad \text{(i)}$$

Now from equation (15), $\frac{\hat{w}}{\hat{r}} = -\frac{1-\theta_{HY}}{\theta_{HY}}$ substituting in (i) above we get,

$$\frac{\hat{\Phi}_Z(w,r)}{\hat{r}} = \theta_{HZ}\left(-\frac{1-\theta_{HY}}{\theta_{HY}}\right) + (1-\theta_{HZ}) = -\frac{\theta_{HZ}}{\theta_{HY}} + 1 < 0, \text{ since } \theta_{HY} < \theta_{HZ}.$$

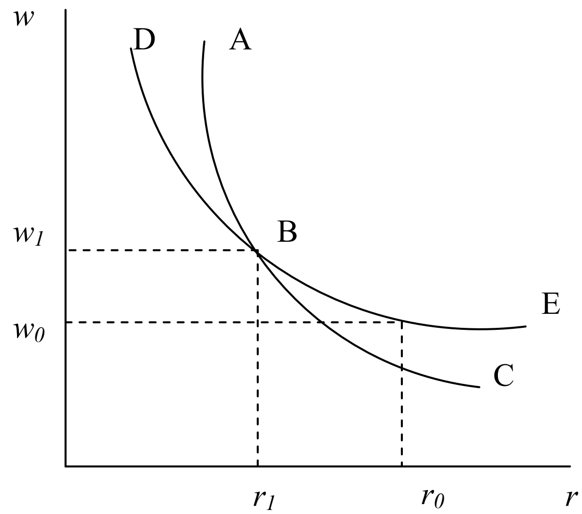

Figure 2-3: Price frontiers

Now notice that if $r > r^*$ (r^* being the value of r on the balance growth path) then it implies that $\frac{\dot{p}}{p} > 0$ so the price level must be growing along the transition path. Therefore price level would be lower than its steady state level throughout the transition, i.e. $\frac{\lambda}{\mu} < p$. Alternatively if $r < r^*$ then $\frac{\dot{p}}{p} < 0$, which implies prices should be falling along the transition path or its value is higher than its steady state throughout the transition i.e. $\frac{\lambda}{\mu} > p$. Figure 2-4 illustrates these cases.

In sum the bang-bang control condition for X in (21) will indeed imply that there will be complete specialization along the transitional path to the balanced growth path. Therefore throughout the transition process, the factor prices should lie on the ABE locus in figure 2-3 (or figure 2-4)[16]. In the other word if $p_2 < p < p_1$ then each economy will completely specialize on the BGP and also on the transition to the BGP.

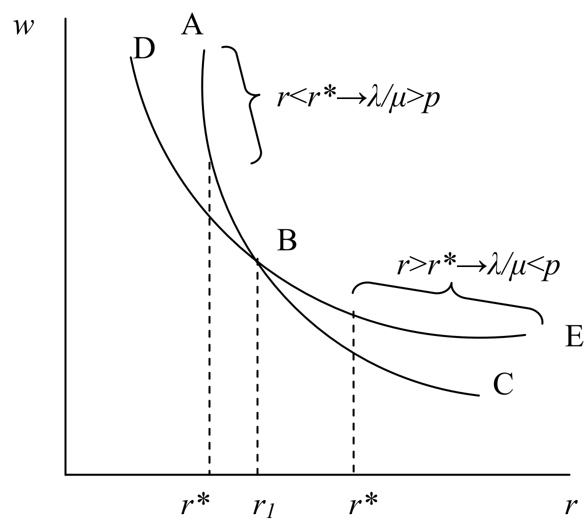

Figure 2-4: Price frontier consistent with the production pattern on the transition path

If $k \equiv \frac{K}{H}$ is the aggregate factor proportions then the full employment requires that $uk_Y(p) + (1-u)k_Z(p) = k$, which yields

$$u(p,k) = \frac{k - k_Z(p)}{k_Y(p) - k_Z(p)} \ ,$$

$$y(p,k) \equiv Y/H = u(p,k) f(k_Y(p)) \, ,$$

[16] If (w_*, r_*) as a point on the frontier which is also consistent with intertemporal arbitrage condition (i.e. it is on the BGP). It could be seen that for the factor prices in the neighborhood of this point, the economy must have the same specialization pattern as on the BGP (except for the case where the factor prices are at the intersection of the price frontiers).

$$z(p,k) \equiv Z/H = [1-u(p,k)]g(k_Z(p)).$$

Now consider and economy that completely specializes in producing *Y* throughout the transition process. Therefore, in this case *u* will be set to 1 and the full employment condition will be $k = k_Y(p)$.

Equation 2.1 and 2.2 for this case will be,

$$\dot{K} = Hf(k) - \delta K - C - X,$$

$$\dot{H} = -\eta + \frac{1}{p}X.$$

The Hamiltonian then can be written as,

$$V = \frac{C^{1-\sigma}-1}{1-\sigma} + \mu\left[Hf(k) - \delta K - C - X\right] + \lambda\left[-\eta H + \frac{1}{p}X\right], \tag{33}$$

and the F.O.C. and the necessary conditions will be,

$$\frac{\partial V}{\partial C} = 0 \rightarrow C^{-\sigma} - \mu = 0, \tag{34.1}$$

$$\frac{\partial V}{\partial X} = 0 \rightarrow -\mu + \frac{1}{p}\lambda = 0, \tag{34.2}$$

$$\dot{\mu} = -\frac{\partial V}{\partial K} + \rho\mu = -\mu\left[f'(k_Y) - \delta\right] + \rho\mu \rightarrow \frac{\dot{\mu}}{\mu} = (\rho + \delta) - f'(k), \tag{34.3}$$

$$\dot{\lambda} = -\frac{\partial V}{\partial H} + \rho\lambda = -\mu\left[f(k) - kf'(k)\right] - \lambda\left[-\eta\right] \rightarrow \frac{\dot{\lambda}}{\lambda} = (\rho + \eta) - \frac{1}{p}\left[f(k) - kf'(k)\right], \tag{34.4}$$

$$\lim_{t\to\infty} e^{-\rho t}\mu(t)K(t) = 0, \tag{34.5}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)H(t) = 0. \tag{34.6}$$

Since all the capital is being used in production of Y then the rate of returns in this case are defined as $r \equiv f'(k)$ and $w \equiv f(k) - kf'(k)$.

Similarly $\frac{\lambda}{\mu} = p$, therefore

$$\frac{\dot{p}}{p} = \frac{\dot{\lambda}}{\lambda} - \frac{\dot{\mu}}{\mu} = f'(k) - \delta - \frac{1}{p}\left[f(k) - kf'(k)\right] + \eta , \tag{35}$$

again consider $r \equiv f'(k)$ and $w \equiv f(k) - kf'(k)$ then we can rewrite (35) as

$$\frac{\dot{p}}{p} = \frac{\dot{\lambda}}{\lambda} - \frac{\dot{\mu}}{\mu} = r(p) - \frac{w(p)}{p} + \eta - \delta . \tag{36}$$

On the BGP we know that the prices will remain unchanged, i.e. $\frac{\dot{p}}{p} = 0$. Therefore (35) could be used to solve for k as a function of price level only. By totally differentiating (35) then we can show that,

$$k'(p) = \frac{-w}{pf''(p+k)} > 0 .$$

If $r = f'(k)$ and $w \equiv f(k) - kf'(k)$ then the factor prices are as well functions of output price alone. Totally differentiating those functions, we get

$$r'(p) = f''(k) \cdot k'(p) < 0 , \tag{37.1}$$

and

$$w'(p) = -kf''(k) > 0 . \tag{37.2}$$

Similarly, using (34.1) and (34.3) the growth rate can be written as,

$$\gamma_C(p) = \frac{1}{\sigma}\left[r(p) - \delta - \rho\right].$$

The dynamic system for this case are described as,

$$\frac{\dot{p}}{p}=\frac{\dot{\lambda}}{\lambda}-\frac{\dot{\mu}}{\mu}=r(p)-\frac{w(p)}{p}+\eta-\delta\,,$$

$$\frac{\dot{c}}{c}=\frac{\dot{C}}{C}-\frac{\dot{H}}{H}=\gamma_C(p)+\eta-\frac{1}{p}x(p,k)\,,$$

$$\frac{\dot{k}}{k}=\frac{\dot{K}}{K}-\frac{\dot{H}}{H}=\frac{y(p,k)-c-x(p,k)}{k}-\delta+\eta-\frac{1}{p}x(p,k)\,.$$

Linearizing the system around the BGP,

$$\begin{bmatrix}\dot{p}/p\\ \dot{c}/c\\ \dot{k}/k\end{bmatrix}=\begin{bmatrix}a_{11} & 0 & 0\\ a_{21} & 0 & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}p-p^*\\ c-c^*\\ k-k^*\end{bmatrix}, \tag{38}$$

where,

$$a_{11}=r'-\left(\frac{w'-w}{p}\right), \tag{38.1}$$

$$\mathrm{a}_{21}=v'_C-\frac{1}{p}\left(\frac{\partial x}{\partial p}-\frac{x}{p}\right), \tag{38.2}$$

$$a_{23}=-\frac{1}{p}\frac{\partial x}{\partial k}, \tag{38.3}$$

$$a_{31}=\frac{1}{k}\left(\frac{\partial y}{\partial p}-\frac{\partial x}{\partial p}\right)-\frac{1}{p}\left(\frac{\partial x}{\partial p}-\frac{x}{p}\right), \tag{38.4}$$

$$a_{32}=-\frac{1}{k}, \tag{38.5}$$

$$a_{33}=-\frac{(y-c-x)}{k^2}+\frac{1}{k}\left(\frac{\partial y}{\partial k}-\frac{\partial x}{\partial k}\right)-\frac{1}{p}\frac{\partial x}{\partial k}. \tag{38.6}$$

To examine the stability of this system we should evaluate the Jacobian matrix of the system in (37), i.e.

$$J_3 = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & 0 & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

The eigenvalues of J_3 are the solutions to its characteristic equation

$$-\gamma^3 + (a_{11} + a_{33})\gamma^2 - (a_{11}a_{33} - a_{23}a_{32})\gamma - a_{11}a_{23}a_{32} = 0, \tag{39}$$

where γ_i is the root of (39)

As we can see from (38.1) and (38.2) it is possible to determine only the signs of a_{11} and a_{32}. The signs of the rest of the elements in the Jacobian matrix however are ambiguous. Therefore it is not possible to use the Routh Theorem in determining the signs of the roots of characteristic equation in (39).

Rewrite the equation in (39) as follow

$$\gamma^3 + B\gamma^2 + C\gamma + D = 0. \tag{40}$$

This 3-degree polynomial equation has either one real and 2 mix roots or 3 real roots. We know $-D = \gamma_1\gamma_2\gamma_3$. Using equation (40), $D = \frac{-a_{11}a_{23}a_{32}}{-1} = a_{11}a_{23}a_{32}$ and therefore $-D = -a_{11}a_{23}a_{32}$. The sign of $-D = \gamma_1\gamma_2\gamma_3$ could be positive or negative depending on the sign of a_{23} and sign of a_{23} depends on the sign of $\frac{\partial x}{\partial k}$. We know that $a_{11} < 0$ and $a_{32} < 0$ could consider two possible cases as follow,

1) if $a_{23} > 0$ then $a_{11}a_{23}a_{32} > 0$ which implies $-D < 0$,

2) if $a_{23} < 0$ then $a_{11}a_{23}a_{32} < 0$ which implies $-D > 0$.

Now, if –D is negative then the characteristic equation specified in (40) will have either one or three roots with negative real parts. However if –D is positive then either all 3 roots will have positive real parts or 1 root will have positive and 2 roots will have negative real parts.

Consider the dynamic system specified in (38), as we can see the dynamic equation of *p* does not depend on *c* or *k* therefore the characteristic equation in (40) contains a real root, $\gamma_1 = a_{11}$ where $a_{11} = r' - \left(\frac{w' - w}{p}\right)$. Using (37.1) and (37.2) we can see that for the case when we have complete specialization in *Y*, a_{11} is always negative. Therefore the dynamic system has at least one negative root. The remaining two roots should satisfy the characteristic equation of the following 2 × 2 subsystem,

$$\begin{pmatrix} \dot{c}/c \\ \dot{k}/k \end{pmatrix} = \begin{pmatrix} 0 & a_{23} \\ a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} c - c^* \\ k - k^* \end{pmatrix}. \tag{41}$$

The characteristic equation for the 2 × 2 Jacobian matrix can be written as

$$\gamma^2 - a_{33}\gamma - a_{23}a_{32} = 0. \tag{42}$$

The discriminant for this equation would be

$$a_{33}^2 - 4(-a_{23}a_{32}) = a_{33}^2 + 4a_{23}a_{32}. \tag{43}$$

Notice that we can not again determine the signs of the terms in the discriminant of equation (30). However we know that a_{32} is always negative. Therefore the followings are the possible cases to consider,

1) If $a_{23} < 0$ therefore $a_{33}^2 + 4a_{23}a_{32} > 0$ which implies that (40) will have 2 real roots of opposite signs ($\gamma_1\gamma_2 = -a_{23}a_{32} < 0$),

2) If $a_{23} > 0$ therefore $a_{33}^2 + 4a_{23}a_{32}$ could be either positive or negative.

Remember when a_{23} is positive then the dynamic system could have either one negative and two positives or three negative roots. Since one of the roots is already negative, the remaining two roots should have the same sign. Now if $a_{33}^2 + a_{23}a_{32}$ is positive then remaining two roots should be of opposite sign, which then can not be the case when a_{23} is positive. Therefore, $a_{33}^2 + a_{23}a_{32}$ can be only negative. In that case then there will be 2 roots with positive real parts if a_{33} is positive.

Combining these results we get,

$$a_{11} < 0 \begin{cases} \text{If } a_{23} < 0 \text{ then (40) will have 2 real negative and 1 positive real root,} \\ \text{If } a_{23} > 0 \text{ then (40) will have either 1 negative real root and 2 roots with} \\ \text{positive real parts (if } a_{33} > 0\text{).} \end{cases}$$

Therefore the dynamic system exhibits saddle path stability regardless of the sign of a_{23}.

Now from (38),

$$\frac{\dot{p}}{p} = a_{11}(p - p^*),$$

multiplying $\frac{p^*}{p}$ to the both sides of the above equation,

$$\frac{dp}{dt}\frac{1}{p}\frac{p^*}{p} = a_{11}\frac{(p - p^*)}{p}p^*,$$

add and subtract $\frac{p}{p}$ from the left hand side,

$$\frac{dp}{dt}\frac{1}{p}\left(\frac{p^*}{p}+\frac{p}{p}-\frac{p}{p}\right)=a_{11}\frac{(p-p^*)}{p}p^*,$$

alternatively,

$$\frac{dp}{dt}\frac{1}{p}\left(1-\frac{p-p^*}{p}\right)=a_{11}\frac{(p-p^*)}{p}p^*$$

we can rewrite this as,

$$\frac{dp}{dt}\frac{1}{p}-\frac{dp}{dt}\frac{1}{p}\left(\frac{p-p^*}{p}\right)=a_{11}\frac{(p-p^*)}{p}p^*. \tag{44}$$

As it can be seen the left hand side of equation (44) is $\frac{d\left(\frac{p-p^*}{p}\right)}{dt}$. Therefore this equation can be integrated to get,

$$\frac{p-p^*}{p}=\left[\frac{p(0)-p^*}{p(0)}\right]e^{a_{11}p^*},$$

where *p(0)* is the initial value of *p*. This equation can be used to solve for *p* as,

$$p=\frac{p^*\cdot p(0)}{p^*e^{a_{11}p^*t}+p(0)\left[1-e^{a_{11}p^*t}\right]}. \tag{45}$$

As it can be seen when $a_{11}<0$, as $t\to\infty, p\to p^*$. Therefore if $p(0)<p^*$ then $\dot{p}>0$ and $p<p^*$ for all t, whereas if $p(0)>p^*$ then $\dot{p}<0$ and $p>p^*$ for all t.

Since the dynamic of price is stable then the continual adjustment of the price along the optimal path is required. Therefore in studying the dynamics of *c* and *k* we should consider the adjustment of price level as well. Since the adjustment of prices are stable we can simplify the dynamics by considering the projected dynamics in (*c* , *k*) space. It is clear that the value function is homogeneous of degree 1-*σ*. Since the costate variables are the derivatives of the value function with respect to the state variables therefore they are homogeneous of degree $-\sigma$. That is $\lambda = \lambda(K, H)$ and $\mu = \mu(K, H)$ are homogeneous of degree $-\sigma$. Therefore $\lambda\left(\frac{K}{H}, \frac{H}{H}\right) = \left(\frac{1}{H}\right)^{-\sigma} \lambda(k,1)$ and $\mu\left(\frac{K}{H}, \frac{H}{H}\right) = \left(\frac{1}{H}\right)^{-\sigma} \mu(k,1)$. Now we know $p \equiv \frac{\lambda}{\mu}$ or we can write $p \equiv \frac{(1/H)^{-\sigma}\lambda(k,1)}{(1/H)^{-\sigma}\mu(k,1)} = \frac{\lambda(k)}{\mu(k)}$. Therefore p is a function of k only and it is a non-decreasing function in k because of the concavity of the value function[17].

To get the projected dynamical system onto (c, k) space we can substitute $dp = p'(k)dk$ into (38). The projected system then will be

$$\begin{pmatrix} \dot{c}/c \\ \dot{k}/c \end{pmatrix} = \begin{pmatrix} 0 & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} c - c^* \\ k - k^* \end{pmatrix}, \tag{46}$$

where

$$b_{12} = a_{23} + a_{21}p'(k) = -\frac{1}{p}\frac{\partial x}{\partial k} + \left[v'_C(p) - \frac{1}{p}\left(\frac{\partial x}{\partial p} - \frac{x}{p} \right) \right] p'(k), \tag{47.1}$$

$$b_{21} = a_{32} = -\frac{1}{k}, \tag{47.2}$$

$$b_{22} = a_{33} + a_{31}p'(k) = \left[-\frac{(y - c - x)}{k^2} + \frac{1}{k}\left(\frac{\partial y}{\partial k} - \frac{\partial x}{\partial k} \right) - \frac{1}{p}\frac{\partial x}{\partial k} \right]$$

$$+ \left[\frac{1}{k}\left(\frac{\partial y}{\partial p} - \frac{\partial x}{\partial p} \right) - \frac{1}{p}\left(\frac{\partial x}{\partial p} - \frac{x}{p} \right) \right] p'(k). \tag{47.3}$$

As it can be seen the projected $\dot{c} = 0$ locus must be vertical. The term b_{12} contains two conflicting forces. If k increases it causes y to raise at constant prices and so it lowers the

[17] see Bond, Wang and Yip (1996).

growth rate of c. On the other hand increase in k makes p to rise, which lowers γ_C. However increase in p will cause y and x to fall which also tend to raise the growth rate of c.

However earlier we showed that the system has a saddle path. Therefore the projection of this saddle path onto (c, k) space implies that the characteristic roots of this projected system (i.e. the system in (46)) should have real parts of opposite sign. This implies that the determinant of the associated Jacobian matrix is negative. As it can be seen the determinant is $-b_{12}b_{21} < 0$. Since $b_{21} < 0$, it fallows that $b_{12} < 0$. Term b_{22} also involves conflicting forces. Similarly increase in k will increase the output of y sector, while increase in p will reduce the relative output of y. However in this case the saddle path property of the system is not sufficient to determine the dominated effects. Therefore b_{22} could be either positive or negative.

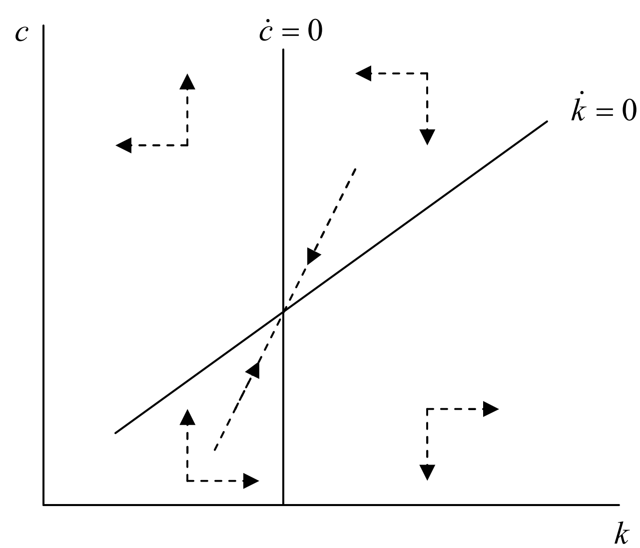

Figure 2-5. Phase diagram when there is complete specialization in Y and $b_{22} > 0$

The slope of the projected $\dot{k} = 0$ locus is $dc/dk = -b_{22}/b_{21}$. If $b_{22} > 0$ then the projected $\dot{k} = 0$ locus will be upward sloping since $dc/dk = -b_{22}/b_{21} > 0$ in this case. The phase diagram for this case is illustrated in Figure 5.

However when $b_{22} < 0$ then $dc/dk = -b_{22}/b_{21} < 0$. In this case the projected $\dot{k} = 0$ locus will be downward sloping. The phase diagram is illustrated in Figure 6.

In both case we can see that in either case when $k_0 < k^*$ then $v_K > v_H$ and $v_C > v_H$. Also along the transition prices are also changing say if *k* is rising then it causes *p* to rise as well. Since the growth rate of consumption is a decreasing function of *p* then along the transition the growth rate of consumption must be falling or along the transition the growth rate of consumption is higher than that on the BGP.

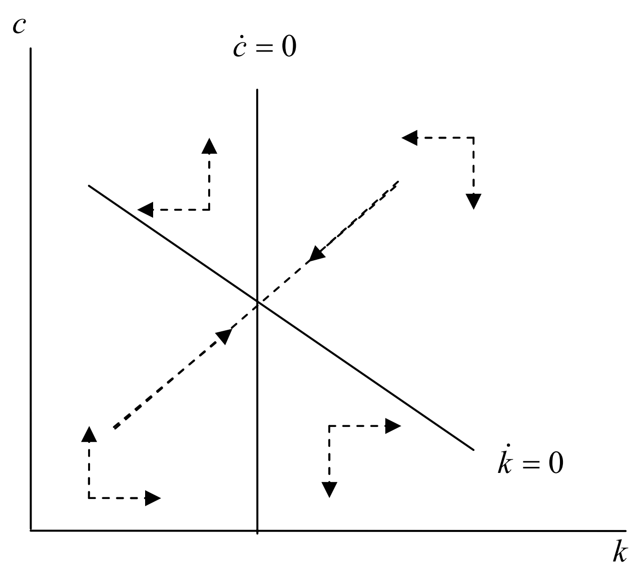

Figure 2-6. Phase diagram when there is complete specialization in Y and $b_{22} > 0$

Now suppose that economy completely specializes in producing Z throughout the transition process. Therefore, in this case u will be set to 0 and the full employment condition will be $k = k_Z(p)$. Therefore evolution equations for K and H will be,

$$\dot{K} = -\delta K - C - X \,,$$

$$\dot{H} = Hg(k) - \eta H + \frac{1}{p} X \,.$$

Similarly we can write the Hamiltonian and its corresponding F.O.C and necessary condition as follow,

$$V = \frac{C^{1-\sigma}}{1-\sigma} + \mu\left[-\delta K - C - X\right] + \lambda\left[H\, g(k) - \eta H + \frac{1}{p} X\right], \tag{48}$$

$$\frac{\partial V}{\partial C} = 0 \rightarrow C^{-\sigma} - \mu = 0 \,, \tag{49.1}$$

$$\frac{\partial V}{\partial X} = 0 \rightarrow -\mu + \frac{1}{p}\lambda = 0 \,, \tag{49.2}$$

$$\dot{\mu} = -\frac{\partial V}{\partial K} + \rho\mu = -\mu\left[-\delta\right] - \lambda\left[g'(k)\right] + \rho\mu \rightarrow \frac{\dot{\mu}}{\mu} = (\rho + \delta) - pg'(k) \,, \tag{49.3}$$

$$\dot{\lambda} = -\frac{\partial V}{\partial H} + \rho\lambda = -\lambda\left[g(k) - kg'(k) - \eta\right] + \rho\lambda \rightarrow \frac{\dot{\lambda}}{\lambda} = (\rho + \eta) - (g - kg') \,, \tag{49.4}$$

$$\lim_{t\to\infty} e^{-\rho t}\mu(t)K(t) = 0 \,, \tag{49.5}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)H(t) = 0 \,. \tag{49.6}$$

As we know $p = \frac{\lambda}{\mu}$, therefore

$$\frac{\dot{p}}{p} = \frac{\dot{\lambda}}{\lambda} - \frac{\dot{\mu}}{\mu} = pg'(k) - \delta - (g - kg') + \eta \,, \tag{50}$$

Let, $r \equiv pg'(k)$ and $w/p \equiv g - kg'$ then (50) could be written as,

$$\frac{\dot{p}}{p} = \frac{\dot{\lambda}}{\lambda} - \frac{\dot{\mu}}{\mu} = r - \delta - \frac{w}{p} + \eta \,. \tag{51}$$

Equation (51) could be used to solve for k as a function of p only. Totally differentiating (51) gives us,

$$k'(p) = \frac{-g'}{(p+k)g''} > 0 \,,$$

also,

$$r'(p) = g' + pg''k' < 0 \,,$$

and

$$\frac{w' - w/p}{p} = -kg''k' > 0 \,.$$

Then the dynamics equations in this case are

$$\frac{\dot{p}}{p} = \frac{\dot{\lambda}}{\lambda} - \frac{\dot{\mu}}{\mu} = r(p) - \delta - \frac{w(p)}{p} - \eta \,, \tag{52.1}$$

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \frac{\dot{H}}{H} = v_C(p) - z(k,p) + \eta - \frac{1}{p} x(p,k) \,, \tag{52.2}$$

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{H}}{H} = -\frac{(c+x)}{k} - \delta - z(k,p) + \eta - \frac{1}{p} x(k,p) \,. \tag{52.3}$$

Linearizing the system around BGP,

$$\begin{pmatrix} \dot{p}/p \\ \dot{c}/c \\ \dot{k}/k \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & 0 & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} p - p^* \\ c - c^* \\ k - k^* \end{pmatrix}, \tag{53}$$

where,

$$a_{11} = r'(p) - \frac{w' - w/p}{p}, \tag{54.1}$$

$$a_{21} = v'_C - \frac{\partial z}{\partial p} + \frac{1}{p^2} x(k,p) - \frac{1}{p}\frac{\partial x}{\partial p}, \tag{54.2}$$

$$a_{23} = -\frac{\partial z}{\partial k} - \frac{1}{p}\frac{\partial x}{\partial k}, \tag{54.3}$$

$$a_{31} = -\frac{1}{k}\frac{\partial x}{\partial p} + \frac{1}{p^2} x(k,p) - \frac{1}{p}\frac{\partial x}{\partial p}, \tag{54.4}$$

$$a_{32} = -\frac{1}{k}, \tag{54.5}$$

$$a_{33} = \frac{c+x}{k^2} - \frac{\partial z}{\partial k} - \frac{1}{p}\frac{\partial x}{\partial k}. \tag{54.5}$$

Similarly here since $a_{11} < 0$ then the adjustment of prices along the transition path is stable. Therefore we can again consider the projected system. To get the projected dynamical system onto (c, k) space we can substitute $dp = p'(k)dk$ into (53). The projected system then will be

$$\begin{pmatrix} \dot{c}/c \\ \dot{k}/c \end{pmatrix} = \begin{pmatrix} 0 & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} c - c^* \\ k - k^* \end{pmatrix}, \tag{55}$$

where

$$b_{12} = a_{23} + a_{21}p'(k) = -\frac{\partial z}{\partial k} - \frac{1}{p}\frac{\partial x}{\partial k} + \left[v'_C(p) - \frac{\partial z}{\partial p} + \frac{1}{p^2} x(k,p) - \frac{1}{p}\frac{x}{p} \right] p'(k), \tag{56.1}$$

$$b_{21} = a_{32} = -\frac{1}{k}, \tag{56.2}$$

$$b_{22} = a_{33} + a_{31}p'(k) = \left[\frac{(c+x)}{k^2} - \frac{\partial z}{\partial k} - \frac{1}{p}\frac{\partial x}{\partial k} \right] + \left[-\frac{1}{k}\frac{\partial x}{\partial p} + \frac{1}{p^2} x(k,p) - \frac{1}{p}\frac{\partial x}{\partial p} \right] p'(k). \tag{56.3}$$

Similar to the previous case both b_{12} and b_{22} contain conflicting effects since change in k and change in price has opposite effect on x and z. In this case however we should notice that this country is exporting Z to obtain Y through trade. In the other word –X is the import of Y. Therefore similar to the previous case we can argue that b_{12} is negative and depending on the sign of b_{22} the locus is upward sloping or downward sloping. Therefore the behavior of the variables and its growth rates along the transition path is similar to the case where the open economy completely specializes in production of Y.

2.4 SUMMARY AND CONCLUSIONS

In this section we considered a two sector endogenous growth model where two goods being produced using two types of reproducible factors of production. One good considered to be a unified consumption and investment while the other good could only be used as a factor of production. Technologies exhibits constant return to scale. We considered a world consistent of two countries each producing both goods under autarky conditions. These countries can trade with each other in both goods while there is no capital mobility allowed. Also countries are considered to be large relative to each other. We examined the condition for the world price under which a balanced growth path exists for the world as well as for individual countries. When this condition holds we showed existence a uniqueness of a balanced growth path at which variables in both countries could grow at the same rate. The production patterns on the balanced growth path however depend on the price under autarky relative to the price after trade. If the price of the country under autarky be equal to the world price after trade then this country continues to produce both goods on the balanced growth path. However when the price under autarky differs from the world price then the country will decide to shut down the production of one of the sectors and produce only one good and trade it to obtain the other good. We also have shown that with trade the growth rate of countries would be higher if both countries completely specialize in producing one good. However if the price level does not change then the growth rate of the country that continue to produce both goods remains unchanged.

We then studied the transition paths of this model and show that countries could decide to complete specialize around the steady state. Considering the complete specialization for each country we showed that the transitional path is saddle path stable.

APPENDIX 2.A

SIMPLIFYING THE PRICE LEVELS

It has been discussed in the previous chapter that the autarky price can be written as

$$p_i^{Au} = \left(\frac{A_i}{B_i}\right)\left(\frac{\alpha_i}{\eta_i}\right)^{\eta_i}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{1-\eta_i}\left[\left(\frac{A_i}{B_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{\eta_i}\right)^{\frac{\eta_i}{1-\alpha_i+\eta_i}}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{\frac{1-\eta_i}{1-\alpha_i+\eta_i}}\right]^{\alpha_i-\eta_i}$$

$$p_i^{Au} = \left[\left(\frac{A_i}{B_i}\right)\left(\frac{\alpha_i}{\eta_i}\right)^{\eta_i}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{1-\eta_i}\right]^{\frac{1-\alpha_i+\eta_i}{1-\alpha_i+\eta_i}}\left[\left(\frac{A_i}{B_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\frac{1}{1-\alpha_i+\eta_i}}\left(\frac{\alpha_i}{\eta_i}\right)^{\frac{\eta_i}{1-\alpha_i+\eta_i}}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{\frac{1-\eta_i}{1-\alpha_i+\eta_i}}\right]^{\alpha_i-\eta_i}$$

$$p_i^{Au} = \left[\left(\frac{A_i}{B_i}\right)^{1-\alpha_i+\eta_i}\left(\frac{\alpha_i}{\eta_i}\right)^{\eta_i(1-\alpha_i+\eta_i)}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{(1-\alpha_i+\eta_i)(1-\eta_i)}\left(\frac{A_i}{B_i}\right)^{\alpha_i-\eta_i}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\alpha_i-\eta_i}\left(\frac{\alpha_i}{\eta_i}\right)^{(\alpha_i-\eta_i)\eta_i}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{(\alpha_i-\eta_i)(1-\eta_i)}\right]^{\frac{1}{1-\alpha_i+\eta_i}}$$

$$p_i^{Au} = \left[\left(\frac{A_i}{B_i}\right)\left(\frac{\alpha_i}{\eta_i}\right)^{\eta_i}\left(\frac{1-\alpha_i}{1-\eta_i}\right)^{(1-\eta_i)}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\alpha_i-\eta_i}\right]^{\frac{1}{1-\alpha_i+\eta_i}}$$

$$p_i^{Au} = \left[\left(\frac{A_i}{B_i}\right)\frac{\alpha_i^{\alpha_i}}{\eta_i^{\eta_i}}\frac{(1-\alpha_i)^{1-\alpha_i}}{(1-\eta_i)^{1-\eta_i}}\right]^{\frac{1}{1-\alpha_i+\eta_i}}$$

Now if $B_1 = B_2$, $\eta_1 = \eta_2$ then,

$$p_1^{Au} = \left[\left(\frac{A_1}{B_1}\right)\frac{\alpha_1^{\alpha_1}}{\eta_1^{\eta_1}}\frac{(1-\alpha_1)^{1-\alpha_1}}{(1-\eta_1)^{1-\eta_1}}\right]^{\frac{1}{1-\varepsilon_1+\eta_1}} = \left[\left(\frac{A_1}{B_2}\right)\frac{\alpha_1^{\alpha_1}}{\eta_2^{\eta_2}}\frac{(1-\alpha_1)^{1-\alpha_1}}{(1-\eta_2)^{1-\eta_2}}\right]^{\frac{1}{1-\varepsilon_1+\eta_2}} = p*$$

$$p_2^{Au} = \left[\left(\frac{A_2}{B_2}\right)\frac{\alpha_2^{\alpha_2}}{\eta_2^{\eta_2}}\frac{(1-\alpha_2)^{1-\alpha_2}}{(1-\eta_2)^{1-\eta_2}}\right]^{\frac{1}{1-\varepsilon_2+\eta_2}}$$

However if $A_1 = A_2$, $\alpha_1 = \alpha_2$ then,

$$p_1^{Au} = \left[\left(\frac{A_1}{B_1} \right) \frac{\alpha_1^{\alpha_1}}{\eta_1^{\eta_1}} \frac{(1-\alpha_1)^{1-\alpha_1}}{(1-\eta_1)^{1-\eta_1}} \right]^{\frac{1}{1-\varepsilon_1+\eta_1}}$$

$$p_2^{Au} = \left[\left(\frac{A_2}{B_2} \right) \frac{\alpha_2^{\alpha_2}}{\eta_2^{\eta_2}} \frac{(1-\alpha_2)^{1-\alpha_2}}{(1-\eta_2)^{1-\eta_2}} \right]^{\frac{1}{1-\varepsilon_2+\eta_2}} = \left[\left(\frac{A_1}{B_2} \right) \frac{\alpha_1^{\alpha_1}}{\eta_2^{\eta_2}} \frac{(1-\alpha_1)^{1-\alpha_1}}{(1-\eta_2)^{1-\eta_2}} \right]^{\frac{1}{1-\varepsilon_1+\eta_2}} = p^*$$

APPENDIX 2.B

DERIVING THE EQUILIBRIUM CONDITION FOR THE PRICE LEVEL.

To get the condition that guarantees world price to stay between autarky prices, we start by deriving the equilibrium condition for prices in Autarky. Then we derive this condition for the case when countries are open to trade and then will describe the conditions that if hold the world equilibrium price will stay between the autarky prices.

2.B.1 CLOSED ECONOMY

Similarly consider a model of endogenous growth in which there are two sectors of production, using two reproducible factors of production, *K* and *H*. The production technologies for each sector are assumed to be different but both exhibit constant return to scale in the reproducible factors of production. Also there is assumed to be perfect competition in final good markets as well as the market for factors of production.

The production technologies for each sector are

$$Y_i = F_i(v_i K_i, u_i H_i) = u_i H_i \; f_i(k_{yi})$$

$$Z_i = G_i\big((1-v_i)K_i, (1-u_i)H_i\big) = (1-u_i)H_i \; g_i(k_{z_i})$$

where $k_{yi} \equiv \dfrac{v_i K_i}{u_i H_i}$ and $k_{zi} \equiv \dfrac{(1-v_i)K_i}{(1-u_i)H_i}$.

One sector produces a unified consumption and one type of capital good (*K*-Type), and the other sector produces another type of capital good (*H*-Type). We denote the unified consumption/*K*-investment good by *Y*, and the other sector by *Z*. Therefore,

$$\dot{K}_i = u_i H_i \; f(k_{yi}) - \delta K_i - C_i , \qquad \text{(B1.1)}$$

$$\dot{H}_i = (1-u_i)H_i g(k_{zi}) - \eta H_i, \tag{B1.2}$$

where δ is the rate of depreciation of K-type capital and η is the rate of depreciation of H-type capital.

A representative agent's optimization problem is

$$\max_{C, u_i, v_i} \int_0^\infty \frac{\mathrm{C(t)}^{1-\sigma}}{1-\sigma} e^{-\rho t},$$

s.t. (1-1) and (1-2),

$$H(0) = H_0 > 0, \quad K(0) = K_0 > 0.$$

Therefore the current value of Hamiltonian for each country is,

$$V = \frac{C^{1-\sigma}}{1-\sigma} + \mu\left[uH\, f(k_Y) - \delta K - C\right] + \lambda\left[(1-u)H\, g(k_Z) - \eta H\right]. \tag{B2}$$

The necessary and F.O.C. are

$$\frac{\partial V}{\partial C} = 0 \rightarrow C^{-\sigma} - \mu = 0, \tag{B3.1}$$

$$\frac{\partial V}{\partial v} = 0 \rightarrow \mu\left[Kf'(k_Y)\right] + \lambda\left[-Kg'(k_Z)\right] = 0,$$

$$\rightarrow f'(k_y) = pg'(k_Z), \tag{B3.2}$$

$$\frac{\partial V}{\partial u} = 0 \rightarrow \mu\left[H\left(f - k_Y f'(k_Y)\right)\right] + \lambda\left[-H\left(g - k_Z g'(k_Z)\right)\right] = 0,$$

$$\rightarrow \left(f - k_Y f'(k_Y)\right) = p\left(g - k_Z g'(k_Z)\right), \tag{B3.3}$$

$$\dot{\mu} = -\frac{\partial V}{\partial K} + \rho\mu = -\mu\left[vf'(k_Y) - \delta\right] - \lambda\left[(1-v)g'(k_Z)\right] + \rho\mu,$$

$$\dot{\mu} = \mu(\rho+\delta) - \mu v f' - \lambda(1-v)g',$$

$$\rightarrow \frac{\dot{\mu}}{\mu} = (\rho + \delta) - vf' - p(1-v)g', \tag{B3.4}$$

$$\dot{\lambda} = -\frac{\partial V}{\partial H} + \rho\lambda = -\mu[uf(k_Y) - uk_Y f'(k_Y)] - \lambda[(1-u)g(k_Z) - (1-u)k_Z g'(k_Y) - \eta] + \rho\lambda,$$

$$\dot{\lambda} = \lambda(\rho + \eta) - \mu u[(f - k_Y f')] - \lambda(1-u)[(g - k_z g'],$$

$$\rightarrow \frac{\dot{\lambda}}{\lambda} = (\rho + \eta) - \frac{u}{p}(f - k_Y f') - (1-u)(g - k_Z g'), \tag{B3.5}$$

$$\lim_{t\to\infty} e^{-\rho t}\mu(t)K(t) = 0, \tag{B3.6}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)H(t) = 0. \tag{B3.7}$$

Notice that (B3.2) and (B3.3) require that the real rates of return on *K* and *H* capital be equalized across sectors. Let $r \equiv f'(k_Y)$ and $w \equiv f(k_Y) - k_Y f'(k_Y)$ denote the market rental on K and H type capital respectively. Therefore,

$$r \equiv f'(k_y) = pg'(k_Z), \tag{B4.1}$$

$$w \equiv (f - k_Y f'(k_Y)) = p(g - k_Z g'(k_Z)). \tag{B4.2}$$

Equations (B4.1) and (B4.2) can be used to solve for the rental rate on *K* and *H* capital and the factor intensity of each sector in terms of the relative price of outputs, *p*.

$$r = r(p), \quad w = w(p),$$

$$k_Y = k_Y(p), \quad k_Z = k_Z(p).$$

Substituting from (B3.2) into (B3.4) we get

$$\frac{\dot{\mu}}{\mu} = (\rho + \delta) - vf' - (1-v)f' = \rho + \delta - f',$$

or alternatively,

$$\frac{\dot{\mu}}{\mu} = (\rho + \delta) - vpg' - p(1-v)g' = \rho + \delta - pg',$$

using $r = f' = pg'$ then it can be written as,

$$\frac{\dot{\mu}}{\mu} = \rho + \delta - r. \tag{B5}$$

Substituting (B3.3) into (B3.5) then we get,

$$\frac{\dot{\lambda}}{\lambda} = (\rho + \eta) - \frac{u}{p}(f - k_Y f') - \frac{(1-u)}{p}(f - k_Y f') = \rho + \eta - \frac{1}{p}(f - k_Y f'),$$

or alternatively,

$$\frac{\dot{\lambda}}{\lambda} = (\rho + \eta) - u(g - k_Z g') - (1-u)(g - k_Z g') = \rho + \eta - (g - k_Z g'),$$

using $w = f - k_Y f' = p(g - k_Z g')$ then it can be written as,

$$\frac{\dot{\lambda}}{\lambda} = \rho + \eta - \frac{w}{p}. \tag{B6}$$

On the BGP:

$$\frac{\dot{p}}{p} = 0 \rightarrow \frac{\dot{\lambda}}{\lambda} = \frac{\dot{\mu}}{\mu}, \tag{B7}$$

substituting from (B4.1) and (B4.2) we get the no arbitrage condition as we have seen before,

$$r(p) - \delta = \frac{w(p)}{p} - \eta,$$

On the BGP for each country the condition of no arbitrage holds

$$r_1(p_1) - \frac{w_1(p_1)}{p_1} = \delta - \eta, \qquad r_2(p_2) - \frac{w_2(p_2)}{p_2} = \delta - \eta,$$

subscribes 1 and 2 are for distinguishing the condition for country 1 and country 2[18].

Following Bond, Wang and Yip (1995) we assume the following condition on technologies. Condition FP (Factor Price):

$$\sup_p (r(p) - \frac{w(p)}{p}) > \delta - \eta > \inf_p (r(p) - \frac{w(p)}{p}),$$

as mentioned in the text, this condition guarantees the existence of a solution for p on the BGP, i.e. (B7) holds.

Using (B4.1) and (B4.2) we can get

$$r(p) - \frac{w(p)}{p} = f'(k_Y) - [g(k_Z) - k_Z g'(k_Z)],$$

taking total differentiation from this equation we get,

$$r'(p) - \frac{w'(p) - w(p)/p}{p} = f'' \cdot k'_Y(p) + k_Z g'' k'_Z(p). \tag{B8}$$

It could be shown that $r(p) - \frac{w(p)}{p}$ is either increasing or decreasing depending on the factor intensity of each sector. To see this, consider first totally differentiating (B2.2) and (B2.3). we get,

$$\text{a) } r'(p) = f'' k'_Y(p), \qquad \text{b) } w'(p) = -k_Y f'' k'_Y(p),$$

(B9)

$$\text{c) } k'_Y(p) = \frac{rk_Z + w}{(k_Z - k_Y) p f''}, \qquad \text{d) } k'_Z(p) = \frac{rk_Y + w}{(k_Z - k_Y) p^2 g''},$$

---

[18] If we assume $\delta = \eta$ then these conditions will become,

$$r_1(p_1) - \frac{w_1(p_1)}{p_1} = 0, \quad r_2(p_2) - \frac{w_2(p_2)}{p_2} = 0.$$

considering the condition in (B9) and using them to evaluate the sign of the equation in (B8) then we get the following two cases,

i) If $k_z - k_Y > 0$, then from part (c) and (d) in equation (B8) $k_Y'(p) < 0$ and $k_Z'(p) < 0$ which in turn imply $f'' \cdot k_Y'(p) + k_Z g'' k_Z'(p) > 0$, and

ii) If $k_z - k_Y < 0$ then from part (c) and (d) in equation (B8) $k_Y'(p) > 0$ and $k_Z'(p) > 0$ which imply $f'' \cdot k_Y'(p) + k_Z g'' k_Z'(p) < 0$.

Figure B-1 shows two possible cases where the price level on the steady state is determined using $r(p) - \delta = \frac{w(p)}{p} - \eta$ by considering 2 cases we describe above in terms of factor intensity of two sectors[19].

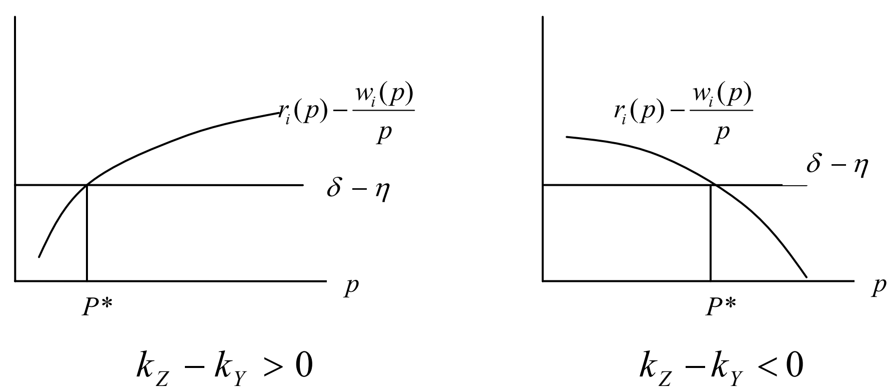

Figure B-1: Intertemporal arbitrage condition for different factor intensities in autarky

[19] Assuming concavity.

As usual we can find the growth rate using the first order condition for consumption, i.e. $\frac{\dot{C}}{C} = -\frac{1}{\sigma}\left(\frac{\dot{\mu}}{\mu}\right)$. Substituting (B5) then $\frac{\dot{C}}{C} = \frac{1}{\sigma}\left(r(p) - \delta - \rho\right)$. It can be shown that on the BGP all the variables that grow will grow at the same rate. Therefore on the BGP,

$$\gamma_i(p_i) = \frac{1}{\sigma}\left(r_i(p_i) - \delta - \rho\right)$$

Now suppose that the production functions are Cobb-Douglas (as we considered in chapter 1),

$$Y = A(vK)^{\alpha}(uH)^{1-\alpha} = (uH)A\left(\frac{vK}{uH}\right)^{\alpha} = (uH)Ak_Y^{\alpha}, \qquad \text{(B10.1)}$$

$$Z = B\left((1-v)K\right)^{\eta}\left((1-u)H\right)^{1-\eta} = \left((1-u)H\right)B\left(\frac{(1-v)K}{(1-u)H}\right)^{\eta}, \qquad \text{(B10.2)}$$

therefore here $f(k_Y) = Ak_Y^{\alpha}$ and $g(k_Z) = Bk_Z^{\eta}$.

Equations (B4.1) and (B4.2) for this example will be,

$$f'(k_Y) = pg'(k_Z) \rightarrow \alpha A k_Y^{\alpha-1} = p\eta B k_Z^{\eta-1}, \qquad \text{(B11.1)}$$

$$(f - k_Y f') = p(g - k_Z g') \rightarrow (1-\alpha)Ak_Y^{\alpha} = p(1-\eta)Bk_Z^{\eta}, \qquad \text{(B11.2)}$$

Equations (B11.1) and (B10.2) can be used to obtain the sectoral capital intensities and the capital returns for each country as follow,

$$k_Y(p) = \left[p\frac{B}{A}\left(\frac{1-\eta}{1-\alpha}\right)^{1-\eta}\left(\frac{\eta}{\alpha}\right)^{\eta}\right]^{\frac{1}{\alpha-\eta}}, \qquad \text{(B12.1)}$$

$$k_Z(p) = \left[p\frac{B}{A}\left(\frac{1-\eta}{1-\alpha}\right)^{1-\alpha}\left(\frac{\eta}{\alpha}\right)^{\alpha}\right]^{\frac{1}{\alpha-\eta}}, \qquad \text{(B12.2)}$$

and,

$$r(p) = \alpha A k_Y^{\alpha-1} = \alpha A\left[p\frac{B}{A}\left(\frac{1-\eta}{1-\alpha}\right)^{1-\eta}\left(\frac{\eta}{\alpha}\right)^{\eta}\right]^{\frac{\alpha-1}{\alpha-\eta}}, \tag{B13}$$

$$w(p) = (1-\alpha) A k_Y^{\alpha} = (1-\alpha) A\left[p\frac{B}{A}\left(\frac{1-\eta}{1-\alpha}\right)^{1-\eta}\left(\frac{\eta}{\alpha}\right)^{\eta}\right]^{\frac{\alpha}{\alpha-\eta}}. \tag{B14}$$

Let $\Omega_1 = \left[\frac{B}{A}\left(\frac{1-\eta}{1-\alpha}\right)^{1-\eta}\left(\frac{\eta}{\alpha}\right)^{\eta}\right]^{\frac{1}{\alpha-\eta}}$, and $\Omega_2 = \left[\frac{B}{A}\left(\frac{1-\eta}{1-\alpha}\right)^{1-\alpha}\left(\frac{\eta}{\alpha}\right)^{\alpha}\right]^{\frac{1}{\alpha-\eta}}$. Therefore (B11) and (B13) can be written as,

$$k_Y'(p) = \frac{1}{\alpha-\eta} p^{\frac{1}{\alpha-\eta}-1}\Omega_1$$

$$r'(p) = \frac{\alpha-1}{\alpha-\eta} p^{\frac{\alpha-1}{\alpha-\eta}-1}\alpha A \Omega_1^{\alpha-1}$$

which then they imply the following two conditions;

a) if $k_Y'(p) > 0$ and $r'(p) < 0$ if $\alpha > \eta$,

b) if $k_Y'(p) < 0$ and $r'(p) > 0$ if $\alpha < \eta$.

Similarly, (B12) and (B13) would become,

$$k_Z'(p) = \frac{1}{\alpha-\eta} p^{\frac{1}{\alpha-\eta}-1}\Omega_2$$

$$w'(p) = \frac{\alpha}{\alpha-\eta} p^{\frac{\alpha}{\alpha-\eta}-1}(1-\alpha) A \Omega_2^{\alpha}$$

they imply the followings;

a) if $k_Z'(p) > 0$ and $w'(p) > 0$ if $\alpha > \eta$,

b) if $k_Z'(p) < 0$ and $w'(p) < 0$ if $\alpha < \eta$.

On the BGP, if we assume $\delta = \eta$, then intertemporal arbitrage condition would become $r(p) = \frac{w(p)}{p}$. Using the solution for *r(p)* and *w(p)* we get

$$r(p) = f'(p) = \alpha A k_Y^{\alpha-1} = \alpha A \left[p \frac{B}{A} \left(\frac{1-\eta}{1-\alpha} \right)^{1-\eta} \left(\frac{\eta}{\alpha} \right)^{\eta} \right]^{\frac{\alpha-1}{\alpha-\eta}},$$

$$\frac{w(p)}{p} = g - k_Z g' = B(1-\eta) k_Z^{\eta} = B(1-\eta) \left[\frac{B}{A} p \left(\frac{1-\eta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\eta}{\alpha} \right)^{\alpha} \right]^{\frac{\eta}{\alpha-\eta}}.$$

Equating these two we obtain,

$$\alpha A \left[p \frac{B}{A} \left(\frac{1-\eta}{1-\alpha} \right)^{1-\eta} \left(\frac{\eta}{\alpha} \right)^{\eta} \right]^{\frac{\alpha-1}{\alpha-\eta}} = B(1-\eta) \left[\frac{B}{A} p \left(\frac{1-\eta}{1-\alpha} \right)^{1-\alpha} \left(\frac{\eta}{\alpha} \right)^{\alpha} \right]^{\frac{\eta}{\alpha-\eta}},$$

from this equality we can solve for the price level at the steady state as,

$$p = \left(\frac{\alpha^{\alpha} (1-\alpha)^{(1-\alpha)} A}{\eta^{\eta} (1-\eta)^{(1-\eta)} B} \right)^{\frac{1}{1-\alpha-\eta}}. \tag{B15}$$

B(15) the is the equation for autarky prices on the balanced growth path for each country as we obtained in chapter 1.

2.B.2 OPEN ECONOMY

Similarly we consider two countries in which 2 goods *Y* and *Z* are being produced using two types of capital. These countries could trade in both goods while there is no capital mobility between countries[20]. If X indicates the export of good Y and p is the price of good Y in terms of Z, then the evolution equations are

$$\dot{K}_i = u_i H_i \ f(k_{yi}) - \delta K_i - C_i - X_i ,$$

$$\dot{H}_i = (1-u_i)\ H_i g(k_{zi}) - \eta H_i + \frac{1}{p} X_i .$$

The representative agent problem is as described in the text. Similarly the Hamiltonian will be,

$$V = \frac{C^{1-\sigma}}{1-\sigma} + \mu\left[uH\ f(k_Y) - \delta K - C - X\right] + \lambda\left[(1-u)H\ g(k_Z) - \eta H + \frac{1}{p} X\right].$$

It could be shown that the F.O.C and necessary condition are as mentioned in equations (4) in the text.

Consequently the growth rates can be obtained from the first order condition for consumption as,

$$\gamma_1(p) = \frac{1}{\sigma}\left(r_1(p) - \delta - \rho\right),$$

$$\gamma_2(p) = \frac{1}{\sigma}\left(r_2(p) - \delta - \rho\right).$$

The equilibrium level of world price, p is at which the trade balance holds. As we argued in the text if trade balance holds then the growth rates of counties should be equal. Therefore we can obtain the equilibrium value of world price by equating the growth rates of the countries on the BGP,

$$\gamma_1(p) = \gamma_2(p) \rightarrow r_1(p) = r_2(p).$$

[20] All the assumptions made in previous section are being held.

On the BGP $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\mu}}{\mu}$ then $r_i(p) - \delta = \frac{w_i(p)}{p} - \eta$. Using this condition the growth rates can also be written as,

$$\gamma_1(p) = \frac{1}{\sigma}\left(r_1(p) - \delta - \rho\right), \tag{B16}$$

$$\gamma_2(p) = \frac{1}{\sigma}\left(\frac{w_2(p)}{p} - \eta - \rho\right). \tag{B17}$$

Therefore the equality of the growth rates on the BGP as described above implies that the equilibrium level of p should also satisfy $r_1(p) - \delta = \frac{w_2(p)}{p} - \eta$ or equivalently $r_1(p) - \frac{w_2(p)}{p} = \delta - \eta$. Notice that we can show that $r_1(p) - \frac{w_2(p)}{p}$ can be

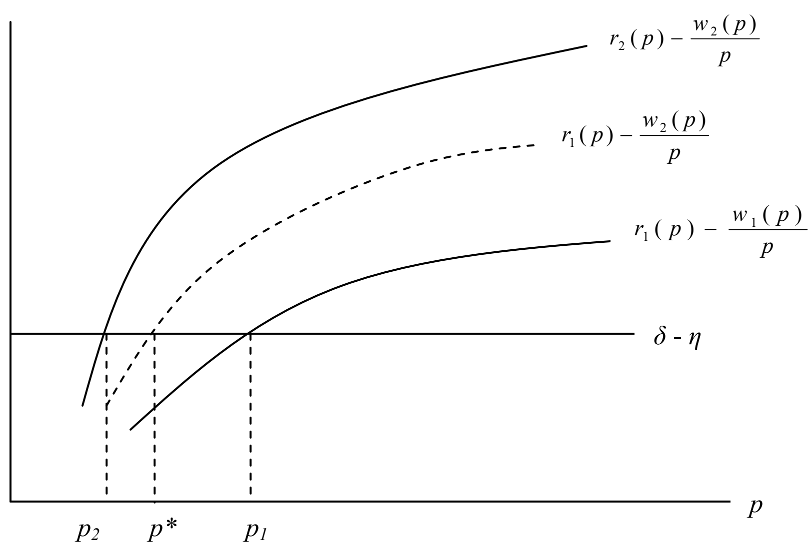

Figure B-2: Intertemporal Arbitrage condition at country's autarky relative to that at trade equilibrium when $k_Z - k_Y > 0$

increasing or decreasing with similar way that we used in previous section.

Remember the value of p that we get from equating the growth rates of two countries could only be the world price if and only if $p \in [p_1 , p_2]$. Figure B-2 illustrates the case where the solution for p can be the equilibrium world price assuming $p_2 < p_1$, and assuming $k_Z - k_Y > 0$.

Remember that on the BGP $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\mu}}{\mu}$. Now from the necessary conditions we know that

$$\frac{\dot{\mu}}{\mu} = \rho + \delta - f' = \rho + \delta - pg', \tag{B18}$$

and ,

$$\frac{\dot{\lambda}}{\lambda} = \rho + \eta - \frac{1}{p}\left(f - k_Y f'\right) = \rho + \eta - (g - k_Z g'). \tag{B19}$$

Therefore when $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\mu}}{\mu}$, using (B18) and (B19) we get,

$$\rho + \eta - \frac{1}{p}\left(f - k_Y f'\right) = \rho + \delta - f',$$

or alternatively,

$$f' - \delta = \frac{1}{p}(f - k_Y f') - \eta. \tag{B20}$$

Now if we again use the Cobb-Douglas production functions and assuming $\delta = \eta$, (B20) will become,

$$\alpha A k_Y^{\alpha - 1} = \frac{1}{p}(1 - \alpha) A k_Y^{\alpha},$$

therefore on the BGP,

$$k_Y = p\frac{\alpha}{1-\alpha}. \tag{B21}$$

Similarly using (B18) and (B19) and $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\mu}}{\mu}$ we can also get,

$$\rho + \eta - (g - k_Z g') = \rho + \delta - pg',$$

which implies

$$pg' - \delta = (g - k_Z g') - \eta. \tag{B22}$$

With Cobb-Douglas production function and δ=η, then (B22) becomes,

$$p\eta B k_Z^{\eta-1} = (1-\eta) B k_Z^{\eta},$$

and on the BGP,

$$k_Z = p\frac{\eta}{1-\eta}. \tag{B23}$$

It has been discussed in the text that the equilibrium level of price is a price at which both countries grow at the same rate on the BGP, that is

$$\gamma_1(p) = \gamma_2(p) \rightarrow r_1(p) - \delta = \frac{w_2(p)}{p} - \eta. \tag{B24}$$

Using the necessary condition of the problem described in the text (equations 4.3 and 4.4) and substituting for the factor prices (B24) becomes,

$$f_1'(k_{1,Y}) - \delta = \left[g_2 - k_Z g_2'(k_{2,Z})\right] - \eta. \tag{B25}$$

For the Cobb-Douglus production function it will become,

$$\alpha_1 A_1 k_{1,Y}^{\alpha_1 - 1} = (1-\eta_2) B_2 k_{2,Z}^{\eta_2},$$

Next substitute for k_Y and k_Z from (B21) and (B23),

$$\alpha_1 A_1\left(p\frac{\alpha_1}{1-\alpha_1}\right)^{\alpha_1-1}=(1-\eta_2)B_2\left(p\frac{\eta_1}{1-\eta_1}\right)^{\eta_2},$$

then the equilibrium price will be,

$$p^*=\left(\frac{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}A_1}{\eta_2^{\eta_2}(1-\eta_2)^{1-\eta_2}B_2}\right)^{\frac{1}{1-\alpha_1+\eta_2}}. \tag{B26}$$

Therefore (B26) shows the level of price that satisfies the intertemporal arbitrage condition on the balanced growth path. Next we try to derive the condition that guarantees the level of world price to be equal to the value in equation (B26).

If the value of price in (B26) will be the world price it should satisfy $p_2 < p^* < p_1$, where p_1 and p_2 are the autarky prices as we described in previous section. Then this inequality can be written as,

$$\left(\frac{\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}A_2}{\eta_2^{\eta_2}(1-\eta_2)^{1-\eta_2}B_2}\right)^{\frac{1}{1-\alpha_2+\eta_2}}<\left(\frac{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}A_1}{\eta_2^{\eta_2}(1-\eta_2)^{1-\eta_2}B_2}\right)^{\frac{1}{1-\alpha_1+\eta_2}}<\left(\frac{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}A_1}{\eta_1^{\eta_1}(1-\eta_1)^{1-\eta_1}B_1}\right)^{\frac{1}{1-\alpha_1+\eta_1}}. \tag{B27}$$

Now assume the equality of factor share across the sectors as well as countries (Seater (2004)). This assumption, as we have seen in chapter 1, implies *v=u* as well. Therefore the production functions for each country will become,

$$Y_i = A_i(u_iK_i)^{\alpha}(u_iH_i)^{1-\alpha}=(u_iH_i)A_ik_i^{\alpha},$$

where, $f(k) = Ak^{\alpha}$ and,

$$Z_i = B_i((1-u_i)K_i)^{\alpha}((1-u_i)H_i)^{1-\alpha}=((1-u_i)H_i)B_ik_i^{\alpha},$$

where $g(k)=Bk^{\alpha}$. Under this assumption as we have seen in chapter 1, equation (B27) will become $\frac{A_2}{B_2}<\frac{A_1}{B_2}<\frac{A_1}{B_1}$. Recall in this case prices under autarky are $p_i=\frac{A_i}{B_i}$ and $p=\frac{A_1}{B_2}$ will be the equilibrium world price if its value stays between the autarky prices. As Seater (2004) argues the condition to guarantee such value is $A_1 > A_2$ and $B_2 > B_1$.

Define the return on capitals as

$$r \equiv f' = pg' = \alpha A k^{\alpha-1}, \tag{B28.1}$$

$$w \equiv f - kf' = p[g - kg'] = (1-\alpha)Ak^{\alpha}, \tag{B28.2}$$

$$\frac{w}{p} \equiv \frac{1}{p}[f - kf'] = g - kg' = (1-\alpha)Bk^{\alpha}. \tag{B28.3}$$

On the BGP we have $\frac{\dot{p}}{p} = \frac{\dot{\lambda}}{\lambda} - \frac{\dot{\mu}}{\mu} = r(p) - \frac{w(p)}{p}$ (assuming equal depreciation rate for both capitals). Taking total differentiation from the latter equation,

$$r'(p) - \left(\frac{w(p)}{p}\right)' = \alpha(\alpha-1)Ak^{\alpha-2}k'(p) - \alpha(1-\alpha)Bk^{\alpha-1}k'(p), \tag{B29}$$

From equation (B29) when $k'(p) > 0$ then $r'(p) - \left(\frac{w(p)}{p}\right)' < 0$, and if $k'(p) < 0$ then $r'(p) - \left(\frac{w(p)}{p}\right)' > 0$.

Now assume $k'(p) > 0$, then $p_2 < p^* < p_1$ requires,

$$r_2(p) - \frac{w_2(p)}{p} < r_1(p) - \frac{w_2(p)}{p} < r_1(p) - \frac{w_1(p)}{p}, \tag{B30}$$

which implies,

$$r_1(p) > r_2(p),$$

$$w_1(p)/p < w_2(p)/p.$$

For Cobb-Douglas production functions, $r_1(p) > r_2(p)$ implies $\alpha A_1 k_1^{\alpha-1} > \alpha A_2 k_2^{\alpha-1}$. Therefore if $\alpha_1 = \alpha_2$ and $k_1 = k_2$ then the latter inequality implies $A_1 > A_2$. Similarly if $w_1(p)/p < w_2(p)/p$, which will be $(1-\alpha)B_1 k_1^{\alpha} < (1-\alpha)B_2 k_2^{\alpha}$ for Cobb-Douglas production function. This condition if $\alpha_1 = \alpha_2$ and $k_1 = k_2$ implies $B_1 < B_2$. Therefore for the case where we have, $k'(p) > 0$, then having $A_1 > A_2$ and $B_1 < B_2$, $p_2 < p^* < p_1$ condition would become $\frac{A_2}{B_2} < \frac{A_1}{B_2} < \frac{A_1}{B_1}$.

In the general case where we drop the assumption of $\alpha_1 = \alpha_2$, if $k_z - k_Y < 0$ then similar condition could be used in order to find the sufficient condition for the world price be equal to the value presented in (B26).

Similarly $p_2 < p^* < p_1$ implies $r_2(p) - \frac{w_2(p)}{p} < r_1(p) - \frac{w_2(p)}{p} < r_1(p) - \frac{w_1(p)}{p}$. This condition requires $r_1(p) > r_2(p)$ and $w_1(p)/p < w_2(p)/p$. Now, $r_1(p) > r_2(p)$ implies $\alpha_1 A_1 k_1^{\alpha_1-1} > \alpha_2 A_2 k_2^{\alpha_2-1}$. Now if $k_1 = k_2$ and $Y_1 > Y_2$,[21] then $A_1 k^{\alpha_1} > A_2 k^{\alpha_2}$ or $A_1 k^{\alpha_1-1} > A_2 k^{\alpha_2-1}$. If $\alpha_1 > \alpha_2$ then $\alpha_1 A_1 k_1^{\alpha_1-1} > \alpha_2 A_2 k_2^{\alpha 2-1}$. Therefore when $k_z - k_Y < 0$ and $\alpha_1 > \alpha_2$, if country 1 has absolute advantage in *Y*, $r_1(p) > r_2(p)$ will be satisfied.

[21] If *k>1*, then $\alpha_1 > \alpha_2$ and $A_1 > A_2$ implies $Y_1 > Y_2$

Similarly, when $w_1(p)/p < w_2(p)/p$ then $(1-\eta_1)B_1k_1^{\eta_1} < (1-\eta_1)B_2k_2^{\eta_2}$. Now consider country 2 having absolute advantage in producing Z. Therefore for $k_1 = k_2$, $Z_1 < Z_2$,[22] which implies $B_1k^{\eta_1} < B_2k^{\eta_2}$. Also if $\eta_1 > \eta_2$ or $(1-\eta_1) < (1-\eta_2)$ then $(1-\eta_1)B_1k_1^{\eta_1} < (1-\eta_1)B_2k_2^{\eta_2}$.

To summarize, we showed that if country 1 has absolute advantage in *Y* and country 2 has absolute advantage in *Z* then when we have $\alpha_1 > \alpha_2$ and $\eta_1 > \eta_2$, also $k_z - k_Y < 0$ then the inequality in (B30) will be guaranteed.

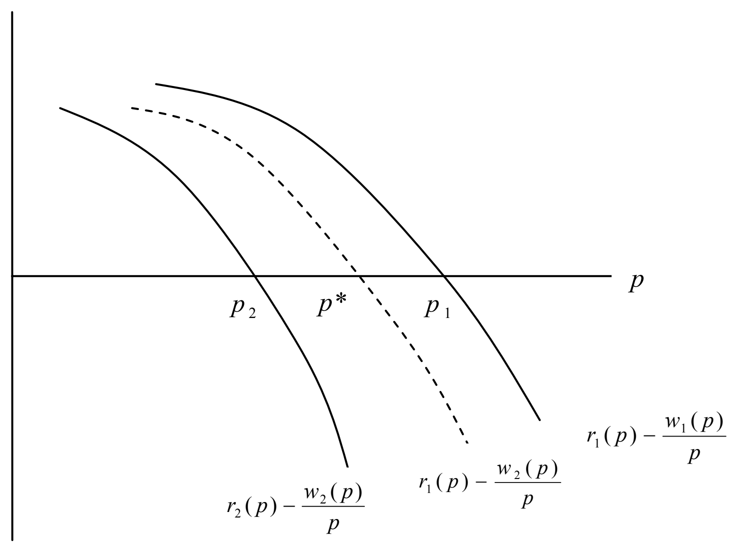

Figure B-3: Intertemporal Arbitrage condition at country's autarky relative to trade equilibrium when $k_z - k_Y < 0$

[22] If k>1, then $\eta_1 > \eta_2$ or $(1-\eta_1) < (1-\eta_2)$ and $B_1 < B_2$ implies $Z_1 > Z_2$

CHAPTER 3

INTERNATIONAL SPECIALIZATION AND ECONOMIC GROWTH

3.1 INTRODUCTION

Over past two and a half decades there have been many studies in which the relationship between trade openness and economic growth has been examined empirically. This list includes Michaely (1977), Krueger (1980), Feder (1982), Ram (1987), World Bank (1987), Dodaro (1991), Salvatore and Hatcher (1991), Baldwin (1992), Dollar (1992), Levine and Rentel (1992), Edwards (1992, 1993) Sachs and Warner (1995), Lee (1995), Sala-i-Martin (1997), Frankel and Romer (1999), Wacziarg (2001), and Wacziarg and Welch (2003). Also there are many studies in which the effect of international trade on productivity level has been considered. Studies such as Coe and Helpman (1995), Coe, Grossman and Hoffmaister (1997), Keller (1998), Xu and Wang (1999), Keller (2002), Alcala and Cicoone (2002) and Halpern, Koren and Szeidl (2005) are few examples. These studies mostly support a strong and positive relationship between international trade and economic growth and therefore show that openness is growth promoting. However the question that one might ask is, whether the effect of trade on growth depends on the type of goods that country is exporting or importing. Most recent international trade theories show that the nature of the specialization of a country is important for its growth performances. For instance Grossman & Helpman (1991), Rivera-Batize and Romer (1991) and Young (1991) provide theoretical foundation that importing goods such as intermediates that could facilitate more research and development or learning-by-doing activities are growth promoting.

Despite that the theory of comparative advantage and specialization and its conclusion on international trade is widely accepted, most empirical studies in growth literature consider mainly the effect of trade openness on growth without taking into account potential sectoral specialization effects. There are however few studies such as Lee (1995), Rauch and Weinhold (1997), Xu and Wang (1999), Bensidoun, Gaulier and Ünal - Kesenci (2001) and Lewer and Van den Berg (2003), that have considered the effect of trade specialization and the nature of goods being imported or exported on growth performance.

Ruach and Weinhold (1997) and Xu and Wang (1999) examine the effect of trade specialization or the nature of importing good on the productivity level. Ruach and Weinhold (1997) use a Herfindahl index of production specialization for the manufacturing sector in series of dynamic panel regression and show that specialization affects the manufacturing productivity growth positively. Also Xu and Wange (1999) estimate Coe and Helpman's (1995) model for productivity growth using capital good weighted R&D spillover variables instead of using the total import and show that the effect of capital good weighted R&D spillover is significantly more than the effect of this variable when considering total import. Lee (1995) shows that those LDCs that spend more on importing foreign capital good enjoy higher growth rates using a cross-section of countries for period of 1960-1985. Lewer and Van den Berg (2003) also examine the effect of the nature of trading goods in a series of time series regression for 28 developed and developing countries.

The purpose of this section is also to examine if the type of importing good and specifically capital good matter for the output growth performances of countries. In the line of the models we considered in previous chapters, here we consider a two sector endogenous growth model in which a country produces two types of good, investment and consumption, using only one type of capital. However technologies between two sectors are different. We show that in this model when countries open to trade, each country completely specializes in producing one good and shut downs the production of the other good. As the result on the balanced growth path the country which imports capital experiences higher growth rates while the growth rate of the country that imports consumption will be unaffected by trade. To examine this implication we use a panel of 92 developing and developed countries for the period of 1965-2000. We use both yearly and non-overlapping five-year averages. To estimate the long-run effect of different types of importing good on growth rates of countries, we introduce different measures of trade specialization in a specification suggested by Bond, Leblebicioglu and Schiantarelli (2006). Our results imply that specializing in consumption goods and therefore importing capital goods affect growth positively however specializing in capital goods does not seem to matter.

In the following sections we first provide a quick review on some of the previous studies. Then in section 3 we present the theoretical model and its result. Section 4 discusses the econometric modeling and method of estimation. In section 5 we introduce the index of specialization. Section 6 presents the results and finally section 7 concludes.

3.2 REVIEW OF SOME LITERATURES

As mentioned earlier most of the empirical studies on growth and trade focus on the effect of international trade in general. However there are few studies that consider the effect of the nature of good being imported or exported. Quah and Rauch (1990) show how increased openness to international trade can lead to increased specialization in models of endogenous growth through learning by doing. These models imply that increased specialization accelerates productivity growth by more fully realizing dynamic economies of scale. They consider learning-by doing Lucas type model where different intermediate goods could be produced using human capital and labor. Each sector accumulates human capital by the process of learning-by doing. The coefficient of learning is different across sectors. Therefore each sector has different level of human capital and productivity. Under the autarky country would have to produce all types of intermediate goods by allocating more labor to the sectors with lower human capital and less labor inputs to the sectors with higher human capital. However international openness allows countries to import the intermediate goods with the lowest learning coefficient in their sector. Therefore this country could enjoy higher productivity growth by reallocating its resources to the other sectors and therefore specialization follows. It is however important to note that in this setting learning-by doing is not subject to diminishing returns. Quah and Rauch consider this feature to be an indication of LDCs, since these countries face continually and exogenously expanding technological frontier as determined by developed countries and that LDCs could never quite catch up in their technological developments. Rauch and Weinhold (1997) then attempt to test this hypothesis. In order to test the hypothesis that specialization increases productivity growth in LDCs, Rauch and Weinhold (1997) first define a Herfindahl index of production specialization for the manufacturing sector in 39 countries. Then they present a series of dynamic panel regressions controlling for country fixed effects. By doing so they could show that in the less developed countries specialization is positively and significantly correlated with increased manufacturing productivity growth, even when variables, such as openness, inflation, government spending, and investment are controlled for.

Coe and Helpman (1995) have related productivity to the import-share weighted R&D of the countries' trade partners. In recent growth literatures Romer (1990) and Grossman and Helpman (1991) have considered innovation to be the engine of technological progress and productivity growth. In this view, innovation depends on knowledge that results from cumulative R&D experience and it also contributes to the stock of knowledge. Consequently an economy's productivity level depends on its cumulative R&D effort and on its effective stock of knowledge, with the two being interrelated. Further, Coe and Helpman (1995) argued that a country's productivity depends not only on its own R&D but also on its trade partner's R&D efforts. Own R&D produces traded and nontraded goods and services that causes more effective use of existing resources and therefore raises a country's productivity level. The country could also benefit from its trade partner's R&D efforts. It could learn about new technologies and materials and production processes of foreign countries. Also it could benefit from goods and services that have been developed by trade partner through importing of such goods and services. Following these theoretical backgrounds, Coe and Helpmen (1995) examine the effect of domestic and foreign R&D capital stock on a country's productivity level. They construct a stock of domestic knowledge based on domestic R&D expenditure and a foreign stock of knowledge based on R&D spending on its trade partner. To construct the foreign R&D capital stock however, they use import-weighted sums of trade partner's cumulative R&D spending. They use data for 21 OECD countries plus Israel during the period 1971-1990. Using pooled time series cross section data they obtain that both domestic and foreign R&D capital stock have important effect on total factor productivity. Also their estimates of the elasticity of TFP with respect to R&D capital stock suggest that the foreign R&D capital stock may be at least as important as the domestic stock in the smaller countries while in the larger countries the domestic stock of R&D capital could be more important.

Coe and Helpman (1995) in their investigation of R&D spillover consider the total import while Xu and Wang (1999) distinguish between capital goods and non-capital goods in measuring foreign R&D spillovers embodied in trade. They argue that capital goods have higher contents of technology than non-capital goods. Therefore capital goods are the major carriers of R&D spillovers embodied in trade flows. Xu and Wang (1999) decompose total imports into capital goods imports and non-capital goods imports. Then they construct two types of foreign R&D capital stock by considering the import of capital and non-capital goods separately. Using these two spillover variables instead of the total import weighted spillover variable that Coe and Helpman (1995) considered, they test the importance of capital versus non capital goods spillovers on a country productivity level. They find that the capital good weighted R&D spillover variable in Coe and Helpman's (1995) regressions for 21 OECD countries (plus Israel) is statistically significant and explains more of the variation in productivity across countries than the total-import weighted spillover variable. Also, the non capital goods weighted R&D spillover variable is statistically insignificant.

Lee (1995) also examines the importance of the role of capital goods import on long run growth in an endogenous growth model. In contrast to the previous literature, which stresses the effects of trade on technological progress, this paper emphasizes on the efficiency of capital accumulation in linking the foreign inputs and growth. He presents a simple model of an open economy by extending the endogenous growth model of Rebelo

(1991). In this model two final goods - one being consumption and one being capital good - are produced. Capital and labor are used in producing consumption good while capital is the only input in producing capital goods. The model of Rebelo implies that the relative price of the capital good decreases over time as capital stock rises and thus the price of the capital good relative to the consumption good is lower in a higher income country, which has a larger capital stock.

Consequently, he considers a global economy in which two countries; one being a less developed country (LDC) and the other one being a developed country (DC) are engaged in trade. The DC grows at the steady state while the LDC is just starting to produce its own capital goods. The model implies that the domestic price of capital good is relatively lower in DC, which has a larger capital stock. Therefore the DC has a comparative advantage in the capital good while the LDC has a comparative advantage in consumption good. By trading with each other, DC imports consumption good and LDC imports capital good. He assumes that the capital sector in LDC could use both imported and domestic capital in its production. Then he shows that for two LDCs which have the same per capita income and trade with the same DC, the country that devotes relatively more of a given portion of its income to the importation of cheap foreign capital goods than to the purchase of domestic capital goods, grows faster than the other country. Next he tests the empirical implication of the model by using cross-country data for the period 1960-85. The results show that investment (i.e., the ratio of imported to domestically-produced capital goods) has a significant positive effect on per capita income growth rates across countries. On the other hand, the share of total imports in GDP has no significant effect on growth. The results, thus, highlight that the composition of investment in addition to its size should be considered important in determining economic growth.

Bensidoun, Gaulier and Ünal - Kesenci (2001) examined the non-neutrality of trade specialization on country's growth performance by introducing indicators reflecting the nature of specialization into a standard equation of conditional convergence. Their study covers 53 countries for six periods of 5 years over 1967-1997. They used the general method of moments in estimating their equations since they were dealing with the dynamic panel data model. Their results show that the specialization variables have the expected sign and are highly significant. Therefore their results provided strong evidence that the growth effects of international trade depend on the type of products countries are specialized on.

Lewer and Van den Berg (2003) also go further in examining the effect of international trade on growth by examining whether the growth effect of trade depend on what a country exports and imports. Their idea is inspired by the work of Mazumdar (1996) who qualified Baldwin's (1992) neoclassical model of trade and growth. Baldwin (1989) argues that increase in international trade creates a one time upward shift in production function which in turn creates not only a short-run static gain from trade, i.e. higher level of output for any given level of input, but also boosts the saving and investment in an open economy with constant saving rate. This latter effect makes economy to move to a higher steady state and therefore creating a further medium-run gain in per capita output. Mazumdar (1996) however shows that although trade liberalization always leads to an increase in the level of real income but growth will depend on the kind of good that is imported. Specifically growth will not occur if the consumption good is being imported and the capital good is being exported. He argues that if a country imports capital goods and exports consumer goods the relative price of capital falls therefore the cost of investment to replace depreciated capital declines. A country that is importing capital good then enjoys lower depreciation costs and so a higher steady-state level of capital and output per worker and its growth rate along the transition to this new steady state will be higher with trade and reverse happens in a country that exports capital goods. To test this hypothesis Lewer and Van den Berg (2003) create two international trade composition variables for 28 developed and developing countries. Their results support the hypothesis that capital importing countries grow faster than countries that export capital goods.

All these studies imply that the nature of the good that a country is trading in, especially the type of good that is importing has an important role in growth performances of the countries. In this paper we provide an alternative approach to further study the importance of the type of trading goods on economic growth. We consider an endogenous growth model where two goods are being produced using physical capital only. One good being purely consumption and the other one is an investment good. There are two countries that produce these goods with different technologies. Countries could trade in both goods with each other. When there is free trade each country produces only one good and trade it to obtain the other good. On the balance growth path the growth rate of capital good importing county increases with trade while the growth rate of the country that imports consumption good remains unchanged. We use an empirical specification developed by Bond, Leblebicioglu and Schiantarelli (2006) in examining the effect of trade specialization on growth in an AK type growth model. We introduce an index of specialization in this specification to capture this effect. Our sample consists of 92 developing and developed countries over the period of 1965-2000. We estimate the model using yearly as well as non-overlapping five-year average panel data. The instrumental variable approach is used to get the consistent estimate of the coefficients. Different lagged values of endogenous variables as well as lagged values of some extra variables that usually is being considered in estimating growth models are being used. In addition when using the five-year averages we report the GMM estimators proposed by Arellano and Bond (1991).

3.3 THE MODEL

Recall that the model we used in previous section is a 2 sector endogenous growth model in which 2 goods are being produced using two reproducible factors of production, K and H, in different sectors. One sector produces a unified consumption and one type of capital good (K-Type), Y-sector, and the other sector produces another type of capital good (H-Type), Z-sector.

Both sectors are assumed to have a constant return to scale in the reproducible factors of production with perfect competition in goods and factor markets. The production technologies for each sector can then be expressed as

$$Y_i = F_i(s_i K_i, u_i H_i) = u_i H_i \; f_i(k_{yi}),$$

$$Z_i = G_i\big((1-s_i)K_i, (1-u_i)H_i\big) = (1-u_i)H_i \; g_i(k_{z_i}),$$

where $k_{yi} \equiv \dfrac{s_i K_i}{u_i H_i}$ and $k_{zi} \equiv \dfrac{(1-s_i)K_i}{(1-u_i)H_i}$.

Now consider a global economy in which there are two countries producing these two goods with different technologies. In this world countries could engage in trade in goods but there are no international lending and borrowing.

Assume X represents the export of good Y, therefore the evolution stock of each type of capital could be written as

$$\dot{K} = uH\ f(k_y) - \delta K - C - X\ ,$$

$$\dot{H} = (1-u)\ Hg(k_z) - \eta H + \frac{1}{p} X\ ,$$

where δ and η are the rate of depreciation for K and H capital respectively and ‘p’ represents the price of good Z in terms of good Y.

The representative agent’s optimization problem then would be

$$\max_{C, u_i, v_i} \int_0^\infty \frac{C(t)^{1-\sigma}}{1-\sigma} e^{-\rho t}\ ,$$

s.t (2.1) – (2.2), $1 \geq u_i \geq 0$, $1 \geq s_i \geq 0$, *i=Y, Z,*

$$H(0) = H_0 > 0\ ,\ K(0) = K_0 > 0\ .$$

As we have shown in the previous section, we can use the first order and necessary condition for this optimization and examine the balanced growth path as well as the dynamic transition of the model. Using the balanced growth conditions we showed that the growth rate could be written as

$$v^* = \frac{1}{\sigma}\left(r(p) - \delta - \rho\right),$$

where p is the world price of good Z in terms of Y in the global economy. Then r(p) is the return on investment in K capital. We showed that as countries start to trade with each other, they will specialize base on their comparative advantage and their growth rate will rise.

Although it is interesting to test this hypothesis but it is difficult given that the K-type and H-type capital do not refer to physical and human capital in this model. Therefore s it is difficult to define two broad types of capital in terms of data availability, we consider different model here.

Assume that instead of 2 sectors, there are 3 sectors in the country. One sector produces an investment good, one producing consumption good and the other one producing education using two types of capital, physical (K) and human (H). In this model however countries can only trade in investment (physical capital) and consumption goods. Bond and Trask (1997) have considered this model and showed the existence, uniqueness and saddle path stability of a balanced growth path for a small open economy. They discussed the existence of 2 types of production pattern on the balanced growth path. If the relative price of traded goods remain the same as the price under autarky they showed that the country would continue to produce both goods. At any price level other than that, the country will completely specialize in production of one type of the traded good and the non-traded good. Further they argued that with international trade the growth rate will be the same as autarky if the country continues to produce both goods. However it will be strictly higher if the country exports the consumption good (i.e. import investment good).

Seater (2007) obtains very similar results by considering a 2 sector model with one reproducible factor of production. One sector produces consumption goods and the other one produces the factors of production. In his model growth rate depends on the total factor productivity parameters (TFP) of the sectors that produce factors of production. He argues that country's growth rate rises when it substitutes another country's higher TFP for its own by importing the factor of production. His results then imply that the nature of the good that is being imported is more important for economic growth than the good that is being exported.

We obtain the same results when we alter our model (presented earlier) to the form that Seater (2007) has considered. Similarly consider a model with two sectors of production, one producing only consumption goods and the other one produces one type of capital good. Also capital is the only factor of production in both sectors. Similarly consider constant return to scale in the reproducible factors of production with perfect competition in goods and factor markets. The production technologies then are

$$Y = F(uH) = C + X \, , \tag{1-1}$$

$$Z = G((1-u)H) = \dot{H} + \delta H - \frac{1}{p} X \, , \tag{1-2}$$

where u is the proportion of capital used in producing Y, X is the export of Y and p is the price of H (Z) in terms of Y. The evolution equation for stock of capital can be written as

$$\dot{H} = G((1-u)H) - \delta H + \frac{1}{p}(F(uH) - C) \, . \tag{2}$$

Representative agent is again maximizing its lifetime utility function subject to (2) and initial condition. Therefore the Hamiltonian would be

$$V = \frac{C^{1-\theta} - 1}{1-\theta} e^{-\rho t} + \varphi \left[G((1-u)H) - \delta H + \frac{1}{p}(F(uH) - C) \right].$$

The necessary and first order conditions are

$$\frac{\partial V}{\partial C} = 0 \rightarrow C^{-\theta} e^{-\rho t} - \varphi = 0 \rightarrow \frac{\dot{C}}{C} = \frac{1}{\theta}\left[\frac{\dot{\varphi}}{\varphi} - \rho\right], \tag{3}$$

$$\frac{\partial V}{\partial u} = 0 \rightarrow \varphi\left[-HG'((1-u)H) + \frac{1}{p}\left(HF'(uH)\right)\right] = 0\,, \tag{4}$$

$$\dot{H} = \frac{\partial V}{\partial \varphi} = G((1-u)H) - \delta H + \frac{1}{p}(HF'(uH))\,, \tag{5}$$

$$\dot{\varphi} = -\frac{\partial V}{\partial H} = \varphi\left[-(1-u)G'((1-u)H) - \delta + u\frac{1}{p}F'(uH)\right]. \tag{6}$$

From equation (4) we get,

$$-G'\left((1-u)H\right) + \frac{1}{p}F'\left(uH\right) = 0\,,$$

which in turn implies

$$p = \frac{F'(uH)}{G'((1-u)H)}\,. \tag{7}$$

Substituting for p from (7) into (6) we obtain

$$\frac{\dot{\varphi}}{\varphi} = G'((1-u)H) - \delta\,. \tag{8}$$

Then using (8) and (3) the growth rate for the will be

$$v = \frac{1}{\theta}\left[G'((1-u)H) - \delta - \rho\right]. \tag{9}$$

If p_1 and p_2 are the autarky prices for country 1 and 2 respectively and p would be the relative world price of traded goods, assume that $p_1 > p$ and $p_2 < p$. As we discussed in chapter 2, now country 1 will completely specialize in producing Y country 2 will produce only Z.[1] In this case country 2 will choose u=0 and uses all its capital to produce and export Y (investment goods) and import consumption goods. Put u=0 and solve the above optimization problem for country 2. The growth rates with trade for this country would then obtain as

$$v_{2,T} = \frac{1}{\theta}\left[G_2' - \delta - \rho\right] = v_{2,Au} \,. \qquad (10)$$

Country 1 in the other hand choose u=1 and produce only consumption good s and imports all the capital it needs. The growth rate with trade for this country could be obtained by choosing u=1 and solving the optimization problem. This growth rate then would be, [2]

$$v_{1,T} = \frac{1}{\theta}\left[\frac{1}{p}F' - \delta - \rho\right] > v_{1,Au} = \frac{1}{\theta}\left[\frac{1}{p_1}F' - \delta - \rho\right]. \qquad (11)$$

As we can see country 1 that imports capital good will enjoy higher growth rate while the growth rate of the country that exports the factor of production i.e. country 2, will remain unchanged with trade.

3.4 INTERNATIONAL SPECIALIZATION AND GROWTH

In section 3 we showed that what is important for economic growth is the nature of good being imported. Importing the means of production, i.e. investment good could raise the

[1] If we consider Cobb-Douglas production function then the model in this section would be an special case of the model that we considered in chapter 1 where $\alpha = \eta = 0$ therefore the production function for each sector would be $Y = A(uH)$ and $Z = B((1-u)H)$ which is exactly the model Seater (2007) considered. We also can consider another reproducible factor of production which then in this case will not be tradable. This would be the model that Bond and Trask (1996) considered. They also showed that in the case of interior solution each country will completely specialize, and the growth rates will rise for both countries. However the growth rate for the country that imports investment good rises more than that for the country that imports consumption good.

[2] Notice that we used $G'((1-u)H) = \frac{1}{p}F'(uH)$ that we obtained from equation (4) to get the autarky growth rate as we presented in equation (11).

growth rate of a country while importing consumption good does not affect the growth rate. In the other word by importing the investment good, trade could allow the country to substitute the other country's higher TFP for its own[3] which causes its growth rate to rise. In the following section we conduct an empirical test that could examine whether the nature of importing goods could be important for the economic growth.

3.4.1 ECONOMETRIC MODEL

It can be seen that the long-run per capita growth rate as described in equation (9) depends on the parameters that determine the willingness to save and the real return to the investment good and therefore productivity of capital. As we discussed in the previous section when we consider trade in our model, trade could affect the long run growth rate of output per capita through affecting the real rate of return to investment. However the effect of trade on long run growth depends on the type of good that is being imported. Importing capital goods can affect growth rates positively while importing consumption good does not have any effect on the growth performances of importing country.

The striking difference between this type of model and the neoclassical growth model therefore concerns the determination of the long-run per capita growth rate. As we know the neoclassical growth models imply that the long-run per capita growth rate is fixed at the exogenous rate of technological change. Consequently the factors that could affect the willingness to save or improvement in the level of technology shows up in the long run as higher levels of capital and output per worker but in no change in the per capita growth rate.

[3] Seater (2007)

Most of the empirical literature that studies economic growth based on the Solow model framework usually relate differences between countries' growth rate of output per worker to their structural characters such as investment rate, population growth rate and their level of technology as well as other economic and political variables such as human capital, public expenditure, rates of openness, revolution, and many more. The typical specification used in the study of economic growth base on the neoclassical framework usually is in the general form of

$$\Delta y_{i,t} = (\alpha - 1) y_{i,t-1} + W_{i,t}\beta + \gamma t + \eta_i + \varepsilon_{i,t} . \tag{12}$$

where $y_{i,t}$ is GDP per worker in country *i*, and in period *t*. $W_{i,t}$ is a row vector of determinants of economic growth, η_i is a country specific effect and the time trend determines a common rate of long-run growth, and $\varepsilon_{i,t}$ is an error term. In equation (12) a significantly negative coefficient of y_{it} implies the existence of conditional convergence as the neoclassical growth model predicts i.e. the countries relatively close to their steady state level of output will experience a slowdown in growth.

The equation specified in (12) then indicates that in the long-run, growth rate of output per worker is the same for all countries and depends on an exogenous rate of technological change.[4]

[4] To see this rewrite equation (12) as

$$y_{i,t} = \alpha \ y_{i,t-1} + W_{i,t}\beta + \gamma t + \eta_i + \varepsilon_{i,t} . \tag{13}$$

Now consider the steady state at which output per worker grows at a constant rate, g, and other variables remain constant (i.e. $y_{i,t} = y_{i,t-1} + g$ and $W_{i,t} = W_i$).

Equation (12) is being tested in many literatures studying growth, using cross-country data of a large sample of developed and developing countries. In these regressions the dependent variable usually is a time-average of growth rates and on the right hand side, a combination of time-averages flows, such as investment rate, rate of government expenditure, and beginning of period stocks, such as the beginning of period GDP per worker as well as measures for openness and international trade are included.

On the other hand however the AK type endogenous growth model has suggested that investment in broadly defined capital has a positive effect on long-run growth. [5] As we discussed earlier, similarly in the model that we considered the long run growth rate also depends on investment.[6] Moreover we showed that trade and specifically importing capital

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Substituting these conditions in equation (13) and $\varepsilon_{i,t} = E(\varepsilon_{i,t}) = 0$, we get,

$$y_{i,t} = \alpha\,(y_{i,t} - g) + W_i \beta + \gamma t + \eta_i,$$

and so,

$$y_{i,t} = \left(\frac{\beta}{1-\alpha}\right) W_i + \frac{\eta_i}{1-\alpha} + \left(\frac{\gamma}{1-\alpha}\right) t - \frac{\alpha g}{1-\alpha}. \qquad (14)$$

Equation (14) implies that at the steady state the level of output per worker depends on the variables in $W_{i,t}$. Taking the first difference from equation (14) will give us the long-run growth rate of output per capita as

$$g = \frac{\gamma}{1-\alpha}. \qquad (15)$$

[5] Romer (1986, 1987), Lucas (1988), and Rebelo (1991).
[6] Consider the two sector model with Cobb-Douglas production functions similar to the one we considered in section one. Remember that the capital accumulation restriction is $\dot{K} = I - \delta K \rightarrow \gamma_K = \frac{I}{K} - \delta$. From the production function for Y-sector we have $Y = A(vK)^{\alpha}(uH)^{1-\alpha} = A\left(\frac{vK}{uH}\right)^{\alpha-1}(vK)$. Also we showed that

can affect this long run growth rate. Consequently the long run growth rate of per capital output that we consider should depend on such variables

$$g_i = f(W_i)$$

where g_i is the country's long run rate of growth and W_i is the steady state value of a vector of variables that could affect the growth i.e. investment and trade in our case. To obtain this property for the growth rate, take the first difference from equation (13) and include a lagged level of explanatory variable (s) as additional explanatory variable (s), we get

$$\Delta y_{i,t} = \alpha\, \Delta y_{i,t-1} + \Delta W_{i,t}\beta + W_{i,t-1}\theta + \gamma + \Delta\varepsilon_{i,t}\,, \qquad (16)$$

Now consider a steady state in which, $W_{i,t} = W_{i,t-1}$ and $y_{i,t} = y_{i,t-1} + g$, then the growth rate of output at this steady state could be obtain as

$$g_i = \frac{\gamma}{1-\alpha} + W_i\left(\frac{\theta}{1-\alpha}\right). \qquad (17)$$

Therefore in the dynamic specification of the growth rate specified in equation (16), the coefficient (s) of $W_{i,t-1}$, i.e. θ (θ being a vector in case of more than one explanatory variable) predicts the long-run effect of the corresponding variable (s) on the steady state level of output per worker while β determines the effect of such variable (s) on the level of output per worker at the steady state.

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on the BGP we have $\frac{vK}{uH} = p\frac{\alpha}{1-\alpha}$. From this latter equation and the production function for Y-sector we can get $K = \frac{Y}{A\left(p\frac{\alpha}{1-\alpha}\right)^{\alpha-1} v}$. Substituting for K in its accumulation restriction, we get

$\gamma_K = A\left(p\frac{\alpha}{1-\alpha}\right)^{\alpha-1} v \cdot \frac{I}{Y} - \delta$. Therefore we can see that the growth rate of capital on the BGP depends on the investment share of physical capital. Since on the BGP all the variables grow at the same rate then the GDP growth rate should also depend on the investment share.

We should however note that there is an important problem with using least squares estimation procedure to estimate the coefficients in equation (16). First, notice that the shocks in equation (16) are the first difference of the shocks introduced in equation (13). Therefore if we assume that the shocks in equation (13) are serially uncorrelated then the shocks in (16) will be serially correlated (MA(1)). Also notice that on the right hand side of equation (16) includes $\Delta y_{i,t-1}$ which in turn will be correlated with the lagged shock, i.e. $\varepsilon_{i,t} - \varepsilon_{i,t-1}$. This implies that the estimates of least squares will be biased and inconsistent. Also variables that usually is included as explanatory such as investment also will be correlated with random shock.

Bond, Leblebicioglu and Schiantarelli (2006) have developed an approach that could be used as a way of obtaining consistent estimates of variables in dynamic growth models of the kind explained in equation (16). We use their framework in developing an econometric model in estimating the growth effect of trade specialization.

Following Bond, Leblebicioglu and Schiantarelli (2006) we start with a very general specification of relationship between the growth of output per worker and investment in physical capital. In addition we include another variable to capture the effect of trade on the growth of output per worker. Consider $y_{i,t}$ to be the logarithm of GDP per worker and $x_{i,t}$ to be the logarithm of the investment to GDP ratio and also $w_{i,t}$ to be the logarithm of trade. We use the share of import (% of GDP) and the share of export (% of GDP) as well as a measure of comparative advantage which we describe in the next section. Now assume the following model of Autoregressive-Distributed Lag of order (p, p) (ADL(p, p)) to describe the behavior of GDP per worker

$$y_{i,t} = c_{i,t} + \alpha_1 y_{i,t-1} + \alpha_2 y_{i,t-2} + \cdots + \alpha_p y_{i,t-p} + \beta_0 x_{i,t} + \beta_1 x_{i,t-1} + \cdots + \beta_p x_{i,t-p}$$

$$+ \rho_0 w_{i,t} + \rho_1 w_{i,t-1} + \cdots + \rho_p w_{i,t-p} + \varepsilon_{i,t} . \qquad (18)$$

In this specification ε_{it} is a mean zero, serially uncorrelated shock assumed to be independent across countries, and c_{it} is considered to be a non-stationary process that determines the behavior of the growth rate of $y_{i,t}$ in the steady state (i.e. rate of return to investment good in our model).

Now take the average of all variables across all countries in the same period,

$$\overline{y}_{.t} = \overline{c}_{.t} + \alpha_1 \overline{y}_{.,t-1} + \alpha_2 \overline{y}_{.,t-2} + \ldots + \alpha_p \overline{y}_{.,t-p} + \beta_0 \overline{x}_{.,t-1} + \beta_1 \overline{x}_{.,t-1} + \ldots + \beta_p \overline{x}_{.,t-p}$$

$$+ \rho_0 \overline{w}_{i,t} + \rho_1 \overline{w}_{i,t-1} + \cdots + \rho_p \overline{w}_{i,t-p} + \overline{\varepsilon}_{i,t} , \qquad (19)$$

where, $\overline{y}_{.t} = \frac{1}{N}\sum_{i=1}^{N} \widetilde{y}_{it}$, $\overline{x}_{.t} = \frac{1}{N}\sum_{i=1}^{N} \widetilde{x}_{it}$, $\overline{w}_{.t} = \frac{1}{N}\sum_{i=1}^{N} \widetilde{w}_{it}$ and $\overline{\varepsilon}_{.t} = \frac{1}{N}\sum_{i=1}^{N} \widetilde{\varepsilon}_{it}$ are the year-specific means.

Subtract (19) from (18) we get

$$\widetilde{y}_{i,t} = \widetilde{c}_{it} + \alpha_1 \widetilde{y}_{i,t-1} + \alpha_2 \widetilde{y}_{i.t-2} + \ldots + \alpha_p \widetilde{y}_{i,t-p} + \beta_0 \widetilde{x}_{i,t-1} + \beta_1 \widetilde{x}_{i,t-1} + \ldots + \beta_p \widetilde{x}_{i,t-p}$$

$$+ \rho_0 \widetilde{w}_{i,t} + \rho_1 \widetilde{w}_{i,t-1} + \cdots + \rho_p \widetilde{w}_{i,t-p} + \widetilde{\varepsilon}_{i,t} . \qquad (20)$$

In this specification $\widetilde{y}_{it} = y_{it} - \overline{y}_{.t}$, $\widetilde{w}_{it} = w_{it} - \overline{w}_{.t}$, $\widetilde{x}_{it} = x_{it} - \overline{x}_{.t}$ and $\widetilde{\varepsilon}_{it} = \varepsilon_{it} - \varepsilon_{.t}$ denote deviation from the year-specific means.

Bond, Leblebicioglu and Schiantarelli (2004) experiment with different specification of the process for c_{it} on how investment affects the steady state growth rate of output per worker. One specification that they have considered is

$$c_{it} = c_{i,t-1} + \theta_0 + \theta_1 x_{it} + \theta_2 w_{it} + e_t , \qquad (21)$$

where the change in c_{it} depends directly on the current share of investment and a measure of trade. As mentioned earlier c_{it} is a non-stationary process that determines the behavior of the growth rate of $y_{i,t}$ in the steady state. If we take the deviation from year-specific means then we get

$$\tilde{c}_{it} = \tilde{c}_{i,t-1} + \theta_1 \tilde{x}_{it} + \theta_2 \tilde{w}_{it}. \tag{22}$$

Now take the first differenced of equation (20)

$$\Delta\tilde{y}_{i,t} = \Delta\tilde{c}_{it} + \alpha_1\Delta\tilde{y}_{i,t-1} + \alpha_2\Delta\tilde{y}_{i.t-2} + \ldots + \alpha_p\Delta\tilde{y}_{i,t-p} + \beta_0\Delta\tilde{x}_{i,t-1} + \beta_1\Delta\tilde{x}_{i,t-1} + \ldots + \beta_p\Delta\tilde{x}_{i,t-p}$$
$$+ \rho_0\Delta\tilde{w}_{i,t} + \rho_1\Delta\tilde{w}_{i,t-1} + \cdots + \rho_p\Delta\tilde{w}_{i,t-p} + \Delta\tilde{\varepsilon}_{i,t} \tag{23}$$

Next if substitute for Δc_{it} from (22) we obtain

$$\Delta\tilde{y}_{i,t} = \alpha_1\Delta\tilde{y}_{i,t-1} + \alpha_2\Delta\tilde{y}_{i.t-2} + \ldots + \alpha_p\Delta\tilde{y}_{i,t-p}\beta_0\Delta\tilde{x}_{i,t-1} + \beta_1\Delta\tilde{x}_{i,t-1} + \ldots + \beta_p\Delta\tilde{x}_{i,t-p}$$
$$+ \theta_1\tilde{x}_{it} + \theta_2\tilde{w}_{it} + \rho_0\Delta\tilde{w}_{i,t} + \rho_1\Delta\tilde{w}_{i,t-1} + \cdots + \rho_p\Delta\tilde{w}_{i,t-p} + \Delta\tilde{\varepsilon}_{it}. \tag{24}$$

Equation (24) indicates the output per worker as a distributed lag of itself and a distribute lag of first-differences of the investment shares and with additional terms in the log level of the investment share. The relationship between growth, investment and trade on the steady state when $\tilde{x}_{it}$ and $\tilde{w}_{it}$ will be constant and $\tilde{\varepsilon}_{it}$ is set to its expected value of zero can be obtained as

$$\tilde{g}_i = \tilde{y}_{it} - \tilde{y}_{it-1} = \frac{\theta_1\tilde{x}_i + \theta_2\tilde{w}_i}{1 - \alpha_1 - \alpha_2 - \cdots - \alpha_p},$$

where $\tilde{g}_{it}$ indicates the steady state growth rate of output per worker. This equation implies that the steady state growth rate of output per worker depends on the country-specific effect of share of investment and trade. Consequently $\frac{\theta_1}{(1-\alpha_1-\alpha_2-...\alpha_p)}=0$ test the long-run effect of investment on growth and $\frac{\theta_2}{(1-\alpha_1-\alpha_2-...\alpha_p)}=0$ test the long-run effect of trade on growth. Also this specification allows for the investment and trade to affect the level of output per worker on the steady state. $\frac{\beta_0+\beta_1+\cdots+\beta_p}{1-\alpha_1-\alpha_2-\cdots-\alpha_p}$ reflects the level effect of investment on the level of output per worker at the steady state and $\frac{\rho_0+\rho_1+\cdots+\rho_p}{1-\alpha_1-\alpha_2-\cdots-\alpha_p}$ reflects the level effect of trade.[7]

Notice that ε_{it} reflects the first-differences of the shocks to the level of output per worker in equation (18). Therefore if the shocks that affect the log level of output per worker are serially uncorrelated in equation (17) then the shocks in equation (27), $\Delta\tilde{\varepsilon}_{it}$ has an MA(1) structure which then will be correlated with the lagged dependent variable $\Delta\tilde{y}_{i,t-1}$. These facts indicate that the least squares estimates of the parameters in (20) will be biased and inconsistent. In addition we have the lagged dependent variable on the right hand side of the equation. However we can use the instrumental variable estimation procedure to obtain consistent estimates.

Notice that in the specification for $\tilde{c}_{it}$ in (20) the only factors that affect the growth are investment share and trade that influence the growth of $\tilde{c}_{it}$. As the result there is no time-invariant country-specific component of the error term in the first-differenced equation

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[7] This could be obtained by imposing $\tilde{y}_{it}=\tilde{y}_{it-1}+\tilde{g}_i$, $\tilde{x}_{it}=\tilde{x}_{it-1}=\tilde{x}_i$, $\tilde{w}_{it}=\tilde{w}_{it-1}=\tilde{w}_i$ and $\tilde{\varepsilon}_{it}$ to its expected value zero in equation (20).

specified in (24). However as we know there might exist unobserved country-specific factors that might affect the steady state growth rates or other variables. To overcome this issue a time-invariant country-specific drift term such as d_i can be included in the process of c_{it} as follows,

$$c_{it} = c_{i,t-1} + d_i + \theta_0 + \theta_1 x_{it} + \theta_2 w_{it} + e_t \,. \tag{25}$$

If we take the deviation from year-specific means then we get

$$\widetilde{c}_{it} = \widetilde{c}_{i,t-1} + \widetilde{d}_i + \theta_1 \widetilde{x}_{it} + \theta_2 \widetilde{w}_{it} \,. \tag{26}$$

Now if we substitute for Δc_{it} from (22) we obtain

$$\Delta\widetilde{y}_{i,t} = \alpha_1 \Delta\widetilde{y}_{i,t-1} + \alpha_2 \Delta\widetilde{y}_{i.t-2} + \ldots + \alpha_p \Delta\widetilde{y}_{i,t-p} \beta_0 \Delta\widetilde{x}_{i,t-1} + \beta_1 \Delta\widetilde{x}_{i,t-1} + \ldots + \beta_p \Delta\widetilde{x}_{i,t-p}$$

$$+ \theta_1 \widetilde{x}_{it} + \theta_2 \widetilde{w}_{it} + \rho_0 \Delta\widetilde{w}_{i,t} + \rho_1 \Delta\widetilde{w}_{i,t-1} + \cdots + \rho_p \Delta\widetilde{w}_{i,t-p} + \widetilde{d}_i + \Delta\widetilde{\varepsilon}_{it} \,. \tag{27}$$

As it can be seen now $\widetilde{d}_i$ is a country-specific component of the error term. Therefore the model in (27) can be estimated consistently by instrumental variables and including country-dummies.

We will estimate the model specified in (27) and (24) using the pooled annual panel data with and without including the country-dummies. In addition, we use the GMM estimation procedure developed by Arellano and Bond (1991) using pooled data for non-overlapping five-year periods to obtain the consistent estimates of the effect of trade on average growth rates. Following Bond, Leblebicioglu and Schiantarelli (2006) we use the following specification,

$$\Delta_5 \widetilde{y}_{it} = (\alpha_1 - 1)\widetilde{y}_{i,t-5} + \theta_1 \widetilde{x}_{it}^A + \theta_2 \widetilde{w}_{it}^A + \beta_0 \Delta_5 \widetilde{x}_{it}^A + \rho_0 \Delta_5 \widetilde{w}_{it}^A + \Delta_5 \widetilde{\varepsilon}_{it} \tag{28}$$

where $\Delta_5 \tilde{y}_{it} = \tilde{y}_{it} - \tilde{y}_{i,t-5}$, and $\tilde{x}_{it}^A$ and $\tilde{w}_{it}^A$ denote the average values of log of the investment share and trade measures over our five-year periods. Also after taking first-differencing, there will be no unobserved fixed effect remaining in the error term. We present both the one-step estimator in Arellano and Bond (1991), as well as the two-step estimator with standard errors that use the finite-sample correction by Windmeijer (2005).

To examine the effect of trade specialization on growth, we first use cross country data on average growth rates for a sample of 92 countries over the period of 1965-2000. We estimated the average growth rate on the beginning of period level of output per worker, average level of investment share and a measure of trade specialization.

However as Pesaran and Smith (1995) discussed the least square estimates of a dynamic model using cross country data would be biased and inconsistent. In addition in cross-section estimates, the intercept cannot be allowed to be specific to individual countries. In panel applications however it is possible to allow for heterogeneous intercepts. Therefore we also use panel data to estimate the specification in (24) and (27). We present IV estimates using the yearly panel data as well as GMM estimates for non overlapping five-year averages over 1965-2000 for 92 countries excluding the oil producing countries. The lagged values dated t-2 and earlier of endogenous variables are used as instruments. Also we include the lagged values of log of government spending (as % of GDP) and log of inflation rate plus one (as GDP deflator) as additional instruments dated t-2 and earlier. Moreover when using the five-year average panel data we also include an additional variable to capture the effect of human capital. Next we briefly explain the data and the index we use to measure the trade specialization.

3.5 DATA SOURCE AND MEASURE OF INTERNATIONAL SPECIALIZATION

The data for GDP per worker and investment share comes from the Penn World Table 6.1 (PWT 6.1) data set. These variables are measured in constant international dollars. The data on government spending as percentage of GDP and inflation (measured using the GDP deflator) are taken from the World Bank World Development Indicators (WDI) 2005. Also we use measures of human capital accumulation from Barro and Lee (2000). To capture the effect of trade specialization on economic growth we use three different measures. We use import share as well as export share for consumption, capital and intermediate goods. However using just import or export share to measure trade specialization might not be enough. Therefore we also use an index that measures trade specialization for consumption, capital and intermediate goods.

To measure a country's specialization in a given sector we use an index of international specialization proposed by Lafay (1992). This measure has been used in many literatures to study the evolution of trade specialization.[8] Although there are several indexes that we can use to measure trade specialization, there are advantages to this particular index that make it a suitable candidate to use in our study. First, Lafay index considers not only exports but also imports. In fact, the Lafay index takes into account the difference between exports and imports in each sector. By doing so, it allows us to control for intra-industry trade flows. Intra industry trade flows have particularly been important in recent years since there are more and more firms delocalizing part or all of their production into other countries through establishing affiliates and by outsourcing agreement with local firms. This generates

[8] For instance, Evolution of trade patterns in the new EU member states (Andrea Zaghini , 2005) and International Specialization Models In Latin America: The Case of Argentina (Paola Caselli and Andrea Zaghini, 2005).

trade flow in both directions. Moreover for fairly aggregate groups of product the size of intra-industry trade flows becomes significant and therefore indexes based only on exports would be a poor indicator.[9] Second, the Lafay index considers the difference between each item's normalized trade balance and the overall normalized trade balance. In this way the Lafay index controls for the influence of cyclical factors, which can affect the size of trade flows in the short run. Finally, it weights each group's contribution according to the respective importance in trade.[10]

The Lafay (LFI) Index for a given country, *i*, and for any given group of products, *j*, is given by

$$LFI_j^i = 100\left[\frac{x_j^i - m_j^i}{x_j^i + m_j^i} - \frac{\sum_{j=1}^{N}(x_j^i - m_j^i)}{\sum_{j=1}^{N}(x_j^i + m_j^i)}\right]\frac{x_j^i + m_j^i}{\sum_{j=1}^{N}(x_j^i + m_j^i)}, \qquad (34)$$

where x_j^i and m_j^i are exports and imports of products of group *j* of country *i,* to and from the rest of the world, respectively, and N is the number of traded groups. As we can see according to this index, the comparative advantage of country *i* in the production of group *j* is

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[9] An example of such index could be the traditional revealed comparative advantage (RCA) indicator proposed by Balassa (1965). This index is defined as $RCA_j^i = \frac{x_j^i \Big/ \sum_{j=1}^{N} x_j^i}{x_j^w \Big/ \sum_{j=1}^{N} x_j^w}$ where x_j^w represents world exports of item *j* .Therefore the RCA index compares the national export structure with the world export structure. Since RCA considers only export flows it sometimes is defined as a single-flow indicator of trade intensity (Iapadre,2003) and the LFI is defined as a net-trade indicator of specialization.

[10] Lafay (1992).

measured by the deviation of the normalized trade balance of group *j* from the overall normalized trade balance multiplied by the share of trade of group *j* on total trade.

Positive values of the Lafay Index indicate the existence of comparative advantages in group *j* for a given country. The larger the value, the higher is the degree of specialization in any given group. The negative values of this index in the other hand represent comparative disadvantage in group *j*.

The data for international trade flows are taken from the International Trade Data, NBER-UN world trade data[11], documented by Robert Feenstra, Lipsey, Deng, Ma and Mo (2005). The data are organized by the 4-digit Standard International Trade Classification, Revision 2 with country codes similar to the United Nations classification. This dataset contains the data on import and export from and to almost all the countries in the world for the period of 1962-2000.

Recall that in our study we consider testing the growth effect of specialization in trading goods which we consider to be capital goods and consumption goods. Therefore we needed to put each product at the 4-digit SITC level into a more aggregated group. To do so first we arranged the data according to the Broad Economic Categories. The BEC is a classification intended to categorize trade statistics into large economic classes of commodities and to supplement the summary data compiled on the basis of the sections of the Standard International Trade Classification. Appendix 3.B presents this classification. As it can be seen this classification is not as aggregate as we need. The United Nation also provides a classification procedure of the System of National Accounts (SNA) which places all traded goods into three main categories: capital goods, intermediate goods and consumer

[11] This data is available at www.nber.org/data.

goods. Using the United Nation's methodology and the corresponding BEC formulas, the categories could be rearranged to obtain aggregates as comparable as possible with those for the three basic classes in the System of National Accounts (i.e. capital, intermediate, and consumption good).[12] Appendix 3.C presents these categories.

Therefore the corresponding product groups in LFI definition, i.e. j in equation (34), would be Capital, Intermediate and Consumption goods. The values of LFI have been calculated for each country using the NBER-UN world trade data. Appendix 3.F contains the graphs of LFI index for all three product groups in each country.

Each graph illustrates the behavior of LFI index for capital, consumption and intermediate products for the period of 1962-2000. It can be seen that the values of LFI for capital are negative for most of the countries in our sample over the period of 1962-2000. This fact in turn implies existence of comparative disadvantage in capital which means most of the countries are indeed the importer of capital goods. There are however few exceptions such as Japan, USA, Italy, Switzerland and Sweden where the values of LFI for capital goods remain mainly positive during the period of 1962-2000. Other countries such as Denmark, France, Netherlands and UK have positive LFI for capital goods for most of the 60s, 70s and 80s but it starts to fall below zero or very close to zero by the end of 80s and beginning of 90s. In contrast there are also countries such as Finland, Korea, Malaysia, Mexico, Philippines and Thailand that have negative values of LFI for capital for most of the period while these values have become positive by mid or late 90s.

On the other hand it can be seen that there are many countries for which the value of LFI for consumption good is positive (negative) during the period of 1962-2000. This then

[12] Lewer and Berg (2003), p86.

implies that there are many countries that export (import) consumption goods. Notice that some of the countries that export (import) consumption i.e. $LFI_i^{Consumption} > 0$ (<0) they also import capital goods i.e. $LFI_i^{Capital} < 0$ (>0). However there are also countries in which the values of LFI for both consumption and capital could be positive (negative). Therefore there are cases where countries are exporting (importing) both types of goods. In these countries as we can see the values of LFI for intermediate goods are usually of the opposite sign. Therefore base on the structure of their economies there are countries that are specializing and exporting (importing) consumption goods and in return they import (export) capital or intermediate goods. Next we try to examine if there is any growth affect of trade specialization in type of goods that are being traded by countries as we discussed above.

3.6 RESULTS

To examine the effect of trade specialization on economic growth we first use the cross-country data for the sample of 92 countries over period of 1965-2000 to estimate the specification in equation (12). Then present the results of estimating the dynamic models for the growth rate of output per worker using the specifications of equation (24) and (27). In addition, we also discuss the results obtained from estimating (28) using pooled data for non-overlapping five-year periods. In all models we use import share, export share as well as LFI index for capital, consumption and intermediate goods as the proxy for trade specialization.

3.6.1 CROSS-COUNTRY RESULTS

We first present the results of estimating equation (12) of average growth rate using cross-country data for the sample of 92 countries over 1965-2000. Tables 1, 2 and 3 present the results of such estimates using the beginning period level of GDP per worker, average logs of share of investment, share of import for capital, consumption and intermediate goods as well as share of export and the Lafay index that was introduced in previous section for such categories.

Table 1 presents the results of such estimates using the import share of the countries. Models (1), (4) and (7) are the estimates of equation (12) when only average share of investment (log) and average share of imports are included, while in models (2), (5) and (8) we have included variables that usually enters in the cross-country estimation of growth models. In addition we have included the total years of schooling as a proxy for human capital in models (3), (6) and (9). [13] As we can see the estimated parameter for the beginning level of output per worker and investment are all highly significant and with expected sign. The import share for capital or consumption is not significant while import shares for intermediates is significant only when we include lagged level of output and investment share. Table 2 reports the estimates of similar models to what we considered in table 1 using the export shares instead of import shares. Similarly the estimated parameters of the share of investment and the beginning of the period are all significant. The export shares for capital goods are as well significant in all the models. However notice that this coefficient might be capturing effect of some omitted variables in the model. Although not significant but our estimates imply that exporting intermediates might have negative effect on growth rates. Again recall that the computed LFI measures for countries indicate that most of the exporting countries of intermediate goods are less developed countries. Therefore that might be the

[13] In all table t-ratios are reported in the parentheses and p-value of the estimates are reported in the brackets.

reason leading such results. Table 3 considers the Lafay index (measuring revealed comparative advantage) as a measure of trade specialization. We have reported the results for the same models that we considered in previous tables using LFI index instead of import or export share. These results also indicate that specializing in intermediate goods and therefore exporting these goods affect the growth rates negatively and they are significant. However since estimated coefficients using ordinary least square in examining the cross country differences of growth rates are biased and inconsistent, next we use pooled panel data and instrumental variable approach to obtain consistent estimates.

3.6.2 RESULTS OF YEARLY PANEL DATA

We estimated the dynamic model for growth rate of output per worker as specified in equations (24) and (27) using yearly panel data. When there is no country-specific factor beside investment and trade specialization in the equation that specifies the behavior of c_{it}, i.e. equation (21) then no country-specific fixed effect was presented in the first-differenced model. However when we consider the case where there are other country specific effect other than investment and trade specialization affecting the steady state growth rate, i.e. equation (25), then such unobserved effects are controlled by introducing country dummies in estimation. The models specified by "w CE" in the tables reporting results of yearly panel estimations are the models where we assumed unobserved heterogeneity across countries in long-run growth rates and so country dummies are used to controlled of such effects. As we discussed in section 4, if the shocks entering the log level of output per worker considered to be serially uncorrelated then the shocks in first differenced models will be of MA(1) form. Therefore the errors in first-difference models will be correlated with endogenous explanatory variable (s) in our specification. Therefore we need to use Instrumental Variable (IV) approach to obtain consistent estimates of the parameters in the model. We used two sets of instruments. In one set (IV1) we used lagged observations dated from t-2 to t-6 on the log of income per worker ($\tilde{y}_{it}$), the log investment share ($\tilde{x}_{it}$) and the log trade (import share, export share and LFI, $\tilde{w}_{it}$). In the second set (IV2) we added the lagged observations dated t-2 to t-6 on the log of government spending as a share of GDP, and log of inflation plus one to the IV1 set of instruments.

Tables 4 to 12 report the estimates of specification in (24) and (27) using the yearly panel data. We presented the results of such estimates using the share of import for capital, consumption and intermediate goods in table 4, 5 and 6. The results of using export shares of capital, consumption and intermediate are presented in tables 7, 8 and 9 respectively. Finally the estimates of underlying specification using the Lafay index of trade specialization (LFI) are reported in tables 10, 11 and 12. In all tables we have presented the estimates of the more general specification, which includes current and lagged investment, trade and two lags of the lagged dependent variable. Following Bond, Leblebicioglu and Schiantarelli (2006) we also report the results of estimating a more parsimonious dynamic specifications. The results of these estimates are being reported using both sets of instruments. In addition we have reported these estimates with and without using the country fixed effect.

For comparison column one of each table reports the results of OLS estimates. The Sargan-Hansen test of overidentifying restrictions does not reject the validity of the instruments that we have used in the estimation procedure. However when we use the larger set of instruments where we included the lagged of government spending and inflation as additional instruments the validity of the instruments are less approved by Sargen-Hansen test. In most of the estimation when we use the IV2 set of instruments, we find strong evidence of negative first-order serial correlation in the residuals. These results are stronger when we use the parsimonious specification of the model. This result is consistent with the use of endogenous variables dated t-2 in the instrument set.

The long run effects of investment and trade specialization on growth rate of output per worker are captured by the coefficients of the log investment and log of trade specialization. The effect of investment on the growth rate of output in steady state is mostly statistically significant as expected. This result is also stronger when we use the parsimonious specification. However the growth effect of trade specialization is only significant in a few of the cases we have considered. The growth effect of export share of capital is significant and positive when using pooled data and when using small instruments without including country specific effects. Also when using the export share of consumption good the growth effect of trade is significant and positive in both OLS and parsimonious model using the IV1 instrument set. This again indicates that exporting consumption good to import some sort of factor of production either capital or intermediate good can affect long run growth rate positively. The latter coefficient however is negative and significant using export share of intermediate goods. These latter results are being consistent with the results using the lafay index as reported in tables 11 and 12. The coefficient for the LFI using consumption goods and intermediates are statistically significant using IV1 instruments in which the growth effect of specializing in consumption good is positive while it is negative when specializing in intermediates. Also when considering the parsimonious model with IV1 set of instruments the growth effect of specialization in intermediates is as well negative and statistically significant.

3.6.3 RESULTS OF FIVE-YEAR AVERAGES PANEL DATA

In this section we present the results of estimating specification in equation (28) using data on average growth rates, investment shares and trade specialization for non-overlapping five-year periods. Our yearly panel data covered 1965-2000 for 92 countries, taking five-year averages will give us data for seven five-year periods. As discussed earlier we use a GMM estimation approach suggested by Arellano and Bond (1991). Tables 13-21 report the results of such estimates. In all tables we included both the one step estimator in Arellano and Bond (1991), denoted by GMM1 as well as the two step estimator denoted by GMM2 with standard errors that use the finite-sample correction proposed by Windmeijer (2005). Again we have used two sets of instruments one set includes the lags of log of GDP per worker, investment share and a measure of trade specialization (import share, export share and Lafay index), each dated t-2, t-3 and t-4. We call this set of instruments, small instrument set. In addition the lags 2, 3 and 4 of log of government investment (as % of GDP) and log of inflation plus one has been considered as additional instruments. The augmented instrument set includes the instruments of the small set plus the additional instruments. Since we are using the five-year averages, following Bond, Leblebicioglu and Schiantarelli (2006), we include the years of schooling as a measure of human capital, using the Barro and Lee (2000) data based. We use total years of schooling in the population 25 and above as a measure of human capital. We allow for human capital to affect both the growth rate and level of output per worker in steady state. Also when include this variable in our model we include lag 2, 3 and 4 of levels of this measure in our instrument set. In each table the first two columns report the one step and two step estimates using the small set of instruments, columns 3 and 4 report these estimates using the augmented set of instruments and finally columns 5 and 6 includes the human capital measure in our specifications. Similarly the Sargan-Hansen test of overidentifying restriction does not reject the validity of the instruments that we use. There is evidence of negative first order correlation in all the models that we have considered. There is no evidence of second order correlation almost in all the cases where we use the small instrument set and when including the human capital in the model. However there are cases of higher order serial correlation in error terms when we use the augmented set of instruments. As discussed earlier the long-run effect of trade specialization on growth rate of output per worker in the underlying specification, is captured by the coefficient of the trade variable. When using the import share however results do not indicate any significant effect of such variable on long-run growth rate of output per worker.

Tables 16, 17 and 18 report the results of estimating the same model using the export share of capital, consumption and intermediates instead of import shares. As we can see only the coefficients of export shares for intermediates when using the small and augmented set of instruments are significant with negative sign which is consistent with the result we obtained using the yearly panel data.

Similarly using the Lafay index, the coefficient will be significant and positive for the consumption goods when we consider the augmented set of instruments with including the years of schooling. This result once more indicates that specializing in consumption good and therefore exporting consumption goods can affect long run rate of growth of output per worker positively.

In sum, although we could not find a significant effect of import share of capital on long run growth rate however when we consider the effect of both import and export by using the index of specialization our results indicate positive and significant effect of specialization in consumption. In terms of the theoretical model that we considered, we argued that the trade can affect growth rate positively depending on the nature of good that is being imported. Similarly we can argue that if a country is specializing in consumption good and exchange that for a factor of production then growth rate could rise positively with trade. This is the result that we obtained in our empirical results. Also our results indicate that specializing in intermediates and therefore exporting intermediate goods affect growth rate negatively. Our model as we described in section 3 did not include the intermediates as factors of production. Therefore this result is hard to be justified in terms of the model we considered. However if we consider the intermediates as the goods that are essential to production and are being used fully in the process of the production in each period and therefore not being accumulated then the result we obtained here could indicate that specializing in this type of products are not growth promoting.

3.6.4 INVESTMENT AND TRADE

Several empirical literature on trade and growth such as Levine and Renelt (1992), Baldwin and Seghezza (1996), Wacziarg (2001) and Wacziarg and Welch (2003) have suggested that investment rate are the main channel linking trade and growth. In this section, we examine whether trade specialization could affect growth through investment. To do this we first run fixed-effect regressions of investment and growth on trade indicators to assess the within-country effect of investment on trade specialization as well as the within-country effect of growth on trade specialization. We use the following specifications,

$$\Delta y_{it} = \alpha + \beta Trade_{it} + \varepsilon_{it} \tag{29}$$

$$Inv_{it} = \alpha + \beta Trade_{it} + \varepsilon_{it} \tag{30}$$

We use only import share and Lafay index as measures of trade specialization in this section. We also include dummy variables to control for country and time fixed effects. Tables 23 to 25 report the estimates of fixed-effect coefficient of investment and growth on import share of capital, consumption and intermediates respectively. The left-hand section of each table is the estimates of investment on import share and the right-hand side reports the estimates of growth on trade. As it can be seen all estimated coefficients of investment on trade are statistically significant while the estimated coefficients of growth on trade are only significant for import share of capital goods. These coefficients are not significant for consumption or intermediate goods. Again this is the result we expected to obtain based on the model specification we considered in section 3.

Using these results, we can get a rough estimate of how much of the effect of trade specialization in capital on growth can be attributed to the investment channel. To see this we run fixed effects regressions of growth on investment rate. Table 23 reports the results from these regressions. As it can be seen the coefficient on investment is 0.0178. In turn, the effect of specialization on investment in corresponding regression is 0.1789. Therefore multiplying these two effects together, the effect of specialization on growth via investment is estimated to be roughly 0.0032. Compare this estimate to the total effect of specialization on growth from table 23, i.e. 0.007 percentage points, then roughly the investment channel accounts for

46% of the effect of specialization in capital on growth. We could get these estimates for consumption and intermediates as well. However as it can be seen the coefficients of trade in growth regression are not statistically significant for consumption or intermediates.

Similarly tables 26, 27 and 28 reports the within-country estimate of the effect of trade on investment and growth. As we can see all the estimated coefficients of LFI on investment are statistically significant. Similarly we can calculate the effect of trade specialization in capital and intermediates on growth through the channel of investment. For instance the effect of LFI for consumption on investment is 0.0073, multiplying this effect by 0.0178 (effect of investment on growth) the effect of LFI for capital on growth through investment would be 0.0001. Compare this effect with the total effect of LFI on growth, i.e. -0.0002, investment channels could accounts for 50 percent of this effect on growth. Therefore investment channel could account for about 46 percent of the effect of importing capital on growth. Also similarly investment could account for about 50 percent of the effect of specializing in consumption i.e. exporting consumption and importing capital on growth.

3.7 CONCLUSIONS

In this chapter we tried to examine the effect of trade specialization on economic performance. First we examined the relationship between growth and trade specialization using the specification suggested by neoclassical growth model where we included an index of trade specialization as an additional explanatory variable. We considered a specialization index proposed by Lafay (1992) as well as import and export share for capital, consumption and intermediates. Our sample consists of 92 developing countries over the period of 1962 to 2000.

We also used a specification developed by Bond, Leblebicioglu and Schiantarelli (2004) to examine the effect of trade specialization on growth in an AK type model. We modified their specification by including a measure of trade specialization as additional variable in the model. We estimated the effect of trade on growth using yearly and non-overlapping five-year averages. To get consistent estimated we used Instrumental variable method when using the yearly panel data, including the lagged values of endogenous variables as well as additional instruments. However when using the non-overlapping five-year averages we used the GMM estimators proposed by Arellano and Bond (1991).

Using yearly panel data, the coefficient on trade specialization indicate that exporting consumption good or specializing in consumption goods has statistically significant positive long run effect on growth rate of output per worker. In addition specializing in intermediate goods or exporting this types of good are not growth promoting. These results are also consistent when using non-overlapping five year averages.

In addition we tried to examine how much of these effects are through the investment channel. To do so we used the within-country estimates of growth on investment, growth on trade measures and investment on trade measures. When using pooled data our results indicate that import share has positive and significant effect on growth, also specializing in capital good affects growth statistically significant positively. Using the within country estimates we obtained that about 46 percent of the effect of importing capital on growth is through investment channel.

APENDIX 3.A

Main Group of Commodities (SITC1)

- 0 - Food and live animals
- 1 - Beverages and tobacco
- 2 - Crude materials, inedible, except fuels
- 3 - Mineral fuels, lubricants and related materials
- 4 - Animal and vegetable oils, fats and waxes
- 5 - Chemicals and related products, n.e.s.
- 6 - Manufactured goods classified chiefly by material
- 7 - Machinery and transport equipment
- 8 - Miscellaneous manufactured articles
- 9 - Commodities and transactions not classified elsewhere in the SITC
    - I - Gold, monetary
    - II - Gold coin and current coin

APPENDIX 3.B

The BEC (Broad Economic Categories) classification is intended to categorize trade statistics into large economic classes of commodities and to supplement the summary data compiled on the basis of the sections of the Standard International Trade Classification.

1 - Food and beverages

- 11 – Primary
  - 111 - Mainly for industry
  - 112 - Mainly for household consumption
- 12 – Processed
  - 121 - Mainly for industry
  - 122 - Mainly for household consumption

2 - Industrial supplies not elsewhere specified

- 21 – Primary
- 22 - Processed

3 - Fuels and lubricants

- 31 – Primary
- 32 – Processed
  - 321 - Motor spirit
  - 322 – Other

4 - Capital goods (except transport equipment), and parts and accessories thereof

- 41 - Capital goods (except transport equipment)
- 42 - Parts and accessories

5 - Transport equipment and parts and accessories thereof

- 51 - Passenger motor cars
- 52 – Other
  - 521 - Industrial
  - 522 - Non-industrial
- 53 - Parts and accessories

6 - Consumer goods not elsewhere specified

- 61 - Durable
- 62 - Semi-durable
- 63 - Non-durable

APPENDIX 3.C

Three main aggregated products group provided by Lewer and Berg(2003) as:

(1) Capital Goods

41	Capital goods (except transport equipment)
521	Transport equipment, industrial

(2) Intermediate goods

111	Food and Beverages, Primarily, Mainly for Industry
121	Food and Beverages, Processed, Mainly for Industry
2	Industrial Supplies Not Elsewhere Specified
31	Fuels and Lubricants, Primary
322	Fuels and Lubricants, Processed (other than Motor Spirits)
42	Parts and Accessories for Capital Goods
53	Parts and Accessories for Transport Equipment

(3) Consumption Goods

112	Food and Beverages, Primary, Mainly for Household Consumption
122	Food and Beverages, Processed, Mainly for Household Consumption
522	Transport Equipment, Non-Industrial
6	Consumer Goods Not Elsewhere Specified

Appendix 3.D

Table 1: Least Square estimates of equation (12) using cross-country data and import share.

Variable	Capital			Consumption			Intermediates		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
y_65	-0.0061 (-3.09)	-0.0048 (-2.37)	-0.0105 (-3.73)	-0.0062 (-3.07)	-0.0045 (-2.14)	-0.0102 (-3.57)	-0.0069 (-3.59)	-0.0055 (-2.68)	-0.0105 (-3.89)
INV	0.0206 (6.55)	0.0255 (7.21)	0.0220 (4.80)	0.0212 (7.25)	0.0251 (7.76)	0.0220 (5.47)	0.0187 (5.92)	0.0237 (6.59)	0.0205 (4.81)
Import Share	0.0017 (0.84)	0.0010 (0.42)	0.0010 (0.39)	0.0013 (0.84)	-0.0012 (-0.72)	-0.0001 (-0.06)	0.0043 (2.12)	0.0016 (0.71)	0.0026 (0.99)
Government		-0.0083 (-1.68)	-0.0114 (-2.27)		-0.0084 (-1.76)	-0.0108 (-2.22)		-0.0095 (-1.99)	-0.0114 (-2.36)
Inflation		-0.0316 (-4.02)	-0.0172 (-1.83)		-0.0330 (-4.02)	-0.0189 (-1.91)		-0.0296 (-3.76)	-0.0155 (-1.69)
Human Capital			0.0024 (2.34)			0.0024 (2.30)			0.0022 (2.10)
Adj R-Sq	0.3799	0.5381	0.5313	0.3799	0.5403	0.5300	0.4053	0.5402	0.5382
#of Countries	92	75	62	92	75	61	92	75	61

Table 2: Least Square estimates of equation (12) using cross-country data and export share.

Variable	Capital			Consumption			Intermediates		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
y_65	-0.0096 (-5.10)	-0.0079 (-3.72)	-0.0128 (-4.99)	-0.0066 (-3.65)	-0.0059 (-3.04)	-0.0104 (-3.91)	-0.0047 (-2.41)	-0.0043 (-2.19)	-0.0097 (-3.54)
INV	0.0129 (4.00)	0.0187 (4.98)	0.0151 (3.66)	0.0199 (7.00)	0.0238 (7.52)	0.0210 (5.27)	0.0222 (7.49)	0.0258 (8.01)	0.0222 (5.56)
Export Share	0.0045 (4.60)	0.0031 (2.81)	0.0039 (3.51)	0.0030 (2.97)	0.0020 (2.15)	0.0018 (1.57)	-0.0019 (-1.12)	-0.0071 (-1.63)	-0.0021 (-1.00)
Government		-0.0112 (-2.47)	-0.0138 (-3.10)		-0.0088 (-1.92)	-0.0101 (-2.13)		-0.0071 (-1.48)	-0.0094 (-1.89)
Inflation		-0.0257 (-3.42)	-0.0085 (-1.02)		-0.0303 (-4.06)	-0.0170 (-1.99)		-0.0309 (-4.09)	-0.0203 (-2.31)
Human Capital			0.0019 (1.98)			0.0022 (2.16)			0.0025 (2.42)
Adj R-Sq	0.4960	0.5845	0.6159	0.4319	0.5659	0.5502	0.3837	0.5542	0.5383
#of Countries	92	75	62	92	75	61	92	75	61

Table 3: Least Square estimates of equation (12) using cross-country data and LFI index.

Variable	Capital			Consumption			Intermediates		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
y_65	-0.0056 (-2.85)	-0.0047 (-2.28)	-0.0102 (-3.71)	-0.0049 (-2.65)	-0.0046 (-2.36)	-0.0098 (-3.62)	-0.0044 (-2.58)	-0.0045 (-2.43)	-0.0184 (-3.63)
INV	0.0215 (7.36)	0.0249 (7.73)	0.0220 (5.41)	0.0213 (7.38)	0.0246 (7.74)	0.0210 (5.17)	0.0185 (6.65)	0.0224 (7.10)	0.0184 (4.57)
LFI	0.0148 (0.05)	-0.1837 (-0.67)	0.0024 (0.01)	0.2376 (1.52)	0.2366 (1.63)	0.2037 (1.18)	-0.2564 (-4.14)	-0.1790 (-3.00)	-0.1776 (-2.62)
Government		-0.0087 (-1.86)	-0.0109 (-2.24)		-0.0082 (-1.77)	-0.0099 (-2.04)		-0.0067 (-1.49)	-0.0083 (-1.78)
Inflation		-0.0316 (-4.08)	-0.0186 (-2.09)		-0.0327 (-1.77)	-0.0207 (-2.36)		-0.0288 (-3.95)	-0.0175 (-2.14)
Human Capital			0.0024 (2.28)			0.0026 (2.48)			0.0025 (2.52)
Adj R-Sq	0.3750	0.5400	0.5300	0.3909	0.5541	0.5416	0.5400	0.5903	0.5820
#of Countries	92	75	62	92	75	61	92	75	61

Table 4: Yearly panel estimates of equation (24) and (27) using share of import of capital (*Kimp*).

	OLS	IV_1	IV_2	IV_1 w CE	IV_2 w CE	Parsimonious			
						IV_1	IV_2	IV_1 w CE	IV_2 w CE
Δy_{t-1}	0.1179 (3.59)	0.0031 (0.01)	0. 5014 (2.05)	0.6932 (2.97)	0.6217 (2.29)	0.1267 (0.46)	0.4879 (3.35)	0.7231 (4.21)	0.5936 (4.33)
Δy_{t-2}	0.0167 (0.56)	0.0218 (0.42)	-0.0112 (-0.24)	-0.1168 (-1.86)	-0.0442 (-0.74)	0.0289 (0.54)	-0.0255 (-0.63)	-0.1157 (-1.89)	-0.0729 (-1.78)
INV_t	0.0139 (5.63)	0.0153 (2.42)	0.0147 (1.78)	0.0313 (1.07)	0.0279 (1.13)	0.0132 (3.02)	0.0147 (3.74)	0.0307 (0.282)	0.0205 (1.35)
ΔINV_t	-0.0065 (-0.62)	-0.0454 (-0.66)	0.0491 (0.71)	0.0064 (0.08)	0.0746 (0.83)				
ΔINV_{t-1}	0.0245 (3.43)	0.0129 (0.25)	0.0409 0.84	0.1248 (1.43)	0.0506 (0.94)	-0.0144 (-0.30)	0.0703 (2.24	0.1251 (0.135)	0.0938 2.54)
$Kimp_t$	0.0007 (0.54)	-0.0007 (-0.36)	-0.0019 (-1.00)	-0.0013 (-0.11)	-0.00003 (-0.00)	-0.0006 (-0.39)	-0.0019 (-1.14)	-0.0002 (0.987)	-0.0037 (-0.40)
$\Delta Kimp_t$	-0.0018 (-0.36)	-0.0458 (-0.92)	0.0268 (0.270)	-0.0131 (-0.23)	0.0785 (0.77)				
$\Delta Kimp_{t-1}$	0.0012 (0.27)	-0.0090 (-0.42)	-0.0597 (-1.42)	0.0013 (0.04)	-0.0655 (-1.31)	-0.0279 (-1.86)	-0.0409 (-2.28)	-0.0009 (0.979)	-0.0319 (-1.14)
Growth effect (*INV*)	0.0161 [0.000]	0.0157 [0.0012]	0.0963 [0.0005]	0.0151 [0.2719]	0.0660 [0.1210]	0.0156 [0.0001]	0.0273 [0.0000]	0.0782 [0.2687]	0.0428 [0.1176]
Level effect (*INV*)	0.0208 [0.1966]	-0.0333 [0.6282]	0.1765 [0.1306]	0.3097 [0.1541]	0.2963 [0.0593]	-0.0171 [0.7504]	0.1312 [0.0777]	0.3186 [0.1679]	0.1957 [0.0104]
Growth effect (*Kimp)*	0.0008 [0.5909]	-0.0007 [0.7152]	-0.0004 [0.2701]	-0.0031 [0.9090]	-0.00007 [0.9975]	-0.0007 [0.6961]	-0.0035 [0.2602]	-0.0005 [0.9871]	-0.0077 [0.7001]
Level effect (*Kimp*)	-0.0007 [0.9322]	-0.0562 [0.1046]	-0.065 [0.5656]	-0.0279 [0.8238]	0.0308 [0.8764]	-0.0330 [0.0450]	-0.0761 [0.0140]	-0.0023 [0.9789]	-0.0665 [0.3009]
Test of first order corrl.		0.47 [0.6394]	-1.50 [0.1327]	-2.62 [0.0088]	-1.75 [0.0809]	-0.05 [0.9614]	-2.42 [0.0156]	-3.45 [0.0006]	-3.66 [0.0003]
Test of second order corrl.		0.05 [0.9630]	-0.96 [0.3389]	-0.61 [0.5451]	-0.88 [0.3773]	0.68 [0.4984]	-0.48 [0.6316]	-0.49 [0.6267]	-0.30 [0.7675]
Test of over identification		0.5106	0.1874	0.1750	0.3085	0.6336	0.1304	0.3175	0.4619
# of countries	92	92	75	92	75	92	75	92	75
# of obs.	3036	2760	2250	2760	2250	2760	2250	2760	2250

Table 5: Yearly panel estimates of equation (24) and (27) using share of import of consumption (*Cimp*).

	OLS	IV_1	IV_2	IV_1 w CE	IV_2 w CE	Parsimonious			
						IV_1	IV_2	IV_1 w CE	IV_2 w CE
Δy_{t-1}	0.1226 (3.80)	0.2007 (0.43)	0.5577 (2.70)	0.7979 (2.35)	0.5397 (3.27)	0.1490 (0.52)	0.5347 (3.37)	0.6958 (2.98)	0.5504 (3.70)
Δy_{t-2}	0.0151 (0.52)	-0.0172 (-0.20)	-0.0613 (-1.03)	-0.1771 (2.30)	-0.0929 (-2.14)	0.0212 (0.38)	-0.0379 (-0.92)	-0.1651 (-2.38)	-0.0907 (-2.48)
INV_t	0.0146 (6.64)	0.0081 (1.17)	0.0126 (2.15)	0.0574 (1.21)	0.0355 (2.09)	0.0126 (3.19)	0.0131 (3.33)	0.0709 (1.90)	0.0338 (2.51)
ΔINV_t	-0.0051 (-0.53)	-0.2116 (-2.17)	-0.0851 (-1.05)	-0.0936 (-0.69)	0.0075 (0.10)				
ΔINV_{t-1}	0.0247 (3.69)	0.0884 (1.03)	0.1248 (1.79)	0.2645 (2.07)	0.1156 (2.16)	-0.0130 (-0.24)	0.0724 (2.27)	0.2388 (2.00)	0.1139 (3.53)
$Cimp_t$	-0.0001 (0.12)	0.0005 (0.34)	-0.0014 (-0.88)	0.0036 (0.24)	-0.0029 (-0.33)	-0.0002 (-0.15)	-0.0012 (-1.03)	0.0021 (0.29)	-0.0018 (-0.29)
$\Delta Cimp_t$	-0.0155 (-2.68)	-0.0061 (-0.13)	-0.0430 (-0.51)	0.0118 (0.16)	-0.0155 (-0.21)				
$\Delta Cimp_{t-1}$	0.0064 (0.147)	0.0046 (0.16)	0.0237 (0.43)	0.0510 (1.36)	0.0123 (0.33)	-0.0161 (-0.82)	-0.0075 (-0.28)	0.0430 (1.43)	0.0094 (0.34)
Growth effect (*INV*)	0.0169 [0.0000]	0.0099 [0.1010]	0.0250 [0.0020]	0.1514 [0.1480]	0.0642 [0.0074]	0.0152 [0.0001]	0.0260 [0.0000]	0.1511 [0.0552]	0.0626 [0.0036]
Level effect (*INV*)	0.0227 [0.1251]	0.1509 [0.0997]	0.0788 [0.5002]	0.4507 [0.2122]	0.2225 [0.0149]	-0.0157 [0.7990]	0.1439 [0.0899]	0.5088 [0.0675]	0.2108 [0.0016]
Growth effect (*Cimp*)	-0.0001 [0.9010]	0.0006 [0.7427]	-0.0028 [0.3616]	0.0095 [0.8048]	-0.0053 [0.7447]	-0.0002 [0.8774]	-0.0024 [0.3007]	0.0045 [0.7679]	-0.0033 [0.7760]
Level effect (*Cimp*)	-0.0106 [0.2436]	-0.0018 [0.9808]	-0.038 [0.7192]	0.1656 [0.5447]	-0.0058 [0.9558]	-0.0194 [0.3411]	-0.0149 [0.7706]	0.0916 [0.2292]	0.0174 [0.7402]
Test of first order corrl.		-0.36 [0.7208]	-1.93 [0.0542]	-2.49 [0.0128]	-2.45 [0.0145]	-0.13 [0.8962]	-2.51 [0.0119]	-1.30 [0.1949]	-2.78 [0.0054]
Test of second order corrl.		-0.49 [0.6220]	-1.52 [0.1277]	-0.61 [0.5404]	-1.15 [0.2519]	0.26 [0.7965]	-0.97 [0.3297]	-0.88 [0.3772]	-0.79 [0.4318]
Test of over identification		0.9369	0.20907	0.7807	0.0827	0.2295	0.0719	0.7808	0.1063
# of countries	92	92	75	92	75	92	75	92	75
# of obs.	3306	2760	2250	2760	2250	2760	2250	2760	2250

Table 6: Yearly panel estimates of equation (24) and (27) using share of import of capital (*Iimp*).

	OLS	IV_1	IV_2	IV_1 w CE	IV_2 w CE	Parsimonious			
						IV_1	IV_2	IV_1 w CE	IV_2 w CE
Δy_{t-1}	0.1211 (3.78)	0.2638 (0.52)	0.6394 (3.67)	0.6280 (2.12)	(0.4768 (2.43)	0.2457 (0.76)	0.6316 (3.71)	0.7185 (3.44)	0.6019 (3.92)
Δy_{t-2}	0.0143 (0.49)	-0.0364 (-0.38)	-0.0608 (-1.19)	-0.1749 (-2.44)	-0.0936 (-1.95)	0.0111 (0.19)	-0.0488 (-1.17)	-0.1481 (-2.51)	-0.0906 (-2.39)
INV_t	0.0133 (5.54)	0.0122 (1.45)	0.0144 (2.30)	0.0671 (1.55)	0.0492 (2.32)	0.0085 (2.10)	0.0125 (2.83)	0.0543 (1.92)	0.0316 (2.39)
ΔINV_t	-0.0018 (-0.18)	-0.1195 (-1.16)	-0.0006 (-0.01)	0.0152 (0.11)	0.1008 (1.13)				
ΔINV_{t-1}	0.0256 (3.87)	0.0846 (0.86)	0.0991 (1.82)	0.2315 (1.95)	0.0956 (2.00)	-0.0138 (-0.22)	0.0865 (2.41)	0.1934 (2.11)	0.1174 (3.53)
$Iimp_t$	0.0019 (0.175)	0.0008 (0.18)	-0.0019 (-0.72)	-0.0079 (-0.34)	-0.0012 (-0.13)	0.0025 (1.07)	-0.0013 (0.60)	0.0144 (1.36)	0.0016 (0.21)
$\Delta Iimp_t$	-0.0277 (-3.55)	-0.1332 (-1.28)	-0.0415 (-0.50)	-0.1690 (-1.08)	-0.0503 (-0.61)				
$\Delta Iimp_{t-1}$	0.0014 (0.792)	0.02473 (0.59)	-0.0056 (-0.14)	0.0518 (1.16)	-0.0085 (-0.20)	-0.0163 (-0.65)	-0.0192 (-0.79)	0.0507 (1.54)	-0.0059 (-0.23)
Growth effect (*INV*)	0.0154 [0.0000]	0.0158 [0.2636]	0.0673 [0.0235]	0.1227 [0.0464]	0.0798 [0.0019]	0.0114 [0.0422]	0.0299 [0.0077]	0.1264 [0.0545]	0.0647 [0.0103]
Level effect (*INV*)	0.0275 [0.0631]	-0.0452 [0.7943]	0.2337 [0.1876]	0.4511 0.0685]	0.3184 [0.0013]	-0.0186 [0.8143]	0.2073 [0.1252]	0.4250 [0.0650]	0.2402 [0.0045]
Growth effect (*Iimp*)	0.0022 [0.1769]	0.0010 [0.8449]	-0.0045 [0.4923]	-0.0144 [0.7254]	-0.0019 [0.9000]	0.0034 [0.1963]	-0.0031 [0.5679]	0.0335 [0.2069]	0.0033 [0.8360]
Level effect (*Iimp*)	-0.0304 [0.0109]	-0.1404 [0.2904]	-0.0852 [0.4171]	-0.2143 [0.3851]	-0.0953 [0.3124]	-0.0219 [0.4404]	-0.0460 0.3696]	0.1180 [0.2336]	-0.0121 [0.8184]
Test of first order corrl.		-1.40 [0.1600]	-2.78 [0.0055]	-2.17 [0.0301]	-1.53 [0.1271]	-0.46 [0.6652]	-2.98 [0.0029]	-1.92 (0.0549)	-3.15 [0.0016]
Test of second order corrl.		-0.49 [0.6259]	-1.71 [0.0880]	-1.36 [0.1751]	-1.16 [0.2452]	0.15 [0.8841]	-0.80 [0.4256]	-1.30 (0.1948)	-0.55 [0.5795]
Test of over identification		0.4490	0.0755	0.6405	0.2671	0.1009	0.1541	0.4077	0.1835
# of countries	92	92	75	62	75	92	75	92	75
# of obs.	3036	2760	2250	2760	2250	2760	2250	2760	2250

Table 7: Yearly panel estimates of equation (24) and (27) using share of export of capital (*Kexp*).

	OLS	IV_1	IV_2	IV_1 w CE	IV_2 w CE	Parsimonious			
						IV_1	IV_2	IV_1 w CE	IV_2 w CE
Δy_{t-1}	0.1158 (3.53)	0.2046 (0.45)	0.9011 (3.90)	0.6374 (2.38)	0.5889 (2.45)	0.4606 (1.72)	0.8490 (4.78)	0.6835 (3.74)	0.6332 (3.75)
Δy_{t-2}	0.0166 (0.56)	-0.0050 (-0.06)	-0.1080 (-1.51)	-0.1387 (-2.09)	-0.1063 (-2.17)	-0.0251 (-0.42	-0.0932 (-1.76)	-0.1420 (-2.39)	-0.1087 (-2.30)
INV_t	0.0099 (3.37)	-0.0093 (-0.75)	0.0099 (1.09)	0.0653 (1.64)	0.0486 (1.90)	0.0044 (0.76)	0.0157 (2.40)	0.0552 (1.85)	0.0428 (2.49)
ΔINV_t	-0.0046 (-0.46)	-0.2692 (-1.27)	-0.1429 (-0.87)	0.0539 (0.32)	0.0419 (0.29)				
ΔINV_{t-1}	0.0266 (3.99)	0.0512 (0.48)	0.2117 (2.01)	0.1887 (1.67)	0.1338 (1.94)	0.0143 (0.20)	0.1473 (2.70)	0.1967 (2.10)	0.1486 (3.52)
$Kexp_t$	0.0020 (3.24)	0.0051 (1.43)	-0.0007 (-0.38)	0.0023 (0.26)	0.00007 (0.01)	0.0017 (0.97)	-0.0016 (-1.10)	0.0007 (0.09)	-0.00002 (-0.00)
$\Delta Kexp_t$	-0.0015 (-0.83)	-0.0019 (-0.10)	-0.0043 (-0.22)	0.0111 (0.68)	0.0046 (0.28)				
$\Delta Kexp_{t-1}$	-0.0001 (-0.07)	0.0011 (0.17)	-0.0027 (-0.35)	-0.0015 (-0.25)	-0.0062 (-1.04)	-0.0021 (-0.67)	-0.0051 (-0.98)	-0.0009 (-0.14)	-0.0053 (-0.89)
Growth effect (*INV*)	0.0114 [0.0002]	0.0116 [0.3543]	0.0478 [0.4270]	0.1303 [0.0163]	0.0939 [0.0206]	0.0078 [0.5178]	0.0643 [0.1858]	0.1204 [0.0308]	0.0900 [0.0290]
Level effect (*INV*)	0.0254 [0.0943]	-0.2724 [0.1032]	0.3325 [0.6132]	0.4839 [0.0145]	0.3396 0.0177]	0.0253 [0.8543]	0.6032 [0.2354]	0.4290 [0.0256]	0.3125 [0.0124]
Growth effect (*Kexp*)	0.0023 [0.0013]	0.0064 [0.0077]	-0.0034 [0.7451]	0.0046 [0.7939]	0.0001 [0.9918]	0.0030 [0.1769]	-0.0066 [0.4468]	0.0015 [0.9268]	-0.00004 [0.9974]
Level effect (*Kexp*)	-0.0018 [0.5673]	-0.0010 [0.9628]	-0.0338 [0.7223]	0.0192 [0.5852]	-0.0031 [0.9205]	-0.0037 [0.5350]	-0.0209 [0.4316]	-0.0020 [0.8869]	-0.0111 [0.4312]
Test of first order corrl.		-0.41 [0.6793]	-2.77 [0.0055]	-1.10 [0.2707]	-2.09 [0.0367]	-1.23 [0.2178]	-3.53 [0.0004]	-2.22 [0.0266]	-0.83 [0.4058]
Test of second order corrl.		-0.84 [0.4027]	-0.63 [0.5295]	-0.73 [0.4667]	-0.91 [0.3616]	-0.58 [0.5590]	-1.15 [0.2501]	-0.67 [0.5013]	-2.64 [0.0084]
Test of over identification		0.4931	0.5769	0.2612	0.1129	0.0162	0.1737	0.3627	0.1603
# of countries	92	92	75	92	75	92	75	92	75
# of obs.	3036	2760	2250	2760	2250	2760	2250	2760	2250

Table 8: Yearly panel estimates of equation (24) and (27) using share of export of consumption (*Cexp*).

	OLS	IV_1	IV_2	IV_1 w CE	IV_2 w CE	Parsimonious			
						IV_1	IV_2	IV_1 w CE	IV_2 w CE
Δy_{t-1}	0.1133 (3.46)	0.3010 (0.67)	0.7643 (2.66)	0.8448 (2.72)	0.6989 (2.67)	-0.0148 (-0.04)	0.5496 (2.88)	0.7739 (3.19)	0.6067 (3.69)
Δy_{t-2}	0.0160 (0.54)	-0.0196 (-0.24)	-0.0865 (-1.26)	-0.1505 (-2.25)	-0.1101 (-2.24)	0.0450 (0.70)	-0.0427 (-0.98)	-0.1528 (-2.26)	-0.0944 (-2.33)
INV_t	0.0131 (6.13)	0.0090 (2.02)	0.0104 (2.07)	0.0496 (1.06)	0.0338 (1.54)	0.0119 (3.27)	0.0134 (3.82)	0.0616 (1.94)	0.0377 (2.59)
ΔINV_t	-0.0055 (-0.56)	-0.0884 (-0.83)	-0.0939 (-0.90)	0.0052 (0.03)	-0.0336 (-0.29)				
ΔINV_{t-1}	0.0249 (3.77)	0.0555 (0.66)	0.1664 (2.30)	0.2047 (1.80)	0.1602 (2.33)	-0.0309 (-0.45)	0.0963 (2.53)	0.2331 (2.06)	0.1412 (3.36)
$Cexp_t$	0.0018 (2.17)	0.0011 (0.54)	-0.0009 (-0.60)	-0.0207 (-1.30)	-0.0129 (-1.21)	0.0021 (1.30)	-0.0005 (-0.43)	-0.0185 (-1.42)	-0.0103 (-1.19)
$\Delta Cexp_t$	-0.0108 (-3.37)	-0.0508 (-0.91)	-0.0319 (-0.69)	-0.0736 (-1.18)	-0.0357 (-0.94)				
$\Delta Cexp_{t-1}$	-0.0089 (-2.03)	-0.0085 (-0.22)	-0.0136 (-0.38)	-0.0108 (-0.25)	-0.0192 (-0.65)	-0.0415 (-2.24)	-0.0347 (-1.83)	-0.0461 (-1.37)	-0.0350 (-1.49)
Growth effect (*INV*)	0.0151 [0.0000]	0.0125 [0.0808]	0.0323 [0.1125]	0.1623 [0.2854]	0.0822 [0.0960]	0.0123 [0.0015]	0.0272 [0.0011]	0.1626 [0.1933]	0.0773 [0.317]
Level effect (*INV*)	0.0223 [0.1339]	-0.0458 [0.7321]	0.2250 [0.5255]	0.6866 [0.3013]	0.3079 [0.1278]	-0.0319 [0.6082]	0.1953 [01250]	0.6152 [02162]	0.2895 [0.0340]
Growth effect (*Cexp*)	0.0021 [0.0297]	0.0015 [0.4985]	-0.0028 [0.6365]	-0.0677 [0.3814]	-0.0314 [0.3769]	0.0022 [0.0212]	-0.0010 [0.6849]	-0.0488 [0.3417]	-0.0211 [0.3240]
Level effect (*Cexp*)	-0.0226 [0.0010]	-0.0825 [0.2210]	-0.1412 [0.3453]	-0.2762 [0.3588]	-0.0549 [0.2933]	-0.0428 [0.0967]	-0.0704 [0.0817]	-0.1217 [0.3482]	-0.0718 [0.2370]
Test of first order corrl.		-0.38 [0.7014]	-2.08 [0.0380]	-1.88 [0.0599]	-2.52 [0.0116]	0.33 [0.7398]	-2.23 [0.0257]	-2.09 [0.0367]	-2.89 [0.0039]
Test of second order corrl.		-0.57 [0.5714]	-0.67 [0.5022]	-0.67 [0.5034]	-0.65 [0.5165]	0.45 [0.6545]	-1.03 [0.3009]	-0.83 [0.4075]	-0.70 [0.4827]
Test of over identification		0.8145	0.5065	0.8525	0.1985	0.3726	0.0820	0.7696	0.1613
# of countries	92	92	75	92	75	92	75	92	75
# of obs.	3306	2760	2250	2760	2250	2760	2250	2760	2250

Table 9: Yearly panel estimates of equation (24) and (27) using share of export of intermediates (*Iexp*).

	OLS	IV_1	IV_2	IV_1 w CE	IV_2 w CE	Parsimonious			
						IV_1	IV_2	IV_1 w CE	IV_2 w CE
Δy_{t-1}	0.1236 (3.79)	0.1104 (0.24)	.05936 (1.91)	0.7417 (2.53)	0.6405 (2.91)	0.2389 (0.90)	0.6969 (2.92)	0.6615 (3.61)	0.6718 (3.55)
Δy_{t-2}	0.0108 (0.37)	0.0005 (0.01)	-0.0428 (-0.68)	-0.1603 (-2.49)	-0.1022 (-2.05)	-0.0014 (-0.03)	-0.0730 (-1.51)	-0.1380 (-2.67)	-0.1236 (-2.82)
INV_t	0.0160 (7.51)	0.0130 (1.77)	0.0147 (1.77)	0.0591 (1.55)	0.0374 (1.82)	0.0144 (3.25)	0.0099 (1.82)	0.0440 (2.16)	0.0331 (2.31)
ΔINV_t	-0.0091 (-0.93)	-0.1768 (-1.55)	0.0945 (0.72)	-0.0105 (-0.06)	0.1221 (1.01)				
ΔINV_{t-1}	0.0238 (3.61)	0.0810 (1.03)	0.0070 (0.07)	0.2283 (2.24)	0.0596 (0.70)	0.0159 (0.37)	0.0990 (2.79)	0.1606 (2.39)	0.1378 (3.53)
$Iexp_t$	-0.0023 (-1.99)	-0.0028 (-1.34)	-0.0015 (-0.65)	0.0173 (1.17)	0.0108 (0.56)	-0.0023 (-1.65)	-0.0003 (-0.18)	0.0073 (0.82)	0.0175 (1.37)
$\Delta Iexp_t$	-0.0128 (-2.25)	0.0425 (0.60)	-0.1826 (-1.47)	0.0919 (1.11)	-0.1093 (-0.81)				
$\Delta Iexp_{t-1}$	0.0112 (2.75)	-0.0332 (-0.71)	0.1019 (1.72)	-0.0002 (-0.00)	0.1128 (1.99)	-0.0012 (-0.06)	0.0487 (1.33)	0.0355 (1.34)	0.0805 (1.62)
Growth effect (*INV*)	0.0185 [0.0000]	0.0146 [0.0010]	0.0327 [0.0028]	0.1412 [0.0869]	0.0752 [0.0438]	0.0189 [0.0000]	0.0263 [0.0011]	0.0923 [0.0272]	0.0733 [0.0317]
Level effect (*INV*)	0.0169 [0.2510]	-0.1077 [0.2009]	0.2260 [0.2712]	0.5203 [0.1255]	0.3935 [0.0365]	0.0209 [0.7313]	0.2632 [0.2147]	0.3370 [0.0398]	0.3050 [0.0356]
Growth effect (*Iexp*)	0.0027 [0.0437]	-0.0031 [0.0653]	-0.0033 [0.4335]	0.0413 [0.3237]	0.0234 [0.5947]	-0.0030 [0.0574]	-0.0008 [0.8522]	0.0153 [0.4278]	0.0387 [0.2764]
Level effect (*Iexp*)	-0.0018 [0.8345]	0.0105 [0.8441]	-0.1797 [0.3348]	0.2191 [0.2890]	0.0076 [0.9762]	-0.0016 [0.9506]	0.1295 [0.3951]	0.0745 [0.2338]	0.1782 [0.2340]
Test of first order corrl.		-0.42 [0.6740]	-2.00 [0.0455]	-1.51 [0.1310]	-2.05 [0.0404]	-0.44 [0.6572]	-2.30 [0.0212]	-2.45 [0.0142]	-2.49 [0.0126]
Test of second order corrl.		-0.09 [0.9280]	-1.01 [0.3124]	-0.90 [0.3660]	-0.59 [0.5565]	-0.38 [0.7022]	-1.15 [0.2513]	-0.55 [0.5846]	-0.75 [0.4525]
Test of over identification		0.8238	0.8595	0.7888	0.7887	0.3184	0.1470	0.5666	0.4974
# of countries	92	92	75	92	75	92	75	92	75
# of obs.	3306	2760	2250	2760	2250	2760	2250	2760	2250

Table 10: Yearly panel estimates of equation (24) and (27) using LFI index for capital (*Klfi*).

	OLS	IV_1	IV_2	IV_1 w CE	IV_2 w CE	Parsimonious			
						IV_1	IV_2	IV_1 w CE	IV_2 w CE
Δy_{t-1}	0.1173 (3.58)	0.2802 (0.53)	0.7156 (3.01)	0 .9154 (1.98)	0.6169 (2.15)	-0.0334 (-0.12)	0.5743 (3.44)	0.6594 (3.63)	0.5943 (3.91)
Δy_{t-2}	0.0173 (0.59)	0.0282 (0.33)	-0.0341 (-0.61)	-0.0790 (-0.92)	-0.0643 (-1.28)	0.0498 0.89)	-0.0439 (-1.00)	-0.1255 (-2.20)	-0.0876 (-2.10)
INV_t	0.0149 (7.53)	0.0095 (1.56)	0 .0084 (1.64)	0.0231 (0.35)	0 .0308 (0.98)	0.0135 (3.10)	0.0109 (3.07)	0.0451 (1.60)	0.0327 (1.99)
ΔINV_t	-0.0107 (-1.03)	-0.0688 (-0.46)	-0.0130 (-0.11)	0.0464 (0.20)	0.0801 (0.58)				
ΔINV_{t-1}	0.0237 (3.40)	-0.0388 (-0.36)	0.0434 (0.54)	0 .0313 (0.18)	0.0436 (0.66)	-0.0514 (0.90)	0.0783 (2.25)	0.1645 (1.87)	0.1209 (3.03)
$Klfi_t$	-0.0002 (-1.39)	-0.0005 (-1.10)	-0.0002 (-0.76)	-0.0039 (-0.74)	-0.0005 (-0.24)	-0.0001 (-0.40)	0.00002 (0.11)	0.0007 (0.41)	0.0010 (0.61)
$\Delta Klfi_t$	-.0012 (-2.08)	-0.0192 (-1.51)	-0.0135 (-1.53)	-0.0283 (-1.22)	-0.0098 (-0.97)				
$\Delta Klfi_{t-1}$	0.0002 (0.58)	0.0058 (1.25)	0.0067 (2.01)	0.0083 (1.17)	0.0050 (1.24)	-0.0001 (0.05)	0.0029 (1.97)	0.0016 (0.67)	0.0026 (1.06)
Growth effect (*INV*)	0.0172 [0.0000]	0.0137 [0.0394]	0.0264 [0.0486]	0.1412 [0.5911]	0.0688 [0.0988]	0.0137 [0.0000]	0.0232 [0.0004]	0.0968 [0.0565]	0.0663 [0.0194]
Level effect (*INV*)	0.0150 [0.3395]	-0.1556 [0.2110]	0.0954 [0.6831]	0.4749 [0.6021]	0.2765 [0.1024]	-0.0523 [0.2925]	0.1667 [0.1132]	0.3529 [0.0492]	0.2451 [0.0055]
Growth effect (*Klfi*)	-0.0003 [0.1634]	-0.0007 [0.4191]	-0.0007 [0.5365]	-0.0239 [0.7768]	-0.0016 [0.8179]	-0.0001 [0.6824]	0.00005 [0.9119]	0.0014 [0.6954]	0.0019 [0.5683]
Level effect (*Klfi*)	-0.0009 [0.2397]	-0.0192 [0.3805]	-0.0021 [0.4901]	-0.1225 [0.7745]	-0.0108 [0.6348]	-0.0001 [0.9578]	0.0062 [0.0726]	0.0033 [0.5387]	0.0053 [0.3453]
Test of first order corrl.		-0.87 [0.3840]	-2.56 [0.0105]	-1.35 0.1768	-1.79 [0.0742]	0.51 [0.6118]	-2.46 [0.0138]	-2.57 [0.0101]]	-3.07 [0.0022]
Test of second order corrl.		1.55 [0.1222]	0.09 [0.9309]	0.69 0.4895	-0.02 [0.9814]	3.29 [0.0010]	-0.63 [0.5265]	-0.33 [0.7449]	-0.33 [0.7380]
Test of over identification		0.8934	0.3098	0.9130	0.2351	0.1953	0.4978	0.0916	0.0873
# of countries	92	92	75	92	75	92	75	92	75
# of obs.	3306	2760	2250	2760	2250	2760	2250	2760	2250

Table 11: Yearly panel estimates of equation (24) and (27) using LFI index for consumption (*Clfi*).

	OLS	IV_1	IV_2	IV_1 w CE	IV_2 w CE	Parsimonious			
						IV_1	IV_2	IV_1 w CE	IV_2 w CE
Δy_{t-1}	0 .1169 (3.64)	0.0359 (0.09)	0.5812 (3.05)	0.6511 (2.63)	0.5021 (2.75)	0.2478 (0.88)	0.5210 (2.91)	0.7255 3.24	0.5384 3.31
Δy_{t-2}	0 .0138 (0.46)	0.0031 (0.05)	-0.0505 (-1.09)	-0.1534 (-2.32)	-0.0905 (-2.21)	-0.0010 (-0.02)	-0.0405 (-0.92)	-0.1558 -2.56	-0.0954 -2.46
INV_t	0.0146 (7.47)	0 .0116 (2.40)	0.0109 (2.67)	0.0787 (1.91)	0.0484 (2.54)	0.0119 (3.10)	0.0122 (3.45)	0.0628 2.11	0 .0391 2.76
ΔINV_t	-0.0076 (-0.77)	-0.1704 (-1.55)	-0.0272 (-0.41)	0.0846 (0.63)	0.0765 (1.03)				
ΔINV_{t-1}	0.0256 (3.87)	0 .0566 (0.87)	0.0994 (2.09)	0.2137 (2.05)	0 .1084 (2.50)	0.0091 (0.18)	0.0801 (2.34)	0.2280 2.35	0.1330 3.59
$Clfi_t$	0.0003 (2.60)	0.0003 (1.51)	0 .0001 (0.46)	-0.0014 (-1.22)	-0.0005 (-0.50)	0.0002 (1.28)	0.0001 (0.68)	-0.0013 -1.42	-0.0001 -0.14
$\Delta Clfi_t$	0.0001 (0.18)	0.0045 (0.84)	-0.0036 (-0.55)	0.0031 (0.47)	0 .00002 (0.00)				
$\Delta Clfi_{t-1}$	-0.0012 (-2.85)	-0.0042 (-1.420	-0.0010 (-0.26)	-0.0045 (-1.20)	-0.0021 (-0.54)	-0.0021 (-1.38)	-0.0028 (-1.51)	-0.0031 -1.18	-0.0016 -0.67
Growth effect (*INV*)	0.0168 [0.0000]	0.0121 [0.0006]	0.0232 [0.0007]	0.1567 [0.0316]	0.0823 [0.0054]	0.0158 [0.0000]	0.0235 [0.0000]	0.1459 [0.0508]	0.0702 [0.0044]
Level effect (*INV*)	0.0207 [0.1666]	-0.1184 [0.1320]	0.1538 [0.2566]	0.5939 [0.0469]	0.3142 [0.0119]	0.0121 [0.8601]	0.1542 [0.1106]	0.5299 [0.0685]	0.2388 [0.0051]
Growth effect (*Clfi*)	0.0003 [0.0088]	0.0003 [0.0432]	0.0001 [0.6252]	-0.0027 [0.3206]	-0.0008 [0.6400]	0.0003 [0.1090]	0.0002 [0.4670]	-0.0030 [0.2808]	-0.0002 [0.8926]
Level effect (*Clfi*)	-0.0011 [0.1788]	0.0003 [0.9374]	-0.0098 [0.3324]	-0.0028 [0.7837]	-0.0035 [0.6537]	-0.0028 [0.2536]	-0.0053 [0.1813]	-0.0072 [0.3775]	-0.0029 [0.5365]
Test of first order corrl.		-0.10 [0.9193]	-2.18 [0.0291]	-0.52 [0.6038]	-1.61 [0.1064]	-0.44 [0.6616]	-2.08 [0.0376]	-1.85 [0.0644]	-2.44 [0.0147]
Test of second order corrl.		-0.19 [0.8500]	-0.76 [0.4502]	-1.42 [0.1543]	-0.92 [0.3560]	-0.13 [0.8940]	-1.15 [0.2499]	-0.72 [0.4692]	-0.69 [0.4910]
Test of over identification		0.6742	0.3900	0.7897	0.1236	0.2881	0.3440	0.8108	0.1039
# of countries	92	92	75	92	75	92	75	92	75
# of obs.	3306	2760	2250	2760	2250	2760	2250	2760	2250

Table 12: Yearly panel estimates of equation (24) and (27) using LFI index for intermediates (*Ilfi*).

	OLS	IV_1	IV_2	IV_1 w CE	IV_2 w CE	Parsimonious			
						IV_1	IV_2	IV_1 w CE	IV_2 w CE
Δy_{t-1}	0.1085 (3.34)	0.2419 (0.46)	0.7908 (2.68)	0.7943 (2.06)	0.5546 (2.21)	0.2545 (0.78)	0.7222 (3.19)	0.7293 3.30	0.6211 3.46
Δy_{t-2}	0.0124 (0.42)	-0.0298 (-0.34)	-0.0911 (-1.40)	-0.1574 (-2.41)	-0.1015 (-2.30)	-0.0061 (-0.11)	-0.0706 (-1.50)	-0.1499 -2.74	-0.1077 -2.70
INV_t	0.0132 (6.73)	0.0079 (1.41)	0.0082 (1.62)	0 .0405 (1.09)	0.0444 (2.05)	0.0103 (2.99)	0.0104 (2.71)	0.0507 1.98	0 .0378 2.77
ΔINV_t	-0.0064 (-0.64)	-0.1562 (-1.11)	-0.1109 (-0.76)	-0.0697 (-0.32)	0.0633 (0.44)				
ΔINV_{t-1}	0.0250 (3.74)	0.0765 (0.59)	0.1738 (1.76)	0.2133 (1.69)	0.1114 (1.55)	0.0064 (0.11)	0.1102 (2.84)	0.1928 2.34	0 .1406 3.89
$Ilfi_t$	-0.0002 (-4.62)	-0.0002 (-1.19)	-0.00001 (-0.11)	0.0003 (0.49)	0.0002 (0.84)	-0.0002 (-1.45)	-0.0009 (-0.01)	0.0004 0.71	0.0003 1.03
$\Delta Ilfi_t$	0.0004 (3.32)	-0.0005 (-0.28)	0.00001 (0.01)	-0.00003 (-0.01)	-0.0009 (-0.68)				
$\Delta Ilfi_{t-1}$	0.0003 (3.11)	0.0008 (0.71)	0.0011 (1.46)	0.0013 (0.83)	0.0015 (2.06)	0.0005 (0.90)	0.0011 (2.04)	0.0014 1.18	0.0013 1.98
Growth effect (*INV*)	0.0150 [0.0000]	0.0100 [0.0101]	0.0273 [0.0760]	0.1115 [0.1534]	0.0812 [0.0058]	0.0137 [0.0002]	0.0299 [0.0160]	0.1205 [0.0569]	0.0777 [0.0108]
Level effect (*INV*)	0.0212 [0.1511]	-0.1011 [0.2560]	0.2095 [0.5317]	0.3955 [0.2027]	0.3194 [0.0116]	0.0085 [0.9127]	0.3163 [0.2251]	0.4584 [0.0767]	0.2889 [0.0140]
Growth effect (*Ilfi*)	-0.0002 [0.0000]	-0.0003 [0.0079]	-0.00004 [0.9101]	0.0008 [0.7051]	0.0004 [0.4595]	-0.0002 [0.0122]	-0.0026 [0.9918]	0.0009 [0.5436]	0.0005 [0.3782]
Level effect (*Ilfi*)	0.0007 [0.0000]	0.0003 [0.8424]	0.0036 [0.5328]	0.0034 [0.6304]	0.0011 [0.6394]	0.0007 [0.4542]	0.0031 [0.2463]	0.0033 [0.3654]	0.0025 [0.1302]
Test of first order corrl.		-0.35 [0.7289]	-2.14 [0.0326]	-2.00 [0.0451]	-2.17 [0.0300]	-0.44 [0.6630]	-2.45 [0.0141]	-2.26 [0.0236]	-2.61 [0.0091]
Test of second order corrl.		-0.58 [0.5606]	-0.83 [0.4080]	-0.48 [0.6328]	0.70 [0.4813]	-0.12 [0.9058]	-1.10 [0.2728]	-0.51 [0.6107]	-0.61 [0.5404]
Test of over identification		0.8395	0.4915	0.7042	0.3961	0.4124	0.2182	0.8057	0.4291
# of countries	92	92	75	92	75	92	75	92	75
# of obs.	3306	2760	2250	2760	2250	2760	2250	2760	2250

Table 13: Five-year average panel data, using import share of capital (*Kimp*).

	Small Instrument Set		Augmented Instrument Set		Augemnted + Human Capital Instrument Set	
	GMM1	GMM2	GMM1	GMM2	GMM1	GMM2
$\Delta_5 y_{t-1}$	-0.4845 (-3.06)	-0.3457 (-2.02)	-0.3826 (-3.70)	-0.4069 (-3.59)	-0.3318 (-3.70)	-0.3216 (-3.01)
INV_t	0.0121 (2.57)	0.0140 (2.40)	0.0110 (2.20)	0.0106 (1.82)	0.0033 (0.72)	0.0024 (0.46)
$\Delta_5 INV_t$	0.0846 (1.04)	0.1711 (2.40)	0.0509 (0.77)	0.0582 (0.83)	0.0266 (0.61)	0.0264 (0.62)
$Kimp_t$	-0.0015 (-0.92)	-0.0021 (-1.07)	-0.0014 (-0.79)	-0.0007 (-0.32)	0.0013 (0.72)	0.0011 (0.47)
$\Delta_5 Kimp_t$	-0.0243 (-0.52)	-0.0352 (-0.53)	0.0101 (0.22)	0.0117 (0.26)	0.0315 (0.72)	0.0365 (0.79)
TYS_t					-0.0046 (-0.79)	-0.0026 (-0.43)
$\Delta_5 TYS_{t-1}$					-0.2477 (-1.49)	-0.1744 (-1.15)
Growth Effect (*INV*)	0.0250	0.0405	0.0287	0.0261	0.0099	0.0076
Level Effect (*INV*)	0.1745	0.4949	0.1330	0.1429	0.0801	0.0822
Growth Effect (*Kimp*)	-0.0030	-0.0061	-0.0036	-0.0017	0.0041	0.0034
Level Effect (*Kimp*)	-0.0502	-0.1019	0.0264	0.0287	0.0949	0.1136
Test of first order correlation	-3.13 [0.002]	-2.13 [0.033]	-3.77 [0.000]	-2.99 [0.003]	-4.09 [0.000]	-3.45 [0.001]
Test of second order correlation	1.01 [0.313]	0.73 [0.463]	-0.42 [0.676]	0.40 [0.687]	0.33 [0.738]	0.36 [0.718]
Test of over identification	0.173	0.173	0.298	0298	0.848	0.848
Number of Countries	92	92	75	75	62	62
Number of Observations	460	460	375	375	310	310

Table 14: Five-year average panel data, using import share of consumption (*Cimp*).

	Small Instrument Set		Augmented Instrument Set		Augemnted + Human Capital Instrument Set	
	GMM1	GMM2	GMM1	GMM2	GMM1	GMM2
$\Delta_5 y_{t-1}$	-0.5361 (-3.51)	-0.4970 (-2.26)	-0.4340 (-4.27)	-0.4779 (-5.19)	-0.3806 (-3.61)	-0.3880 (-3.18)
INV_t	0.0167 (2.91)	0.0174 (2.45)	0.0146 (2.59)	0.0141 (2.55)	0.0093 (1.68)	0.0099 (1.79)
$\Delta_5 INV_t$	0.1319 (1.51)	0.1632 (2.11)	0.0899 (1.21)	0.0943 (0.1.50)	0.0693 (1.71)	0.0646 (1.71)
$Cimp_t$	-0.0024 (-1.43)	-0.0023 (-1.18)	-0.0007 (-0.45)	0.00004 (0.03)	0.0002 (0.11)	-0.0006 (-0.30)
$\Delta_5 Cimp_t$	-0.0887 (--1.82)	-0.0825 (-1.41)	-0.0632 (-1.23)	-0.0465 (-0.98)	-0.0501 (-0.98)	-0.0545 (-1.11)
TYS_t					-0.0045 (-0.67)	-0.0021 (-0.33)
$\Delta_5 TYS_{t-1}$					-0.2421 (-1.33)	-0.1790 (-0.97)
Growth Effect (*INV*)	0.0311	0.0350	0.0337	0.0294	0.0244	0.0255
Level Effect (*INV*)	0.2461	0.3284	0.2072	0.1973	0.1822	0.1666
Growth Effect (*Cimp*)	-0.0044	-0.0046	-0.0016	0.00008	0.0006	0.0017
Level Effect (*Cimp*)	-0.1655	-0.1661	-0.1455	-0.0974	-0.1317	-0.1404
Test of first order correlation	-2.56 [0.010]	-1.58 [0.114]	-3.52 [0.000]	-2.84 [0.004]	-3.63 [0.000]	-2.71 [0.007]
Test of second order correlation	1.02 [0.309]	0.81 [0.418]	-0.53 [0.599]	-0.43 [0.667]	-0.09 [0.630]	-0.09 [0.925]
Test of over identification	0.090	0.090	0.557	0.557	0.833	0.833
Number of Countries	92	92	75	75	62	62
Number of Observations	460	460	375	375	310	310

Table 15: Five-year average panel data, using import share of intermediates (*Iimp*).

	Small Instrument Set		Augmented Instrument Set		Augemnted + Human Capital Instrument Set	
	GMM1	GMM2	GMM1	GMM2	GMM1	GMM2
$\Delta_5 y_{t-1}$	-0. 5356 (-3.30)	-0.5226 (-2.13)	-0.4280 (-3.79)	-0.4427 (-4.54)	-0.3698 (-3.72)	-0.3797 (-4.20)
INV_t	0. 0126 (1.82)	0.0149 (1.83)	0.0101 (1.49)	0.0093 (1.54)	0.0049 (0.91)	0.0055 (1.00)
$\Delta_5 INV_t$	0. 0612 (0.70)	0.0945 (1.26)	0.0256 (0.45)	0.0255 (0.36)	0.0154 (0.38)	0.0158 (0.39)
$Iimp_t$	-0. .0007 (-0.29)	-0.0015 (-0.50)	0.0008 (0.33)	0.0011 (0.44)	0.0012 (0.39)	0.0004 (0.11)
$\Delta_5 Iimp_t$	-0..0126 (-1.18)	-0.0962 (-1.13)	-0.0315 (-0.40)	-0.0228 (-0.29)	-0.0343 (-0.53)	-0.0385 (-0.64)
TYS_t					-0.0021 (-0.37)	-0.0004 (-0.07)
$\Delta_5 TYS_{t-1}$					-0.2058 (-1.33)	-0.1676 (-1.17)
Growth Effect (*INV*)	0.0235	0.0286	0.0236	0.0212	0.0133	0.0146
Level Effect (*INV*)	0.1143	0.1808	0.0598	0.0575	0.0417	0.0416
Growth Effect (*Iimp*)	-0.0014	-0.0028	0.0018	0.0025	0.0032	0.0009
Level Effect (*Iimp*)	-0.1470	-0.1841	-0.0736	-0.0514	-0.0927	-0.1013
Test of first order correlation	-2.52 [0.012]	-1.40 [0.161]	-3.49 [0.000]	-3.08 [0.002]	-3.58 [0.000]	-3.38 [0.001]
Test of second order correlation	1.01 [0.312]	0.84 [0.399]	-0.35 [0.729]	-0.32 [0.751]	0.15 [0.881]	0.12 [0.903]
Test of over identification	0.083	0.083	0.560	0.560	0.847	0.847
Number of Countries	92	92	75	75	62	62
Number of Observations	460	460	375	375	310	310

Table 16: Five-year average panel data, using export share of capital (*Kexp*).

	Small Instrument Set		Augmented Instrument Set		Augemnted + Human Capital Instrument Set	
	GMM1	GMM2	GMM1	GMM2	GMM1	GMM2
$\Delta_5 y_{t-1}$	-0.4754 (-4.13)	-0.4992 (-3.44)	-0.3167 (-3.22)	-0.3439 (-4.02)	-0.2459 (-3.25)	-0.2629 (-2.55)
INV_t	0.0066 (0.90)	0.0087 (1.03)	0.0062 (1.20)	0.0058 (1.27)	0.0041 (0.93)	0.0038 (0.83)
$\Delta_5 INV_t$	0.0460 (0.51)	0.0632 (0.51)	0.1071 (1.56)	0.0901 (1.29)	0.0881 (1.90)	0.0808 (1.70)
$Kexp_t$	0.0004 (0.34)	0.00002 (0.01)	0.0003 (0.31	0.0003 (0.43)	0.0009 (0.96)	0.0006 (0.81)
$\Delta_5 Kexp_t$	-0.0892 (-1.15)	-0.0799 (-1.66)	-0.0337 (-1.03)	-0.0305 (-0.90)	-0.1022 (-0.79)	-0.0215 (-1.10)
TYS_t					-0.0028 (-0.53)	-0.0014 (-0.31)
$\Delta_5 TYS_{t-1}$					-0.0102 (-0.79)	-0.0823 (-0.70)
Growth Effect (*INV*)	0.0138	0.0175	0.0196	0.0170	0.0165	0.0144
Level Effect (*INV*)	0.0968	0.1266	0.3383	0.2621	0.3581	0.3075
Growth Effect (*Kexp*)	0.0009	0.00003	0.0009	0.0010	0.0035	0.0025
Level Effect (*Kexp*)	-0.1877	-0.1602	-0.1063	-0.0887	-0.0831	-0.0821
Test of first order correlation	-2.51 [0.012]	-1.98 [0.047]	-3.24 [0.001]	-2.67 [0.008]	-4.36 [0.000]	-3.60 [0.000]
Test of second order correlation	1.24 [0.214]	1.26 [0.208]	0.96 [0.335]	0.97 [0.334]	0.05 [0.963]	0.07 [0.945]
Test of over identification	0.132	0.132	0.554	0.554	0.890	0.890
Number of Countries	92	92	75	75	310	310
Number of Observations	460	460	375	375	62	62

Table 17: Five-year average panel data, using export share of consumption (*Cexp*).

	Small Instrument Set		Augmented Instrument Set		Augemnted + Human Capital Instrument Set	
	GMM1	GMM2	GMM1	GMM2	GMM1	GMM2
$\Delta_5 y_{t-1}$	-0.4825 (-3.37)	-0.4136 (-2.12)	-0.3494 (-3.05)	-0.3497 (-2.93)	-0.2402 (-2.67)	-0.2144 (-1.61)
INV_t	0.0108 (2.66)	0.0124 (2.69)	0.0087 (2.76)	0.0093 (3.00)	0.0057 (1.53)	0.0049 (1.22)
$\Delta_5 INV_t$	0.0957 (1.57)	0.1538 (2.34)	0.1282 (2.54)	0.1323 (2.46)	0.0957 (2.11)	0.1030 (2.47)
$Cexp_t$	-0.0005 (-0.40)	-0.0011 (-0.89)	-0.0133 (-0.46)	-0.0007 (-0.72)	-0.0011 (-0.81)	-0.0008 (-0.55)
$\Delta_5 Cexp_t$	-0.0679 (-1.49)	-0.0787 (-1.37)	-0.00009 (-0.10)	-0.0194 (-0.57)	0.0163 (0.40)	0.0193 (0.33)
TYS_t					-0.0013 (-0.29)	0.00003 (0.01)
$\Delta_5 TYS_{t-1}$					-0.1509 (-0.92)	-0.0789 (-0.58)
Growth Effect (*INV*)	0.0225	0.0299	0.0248	0.0267	0.0236	0.0229
Level Effect (*INV*)	0.1984	0.3717	0.3670	0.3784	0.3986	0.4802
Growth Effect (*Cexp*)	-0.0010	-0.0027	-0.0003	-0.0021	-0.0044	-0.0036
Level Effect (*Cexp*)	-0.1407	-0.1903	-0.0379	-0.0554	0.0677	0.0901
Test of first order correlation	-3.09 [0.002]	-1.87 [0.062]	-3.36 [0.001]	-2.42 [0.016]	-4.04 [0.000]	-3.27 [0.0001]
Test of second order correlation	1.11 [0.269]	0.84 [0.401]	0.91 [0.362]	0.85 [0.397]	0.01 [0.994]	0.00 [0.997]
Test of over identification	0.213	0.213	0.618	0.618	0.884	0.884
Number of Countries	92	92	75	75	310	310
Number of Observations	460	460	375	375	62	62

Table 18: Five-year average panel data, using export share of intermediates (*Iexp*).

	Small Instrument Set		Augmented Instrument Set		Augemnted + Human Capital Instrument Set	
	GMM1	GMM2	GMM1	GMM2	GMM1	GMM2
$\Delta_5 y_{t-1}$	-0.7494 (-4.20)	-0.8760 (-4.32)	-0.6086 (-3.90)	-0.6943 (-4.71)	-0.3777 (-3.82)	-0.3560 (-2.96)
INV_t	0.0299 (4.96)	0.0313 (5.20)	0.0254 (5.08)	0.0265 (4.04)	0.0051 (0.93)	0.0045 (0.64)
$\Delta_5 INV_t$	0.0712 (0.71)	0.0173 (0.14)	0.0881 (1.03)	0.0801 (0.94)	0.0255 (0.51)	0.0305 (0.60)
$Iexp_t$	-0.0056 (-2.39)	-0.0051 (-2.22)	-0.0045 (-2.29)	-0.0036 (-1.91)	0.0017 (0.79)	0.0015 (0.65)
$\Delta_5 Iexp_t$	-0.3344 (-3.60)	-0.3427 (-4.24)	-0.2852 (-3.72)	-0.2562 (-3.69)	0.0057 (0.12)	0.0111 (0.21)
TYS_t					-0.0069 (-0.96)	-0.0056 (-0.73)
$\Delta_5 TYS_{t-1}$					-0.3523 (-2.03)	-0.3091 (-1.69)
Growth Effect (*INV*)	0.0399	0.0357	0.0417	0.0381	0.0135	0.0127
Level Effect (*INV*)	0.0951	0.0198	0.1448	0.1154	0.0677	0.0858
Growth Effect (*Iexp*)	-0.0074	-0.0058	-0.0075	-0.0052	0.0046	0.0042
Level Effect (*Iexp*)	-0.4462	-0.3913	-0.4686	-0.3689	0.0150	0.0311
Test of first order correlation	-1.30 [0.195]	-0.65 [0.517]	-2.10 [0.036]	-1.51 [0.131]	-3.72 [0.000]	-3.16 [0.002]
Test of second order correlation	1.51 [0.132]	1.37 [0.172]	1.60 [0.110]	1.62 [0.105]	0.22 [0.826]	0.16 [0.876]
Test of over identification	0.438	0.438	0.500	0.500	0.772	0.772
Number of Countries	92	92	75	75	310	310
Number of Observations	460	460	375	375	62	62

Table 19: Five-year average panel data, using LFI index for capital (*Klfi*).

	Small Instrument Set		Augmented Instrument Set		Augemnted + Human Capital Instrument Set	
	GMM1	GMM2	GMM1	GMM2	GMM1	GMM2
$\Delta_5 y_{t-1}$	-0.5300 (-3.93)	-0.4587 (-2.91)	-0.4070 (-3.81)	-0.3917 (-3.18)	-0.3463 (-3.92)	-0.3734 (-4.24)
INV_t	0.0101 (2.63)	0.0113 (2.52)	0.0095 (3.27)	0.0086 (2.86)	0.0045 (1.19)	0.0047 (1.23)
$\Delta_5 INV_t$	0.0434 (0.60)	0.0643 (0.93)	0.0128 (0.20)	0.0150 (0.23)	0.0044 (0.08)	-0.0099 (-0.20)
$Klfi_t$	-0.0002 (-0.78)	-0.0003 (-1.09)	-0.0007 (-0.34)	-0.00001 (-0.06)	0.00002 (0.09)	-0.000005 (-0.02)
$\Delta_5 Klfi_t$	-0.0066 (-0.78)	-0.0128 (-1.56)	0.0092 (-1.62)	-0.0105 (-1.72)	-0.0098 (-1.13)	-0.0097 (-1.73)
TYS_t					-0.0027 (-0.43)	-0.0024 (-0.40)
$\Delta_5 TYS_{t-1}$					-0.2221 (-1.56)	-0.2142 (-1.83)
Growth Effect (*INV*)	0.0191	0.0246	0.0235	0.0219	0.0129	0.0125
Level Effect (*INV*)	0.0819	01402	0.0315	0.0383	0.0127	-0.0265
Growth Effect (*Klfi*)	-0.0004	-0.0007	-0.0001	-0.00002	0.00007	-0.00001
Level Effect (*Klfi*)	-0.0124	-0.0279	-0.0023	-0.0027	-0.0028	-0.0259
Test of first order correlation	-3.24 [0.001]	-2.25 [0.025]	-3.62 [0.000]	-3.12 [0.002]	-4.19 [0.000]	-3.32 [0.001]
Test of second order correlation	1.37 [0.171]	1.32 [0.186]	0.07 [0.941]	0.12 [0.902]	0.85 [0.393]	0.81 [0.421]
Test of over identification	0.070	0.070	0.314	0.314	0.858	0.858
Number of Countries	92	92	75	75	62	62
Number of Observations	460	460	375	375	310	310

Table 20: Five-year average panel data, using LFI index for consumption (*Clfi*).

	Small Instrument Set		Augmented Instrument Set		Augemnted + Human Capital Instrument Set	
	GMM1	GMM2	GMM1	GMM2	GMM1	GMM2
$\Delta_5 y_{t-1}$	-0.4336 (-2.54)	-0.3159 (-1.48)	-0.3468 (-3.76)	-0.3685 (-3.93)	-0.3661 (-3.24)	-0.3999 (-3.00)
INV_t	0.0095 (2.04)	0.0094 (1.67)	0.0102 (3.45)	0.0099 (3.27)	0.0059 (1.43)	0.0053 (1.36)
$\Delta_5 INV_t$	0.0841 (1.54)	0.1275 (2.43)	0.0477 (0.76)	0.0493 (0.72)	0.0252 (0.43)	0.0184 (0.36)
$Clfip_t$	0.0001 (0.58)	0.00002 (0.13)	-0.00005 (-0.41)	-0.00007 (-0.57)	0.00004 (1.83)	0.00002 (1.70)
$\Delta_5 Clfi_t$	0.0021 (0.25)	0.0028 (0.37)	0.0068 (1.25)	0.0056 (1.04)	0.0102 (1.61)	0.0097 (1.66)
TYS_t					0.0006 (0.11)	0.0010 (0.17)
$\Delta_5 TYS_{t-1}$					-0.1521 (-0.95)	-0.1620 (-1.23)
Growth Effect (*INV*)	0.0220	0.0298	0.0294	0.0271	0.0161	0.0134
Level Effect (*INV*)	0.1938	0.4037	0.1376	0.1337	0.0689	0.0461
Growth Effect (*Clfi*)	0.0002	0.00009	-0.0001	-0.0002	0.0001	0.00005
Level Effect (*Clfi*)	0.0047	0.0089	0.0196	0.0015	0.0278	0.0243
Test of first order correlation	-3.37 [0.001]	-2.11 [0.035]	-4.23 [0.000]	-3.52 (0.000)	-3.53 [0.000]	-2.74 [0.006]
Test of second order correlation	1.01 [0.311]	0.85 [0.393]	0.32 [0.746]	0.31 (0.757)	0.53 [0.596]	0.51 [0.609]
Test of over identification	0.140	0.140	0.370	0.370	0.912	0.912
Number of Countries	92	92	75	75	310	310
Number of Observations	460	460	375	375	62	62

Table 21: Five-year average panel data, using LFI index for intermediates (*Ilfi*).

	Small Instrument Set		Augmented Instrument Set		Augemnted + Human Capital Instrument Set	
	GMM1	GMM2	GMM1	GMM2	GMM1	GMM2
$\Delta_5 y_{t-1}$	-0.4626 (-2.99)	-0.4169 (-2.30)	-0.3438 (-3.49)	-0.3663 (-3.90)	-0.3414 (-3.43)	-0.3590 (-3.56)
INV_t	0.0061 (1.92)	0.0063 (1.83)	0.0087 (3.66)	0.0084 (3.42)	0.0044 (1.05)	0.0037 (1.03)
$\Delta_5 INV_t$	0.0295 (0.40)	0.0469 (0.50)	0.0566 (0.85)	0.0407 (0.56)	0.0333 (0.68)	0.0370 (0.69)
$Ilfi_t$	0.000005 (0.06)	0.00004 (0.48)	-0.00003 (-0.57)	-0.00002 (-0.52)	0.000002 (0.04)	-0.00002 (-0.37)
$\Delta_5 Ilfi_t$	0.0038 (1.34)	0.0036 (1.47)	0.0001 (0.09)	0.0029 (0.25)	0.0005 (0.39)	0.0012 (1.01)
TYS_t					-0.0016 (-0.34)	-0.0009 (-0.20)
$\Delta_5 TYS_{t-1}$					-0.2068 (-1.36)	-0.1825 (-1.48)
Growth Effect (*INV*)	0.0131	0.0150	0.0252	0.0230	0.0128	0.0103
Level Effect (*INV*)	0.0637	0.1125	0.1646	0.1110	0.0975	0.1032
Growth Effect (*Ilfi*)	0.00001	0.0001	-0.00008	-0.00007	0.000007	-0.00008
Level Effect (*Ilfi*)	0.0083	0.0087	0.0004	0.0008	0.0014	0.0032
Test of first order correlation	-2.85 [0.004]	-1.96 [0.050]	-3.84 [0.000]	-3.54 [0.000]	-3.50 [0.000]	-3.26 [0.001]
Test of second order correlation	0.66 [0.509]	0.64 [0.519]	0.47 [0.642]	0.43 [0.670]	0.14 [0.890]	0.05 [0.957]
Test of over identification	0.101	0.101	0.260	0.260	0.883	0.883
Number of Countries	92	92	75	75	310	310
Number of Observations	460	460	375	375	62	62

Table 22: Fixed-Effects Regressions of Growth on *INV*

Dependent variable: GDP per worker Growth rate 1965-2000			
Variable	Country fixed effect	Country and year fixed effect	Country fixed effect with trend
INV	0.0181 (4.33)	0.0178 (4.24)	0.01663 (3.98)
Adj R-Sq	0.0984	0.1421	0.1109
# of Countries	92	92	92
# of Observations	3312	3312	3312

Table 23: Fixed-Effects Regressions of *INV* and growth on Import Share (*Capital*)

Independent Variable:	**Investment 1965-2000**			**Growth rate of GDP per worker 1965-2000**		
Variable	Country fixed effect	Country and year fixed effect	Country fixed effect with trend	Country fixed effect	Country and year fixed effect	Country fixed effect with trend
Kimp	0.1468 (11.62)	0.1789 (11.54)	0.1886 (12.76)	0.0004 (0.22)	0.0070 (2.95)	0.0057 (2.50)
Adj R-Sq	0.7725	0.7831	0.7792	0.0872	0.1351	0.1040
#of Countries	92	92	92	92	92	92
# of Obs.	3312	3312	3312	3312	3312	3312

Table 24: Fixed-Effects Regressions of *INV* and growth on Import Share (*Consumption*)

Independent Variable:	**Investment 1965-2000**			**Growth rate of GDP per worker 1965-2000**		
Variable	Country fixed effect	Country and year fixed effect	Country fixed effect with trend	Country fixed effect	Country and year fixed effect	Country fixed effect with trend
Cimp	0.1315 (10.81)	0.1571 (10.33)	0.1606 (10.96)	-0.0020 (-1.02)	0.0020 (1.40)	0.0024 (1.10)
Adj R-Sq	0.7688	0.7789	0.7731	0.0875	0.1321	0.1018
#of Countries	92	92	92	92	92	92
# of Obs.	3312	3312	3312	3312	3312	3312

Table 25: Fixed-Effects Regressions of *INV* and growth on Import Share (*Intermediates*)

Independent Variable:	**Investment 1965-2000**			**Growth rate of GDP per worker 1965-2000**		
Variable	Country fixed effect	Country and year fixed effect	Country fixed effect with trend	Country fixed effect	Country and year fixed effect	Country fixed effect with trend
Iimp	0.1558 (9.32)	0.1603 (7.80)	0.1794 (9.75)	-0.0047 (-1.86)	0.0007 (0.23)	-0.0010 (-0.36)
Adj R-Sq	0.7686	0.7753	0.7749	0.0884	0.1316	0.1010
#of Countries	92	92	92	92	92	92
# of Obs.	3312	3312	3312	3312	3312	3312

Table 26: Fixed-Effects Regressions of *INV* and growth on *LFI(Capital)*

Independent Variable:	**Investment 1965-2000**			**Growth rate of GDP per worker 1965-2000**		
Variable	Country fixed effect	Country and year fixed effect	Country fixed effect with trend	Country fixed effect	Country and year fixed effect	Country fixed effect with trend
$LFI^{Capital}$	-0.0115 (-6.54)	-0.0161 (-8.51)	-0.0139 (-7.38)	-0.0005 (-1.72)	-0.0010 (-3.44)	-0.0010 (-3.37)
Adj R-Sq	0.7628	0.7741	0.7652	0.0882	0.1357	0.1054
#of Countries	92	92	92	92	92	92
# of Obs.	3312	3312	3312	3312	3312	3312

Table 27: Fixed-Effects Regressions of *INV* and growth on *LFI(Consumption)*

Independent Variable:	**Investment 1965-2000**			**Growth rate of GDP per worker 1965-2000**		
Variable	Country fixed effect	Country and year fixed effect	Country fixed effect with trend	Country fixed effect	Country and year fixed effect	Country fixed effect with trend
$LFI^{Consumption}$	0.0089 (6.12)	0.0073 (4.82)	0.0085 (5.67)	0.0002 (1.03)	0.0002 (2.01)	0.0001 (2.34)
Adj R-Sq	0.7621	0.7692	0.7628	0.0875	0.1318	0.1014
#of Countries	92	92	92	92	92	92
# of Obs.	3312	3312	3312	3312	3312	3312

Table 28: Fixed-Effects Regressions of *INV* on *LFI (Intermediates)*

Independent Variable:	**Investment 1965-2000**			**Growth rate of GDP per worker 1965-2000**		
Variable	Country fixed effect	Country and year fixed effect	Country fixed effect with trend	Country fixed effect	Country and year fixed effect	Country fixed effect with trend
$LFI^{Intermediates}$	-0.0013 (-3.59)	-0.0015 (-3.68)	-0.0016 (-4.18)	0.0012 (1.04)	0.00002 (0.40)	0.00004 (0.61)
Adj R-Sq	0.7597	0.7682	0.7611	0.0880	0.1316	0.1051
#of Countries	92	92	92	92	92	92
# of Obs.	3312	3312	3312	3312	3312	3312

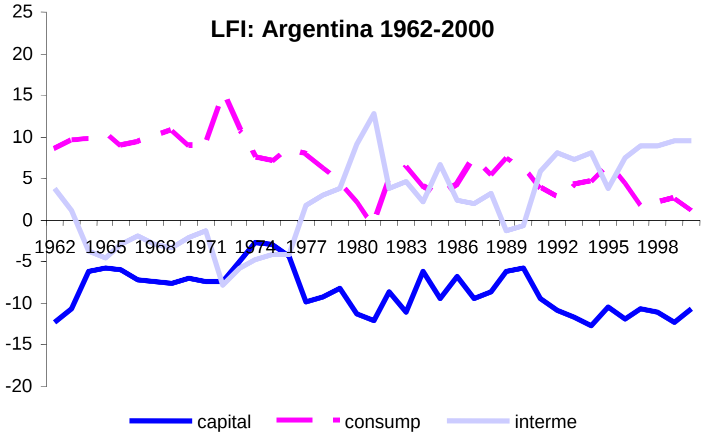
APPENDIX 3.E

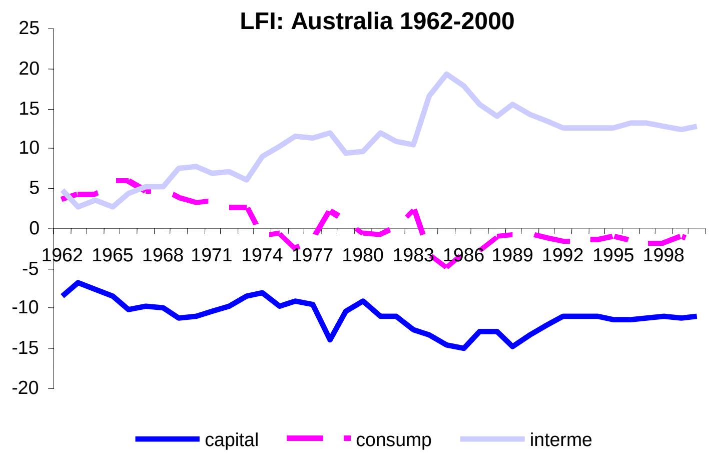
List of Countries

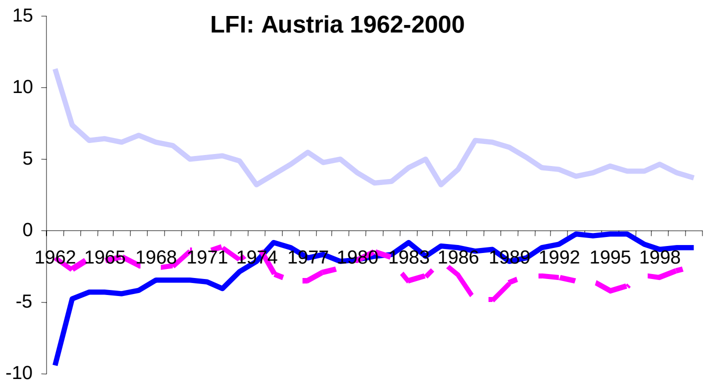
Argentina	El Salvador	Mauritius	Sweden
Australia	Finland	Mexico	Switzerland
Austria	France	Morocco	Syria
Barbados	Gabon	Mozambique	Taiwan
Benin	Gambia	Nepal	Tanzania
Bolivia	Ghana	Netherlands	Thailand
Brazil	Greece	New Zealand	Togo
Burkina Faso	Guatemala	Nicaragua	Trinidad & Tbg
Burundi	Guinea	Niger	Tunisia
Cameroon	Honduras	Nigeria	Turkey
Canada	Hong Kong	Norway	UK
Central Africa	Iceland	Pakistan	USA
Chad	India	Panama	Uganda
Chile	Ireland	Papua New Guinea	Uruguay
China	Israel	Paraguay	Venezuela
Colombia	Italy	Peru	Zambia
Congo	Jamaica	Philippines	Zimbabwe
Costa Rica	Japan	Portugal	
Cote d'Ivoire	Jordan	Romania	
Cyprus	Kenya	Rwanda	
Congo, Dem	Korea Rep	Senegal	
Denmark	Malawi	Sierra Leone	
Dominican Rep	Malaysia	Singapore	
Ecuador	Mali	Spain	
Egypt	Mauritania	Sri Lanka	

APPENDIX 3.F

FIGURES OF LFI INDX FOR EACH COUNTRY IN THE SAMLE







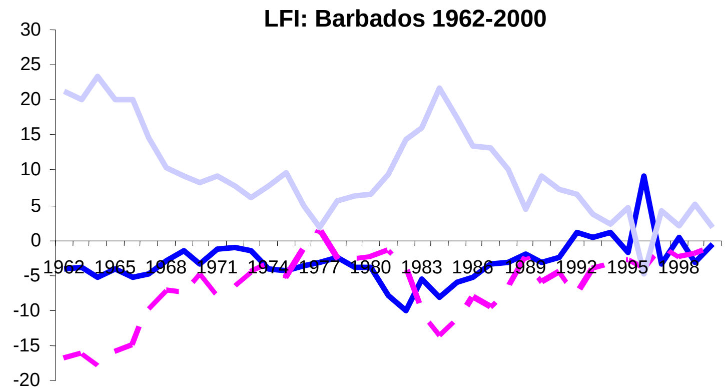
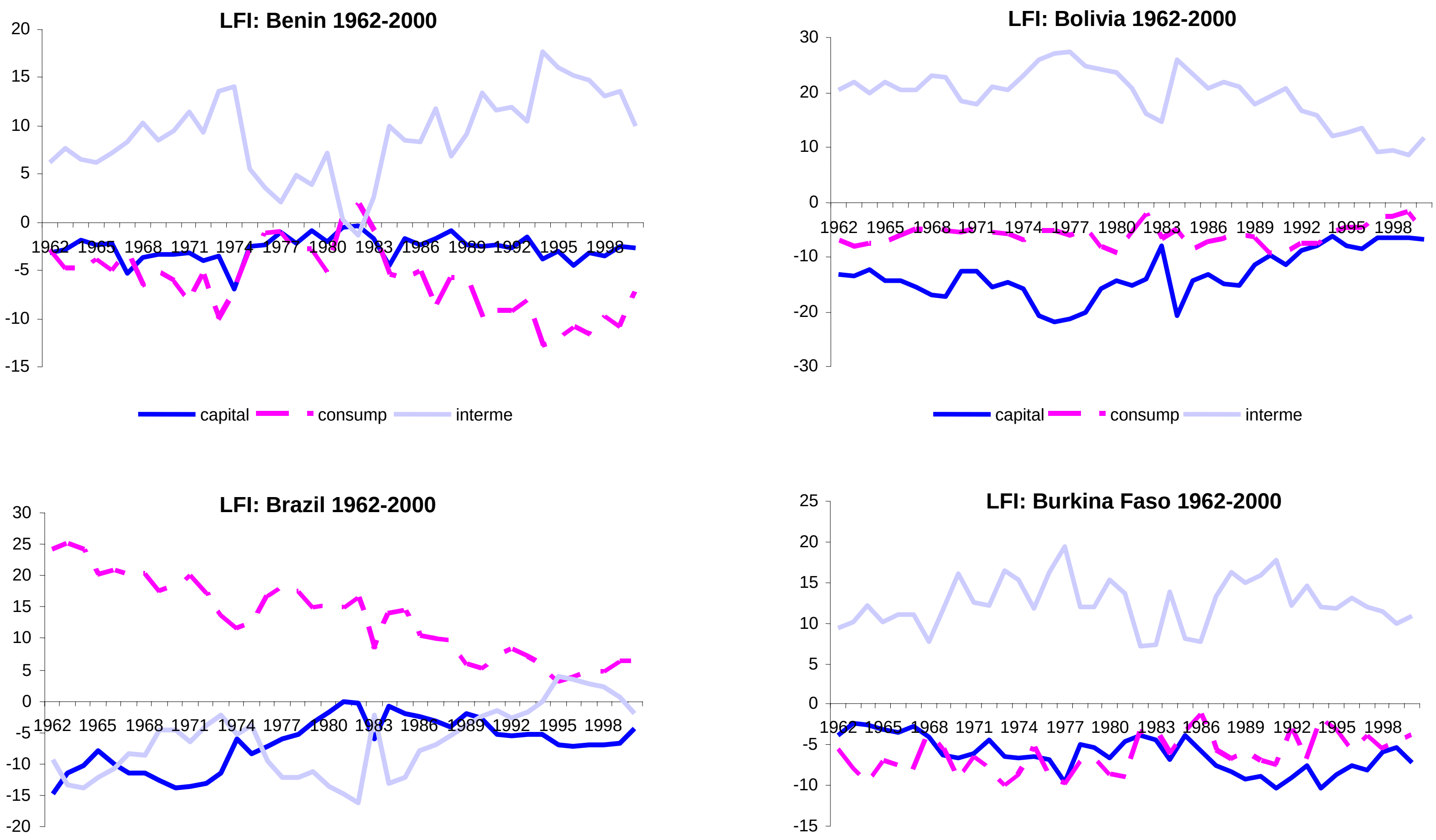


Figure F-1: Figures of LFI index for each country in the sample



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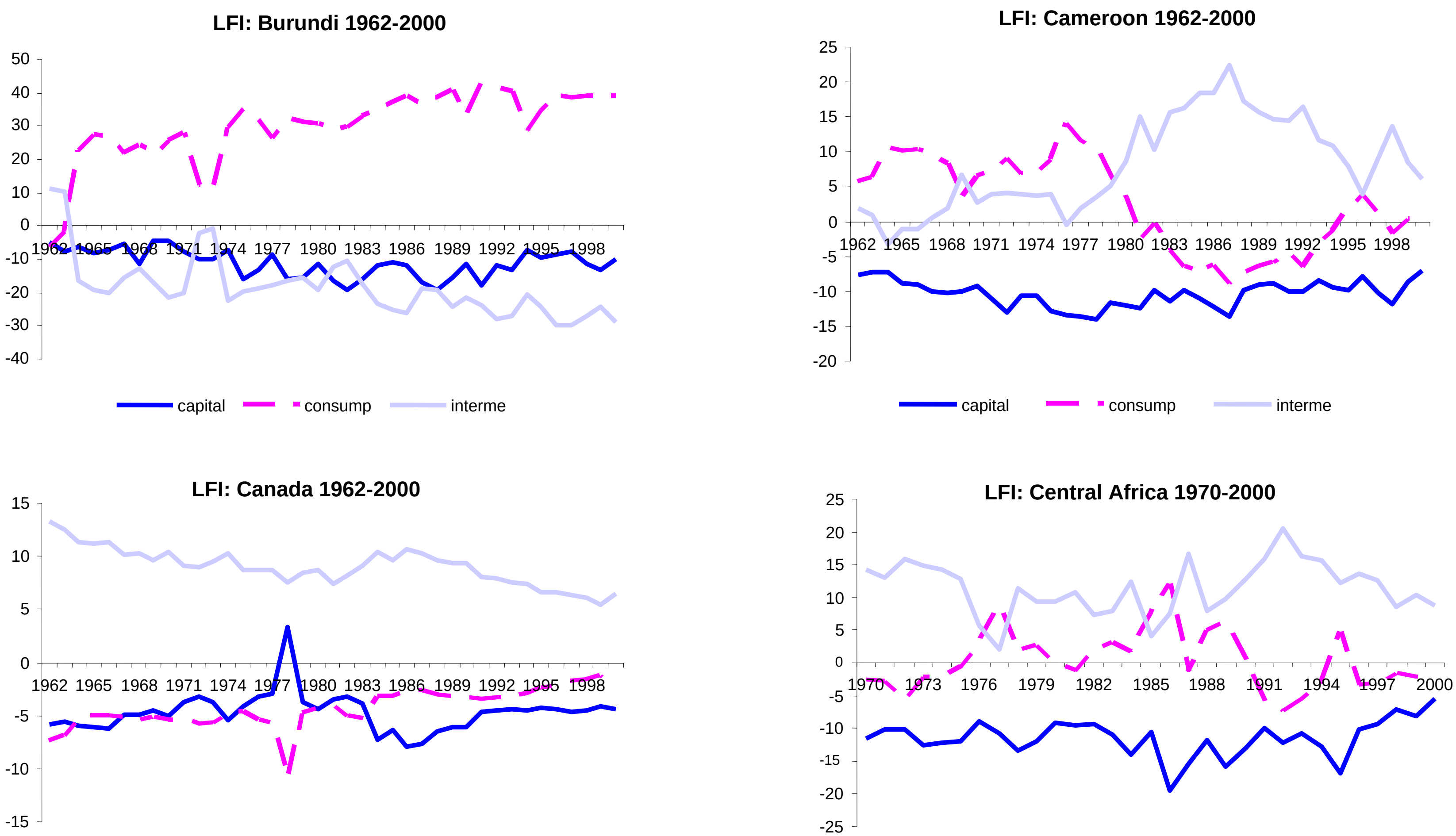

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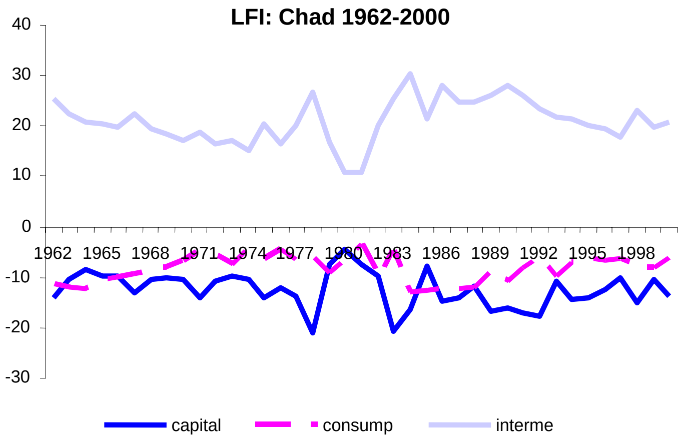

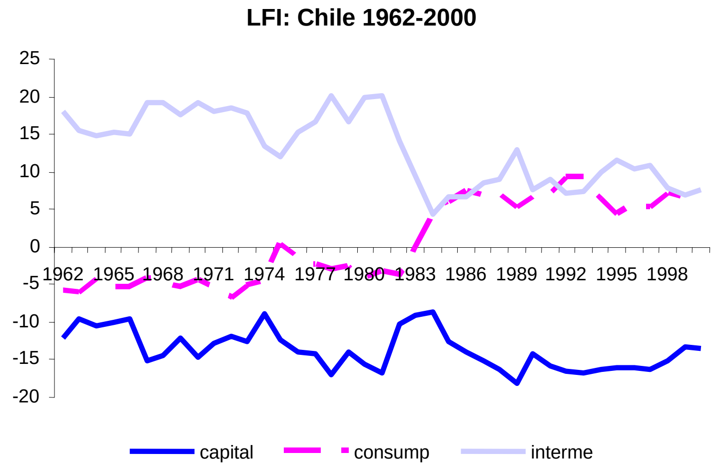

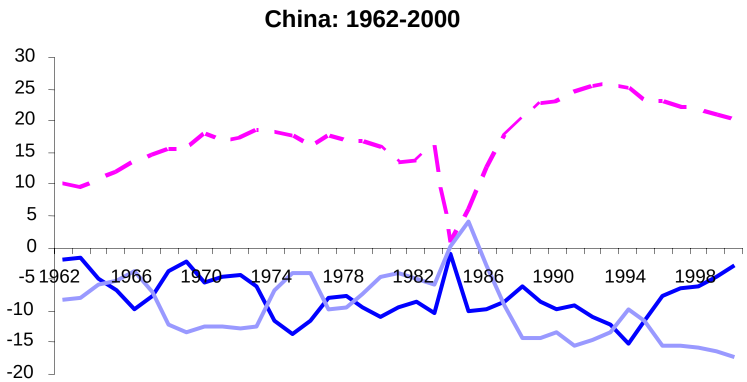

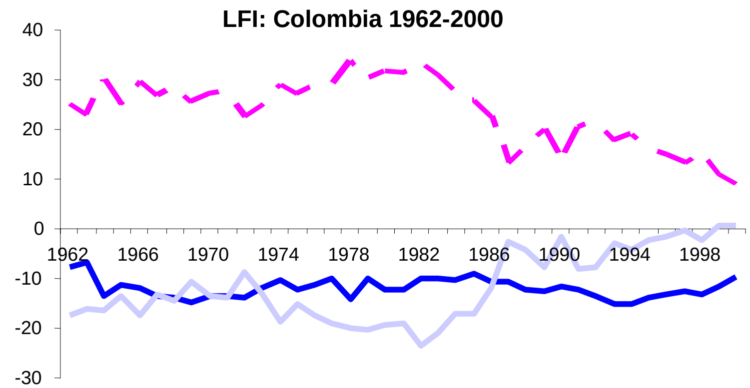

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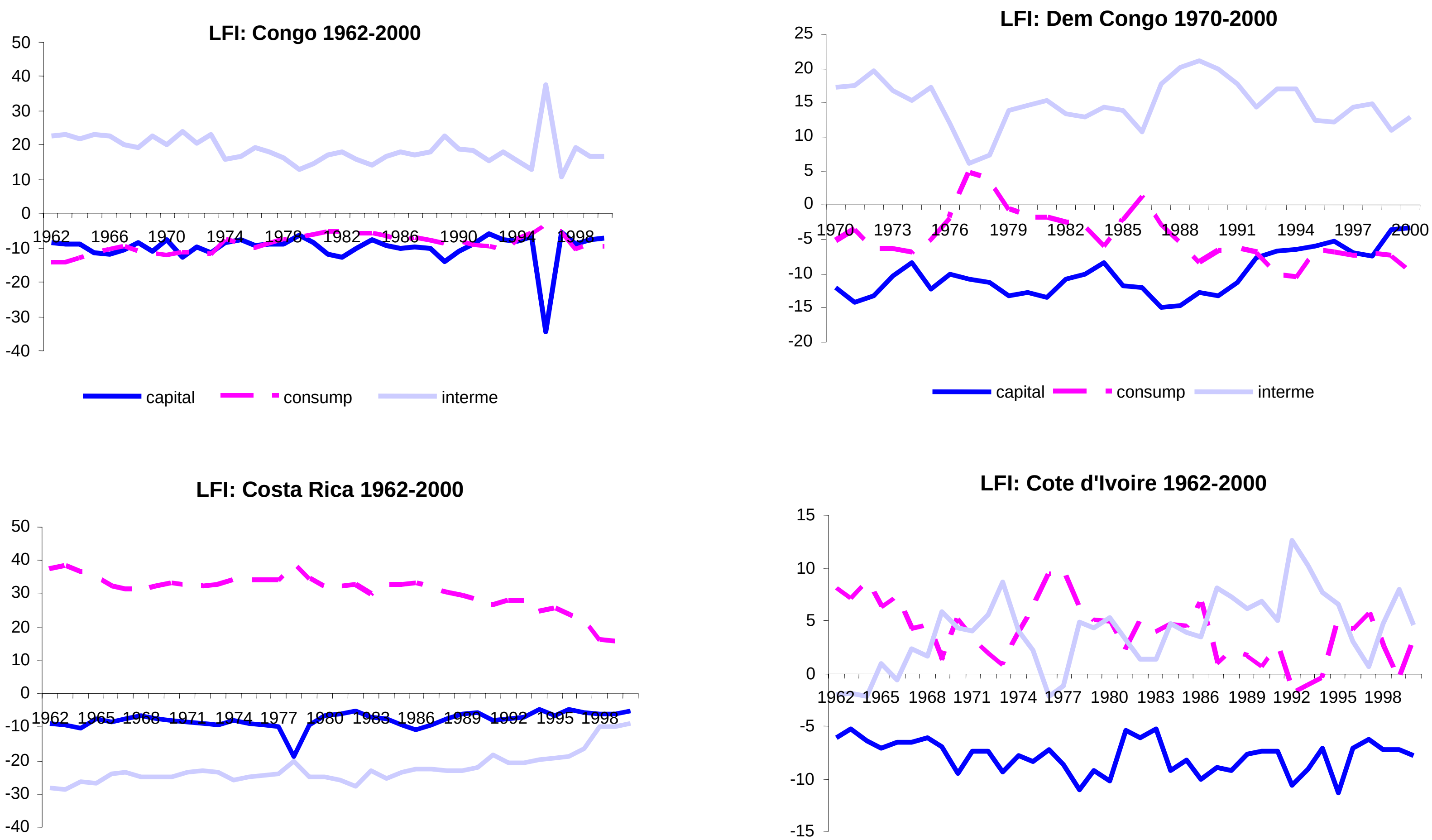

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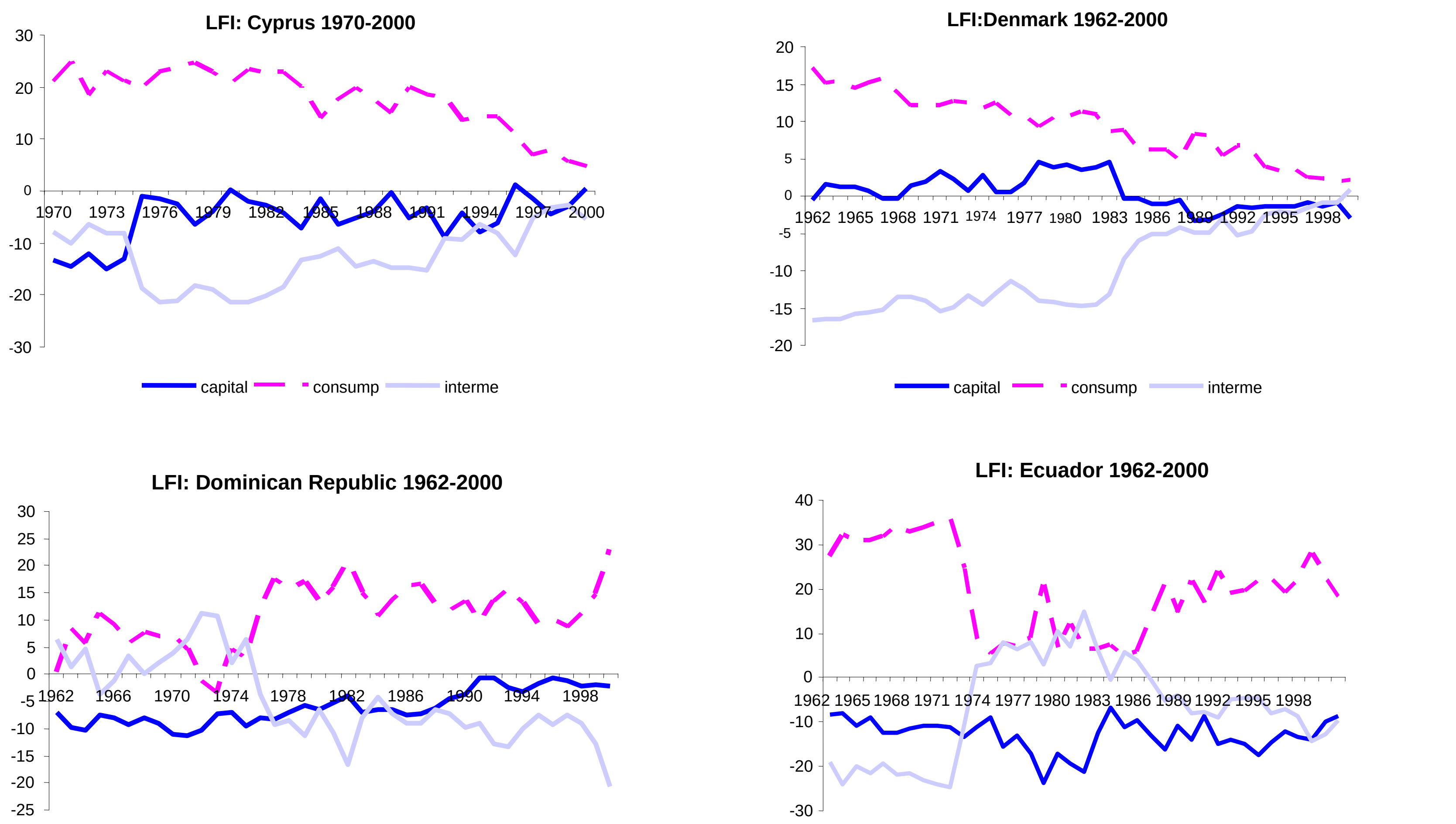

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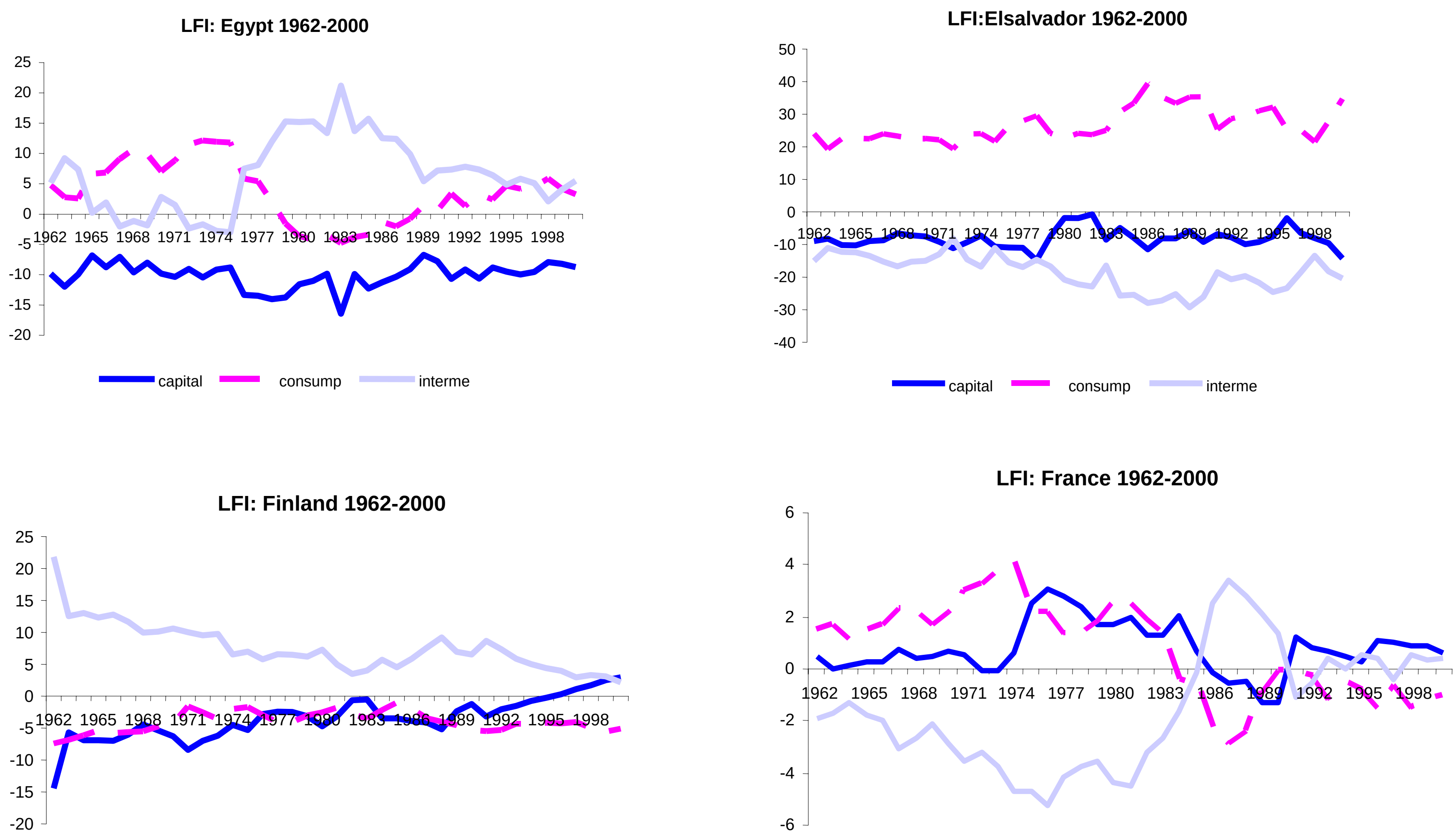

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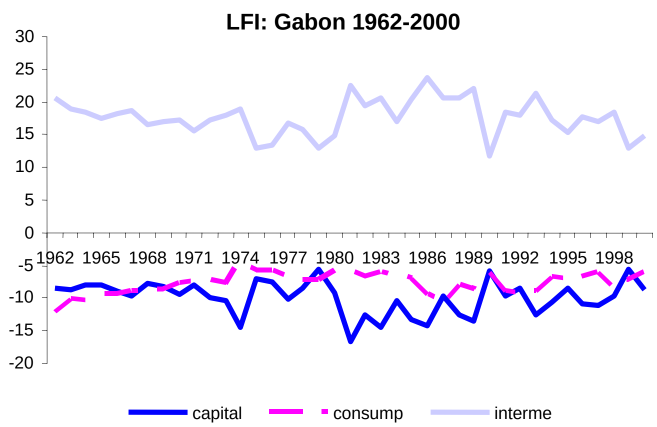

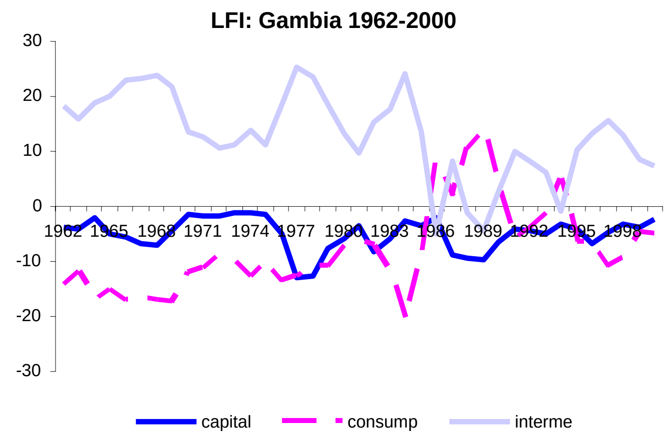

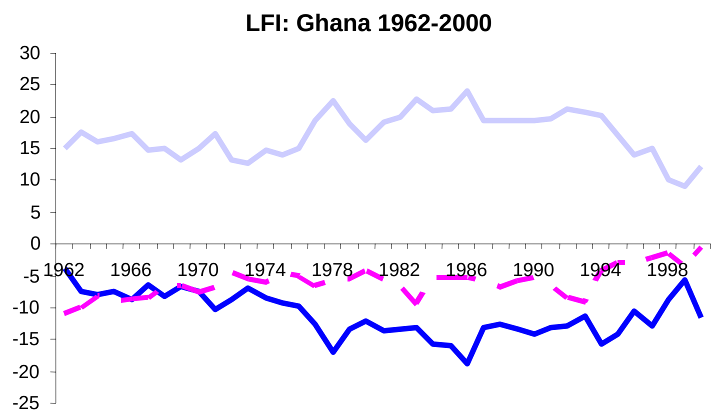

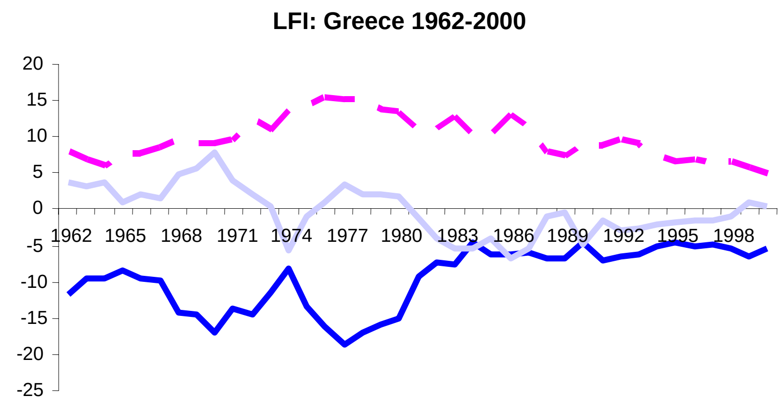

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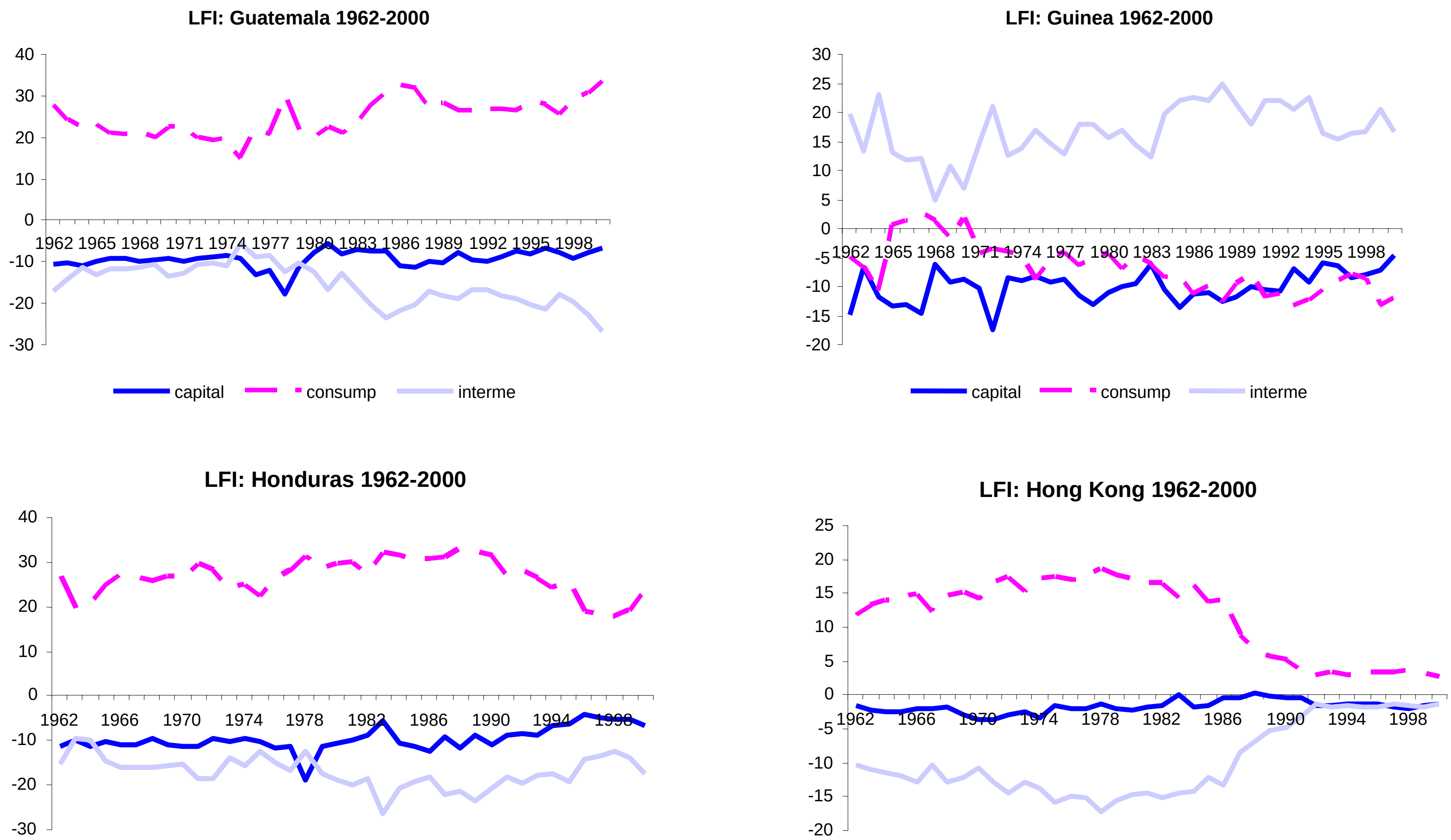

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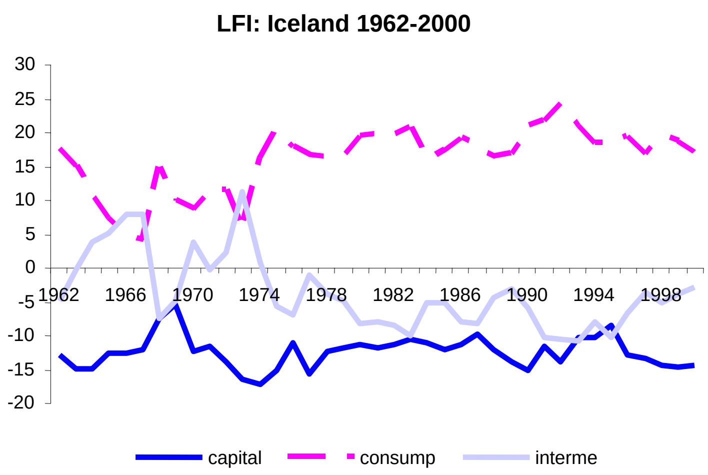

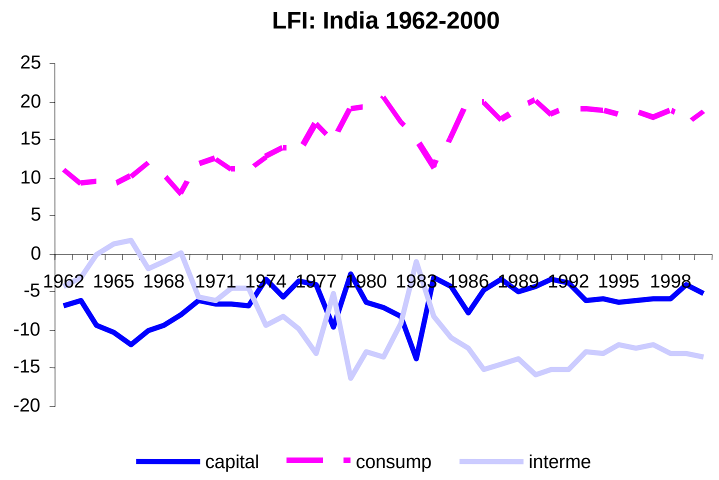

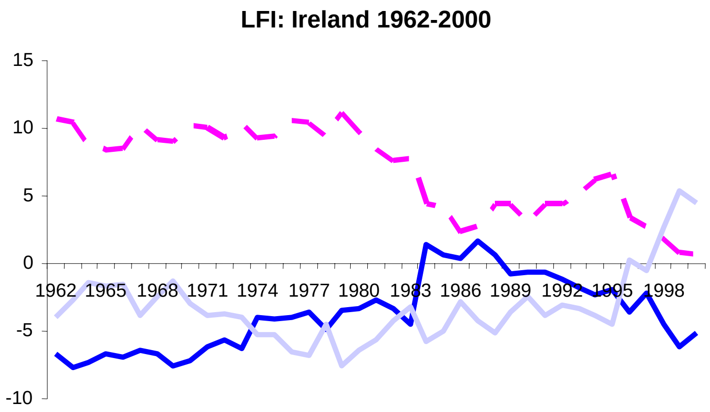

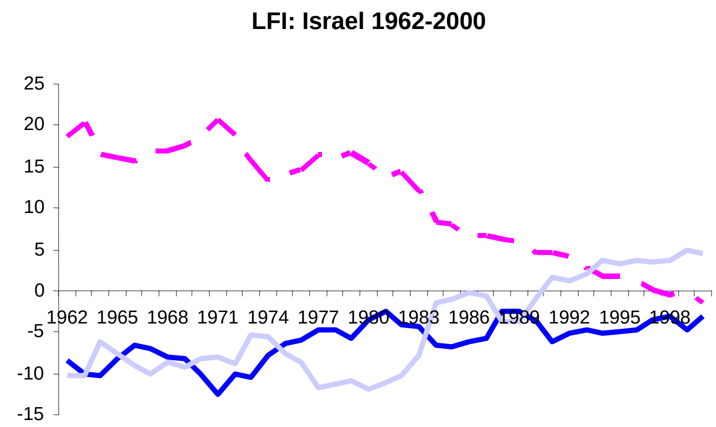

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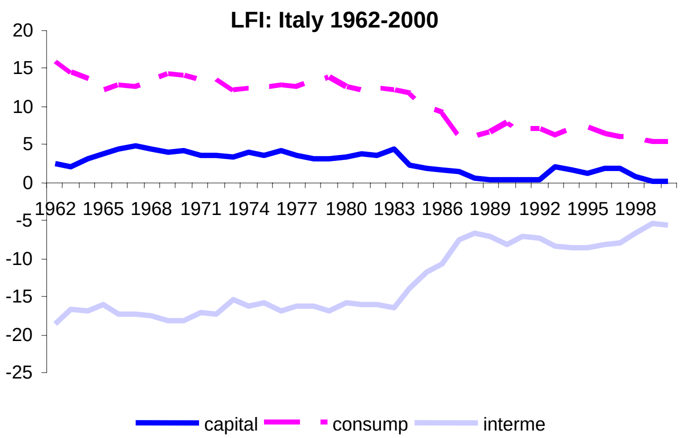

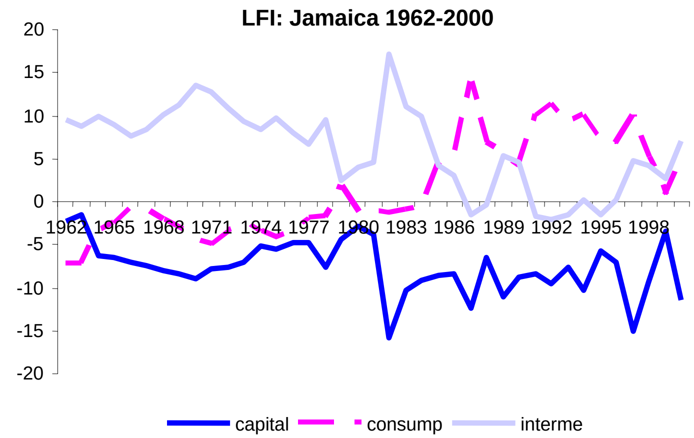

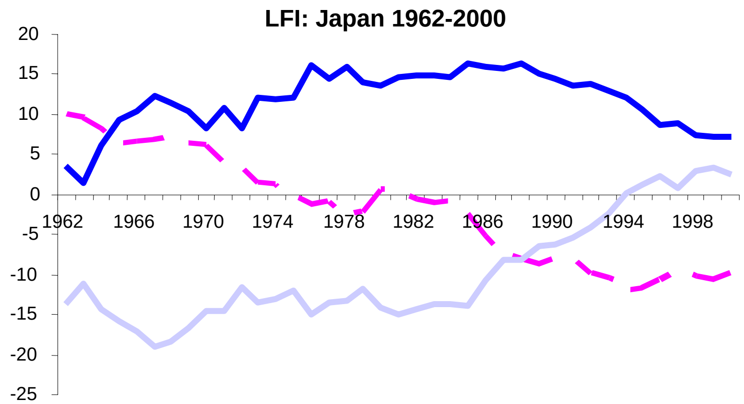

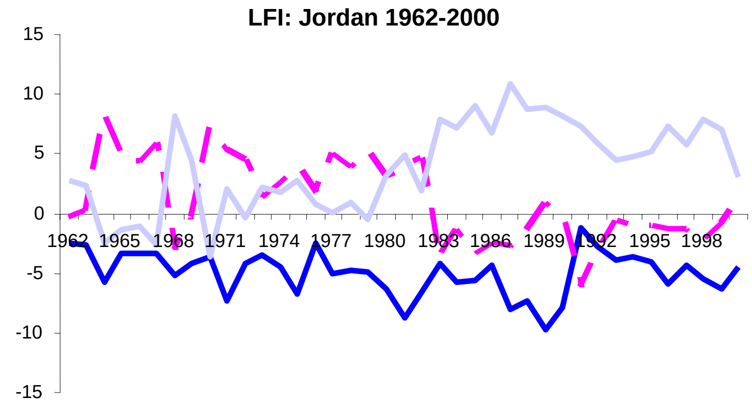

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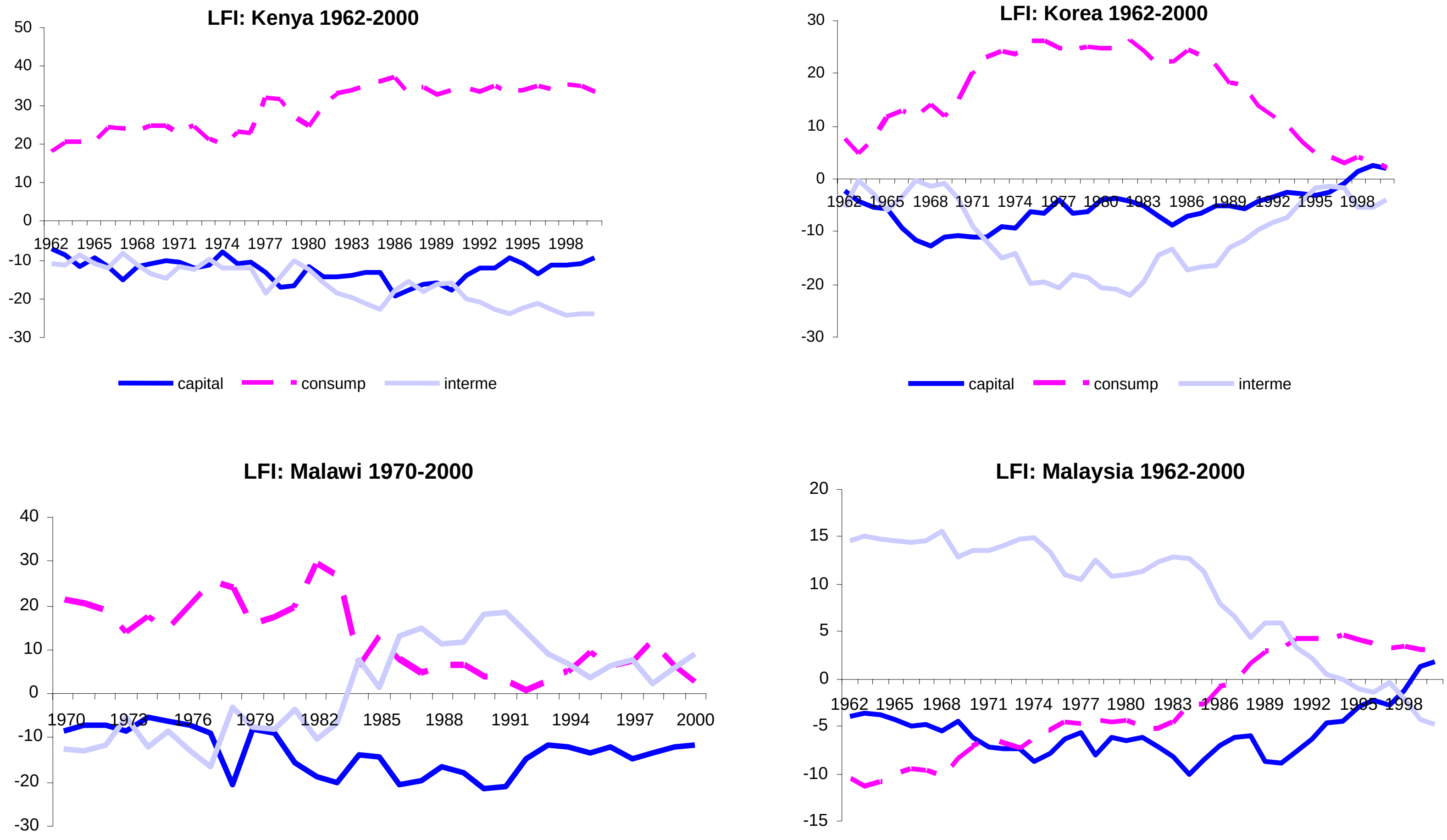

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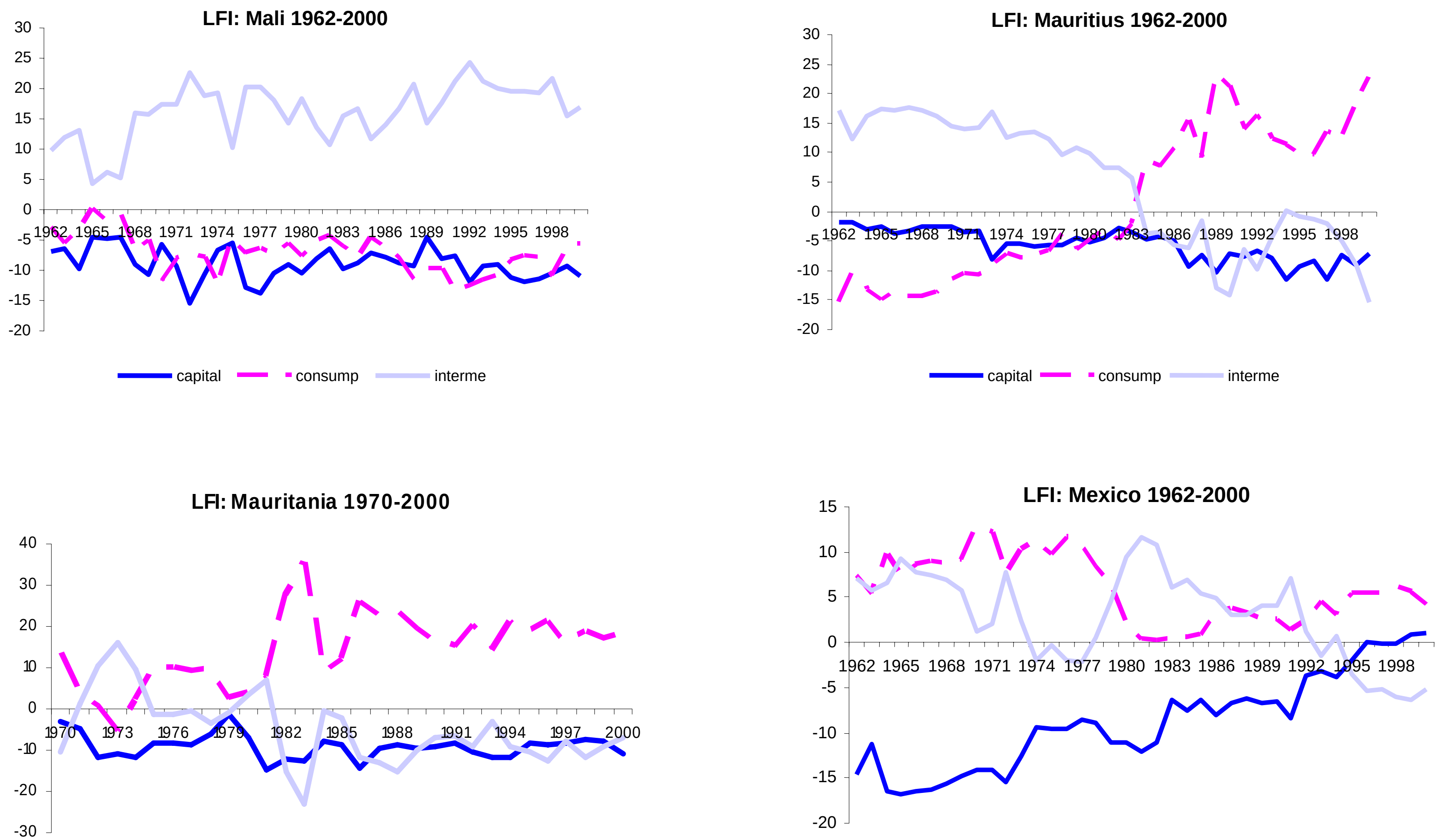

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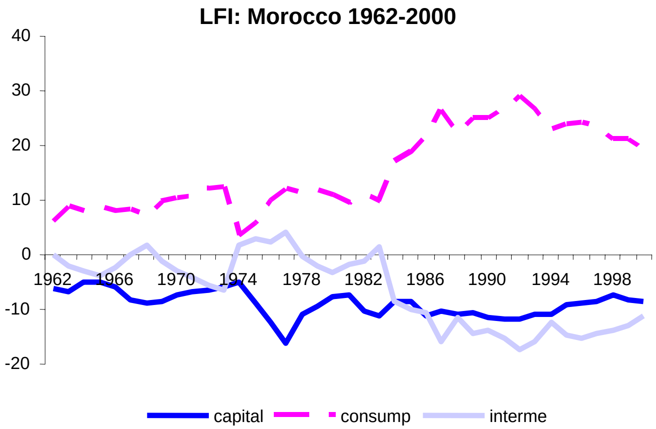

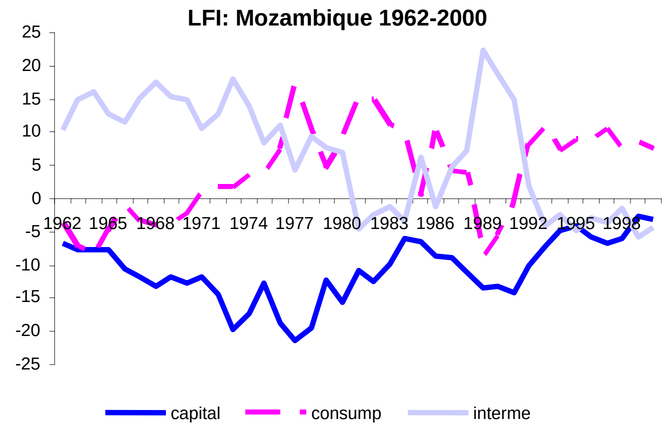

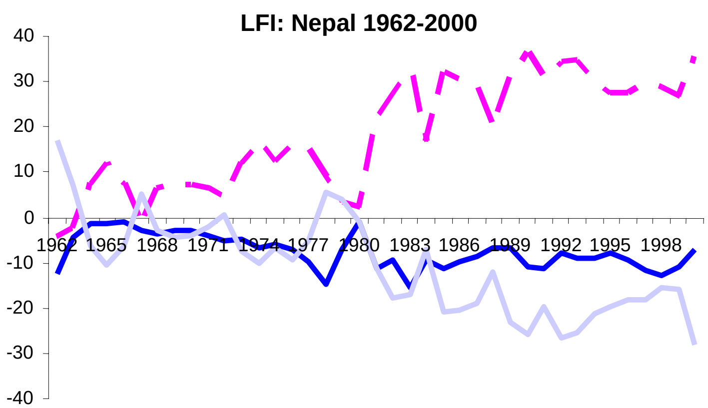

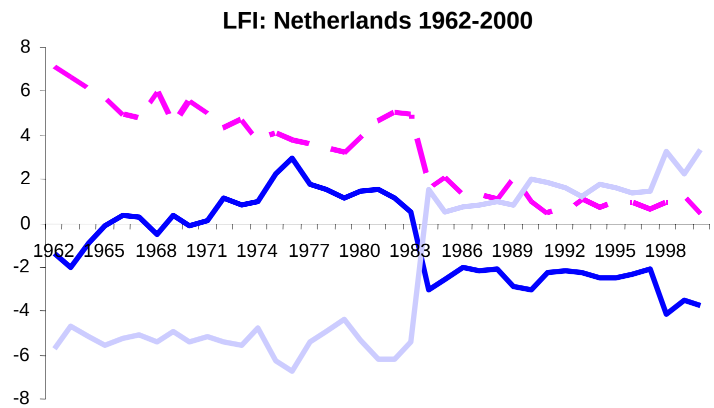

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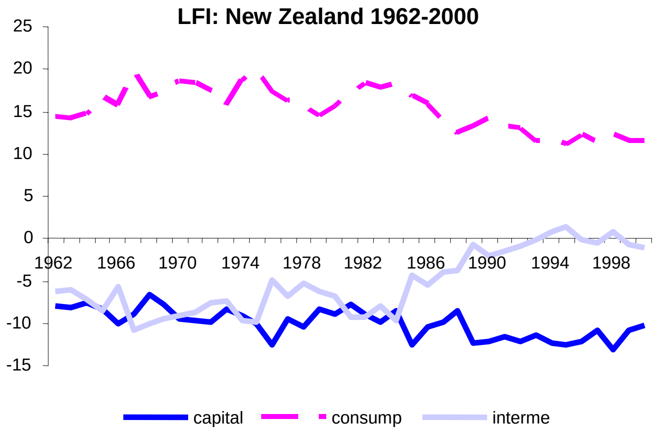

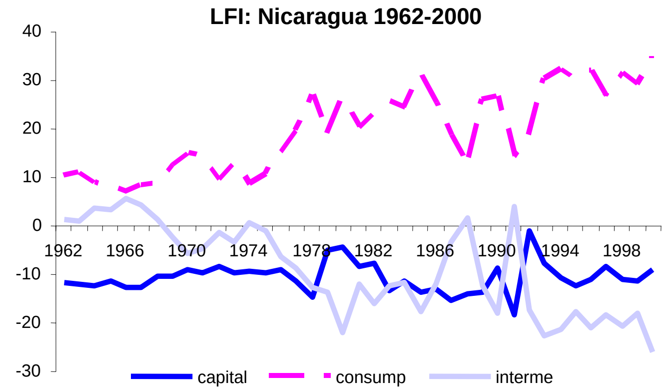

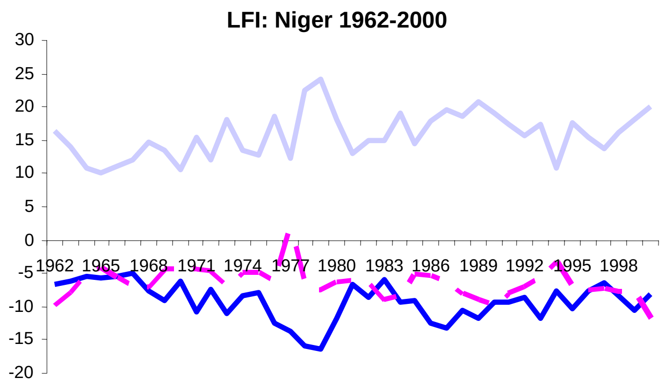

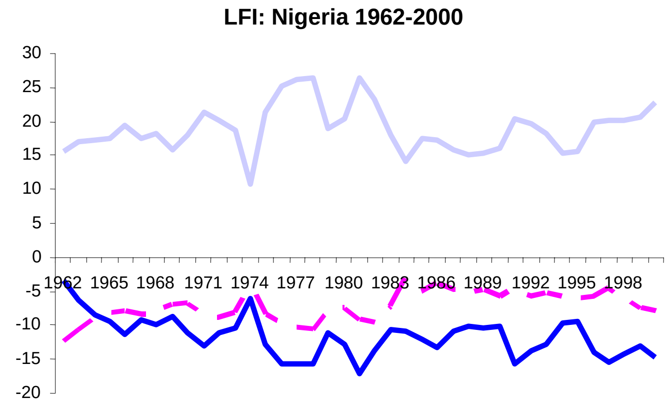

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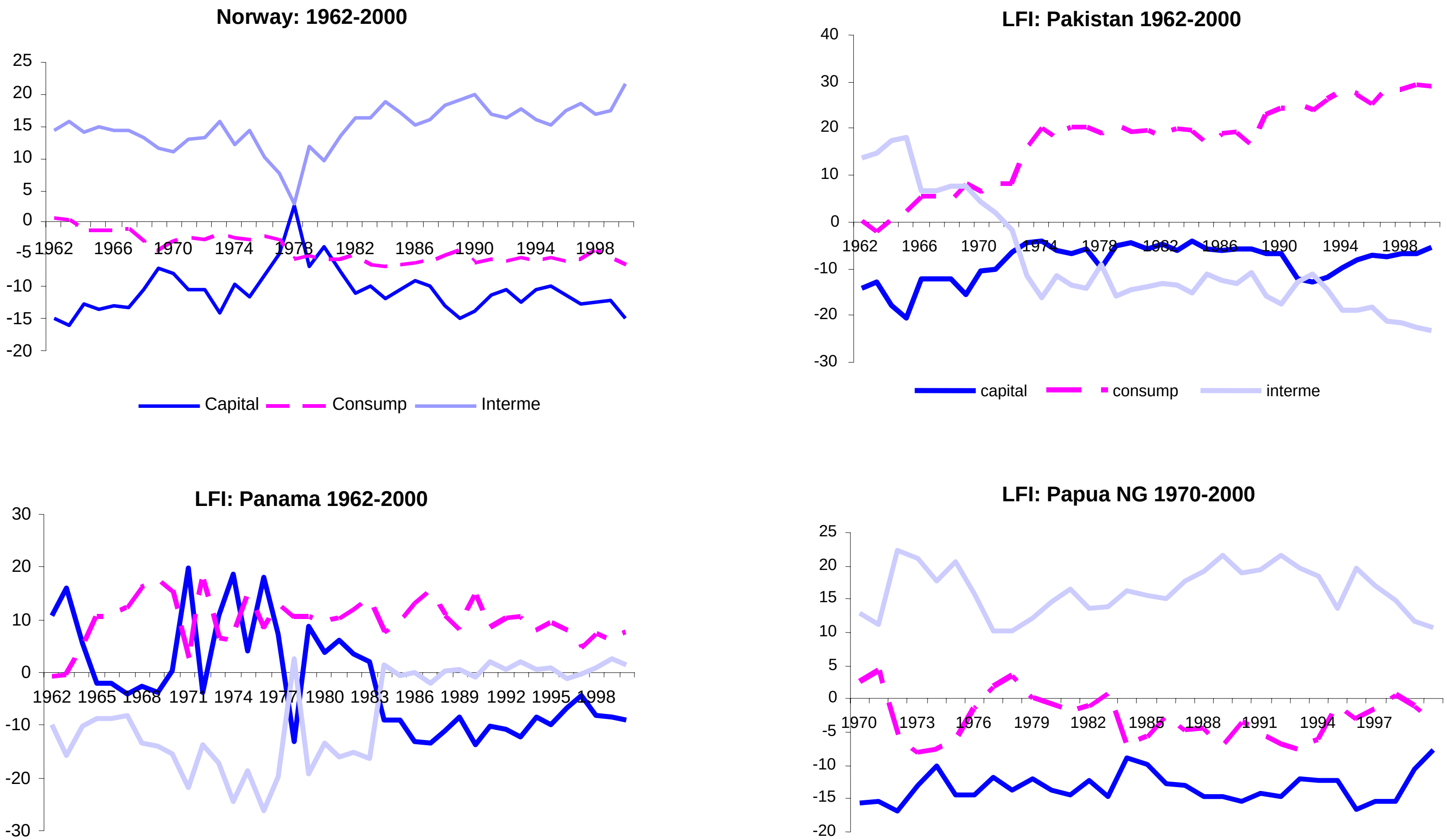

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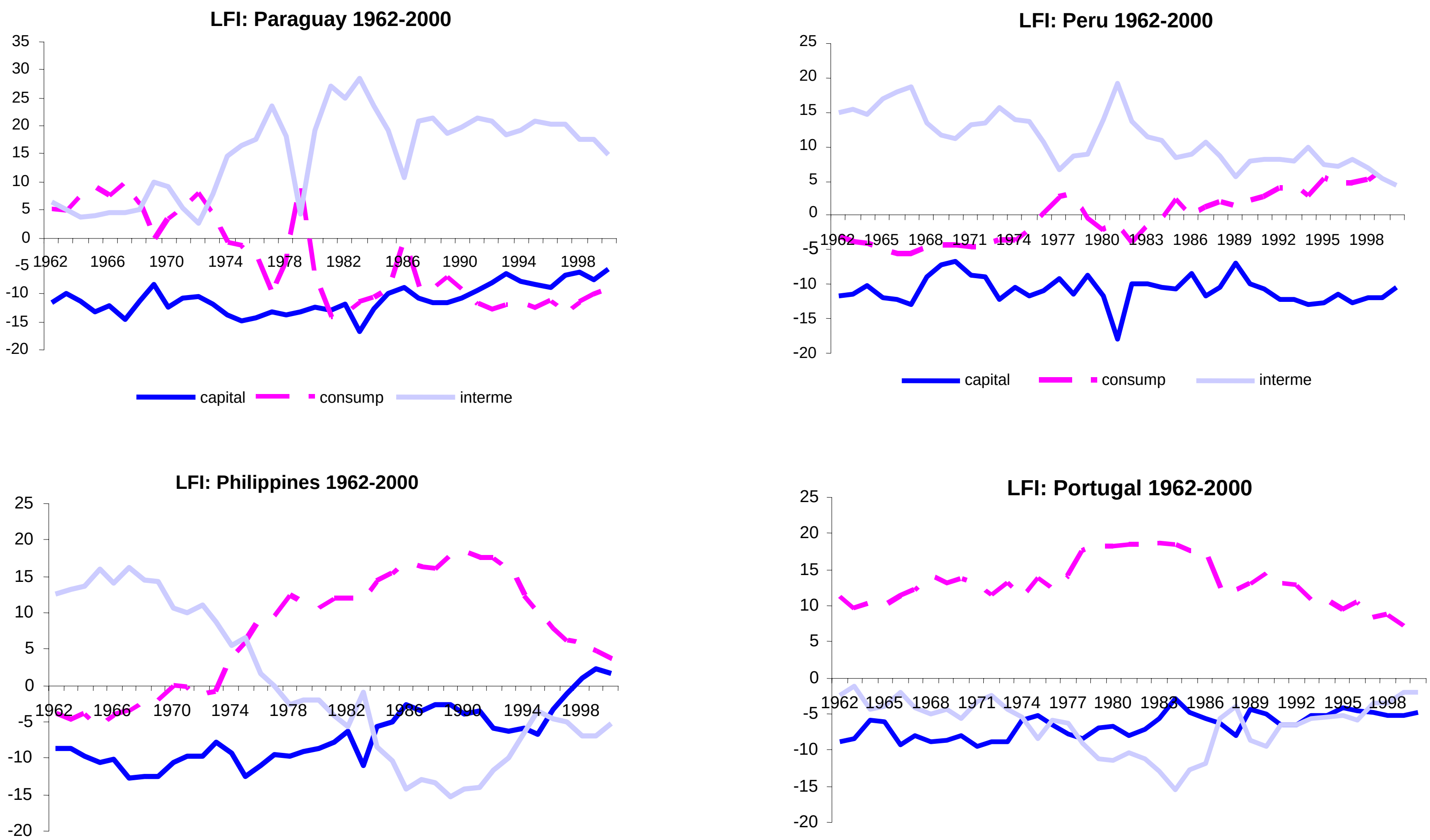

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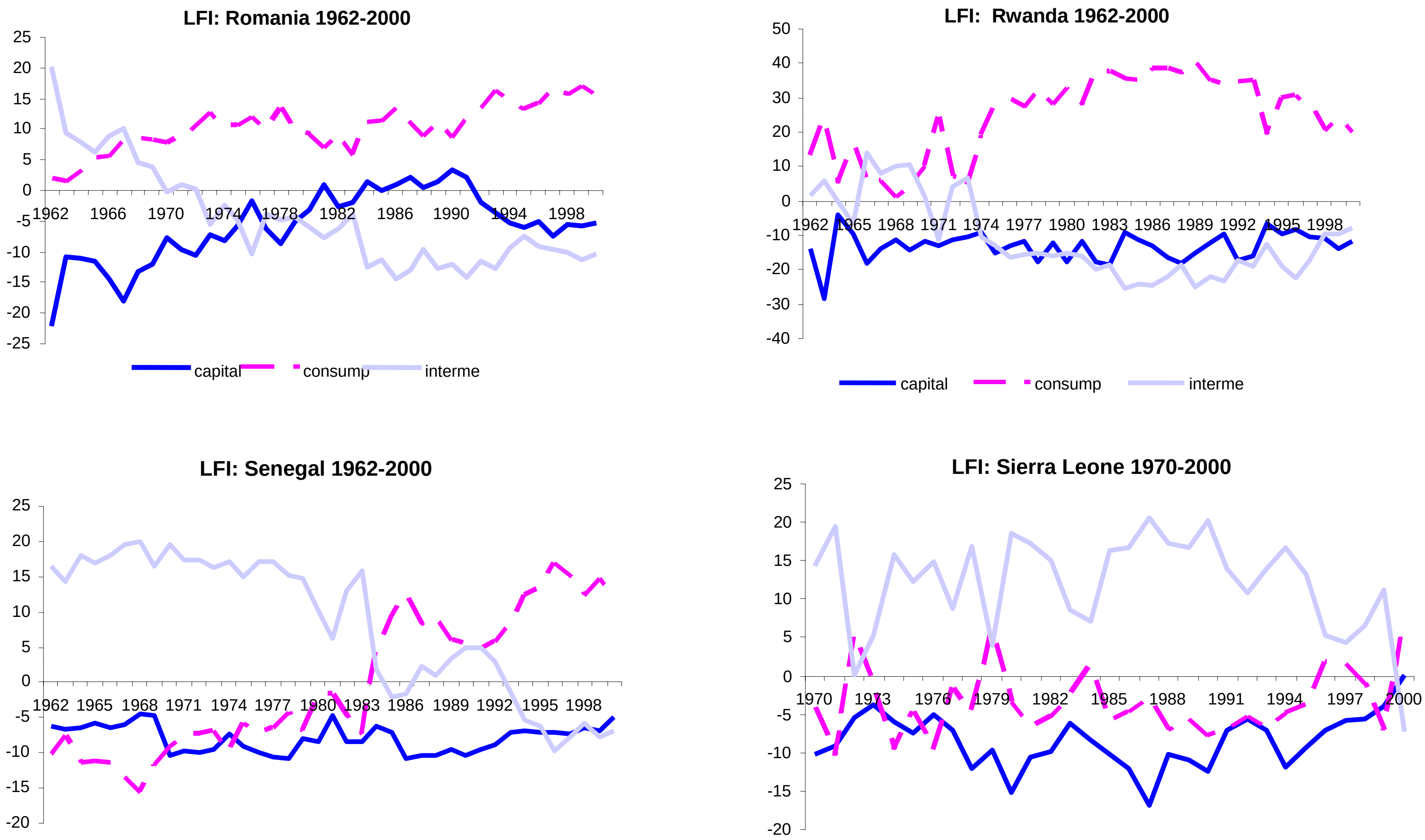

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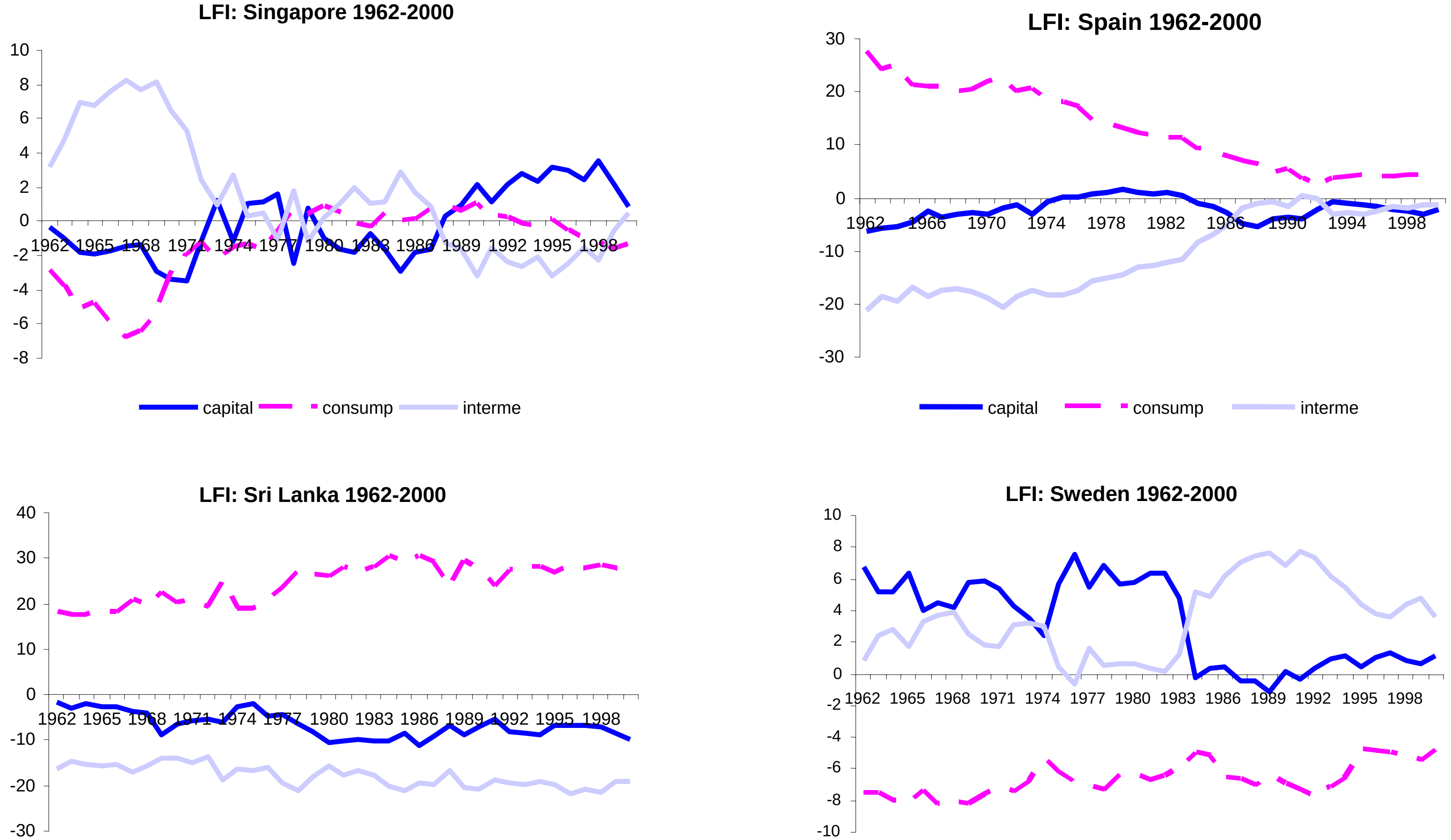

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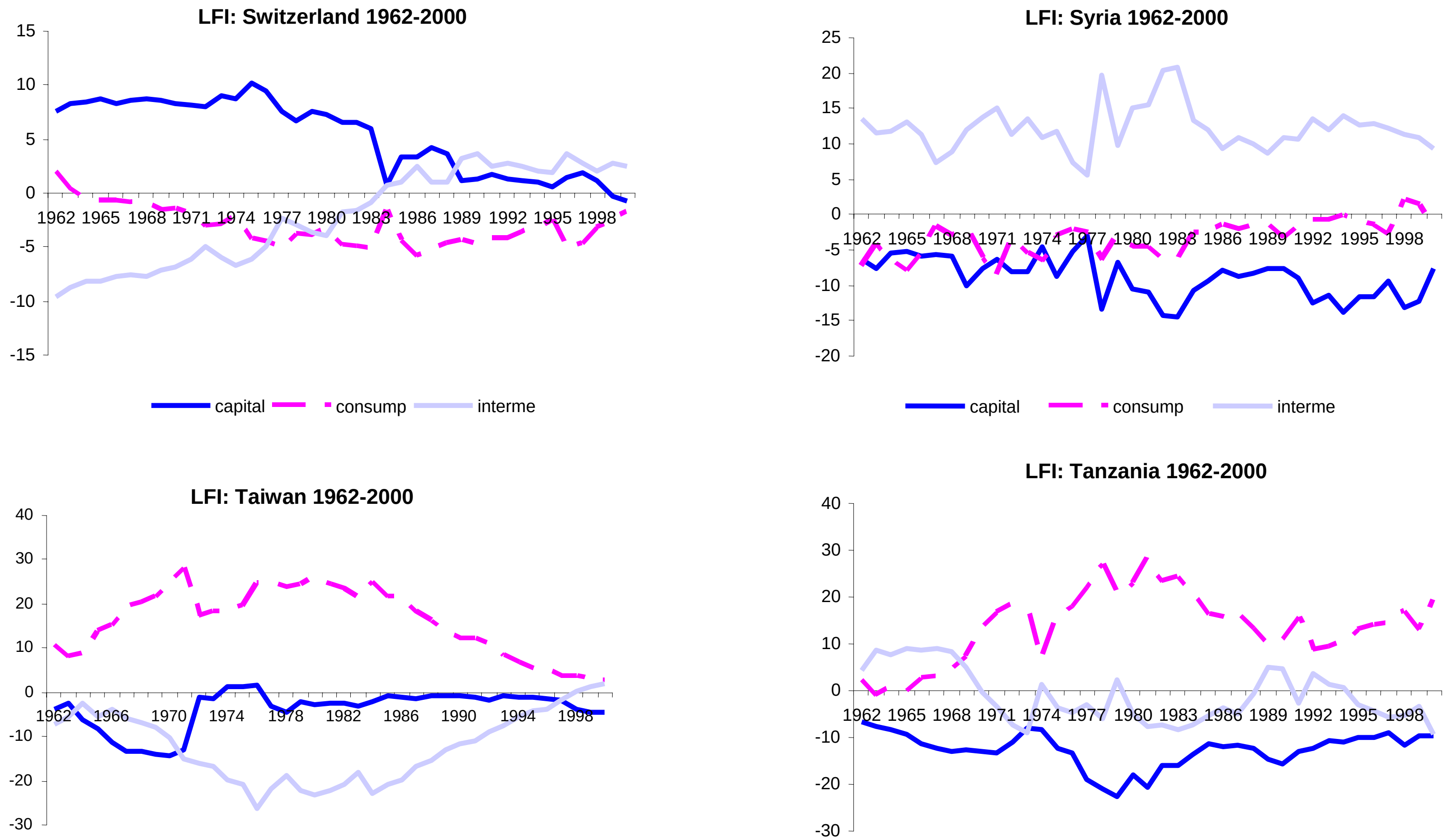

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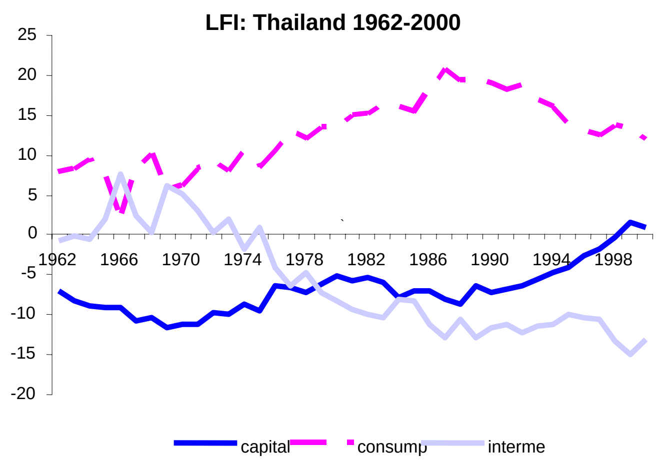

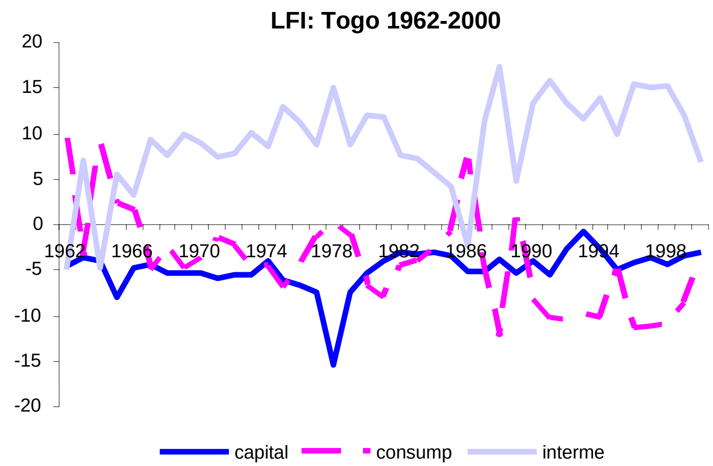

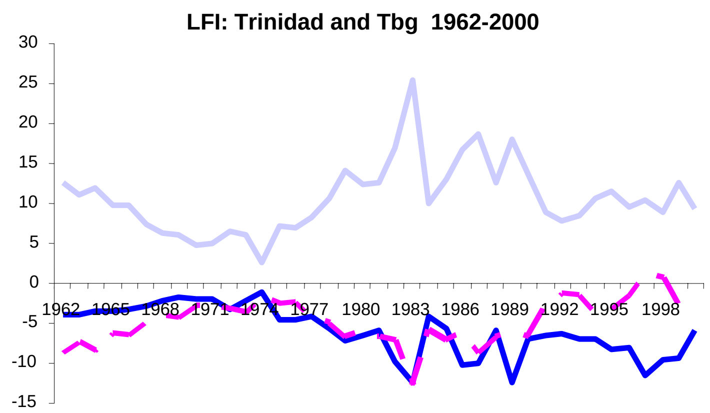

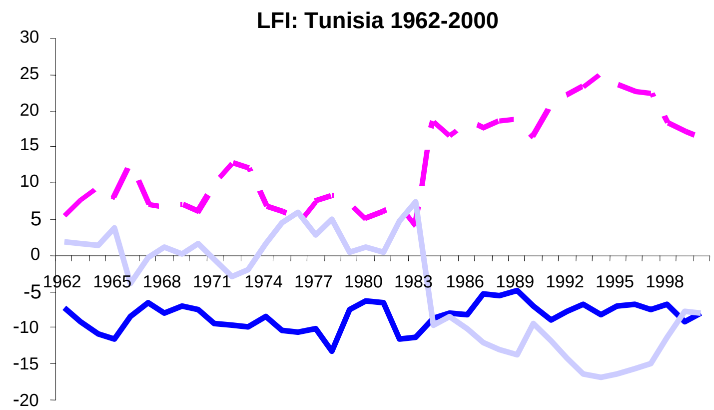

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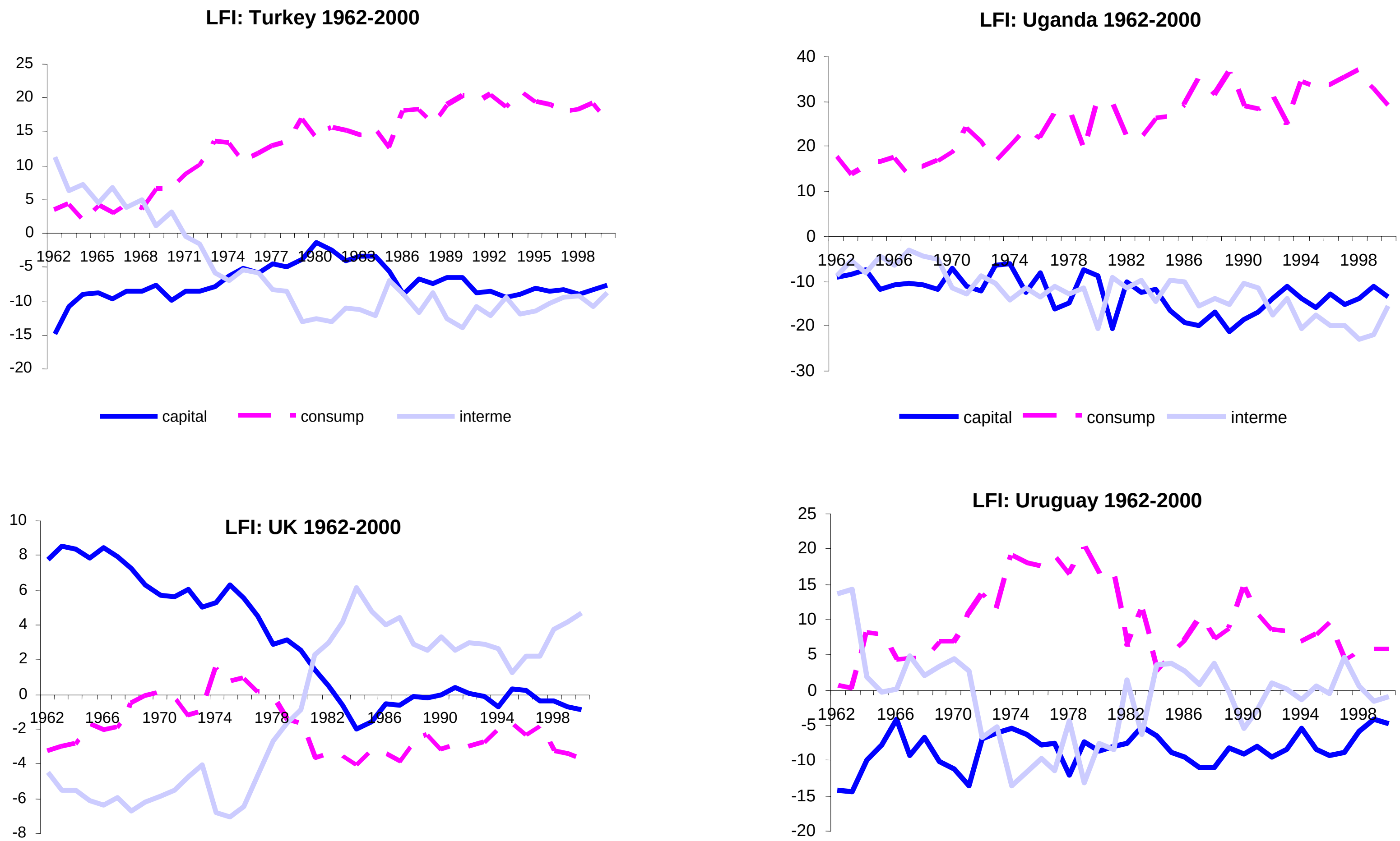

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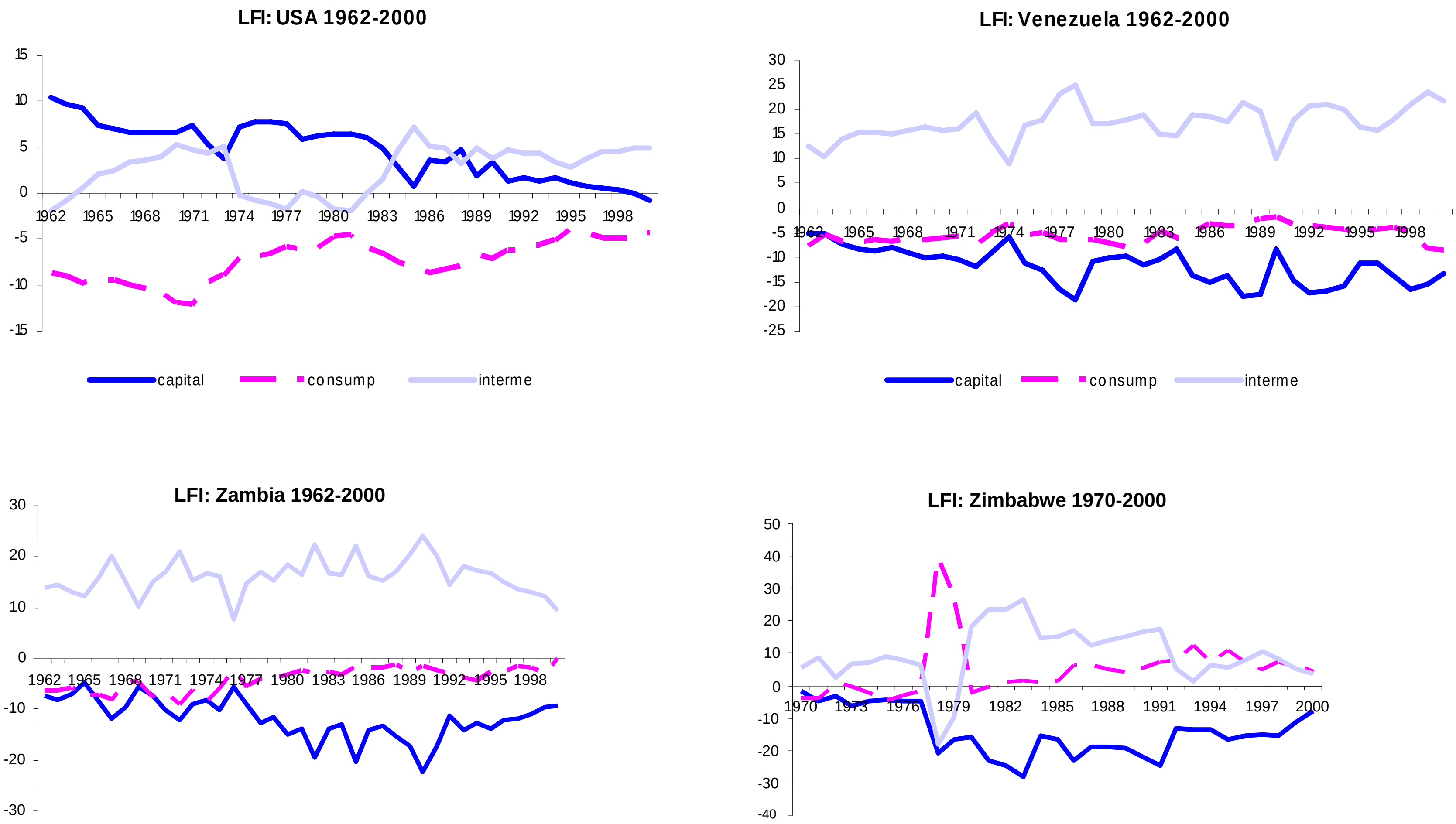

Figure F-1: Figures of LFI index for each country in the sample, continued

CHAPTER 4

SUMMARY AND RECOMMENDATION FOR FUTURE RESEARCH

In this research we tried to study the effect of trade on growth in a two sector endogenous growth model. In this model there are two sectors of production producing two different types of goods, a unified consumption-investment good and the other type is purely an investment good of other type. In this model labor is being normalized to one and therefore two reproducible factors of production are being used in the production process. Output is assumed to be produced under condition of constant returns to scale and perfect competition, so that the model exhibits endogenous growth because it has constant returns to scale in the reproducible factors. When the technologies assumed to be of Cobb-Douglas form, the model has been studied by Barro and Sala-I-Martin (1995, ch5) when economy is closed. Seater (2007) extended a version of this model to include trade. In his work he assumed two countries trading with each other in both types of good. However he assumes that the factor shares between sectors and across countries are the same. His model provide a mechanism in which trade could raise growth through the mechanism of comparative advantage similar to the one where trade raises income level of countries. However this result depends on the absolute advantage of countries in production of goods. When countries have each absolute advantage in producing something then trade raises growth rate of both countries. We showed in the first chapter of this dissertation that similar results hold when we assumed the technology differences across sectors and between countries come from both total factor productivity parameters as well as the factor shares. Although we could not show that the effect of trade on growth depends on the absolute advantage of the countries however when the equilibrium price of the world markets falls between the autarky prices then with trade each country on the balanced growth path completely specialize and their growth rates will be higher with trade.

In chapter two we generalized the model even more by dropping the assumption of Cobb-Douglass production function. This model when economies are closed has been studied by Bond, Wang and Yip (1996) which examine how interaction between the stocks of physical and human capital affects the growth process. We extended this model to include the trade. Although this model is related to the 2×2 Heckscher-Ohlin model but when both factors are reproducible the model has Ricardian flavor in the long run. We showed that there exists a unique price at which the open country incomplete specializes on the balanced growth path. The price associated with the incomplete specialization is the autarky price. When the price is not equal to this unique price then countries will complete specialization on the balanced growth path and trade raises the growth rates of countries. Also we showed that on the transition to the balanced growth countries could also complete specialize. When countries completely specialize then we showed the transition path is saddle stable.

In the last chapter, in a similar two sector endogenous growth model where one good is being pure consumption and the other being pure investment good we showed that similarly on the balanced growth path countries complete specialize. However the effect of trade of growth depends on the type of good that country imports. If country imports capital good then its growth rate will rise while its growth rate will be unaffected by trade if it imports consumption good. We examined this result by using an extended model developed by Bond, Leblebicioglu and Schiantarelli (2006) to estimate the long-run effect of trade specialization on growth. We used yearly panel data of 92 countries for period of 1965-2000 as well as non-overlapping five year averages. Our results shows that specializing in consumption and also consumption export statistically significant affects the growth rate positively while we could not find any effect of specializing in producing capital good on growth rates.

There are possible extensions to the model that we considered in this dissertation. The model we considered did not have any scale effect, technology transfer, research or development or even we did not allow for the foreign investment or capital mobility. These simplifications deliberately considered to concentrate on studying the effect of trade through the mechanism of comparative advantage only. However we could allow for some of these effects in the model and investigate how the results might change. For example we can include foreign investment. In the case of foreign investment the country produces a good and exchanges it for the other good but keeps it in the host country instead of bring it home as it is the case with trade. When considering the foreign investment in this way, we expect to obtain similar result to what we obtained in without the foreign investment. However it is interesting to actually set up the model.

Also in the model we considered in this research productions are being processed with domestic capital only. Another possible extension then might be to allow for capital mobility across country and see if we still get the complete specialization result on the balanced growth and on the transition path. Another possible extension could be to include intermediate goods in the model. Acemuglu and Ventura (2002) have considered intermediate goods in their model where growth arises from capital accumulation.

However in their model comparative advantage is exogenous and therefore in their model they just imposes specialization. In contrast in the model that we considered here specialization arises endogenously. Therefore it could be also useful to generalize the model Acemuglu and Ventura (2002) considered by modifying model to let specialization arises endogenously. Finally we could explore the positive and normative implications of taxes, tariffs and other trade barriers.

CHAPTER 5

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