FIELDS ANALYSIS OF TFR 604 COPPER COILS

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This paper describes the analysis of the toroïdal Bitter type coils of TFR-604 fusion device. Electric, magnetic and mechanical fields have been investigated. The major difficulty arise from the quite complicated shape of the structure.

The coil is built out of 35 imbricated helicoïdal plates maintained by pressing threaded rods. The outer boundary is a 1000 x 1000 mm square, the center circular hole is 604 mm diameter and a lot of small holes for the rods and for cooling water circuit completes the geometrical description of the structure.

. First the electric current j-vector field and the electric potential are calculated. For this purpose, a thermal computing code is used taking into account the similarity between formulations and equations of these two problems. Electric potential is prescribed along meridional boundary ; electric current is assumed to remain tangent to the other edges.

. In a second step, the magnetic field distribution inside the coil is computed using Ampere's law. The total cut current by any enclosing line is determined everywhere, and thus one gets the B vector field, the intensity of which varies from 8 to 4 Teslas.

. The magnetic pressure distribution can then be defined by the cross vector product $\vec{j} \times \vec{B}$.

. Then a mechanical calculation provides the stress componants and intensities everywhere inside the coils. The resulting forces applied to the coil are obtained by this way and compared with previous hand-calculations. The stress concentration appearing near the openings are in good agreement with tabulated formulas. The displacement is fairly small to ensure a satisfactory mechanical functioning of the supporting system.

. Finally a thermal analysis is carried out. First the thermal sources induced by electric current are determined. In a steady state global computation without thermal conduction, the results appear too pessimistic. So a thermal transient analysis with free conduction boundary condition is performed on a isolated element of structure involving one circular opening, thus the magnitude of induced stress can be evaluated.

All the computations have been performed with the finite element method computing code CASTEM built by Commissariat à l'Energie Atomique COCO code as mesh generator, INCA and BILBO for thermal, electric and mechanical computations. A lot of benchmark test problems have been solved by analytical way in order to make comparisons and to ensure the validity of the choosen mesh. The whole structure involves 2380 nodes and 1050 six nodes isoparametric triangular elements.

Thermal and mechanical stresses are then superimposed in the structure. The highest stress level obtained and the stress concentration remains reasonable in view of a fatigue analysis. Nevertheless, some geometrical perturbations, introduced for neutral injection devices created several critical areas from a stress point of view.

1. Introduction

The TFR 604 (Tokamak Fontenay aux Roses) magnetic structure is built out of 32 toroïdal Bitter type coils; each of them contains 35 sliced copper magnet plates; the general outline is a square (1000 mm x 1000 mm) with a 604 mm diameter central hole. The main parameters are:

. plasma diameter 604 mm

. current intensity 35000 A

. magnetic field 60 KG

The wrapped copper plates are separated by insulating sheet and perforated with holes: the circular ones enable the tightening of the coils, the oblong ones are designed for cooling water system.

Due to penetration of neutral injectors and of measurements diagnostics, these coils present two different geometries : coils with inner notch, called number one, and outer notch, called number two, are studied separately.

2. Assumptions and modelization

Calculations have been done upon these assumptions :

. The coils are supposed similar on both sides of the median plan and only half a coil is considered.

. The material is supposed isotropic.

. Calculations are based on linear theories and the problem is reduced to a plane one.

Six nodes isoparametric and triangular finite elements are used. One obtains 1056 elements and 2383 nodes for the first coil and 1050 elements and 2372 nodes for the second one. Figure 1. This kind of mesh seems to be an accurate one : the geometry is fairly well respected with a restricted number of shells.

By another way, this choice is justified insofar as we perform comparisons between the results obtained with a small plate which has been meshed in the same was as the TFR coils and the theoretical following calculations.

3. Benchmark comparisons

An infinite elastic plate is considered (fig. 2,1). It is loaded by a uniform axial stress field and electrical current (fig. 22).

The disturbance created by a circular hole (radius a and center 0) modifies the following fields :

- stresses (reference 2)
- distribution of the electric current field
- thermal induced phenomena

3.1 Mechanical problem

The stress field is given by

$$\begin{cases} \partial_x \sigma_{xx} + \partial_y \sigma_{xy} = 0 \\ \partial_x \sigma_{xy} + \partial_y \sigma_{yy} = 0 \end{cases}$$

σ_{xx} , σ_{yy} and σ_{xy} can be written as :

$$\sigma_{xx} = \frac{\partial^2 \phi}{\partial y^2} \qquad \sigma_{yy} = \frac{\partial^2 \phi}{\partial x^2} \qquad \sigma_{xy} = \frac{-\partial^2 \phi}{\partial x \partial y}$$

The energy minimum theorems lead to the equation $\Delta\Delta\phi = 0$

Considering the boundary conditions and using polar coordinates :

$$\phi(r,\theta) = f_o(r) + f_2(r) \cdot \cos 2\theta$$

with, by integration :

$$\begin{cases} f_o(r) = A_o + B_o \cdot r^2 + Co.\mathrm{Log}\, r + Do.r^2.\log r \\ f_2(r) = A_2 r^4 + B_2 r^2 + C_2 + D_2 / r^2 \end{cases}$$

From there, the stress values :

$$\begin{cases} \sigma_{rr}(r,\theta) = \frac{\sigma}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma}{2}\left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right)\cos 2\theta \\ \sigma_{\theta\theta}(r,\theta) = \frac{\sigma}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta \end{cases}$$

(figure 2.3)

3.2 Electrical problem

Let $\vec{j}_\infty$ the electric current field far from the hole, the electric potential is :

$$\Delta V = 0 \qquad \forall\ (r,\theta)$$

$$\overrightarrow{\mathrm{grad}}\, V.\, \vec{e}_r = 0 \qquad \forall\theta\ ,\ r = a$$

$$\vec{j}_\infty = \frac{1}{\rho}\ \overrightarrow{\mathrm{grad}}\, V \qquad \forall\theta\ ,\ r_\infty$$

Considering periodicity and parity besides the boundary conditions, one can write :

$$V(r,\theta) = f(r) \cdot \cos\theta$$

and the derivative :

$$\vec{j} = j_\infty \left[\vec{e}_x - \frac{a^2}{r^2}\left(2\,\vec{e}_r \cos\theta - \vec{e}_x\right)\right]$$

on the edge of the hole

$$V(\theta) = 2\,.\,\rho\,.\,j_\infty\,.\cos\theta$$

$$\vec{j}(\theta) = -2\, j_\infty\,.\sin\theta\,.\,\vec{e}_\theta \qquad \text{(fig. 2.4)}$$

The associated ohmic effects are

$$T = \alpha\cdot(\overrightarrow{\mathrm{grad}}\, V)^2$$

$$T(r,\theta) = \alpha\,.\,\rho^2\,.\,j_\infty^2 \left(1 + \frac{a^4}{r^4} - \frac{2a^2}{r^2}\cos 2\theta\right)$$

It is obvious that the temperature ranges between 0 and 4 $(\alpha\, \rho^2\, j_\infty^2)^{\circ}$ along the hole.

4. Magnetical computation

The electromagnetic calculation is carried out with BILBO, part of the finite elements system CEASEMT which is usually used for thermal problems.

Indeed, when the voltage is applied to the coil, a current $\vec{j}$ appears in the coil, $\vec{j}(M) = \frac{1}{\rho}\,\overrightarrow{\mathrm{grad}}\, V$ is the electric current vector. The voltage is, as we consider half a coil, Vo/2. The electrical equilibrium equations are

$$\begin{cases} \Delta V = 0 & \text{in the coil} \\ V(M) = Vo/2 & \text{along } \overrightarrow{AB} \\ V(M) = 0 & \text{along } \overrightarrow{CD} \\ \overrightarrow{\mathrm{grad}}\, V\,.\,\vec{n} = 0 & \text{along the other edges} \end{cases}$$

The problem is exactly the same as a thermal one : determination of temperature field when given temperatures or given flux exchange is prescribed along the edges. So the distribution of the electric current and the potential are determinated (fig. 4) considering the Ampere's law and the axial symmetry, it can be written

$$2 . \pi . R\ (M) . B\ (M) \;=\; \mu_o . N . I\ (M)$$

I (M) : induced current

B (M) : the magnetic field vector depends on the position of the point M at a distance R(M) from the radius center.

I (M) is calculated step by step according to the following relation

$$I\ (M) \;=\; I\ (P) + \int_P^M \vec{j}\ (M) \cdot \vec{d}_m$$

Considering I(A) = 0 on the external edge of the structure.

In another way, the electric resistance of the coil can be calculated

. first as if the magnet was a thread, one obtains : $R = \rho \frac{L}{S} = \rho \frac{L^2}{V}$

V is the volume coil.

L is an average length

. and then by integration in the area as it has been done for a similar structure : reference 1.

So, two values have been calculated which are in good agreement with the finite elements computation : R = U/I U is prescribed and I can be estimated as $\int j \cdot d_s$ along, for example, the symmetry axis.

The isovalues curves of the magnetic field reveal the 1/R term given by the Ampere's law in the B expression : figure 5; these curves have been drawn by using another part of the CEASEMT system : ESPACE.

5. Mechanical calculation

A hand calculation gives an approximate value of the forces : the coil is assimilated as a thread in which exists a current intensity of 35000 A, the Laplace's law gives the expression of the distributed pressure forces : $df = B \wedge I . dl$

for a circular thread, for 35 magnets which are 2,8 mm thick and for a current intensity of 35000 A, f_x is worth 195 tons.

By another way, the magnetic pressure is given by

$$p = \frac{B^2}{2\mu o}$$

$$f_x = \int_o^{2\pi} \frac{B^2}{2\mu o} \cdot a \cdot d\alpha \cdot \cos\alpha$$

with $\begin{cases} B = Bo \dfrac{r_o}{r} \\ r = r_o - a . \cos\alpha \end{cases}$

a mechanical calculation provides the nodal displacements, the stress field at any point and the resulting forces, when the magnetic pressure distribution given by the cross product $\vec{P} = \vec{j} \wedge \vec{B}$ is applied to the coil.

Computations of stresses, and displacements are performed with the finite elements code INCA which is based ou plane stress assumption.

The obtained results are in good agreement with the previous approximate values :

- coil number one

F_x = 190,7 tons $\quad$ F_y = 272 tons

- coil number two

F_x = 201,9 tons $\quad$ F_y = 290 tons

Concerning stresses, average Von Mises is about 6 kgf/mm2.

The stresses are the highest near the openings where B is maximum.

6. Thermal computation

The current flow induces inside the coils rise in temperature.

For every unit of time and volume, a thermal evaluation gives the heat quantity

$$dQ = \rho j^2 \cdot dt \cdot dV$$

Following the mechanical one, a thermo elastic calculation is carried out considering a temperature repartition in the coil number one.

This calculation does not make allowance of thermal conduction, the results give highest stresses near the small holes as it has been shown by the previous analytical calculation when a computation has been performed with a small part of the TFR mesh; it is a transient thermal analysis which reveals smoothing in the temperature distribution : the heat moves from the hot places to colder ones.

Nevertheless, these results are only qualitative ; a transient thermal analysis concerning the whole structure would be necessary, taking into account good boundary conditions.

7. Conclusion

A lot of different problems have been shown for the analysis of the coil: electrical, thermal, magnetic, mechanical problem.

It is remarkable that all these problems have been solved using only one finite elements general code CASTEM with various and original tricks.

The results, in good agreement with hand calculations, good engineering practice and theoretical developpment show that this general procedure for coil analysis is fairly accurate.

References

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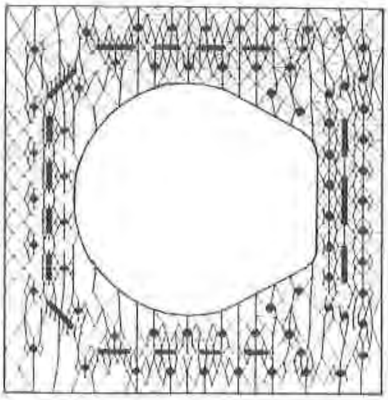
P.H. REBUT, J.L. BERAUD, M. HUGUET, Z. SLEDZIEWSKI, A. TOROSSIAN

[2] "Photoelastic and strain measurements of the effects of circular notes on the distribution of stress in tension members"
Engineering LONDON

[3] "Système CEASEMT - Notices d'utilisation des programmes COCO, BILBO, INCA, ESPACE"

Fig. 1 : Mesh of coils



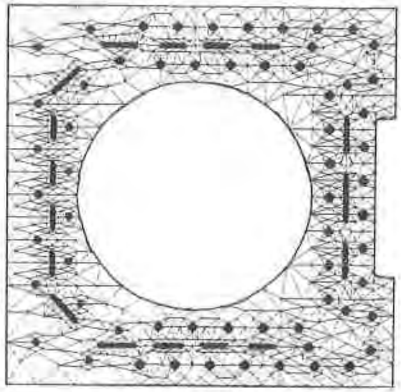


Fig. 3 : Boundary conditions for thermal computation

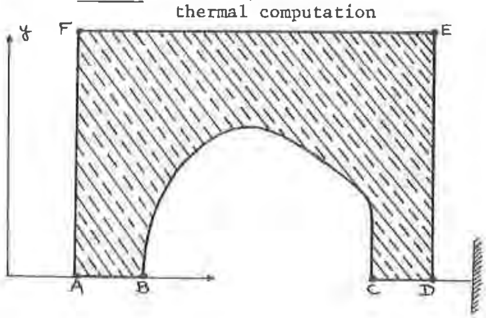


Fig. 5 : Isovalues : magnetic field

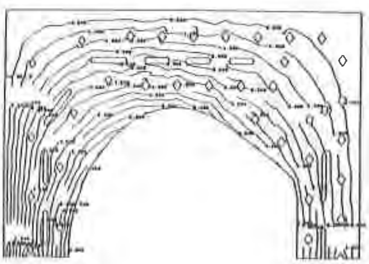


Fig. 4 : Isovalues : potential

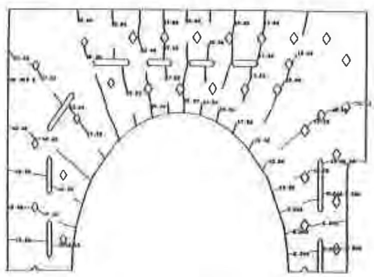


Fig. 2.1 : Small part of the TFR mesh

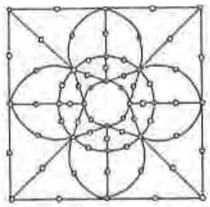


fig. 2.2 : uniform axial stress

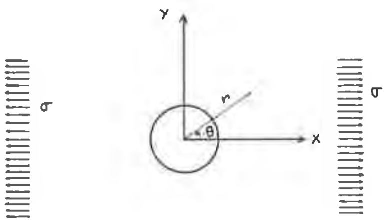


fig. 2.3 : Stress distribution

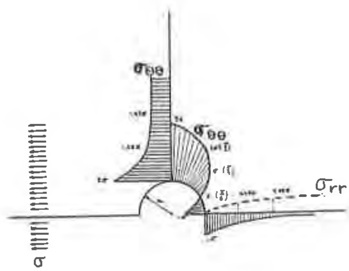


fig. 2.4 : Potential distribution