Generation of Target Power Spectral Density using Design Spectrum Compatible Artificial Time Histories

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ABSTRACT

Design regulations for nuclear power facilities require that the Time History (TH) used as input motion in seismic analysis of structures should envelop a proper target Power Spectral Density (PSD) in addition to a Design Response Spectrum (DRS). Standard Review Plan (SRP) Section 3.7.1 of United States Nuclear Regulatory Commission (USNRC) states only a target PSD function corresponding to the DRS of USNRC Regulatory Guide (RG) 1.60 and describes the equation calculating PSD from the TH. However, engineers sometimes face the problem of defining a valid target PSD related to a specific DRS as well as input TH. This paper proposes a procedure to develop the target PSD corresponding to an arbitrary DRS whose shape is different from the DRS of RG 1.60. The target PSD is generally calculated through statistical methods using many THs compatible with the specific DRS. Since TH has a crucial effect on the shape of the target PSD, artificial THs closely compatible with the DRS are used instead of real earthquake motions or roughly generated THs using conventional methods. The elaborately generated TH gives more intimate dynamic response with respect to the given DRS. Final target PSD can be established by applying the smoothening process such as averaging, windowing and linearization to the PSD of each TH. In a numerical example analysis using a DRS, named CMS1+, partially modified from the RG 1.60, the calculated PSD values are compared with those of SRP Section 3.7.1. Consequently, the procedure of this study provides reasonable target PSD related to the given DRS with fewer THs.

INTRODUCTION

Design regulations require the Acceleration Time History (ATH) used in the seismic analysis of nuclear power plant structures to meet a target PSD function in addition to a DRS. The revision of USNRC SRP Section 3.7.1 describes the procedure for calculating PSD functions from a given TH, but the specific shape of the target PSD related to other DRS whose shape is different from the DRS in USNRC RG 1.60 is not mentioned. That kind of PSD should be constructed using ATHs compatible with the DRS [1]. In general, PSD functions of the ATHs have seriously fluctuated peaks due to their randomness, which is one of the main factors for the convergence of the target PSD curve to a simple and consistent form. This paper presents a procedure constructing target PSD function using ATHs compatible with a specific DRS in order to reduce these variations. The PSD functions calculated by this method have reduced fluctuation compared to those from the ATHs incompatible with the DRS. The averaged target PSD function obtained through smoothening process gives definite form that can be easily converted into an equivalent linear curve. Furthermore, such a process can be completed within only twenty or thirty THs. The effectiveness of the proposed procedure in this study is identified through an example using numerical analysis.

CALCULATION OF ATH COMPATIBLE WITH DRS

The ATH $\ddot{Z}(t)$ can be expressed in any periodic function containing sinusoidal waves with random phase angles as follows [2]:

$$\ddot{Z}(t) = I(t)\sum_{i=1}^{n} A_i \sin(\omega_i t + \varphi_i) \tag{1}$$

where A_i, ω_i, and φ_i are the amplitude, the frequency, and the phase angle of the *i*-th contributing sinusoid, respectively; $I(t)$ is the deterministic intensity function which changes spectral responses of the stationary random wave into the non-stationary wave form; *t* represents time.

Spectral amplitudes of the time history obtained from Eq. (1) are modified in a time domain to match design spectra as closely as possible. The response of a single-degree-of-freedom to an initial time history is described by:

$$\ddot{X}(t) + 2\omega_i \xi \dot{X}(t) + \omega_i^2 X(t) = -\ddot{Z}(t) \tag{2}$$

in which $X(t)$ is the time history of relative displacement response; dots over $X(t)$ denote its time derivatives; ω_i is the *i*-th natural frequency of the oscillator; ξ is the damping expressed as a fraction of critical damping.

Let the spectral response of the time history be $S_R(\omega_i, \xi)$ and let the design spectrum be $S_T(\omega_i, \xi)$ at frequency ω_i for damping ξ. The spectral response needs to be corrected to match the design spectrum by the amount computed by:

$$\delta S(\omega_i, \xi) = S_R(\omega_i, \xi) - S_T(\omega_i, \xi) \tag{3}$$

Based on the assumption that the time at which the spectral response occurs is not perturbed by a small adjustment of the time history, $\delta S(\omega_i, \xi)$ can be related to the small adjustment in the initial time history, $\delta \ddot{Z}(t)$, through the following Duhamel's integral:

$$\delta S(\omega_i, \xi) = \int_0^{t_{mi}} \delta \ddot{Z}(\tau) h_i^k (t_{mi} - \tau) d\tau \tag{4}$$

Where

$$h_i\ (t) = -\frac{1}{\omega_{Di}} \exp(-\xi \omega_{Di} t) \sin \omega_{Di} t \tag{5}$$

is the acceleration impulse response function of a single-degree-of-freedom system having frequency ω_i and damping ξ ($\omega_{Di} = \omega_i \sqrt{1-\xi^2}$); t_{mi} (abbreviated form of $t_m(\omega_i)$) is the time at which spectral response occurs at frequency ω_i; τ is the time lag. The correcting function $\delta \ddot{Z}(t)$ from Eq. (4) is needed to adjust the spectral response of the initial time history, and can be expressed as follows [3]:

$$\delta \ddot{Z}(t) = b' f'(t) \tag{6}$$

where b' is the unknown constant coefficient to be determined at the frequency where the maximum spectral response difference is given, $f'(t)$ is the acceleration impulse response function. This adjustment is applied only for the spectral matching frequency having the maximum spectral response difference at spectral matching frequencies for all damping values. The acceleration impulse response function can be specified as follows:

$$f'(t) = h_i(t_{mi} - t) \tag{7}$$

Substituting Eq. (6) into Eq. (4), the maximum spectral response difference in every iteration cycle $\delta S^{\max}$ can be expressed as follows:

$$\delta S^{\max} = C'b' \tag{8}$$

where

$$C' = \int_0^{t_{mi}} h_i(t_{mi} - \tau)\, f'(\tau)\, d\tau \tag{9}$$

Using Eqs. (7) and (9), Eq. (8) is expressed as follows:

$$\delta S^{\max} = b' \int_0^{t_{mi}} [h_i(t_{mi} - \tau)]^2\, d\tau \tag{10}$$

The unknown constant coefficient b' can be obtained from Eq. (10), and the adjustment to remove the maximum spectral response difference that occurred in every iteration cycle can be determined from Eq. (6). The adjusted time history for cycle $(n+1)$ can be obtained from the time history of the previous cycle (*n*) as follows:

$$\ddot{Z}_{n+1}(t) = \ddot{Z}_n(t) + \delta \ddot{Z}_n(t) \tag{11}$$

The above iterative scheme is repeated until the desired level of accuracy for matching the design spectrum values and spectral responses of the time history can be achieved.

PROCEDURE FOR CALCULATION OF TARGET PSD

The ATHs representing dynamic response characteristics of a DRS can be generated using the above scheme. The one-sided PSD function $S_i(\omega)$ using the Fourier amplitude $|F(\omega)|$ of each individual time history is defined as follows [1]:

$$S_i(\omega) = \frac{2|F(\omega)|^2}{2\pi T_D^i}, \quad i = 1, 2, \cdots, N \tag{12}$$

in which T_D^i is the strong motion duration over which $F(\omega)$ is evaluated. *N* is the total number of time histories generated to construct target PSD function. The mean PSD function of the time histories, designated as $S_m(\omega)$, which corresponds to the averaged response spectra of the time histories is calculated at each frequency as follows:

$$S_m(\omega) = \frac{1}{N}\sum_{i=1}^{N} S_i(\omega) \tag{13}$$

T_D^i representing the strong motion duration of each time history $a_i(t)$, is defined to be the duration over which the slope (power) of the cumulative energy time history plot is nearly constant and near maximum [4]. Thus, let $E_i(t_p)$ be the cumulative energy at the time t_p, in which *p* is the percentage of the total cumulative energy, $E_i(t_p)$ can be computed as follows:

$$E_i(t_p) = \int_0^{t_p} a_i^2(t)dt, \quad i = 1, 2, \cdots, N \tag{14}$$

Based on the plot of Eq. (14), t_p is taken to be at p=5% and 75%, respectively, for the lower and upper bounds. Thus, T_D^i of $a_i(t)$ can be obtained as follows:

$$T_D^i = t_{75\%}^i - t_{5\%}^i, \quad i = 1, 2, \cdots, N \tag{15}$$

The common equivalent-stationary duration T_D is determined as the mean values of individual T_D^i calculated from Eq. (15), i.e.,

$$T_D = \frac{1}{N}\sum_{i=1}^{N} T_D^i, \quad i = 1, 2, \cdots, N \tag{16}$$

The mean PSD function of the time histories, $S_m(\omega)$, as determined from Eq. (13) is smoothed using the moving average technique over a ±20% frequency-bandwidth centered at the frequency of the PSD function value to be smoothed within the bandwidth, in accordance with the recommendation in NUREG/CR-5347. The resulting smoothed mean PSD function is designated herein as $\tilde{S}_m(\omega)$.

To be consistent with the DRS which is piecewise log-linear response spectrum over several frequency bands, the smoothed mean PSD function $\tilde{S}_m(\omega)$ is again smoothed using log-linear functions over the same frequency-bands or additionally subdivided frequency-bands in order to more closely represent the computed PSD function. The criteria used in obtaining the segmentally log-linear mean PSD function, designated as $\overline{S}_m(\omega)$, are that (1) the segmentally-smoothed log-linear mean PSD function is connected over the same frequency-bands as defined in the DRS, and (2) the cumulative total power of the non-segmentally-smoothed log-linear PSD function over the entire frequency range of interest is equal to or greater than the corresponding cumulative total power of the non-segmentally-smoothed PSD function.

The computed log-linear PSD function is equal to the mean plus one standard deviation (m+σ) level target PSD function which corresponds to the (m+σ) level DRS. The PSD function of an ATH, having its time-history response spectra compatible with and enveloping the corresponding design response spectra, should generally envelop the "minimum-required target PSD function" consistent with the DRS. The minimum-required target PSD function is defined to be 80% of the (m+σ) level target PSD function, i.e.,

$$\hat{S}_m(\omega) = 0.8\overline{S}_m(\omega) \tag{17}$$

where $\hat{S}_m(\omega)$ is the minimum-required target PSD function.

NUMERICAL EXAMPLE

The analysis example shows the calculation of a target PSD function related to the modified DRS from RG 1.60. The modified DRS, called Control Motion Spectrum 1+ (CMS1+), complies with the DRS in RG 1.60 and is enriched in the high frequency range. The high frequency enrichment is accomplished in the following manner:

(a) Increase the acceleration of the DRS in RG 1.60 at 25 Hz by the factor of 1.30.
(b) Linearly vary the DRS, on log-log-scale, from the DRS in RG 1.60 from 9 Hz to the amplified spectrum at 25 Hz.
(c) Linearly vary the DRS, on log-log-scale, from the amplified spectrum at 25 Hz to the PGA at 40 Hz.

Table 1 shows the spectral amplification values on the horizontal component of the CMS1+ at the following control frequencies, 0.25, 2.5, 9, 25 and 40 Hz, for damping values of 2%, and its spectral curve is plotted in Fig. 1.

Fifty ATHs roughly compatible with a 2% damped DRS are generated, and their spectral accelerations are plotted in Fig. 2. Roughly matched spectral shapes of the ATHs are again modified to be compatible with the given DRS more closely. After matching spectral acceleration of the ATH with the 2% damped DRS, the Root Mean Square (RMS) difference of spectral accelerations between ATH and DRS is 0.5%, and their final spectral shapes of the ATHs are also illustrated in Fig. 3. Raw PSD functions of each ATH calculated using Eq. (12) are shown in Fig. 4. The Raw PSD values calculated at each frequency step are averaged in order to remove spikes of the raw PSD functions, and the averaged PSD values obtained using Eq. (13) to the case of 10 and 50 ATH sets are plotted in Fig. 5. According to the regulation of USNRC SRP Section 3.7.1 that the calculated PSD function should be smoothed using the moving average technique over a ±20% frequency-bandwidth centered at the frequency of the PSD function value, the averaged PSD function is smoothed by the moving average technique. The smoothed PSD functions to the 10, 20, 30, 40 and 50 ATH sets shown in Fig. 6 give almost the same PSD values.

To be consistent with the log-linear DRS over several frequency bands, the smoothed PSD function is segmentally linearized using log-linear functions over the same frequency-bands or additionally subdivided frequency-bands. The lowest and highest frequencies for the PSD function, 0.3 Hz and 24 Hz respectively, are the same values defined in the regulation of USNRC SRP Section 3.7.1. Three intermediate control frequencies, 2.5 Hz, 9 Hz, and 16 Hz, are selected to linearize the smoothed PSD function. The linearized PSD curves in Fig. 7 to the five different ATH sets are almost accorded with one another. Finally, the linearized PSD function is scaled down by multiplying scale factor of 0.8 given in USNRC SRP Section 3.7.1 in order to obtain the minimum required level of mean PSD function. The PSD functions calculated from all ATH sets except 10 ATH set give almost the same results as being found from Fig. 6 and Fig. 7. Figure 8 shows the comparison of the PSD functions by USNRC SRP Section 3.7.1 and CMS1+. The PSD values of the above 9 Hz as shown in Fig. 8 are enriched in accordance with the DRS amplified above 9 Hz.

Table 1. Spectral Amplitude of Horizontal CMS1+ for Control Points

Damping Ratio (%)	Amplification Factor for Control Points						Remarks
	0.2 Hz	0.25 Hz	2.5 Hz	9 Hz	25 Hz	40 Hz	
2	0.37	0.57	4.25	3.54	1.70	1.00	Scaled to 1.0 g

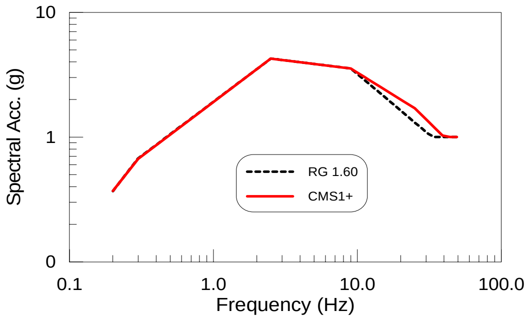

Figure 1. Comparison of DRS in RG 1.60 and modified DRS used for analysis example

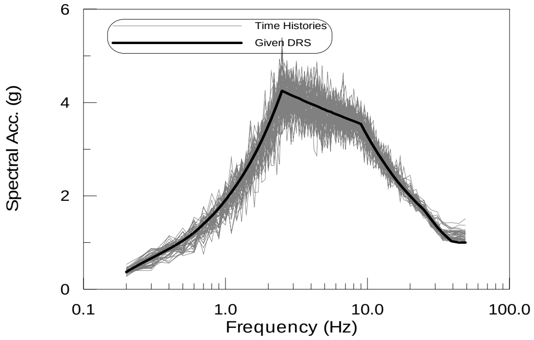

Figure 2. Roughly matched spectral acceleration shapes of 50 ATHs with 2% damped DRS in RG 1.60

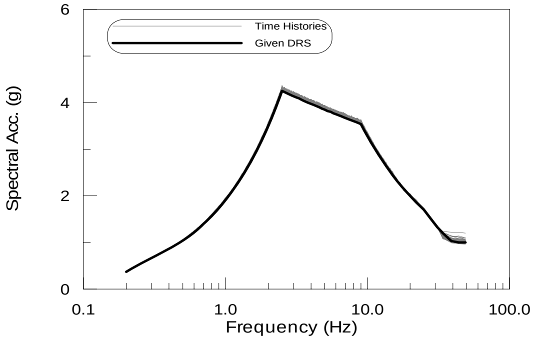

Figure 3. Closely matched spectral acceleration shapes of 50 ATHs with 2% damped DRS in RG 1.60

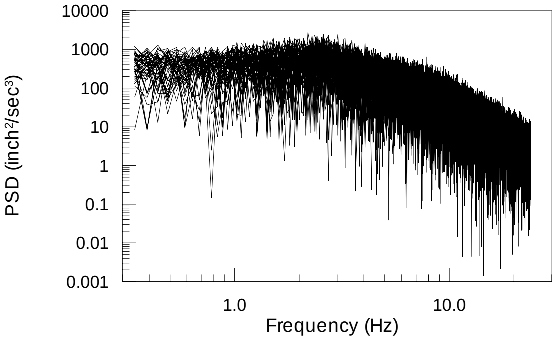

Figure 4. Raw PSD function of 50 ATH set

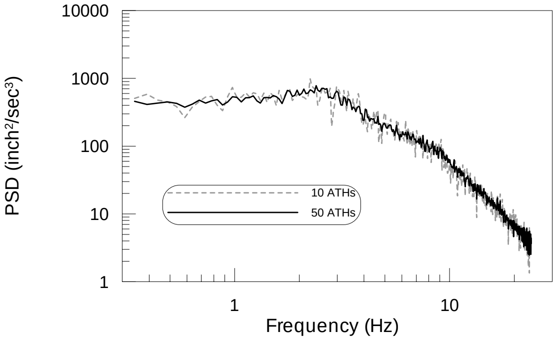

Figure 5. PSD functions averaging 10 ATH set and 50 ATH set

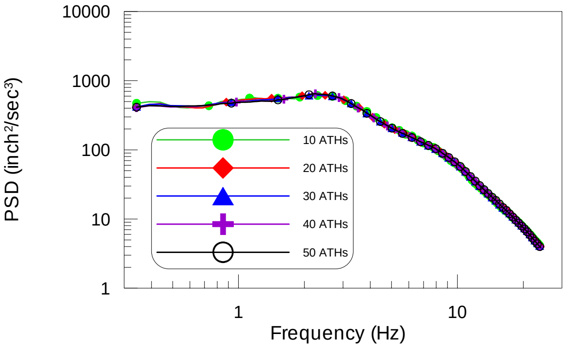

Figure 6. PSD functions smoothed by moving average technique

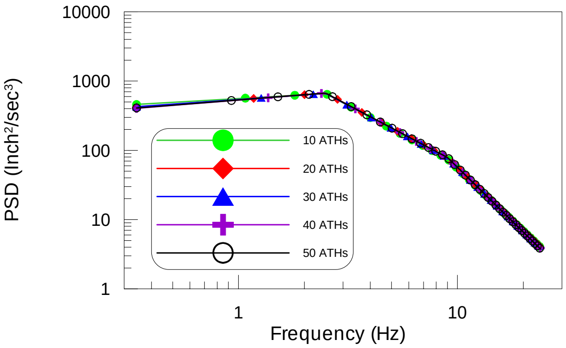

Figure 7. Linearized PSD functions

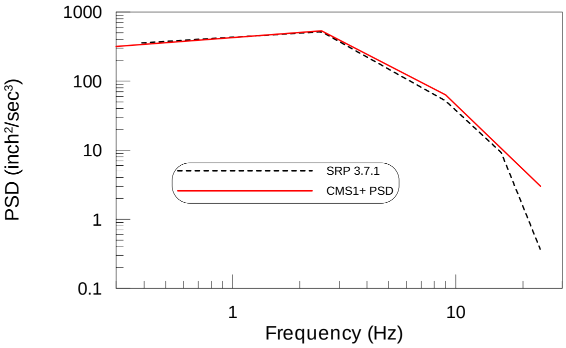

Figure 8. Comparison of PSD functions corresponding to DRS in RG 1.60 and DRS of CMS1+

CONCLUSIONS

In accordance with USNRC SRP Section 3.7.1, this paper proposes a procedure for calculating the target PSD corresponding to an arbitrary DRS whose shape is different from that of RG 1.60. Since time histories have crucial effect on the specific shape of the final target PSD, precisely generated time histories compatible with the DRS are used instead of real earthquake motions or roughly generated time histories by conventional methods. This procedure satisfies the requirements of USNRC SRP Section 3.7.1, and the resultant target PSD is consistent with the given DRS. Using ATHs closely compatible with a DRS leads to more effective procedure to obtain a corresponding PSD level than using real earthquakes or roughly generated THs. Consequently, reasonable target PSD curve of a specific DRS can be obtained by using 20 ~ 30 THs compatible with the DRS.

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