On the Influence of Geometric Imperfections and Thermal Gradients on Elastic-Plastic Buckling of Shells

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Abstract

This paper is aimed at the fast prediction of buckling loads, account being taken of the geometric imperfections and thermal gradients (if any).

On a very simple case (double pin-ended beam) it is shown how the collapse load can be assessed taking into account the present shape imperfection and assuming elastic-plastic behaviour. To compute this, it is necessary to :

- establish the nonlinear moment-curvature relationship,
- write the equilibrium equations "on the deformed body".

The moment-curvature relationship depends on the elastic-plastic properties of the material and the shape of the cross section (of the beam). No simplifying hypothesis is necessary at this stage. Equilibrium "on the deformed body" is a convenient way of introducing large displacements non-linearity.

In this way it is possible to construct a diagram showing the interaction between buckling and plastic collapse. This diagram is obtained by assuming either a triangular shape or a sinusoïdal one for the deformed state.

It consists of a set of curves plotted in $\frac{P}{P_L}$, $\frac{P}{P_E}$ coordinates (P_L : limit load ; P_E : Euler buckling load), each curve corresponding to a certain amplitude of the initial imperfection.

This approach can be extended to the case of shells. To achieve that, it is necessary to generalise :

- The law predicting the amplification of the defect as a result of large displacements,
- The law giving the increase of bending stresses in function of the existing membrane stresses,
- The criterion of collapse.

This diagram can assist the designer when dealing with structures prone to elastic-plastic buckling, allowing a quick assessment of the influence of the different factors : slenderness, geometric imperfections, thermal or residual stresses.

1. Introduction

By comparison with the "classical" Euler buckling, plasticity, geometric imperfections and thermal loadings give rise to extensive changes in structural behaviour and generally lower the collapse load.

At the present time, predicting such phenomena on real structures requires long and costly computations. There is consequently a need for simple methods allowing quick assessment of the sustainable load when plasticity, shape imperfections and thermal gradients are present. Since literature is rather scarce on the subject, we considered it necessary to get a better understanding of the effects involved by studying simple models.

2. Interaction between buckling and plastic collapse

When a real structure buckles in the elastic-plastic field, two collapse modes interact :

- the elastic buckling, characterized by the Euler buckling load P_E,
- the plastic collapse, characterized by the limit load P_L.

In order to quantify this interaction, the actual collapse load P_c may be related to both P_E and P_L and if we plot $\frac{P_c}{P_L}$ versus $\frac{P_c}{P_E}$ for various thicknesses and imperfection on thickness (δ_o/h) ratios we can get a set of curves that we call an interaction diagram. It is easy to see that in such a diagram a given structure is represented by a straight line passing through the origin with a slope $\frac{P_E}{P_L}$.

This interaction diagram was first constructed for a simple model : a double pin-ended beam with a rectangular cross-section (see figure 1), made of an elastic-perfectly-plastic (EPP) (no strain-hardening) material. This diagram was obtained by three different ways corresponding to increasing degrees of refinement regarding the basic assumptions:

- The first approach relies on the following two assumptions :
  - the relationship between the deflection δ and the applied load P has the same expression $\delta = \frac{\delta_o}{1 - \frac{P}{P_E}}$ as in the elastic field.
  - the beam collapses when P reaches a value such that the mid-length cross section is fully plastic as a result of the applied force and bending moment.
- The second approach admits the same first assumption but here the collapse load is considered as beeing the maximum sustainable load taking into account the actual through-thickness stress distribution at mid-length. Thus this section is no longer required to be fully plastic.
- The third approach is very similar but also takes into account the stress distribution and the plasticity over the length of the beam.

Figures 2, 3, 4 show these three diagrams. The comparison between them leads to the following remarks :

- small imperfections ($\delta_o/h \ll 1$) lower the collapse load more than could be anticipated from a simplified analysis (i.e. according to the first diagram).
- for large imperfections ($\delta_o/h \approx 1$) all three diagrams are in agreement, which is remarkable if one keeps in mind the simplicity of the first approach.
- the influence of imperfections is maximum when $P_E \approx P_L$.

A more detailed presentation of these diagrams is given in ref. [1] . These diagrams can be related to those used in steel construction (see for instance references [2], [3], [4]).

3. Influence of thermal gradients

Thermal gradients may have two kinds of effects on buckling : on the one hand, if the structure is sufficiently free, they create additional deflections and act as initial imperfections; on the other hand if the structure is prevented from deforming, they create additional stresses, which may further the onset of plasticity. In what follows, we shall only deal with the latter case.

Again, we studied a very simple model : a double clamped beam (see figure 5), with rectangular section, made of the same material as in § 2. It was subjected to a compressive force P and a uniform through-thickness gradient Δt. This loading was applied following different paths, which are shown on figure 6 :

- P alone
- Δt first, then P
- P first, then Δt (with or without cycling).

Computations were performed by the finite element method, using the INCA code (CEA-SEMT system, described by HOFFMANN [5]). We used both geometric linear bifurcation analysis on a perfect model and geometrically non-linear step-by-step computation on an imperfect model with an imperfection amplitude equal to its thickness.

The main results can be summarized as follows :

- the collapse point (figure 6) is strongly dependent on the loading path.
- the thermal gradients considerably lower the collapse load.
- the model collapses for even smaller gradients if they are cycled.
- the relationship between load and deflection at collapse does not appear to depend on the thermal loadings.

The interpretation of the first fact is easy : the gradient creates bending stresses that contribute to the initiation of plastic yielding.

The influence of the order of application of P and Δt is related to the state of stress created by the loading path : applying the gradient first and then the force causes full stress reversals, whereas this is not the case when dealing conversely. Consequently the related stress states are very different from each other, resulting in different flexural stiffnesses, which govern the phenomenon.

The collapse under an imposed force and a cycled thermal gradient is illustrated by figures [7], [8] where the increase in deflection is plotted against the number of cycles. This collapse comes from the interaction between buckling and thermal ratchetting.

NGUYEN et al. [6] are currently reviewing the theoretical aspects of this interaction. The comparison (see figure 10) of our numerical results with the ASME Code [7] rules preventing ratchetting and with BREE [8] shows that these rules cannot account for this phenomenon because of two facts :

- the primary stresses are predominantly bending stresses and not membranes stresses as in the classic ratchetting models
- the primary bending stresses increase with each cycle.

If one does not take into account the latter effect, collapse will not occur.

The P - δ curve (figure 9) in the first case (P alone) seems to be intrinsic, i.e. for a given value of P, the beam collapses roughly for the same deflection, δ whatever the loading path. If one imposes a load P smaller than the maximum sustainable load P_{max} of fig. 9, the effect of thermal loading will be to allow jumping from the ascending portion of the curve to the descending one.

If the gradient initially applied is maintained throughout, it is possible to find an imperfection equivalent to this gradient as is done in steel construction for residual stresses.

From the above considerations, it is possible to draw the following conclusions :

- thermal stresses have a rather complicated influence on elastic-plastic buckling. They are not, in general, similar to residual stresses.
- thermal gradients can be made equivalent to initial imperfections only if they are maintained constant after beeing applied.
- otherwise their influence depends strongly on the loading history : for the same compressive force P, buckling can be obtained for very different values of Δt, according to the loading path followed.
- in particular, cycling the gradient can lead to collapse.

In other words there is a "progressive buckling". This is a very important effect because thermal loads are unlikely to remain constant in reactor structures.

4. Extension to shells

It is possible to extend the considerations of § 2 to the case of shells under the following assumptions :

- the Euler buckling load is independent of the amplitude of the initial imperfections.
- the buckling mode does not involve membrane strains. (Consequently the presence of a defect will not affect the membrane state of stress).
- plastic flow folllows the Hencky law.

The first assumption permits use of the same load-deflection relationship as previously : $\delta = \frac{\delta_0}{1 - \frac{P}{P_E}}$. As a consequence of the second assumption the membrane state of stress may be calculated on the perfect shell. The third assumption is used to formulate a collapse criterion based on the generalized stresses M, N : f (M, N) = σ_Y (further information on Hencky's law and collapse criteria for shells may be found in HODGE [9].

Under these assumptions, the equation governing the stress state of the imperfect shell takes a simpler form and admits $M = \delta N_0$ as a particular solution. This solution is introduced in the collapse criterion and, in this way, it is possible to construct a diagram similar to those of § 2. These considerations can be found in greater detail in reference [10].

Unfortunately, such assumptions are unlikely to be confirmed for real shells, particularly the second one (no membrane strain in the buckling mode). This problem arises from the fact that membrane and bending equations are almost always coupled for shells. So it is generally not possible to modify the shape of a perfect shell (by imposing large defects) without considerably changing its plastic buckling behaviour.

5. Conclusions

There are two main interests in having established interaction diagrams for beams :

- they show the large reduction in collapse load due to geometric imperfections and the intricate effects of thermal gradients.
- they lead to some understanding of these effects.

Such results can be directly transposed to shells only in rather rare cases. Actual shell behaviour analysis requires further consideration of :

- the influence of initial imperfections on the membrane force and the Euler buckling load,
- the incidence of multiple buckling modes,
- the stability of bifurcated solutions.

These points are still under investigation.

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Fig. 1 : The double pin-ended beam model

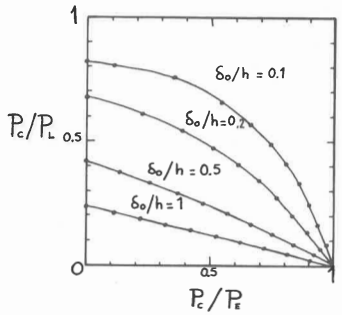

Fig. 2 : Interaction diagram n° 1

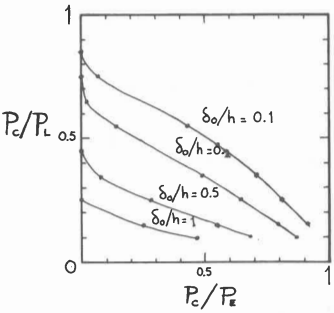

Fig. 3 : Interaction diagram n° 2

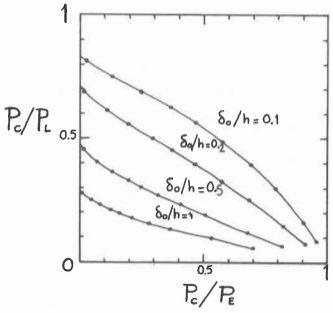

Fig. 4 : Interaction diagram n° 3

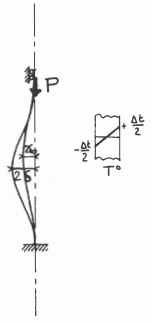

Fig. 5 : The double clamped beam model

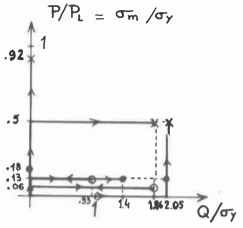

Fig. 6 : Loading paths

σ_m : membrane stress

σ_y : yield strength

Q : thermal stress

× : bifurcation (geometrically linear)

● : collapse (geometrically non-linear)

o : collapse (under cycled gradient)

: thermal cycling

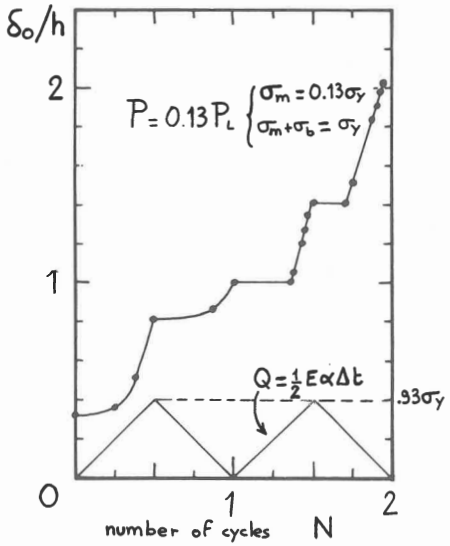

Fig. 7 : Deflection versus number of cycles - case 1

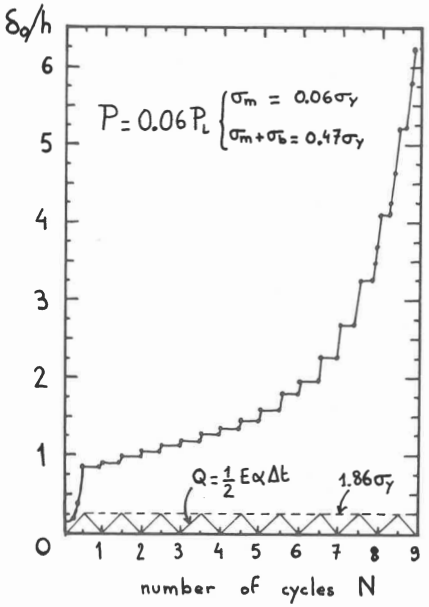

Fig. 8 : Deflection versus number of cycles - case 2

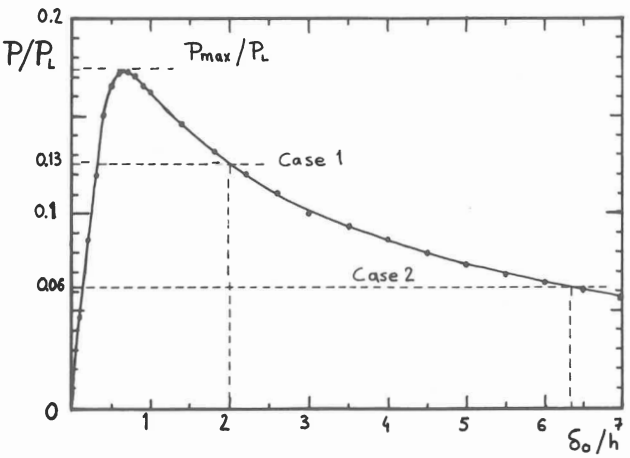

Fig. 9 : The "intrinsic" collapse curve

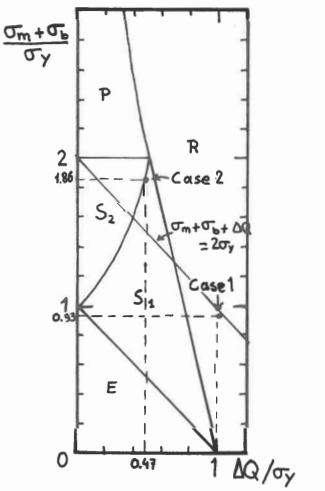

Fig. 10 (from reference 7) : Ratchetting-buckling compared with ASME rules

E : elastic behaviour

P : plastic cycling

S_1 and S_2: shake down

R : ratchetting