Lagrangian Finite Element Formulation for Fluid-Structure Interaction and Application

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ABSTRACT

The aim of this communication is to present a new finite element software (FLUSTRU) for fluid-structure interaction in a lagrangian formulation. The stiffness and damping matrices of the fluid are computed from the governing laws of the medium : the fluid is supposed to be viscous and compressible (Stokes' equations). The elementary damping matrix is computed by evaluating the power dissipated by viscosity :

$$[C]_e = \int_E \mu [B]^t [S] [B] \, dE \qquad \mu = \text{dynamical viscosity}$$

The stiffness matrix is given by :

$$[K]_e = \int_E k [B]^t [D] [B] \, dE \qquad k = \text{bulk modulus}$$

An additionnal stiffness is applied to the free surface's nodes. The mass matrix is computed identically as the mass matrix of the structure.

The main problem stated by the lagrangian formulation of the fluid is the presence of spurious free-vibration modes (zero energy modes) in the fluid. Those modes are generated by the particular form of the matrix [D] . These spurious modes have been examined and two particular methods to eliminate them have been developed : industrial applications prove the efficiency of the proposed methods.

1. Introduction

Problems of fluid-structure interaction appear in various and diversified dynamical analysis : in fact, everywhere a structure and a fluid vibrate together :

- nuclear engineering
- explosion problems in gaz reservoirs
- acoustic problems such as noise in electrical transformer
- seismic loading of a dam reservoir
- ship building
- ......

A dynamical analysis of a fluid-structure system is generally achieved by using two models (a structural model and a fluid model) and by coupling them. The structural model is usually based on a lagrangian formulation. For the fluid model, two formulations can be used : a lagrangian formulation or an eulerian formulation. A lagrangian formulation of the fluid medium is easier to couple with an existing structural model but this formulation unfortunately generates spurious modes in the fluid mesh : these modes need a special treatment otherwise the mathematical model is unstable.

After finite element discretization, the dynamical behaviour of a structure is governed by the well-known equation :

$$[M]\left\{\ddot{q}\right\} + [C]\left\{\dot{q}\right\} + [K]\left\{q\right\} = \left\{F(t)\right\} \qquad (1.)$$

with the initial conditions on the nodal displacements $\left\{q_\circ\right\}$
and on the nodal velocities $\left\{\dot{q}_\circ\right\}$

The problem is now to compute the elementary matrices of the fluid mesh, to treat the spurious modes and to couple the two models.

2. Structural Model

The structural model is a classical isoparametric shell model (6 to 16 nodes, 2D and 3D) and consequently, it does not need more explanation. The shell elements used in FLUSTRU are described by C.R. Wouters in (1).

3. Fluid Model

3.1. Basic Assumptions

The fluid is supposed to be viscous and compressible. The governing laws of the fluid medium are the Stokes' equations :

$$\tau_{ij} = -p\delta_{ij} + 2\mu(V_{ij} - 1/3\ \delta_{ij}V_{kk}) \qquad (2.)$$

with summation on the subscript k

where τ_{ij} = stress tensor

p = pressure

V_{ij} = strain velocities tensor

μ = dynamical viscosity

δ_{ij} = Kronecker first symbol

The pressure is related elastically to the fractional decrease in volume $-\delta_v = -\varepsilon_{kk}$ by

$$p = - k\delta_v \quad \text{or} \quad \tau_{kk} = - 3k\varepsilon_{kk} \tag{2 bis}$$

where k = bulk modulus.

3.2. Elementary Stiffness Matrix

The general expression of the elementary stiffness matrix is :

$$[K]_e = \int_E [B]^t \, [D] \, [B] \, dE \tag{3.}$$

where $[B]$ = matrix of the derivatives of the shape functions

$[D]$ = matrix of the elastic coefficients

The fluid elements are quadratic and isoparametric (8nodes for 2D problems, 20 nodes for 3D problems). For 2D problems, the matrix $[D]$ is :

$$[D] = \begin{bmatrix} k & k & 0 \\ k & k & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.}$$

where k = bulk modulus of the fluid

The particular form of the matrix $[D]$ introduces spurious free vibration modes (zero energy-zero frequency) in the fluid, because of the absence of shear resistance in the stress-strain equation (2 bis).

An additional stiffness is applied to the free surface's nodes. Let us consider a free surface at time t=0 :

$p_A = \rho g h$ where p_A = pressure at point A, ρ = specific mass of the fluid, g = gravity,

at time $t > 0$:

$$p_A = \rho g(h + \delta_t)$$

The displacement of the free surface generates a proportionnal pressure $\rho g \delta_t$. This phenomenon can be compared to the problem of an elastic continuous support. The additional stiffness matrix is then :

$$[K]_f = \int_{S_f} [N]^t \left\{g\right\} \left\{g\right\}^t [N] \, dS_f \tag{5.}$$

where S_f = free surface

$[N]$ = shape functions

$\left\{g\right\}$ = gravity vector

The detection and the treatment of the free surface(s) of the fluid mesh are fully automatic.

3.3. Elementary Damping Matrix

Let us consider again the Stokes'equations. The shear velocity $\dot{\gamma}_{ij}$ $(i \neq j)$ is given by

$$\dot{\gamma}_{ij} = 2\, V_{ij} \quad \text{with } i \neq j,\ V_{ij} = \dot{\varepsilon}_{ij} \tag{6.}$$

The viscous part of the Stokes'equations takes a matricial form :

$$\{\sigma\}_{visc} = \mu [S]\ \{\dot{\varepsilon}\} \tag{7.}$$

The power dissipated by viscosity in a newtonian fluid is equal to :

$$\mathcal{P}_{visc} = \int_V \{\dot{\varepsilon}\}^t\ \{\sigma\}_{visc}\ dV \tag{8.}$$

or

$$\mathcal{P}_{visc} = \mu \int_V \{\dot{\varepsilon}\}^t\ [S]\ \{\dot{\varepsilon}\}\ dV \tag{9.}$$

The strain tensor depends on the nodal displacements as follows :

$$\{\varepsilon\} = [B]\ \{q\} \tag{10.}$$

and then :

$$\{\dot{\varepsilon}\} = [B]\ \{\dot{q}\} \tag{11.}$$

which gives :

$$\mathcal{P}_{visc} = \int_V \mu \{\dot{q}\}^t\ [B]^t\ [S]\ [B]\ \{\dot{q}\}\ dV \tag{12.}$$

The power $\mathcal{P}_{visc}$ is also given by :

$$\mathcal{P}_{visc} = \{\dot{q}\}^t\ [C]\ \{\dot{q}\} \tag{13.}$$

We find so the elementary damping matrix :

$$\boxed{[C]_e = \mu \int_E [B]^t\ [S]\ [B]\ dV} \tag{14.}$$

3.4. Elementary Mass Matrix

The elementary mass matrix does not depend on the elastic properties of the medium. As in the structural mesh, the elementary mass matrix is given by :

$$[M]_e = \int_E [N]^t \rho [N]\ dE \tag{15.}$$

3.5. Treatment Of The Spurious Modes

It is known that outside the boundary layer, the fluid behaves as perfect, and the vorticity is confined in the boundary layer. In a perfect fluid initially at rest, the vorticity remains zero. The spurious modes correspond, in a modal analysis, to vorticity modes associated with eigenvalues zero, they have no physical significance (fig. 2)

m	n	Theory	ANSYS-3D	FLUSTRU	ε-ANSYS (°/₀₀)	ε-FLUSTRU (°/₀₀)
1	0	2,28651	2,299	2,28653	5,4	0,01
0	1	2,28651	2,299	2,28653	5,4	0,01
1	1	2,86590	2,829	2,86536	13,0	0,19
2	0	3,48413	3,585	3,48521	28,1	0,3
0	2	3,48413	3,585	3,48521	28,1	0,3
2	1	3,69485	3,670	3,69327	6,8	0,4
1	2	3,69485	3,670	3,69327	6,8	0,4
2	2	4,16721	4,135	4,15808	7,8	2,2
3	0	4,29298	4,505	4,30780	47	3,4
0	3	4,29298	4,505	4,30780	47	3,4

Table 1

A first method climinates the spurious modes in the modal analysis by a shift in the eigenvalue spectrum (see M. Hautfenne [2] and A. Cormeau [3]). Table 1 gives the eigenfrequencies (sloshing modes) of a parallelipipedic reservoir (5" x 5" x 2") filled with water, compared to the theoretical values and to the ANSYS results [4]with a mesh of 5 x 5 x 2 = 50 elements (linear shape function for ANSYS and quadratic for FLUSTRU). There are 145 spurious modes in this mesh. This first method gives very good results in low DOF systems. In higher DOF systems, the spurious eigenfrequencies are not exactly zero due to roundoff errors and the low eigenfrequencies become difficult to distinguish from an approximate spurious mode which can no more be eliminated by a shift method. A second method has consequently been developped for higher DOF systems.

This second method consists to lock these modes with an additional stiffness matrix $[K]_{rot}$ such that the total stiffness is given by :

$$[K]_{tot} = [K]_e + [K]_f + [K]_{rot} \qquad (16.)$$

The locking of the spurious modes increases considerably their very small eigenfrequencies and allows their separation from the low frequencies of the free surface modes or the acoustic modes. $[K]_{rot}$ is defined by the energy associated with the rotational motions of the spurious modes :

$$E_{rot} = \frac{1}{2} \{q\}^t [K]_{rot} \{q\} = \frac{\alpha}{2} \int_E (\mathrm{curl}\ \vec{u})^2 \, dE \qquad (17.)$$

with $\vec{u}$ = displacements, and curl $\vec{u} = [R]\left\{q\right\}$

Hence

$$[K]_{rot} = \frac{\alpha}{2}\int_E [R]^t\,[R]\,dE \qquad (18.)$$

where α is a parameter (dim = pressure) whose magnitude controls the shift of the spurious eigenfrequencies by increasing the locking stiffness of these modes. Numerical experiments (see M. Hautfenne [5]) show :

- for acoustic modes $\alpha/k = 1$ to 5
- for free surface motions $\alpha/k = 10^{-4}$ to 10^{-3}

4. Fluid-Structure Coupling.

The fluid-structure coupling is performed by imposing a sliding condition between the structural and the fluid parts of the mesh. This is done by creating interface elements between the fluid and the structure. All the treatment of the fluid-structure interface is automatic. The main steps are :

1) detection of the nodes common to the structural and the fluid parts of the mesh.

2) creation of new nodes in the fluid elements in contact with the shell.

3) update of the fluid connectivity.

4) computation of the average normal to the shell at the interface nodes.

5) creation of a spring with "infinite stiffness" normal to the shell.

The displacements of the interface nodes must be computed in local axes (normal and tangent to the shell) for a good conditionning of the stiffness matrix. If an interface node belongs to two non-smooth shell elements, two different normals are defined and the corresponding fluid node slides along the edge defined by these two elements.

5. Application - Noise Problem in Electrical Transformers.

The noise problem in electrical transformers is, by one of its aspects, a fluid-structure interaction problem. The noise of a transformer is caused by the magnetostriction of its magnetical core. The noise can propagate to the external shell through :

1) the fixations between the magnetical core and the shell

2) the dielectrical oil which surrounds the magnetic core. (fluid-structure interaction).

The aim of this communication is to bring to light the basic fluid-structure phenomena as easily as possible. So, the problem has been reduced to a simplified 2D problem illustrated by figure 3. The loading of the system is an horizontal harmonic motion of the rigid magnetical core. The frequency of the load is 100 Hz because the magnetostriction is proportionnal to the square of the induction (50 Hz in Europe). The damping is neglected

Figure 4 shows the displacements field and figure 5 shows the corresponding pressure field in oil. The most important phenomenon is the presence of pressure peaks near the shell, just where the displacement amplitude is large.

6. Conclusion.

A new software for fluid-structure interaction using F.E.M. (FLUSTRU) is presented. The structure is supposed to be elastic and the fluid is supposed to be viscous and compressible. The model is expressed in a lagrangian formulation. The main advantage of this formulation is an easy coupling of the fluid and the structural parts of the mesh. A lagrangian formulation of the fluid phase generates spurious modes which must be treated before any computation : two different methods are presented.

7. Acknowledgments.

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8. References.

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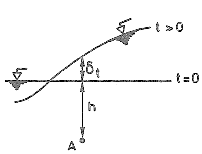

fig. 1

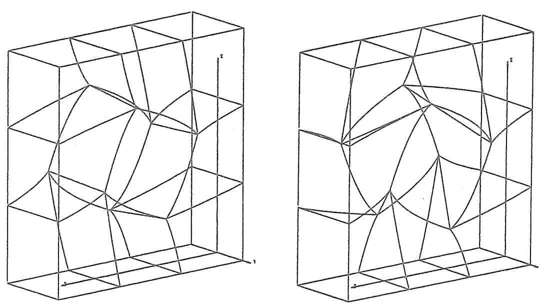

fig. 2

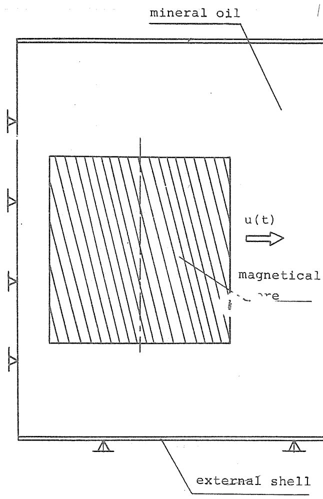

fig. 3

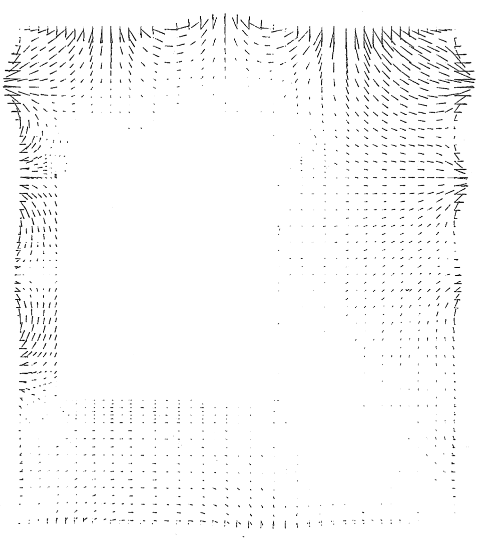

fig. 4

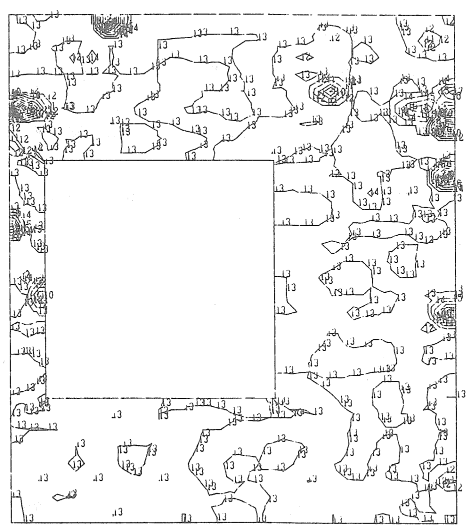

fig. 5