A Numerical Analysis Method on Thermal and Shrinkage Stress of Concrete

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1 INTRODUCTION

It is well known that thermal stresses often cause cracks in large scale concrete such as the one for dam construction. Drying shrinkage of concrete causes cracks in concrete structure. The thermal stress and the drying shrinkage stress may be the main reasons the cracks occur in concrete, however, there are few researches in which they are dealt together. The problems on the thermal stress and on the drying shrinkage are not independent and should be dealt together, because both the temperature and the water content of concrete affect hydration reaction, and the degree of hydration determines all characteristics of concrete at early age. In this paper, the degree of hydration is formulated experimentally, and a numerical stress analysis method considering the hydration reaction is presented.

2 FORMULATION OF THE VELOCITY OF HYDRATION REACTION

The velocity of hydration reaction is assumed to be formulated by the following differential equation.

$$dw_n/dt = a(T)\cdot(W_n(w)-w_n)^2 \tag{1}$$

where dwn/dt; increment of combined water per unit time, $a(T)$; rate coefficient, function of temperature, wn; amount of combined water per unit volume of concrete (kg/m^3), w; water content (kg/m^3), $Wn(w)$; ultimate combined water (kg/m^3), function of water content of concrete.

Assuming that the velocity of hydration reaction of concrete is similar to that of cement paste of the same water-cement ratio, $a(T)$ and $Wn(w)$ were examined on the cement paste.

The relationship between temperature and rate coefficient $a(T)$ are shown in Fig.-1. Solid circles and triangles represent the experimental results. the real and broken lines in Fig.-1 represent the following equations presented by Bazant (ref.[2]).

$$a(T) = a(T_0)\cdot\beta_T(T) \tag{2}$$

$$\beta_T(T) = \exp(Qh/RT_0-Qh/RT) \tag{3}$$

where $Qh/R=2500K$ (ref.[2]), T_0; standard temperature, $a(T_0)$; empirical rate coefficient at T_0.

Experimental values approximately agree with the ones calculated by eqs.(2)

and (3), and therefore it can be said that a(T) is given by eqs.(2) and (3).

In order to examine the influence of water content on the velocity of hydration reaction, the following experiment was carried out. Specimens were cylinders of 5cm in diameter and 10cm in height, and made of cement paste. They were desiccated in a vacuum drying machine from 6 hours after placing, and were controlled to contain designated water. The relationships between the drying time and the amount of desiccated water are described in Tables-1 and 2. In the tables, W, c, wd and w are the amount of mixing water, cement, desiccated water and water content after drying, respectively. The amount of combined water was measured at the age of 1, 2, 3, 7 and 14 days. The ultimate combined water was calculated by the least squares method. The result is shown in Fig.-2. The vertical axis represents the ultimate combined water-cement ratio, and the horizontal axis represents the apparent water-cement ratio after drying. The solid and void circles are the empirical values of water-cement ratio at the mixing, 30% and 50%. The influence of water content on the ultimate combined water Wn can be given by the following equations.

$$W_n(w)/c = 0.29 - 0.09/(w/c+0.31) \qquad (W/c=30\%) \qquad (4)$$

$$W_n(w)/c = 0.36 - 0.12/(w/c+0.34) \qquad (W/c=50\%) \qquad (5)$$

As for the results of water-cement ratio 50% at the mixing, the calculated value approximately agrees with the experimental value. Therefore the present equations is considered to be applicable for concrete with the common mix proportion.

Table-1 Drying time and dessication of water(W/C=30%)

	W(g)	c(g)	w_d(g)	w(g)	w/c(%)	w/W(%)
In water	95	315	-9.6	104.6	33.2	110.1
Sealed curing			1.3	93.7	29.7	98.6
5 hours			14.8	80.2	25.5	84.4
10 hours			24.7	70.3	22.3	74.0
19 hours			32.3	62.7	20.0	66.0
42 hours			43.7	51.3	16.3	54.0

Table-2 Drying time and dessication of water(W/C=50%)

	W(g)	c(g)	w_d(g)	w(g)	w/c(%)	w/W(%)
In water	117	233	-6.8	123.8	53.1	105.8
Sealed curing			0.8	116.2	49.9	99.3
4 hours			13.3	103.7	44.5	88.6
16 hours			26.7	90.3	38.8	77.2
24 hours			42.6	74.4	32.0	63.6
41 hours			60.1	56.9	24.4	48.6

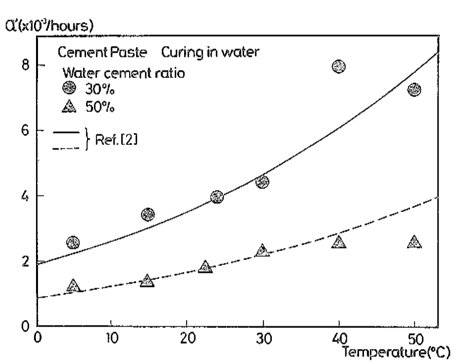

Fig.-1 Rate constants of cement paste

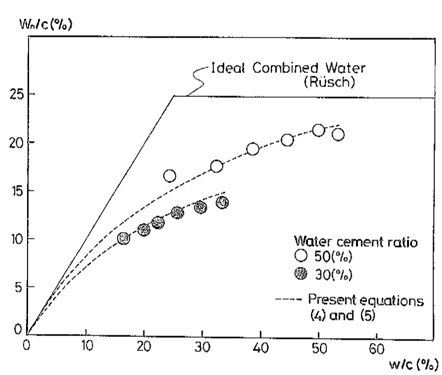

Fig.-2 Ultimate combined water

3 THERMAL AND DRYING SHRINKAGE STRESS ANALYSIS METHOD

Temperature of concrete and water diffusion in concrete are assumed to be expressed by the following differential equations.

$$C \cdot \rho \cdot \frac{dT}{dt} = \lambda \cdot \nabla^2 T + \mathring{Q} \tag{6}$$

for boundary

$$\lambda \cdot \nabla T = \alpha_c \cdot (T - T_\infty) \tag{7}$$

$$\frac{dwe}{dt} = k \cdot \nabla^2 w_e - \mathring{w}_n \tag{8}$$

for boundary

$$k \cdot \nabla w_e = f \cdot (w_e - w_\infty) \tag{9}$$

where C; specific heat of materials, ρ; density of materials, T; temperature, t; time, λ; thermal conductivity, $\mathring{Q}$; heat quantity per unit time, αc; heat transfer coefficient, T_∞; atmospheric temperature, we; evaporable water, k; diffusion coefficient, $\mathring{w}n$; the water combined per unit time, f; surface coefficient and W_∞; water content on the surface determined by the relative humidity. (see Fig.-3)

$\mathring{w}n$ and $\mathring{Q}$ are given by the following equations,

$$\mathring{w}_n = a(T) \cdot (W_n(w) - w_n)^2 \tag{10}$$

$$\mathring{Q} = Q_0 \cdot \mathring{w}_n / \alpha \tag{11}$$

where eq.(10); the same equation as eq.(1), Q_0; is the heat quantity when the unit weight of cement combines, α; water-cement ratio when the cement and the water perfectly combine.

The theoretical equations (6)-(9) were changed to the finite element equations based on Galerkin's method, and the distribution of temperature and water content could be solved step by step numerically. Assuming that the volumetric strain $\Delta\varepsilon_0$ is expressed as follows using the increment of temperature ΔT and the increment of desiccated water Δwd, stress analysis was carried out.

$$\Delta\varepsilon_0 = \alpha_T \cdot \Delta T - \alpha_s \cdot \Delta w_d \tag{12}$$

where αT; coefficient of linear expansion and αs; coefficient of drying shrinkage.

Young's modulus (E) and poisson ratio (ν) of concrete are decided by the following equations.

$$E \doteqdot 3 \cdot K \cdot G / (K+G) \tag{13}$$

$$\nu = (K - 2 \cdot G) / (K+G) / 2 \tag{14}$$

where K; bulk modulus, constant in spite of the rate of hydration reaction, G; shear modulus, given by the linear equation of combined water wn as follows.

$$G = G_0 \cdot w_n / W_n \tag{15}$$

where G_0; the ultimate shear modulus and Wn; the ultimate combined water.

Creep of concrete is not considered in this analysis. The values of the physical constants and the coefficients are shown in Table-3.

Table-3 List of physical constants of concrete

density of materials	ρ	cement water sand gravel	3.15×10^3 1.0×10^3 2.3×10^3 2.6×10^3	kg/m^3
specific heat of materials	C	cement water sand gravel	0.061 1.0 0.2 0.2	kcal/(kg·°C)
thermal conductivity	λ	2.0		kcal/(m·hour·°C)
heat transfer coefficient	α_c	10.0		kcal/(m^2·hour·°C)
diffusion coefficient	k	5.0×10^{-6}		m^2/hour
surface coefficient	f	5.0×10^{-4}		m/hour
rate of water perfectly combined to cement	α	0.25		
heat quantity of unit weight of cement	Q_∞	90		kcal/kg
coefficient of linear expansion	α_T	1.0×10^{-5}		1/°C
coefficient of drying shrinkage	α_S	2.0×10^{-6}* 6.0×10^{-6}**		m^3/kg
bulk modulus	K	3.75×10^5		kgf/cm^2
ultimate shear modulus	G_θ	1.07×10^5		kgf/cm^2

* during capillary water drying
** during gel water drying

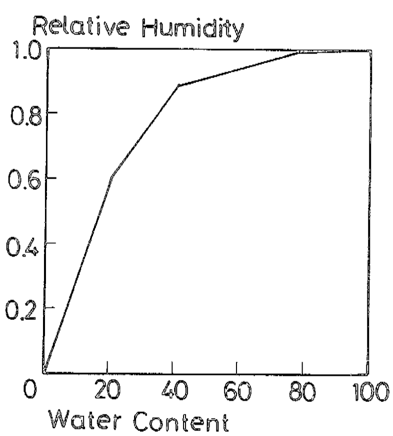

Fig.-3 Relationship between relative humidity and water content

Table-4 Mix proportion of concrete

W/C (%)	Mix proportion (kg/m³) water	cement	sand	gravel	Air (%)
50	180	360	783	956	4

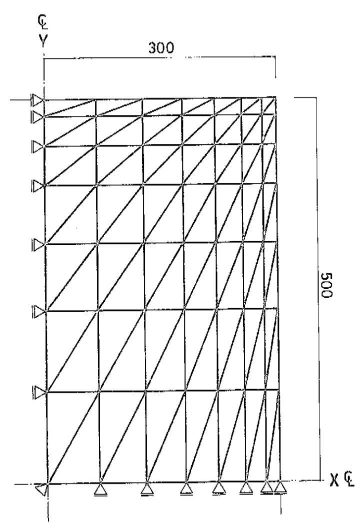

Fig.-4 Analytical model

4 AN ANALYTICAL RESULT OF A CONCRETE COLUMN

An analytical model and its finite element division are illustrated in Fig.-4. It is a quarter of the section of the column of 60cm x 100cm. The mix proportion of concrete is shown in Table-4.

The following figures represent the results when the initial temperature of concrete is 20 °C, atmospheric temperature 20 °C, relative humidity 60%, and when the drying starts at the age of 48 hours. Figures-5 and 6 show the histories of the temperature, the water content and the combined water at the corner and the center of the column section. It was found that the degree of hydration reaction differed in the section of column by reason of the drying and the heat of hydration. Figure-7 shows the history of normal stress in the direction of x-axis. The distribution of the normal stress in the direction of x-axis is shown in Fig.-8. It was known that the stresses caused by the drying was larger than that by the heat of hydration reaction. The stress occurred around the surface of the column was almost equal to the tensile strength of the common concrete.

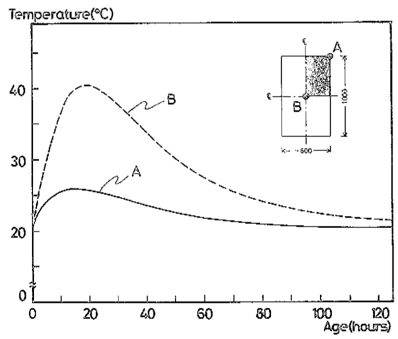

Fig.-5 History of concrete temperature

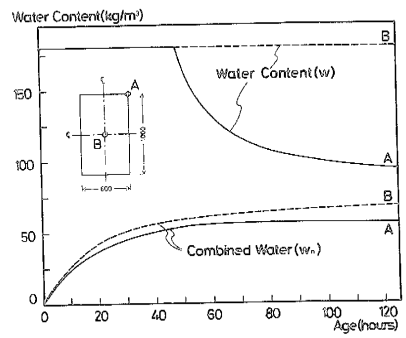

Fig.-6 Histories of water content and combined water

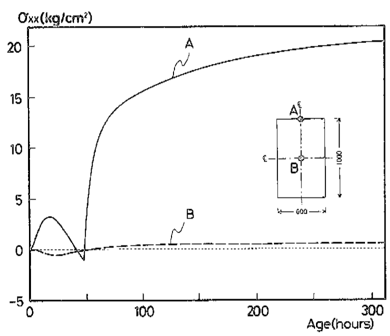

Fig.-7 History of normal stress

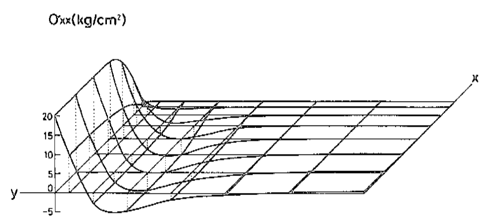

Fig.-8 Distribution of normal stress

5 INFLUENCE OF CONTINUOUS LOAD ON TENSILE STRENGTH OF CONCRETE

It goes without saying that the stress analysis in concrete finally aims at predicting the occurrence of cracks in concrete. The existence of cracks in concrete is generally judged by tensile strength and principal tensile stress. The tensile stress may be relaxed by creep of concrete. Therefore the creep law of concrete is important for the stress analysis. However, it is expected that the continuous stress affects tensile strength of concrete, and the existence of cracks cannot be judged by merely comparing the tensile strength with tensile stress considering relaxation of stress.

The details and the results of the experiment on the influence of continuous load on tensile strength of cement paste is described in the following pages. Test apparatus is illustrated in Fig.-9. Specimens were made of cement paste whose water-cement ratio was 30%. Their dimensions were 4cm x 4cm x 16cm. They were cured in water for 4 weeks. after curing, The continuous loads of 140kgf, 160kgf and 180kgf were applied to the specimens, and the flexural strengths were examined 2 weeks later. When the continuous loads of 140kgf, 160kgf and 180kgf were applied to the specimens, The apparent tensile stresses at the edge of the specimen were equivalent to about 60, 70 and 80% of the flexural strength at the age of 4 weeks. The flexural strength were also examined at the age of 4 weeks and 6 weeks. The distributions of the flexural strengths are shown in Fig.-10. It is obvious that the continuous load has much effect on the tensile strength.

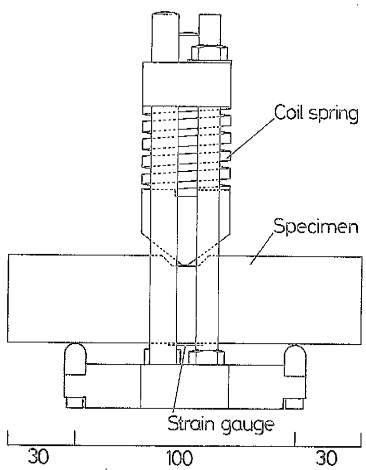

Fig.-9 Test apparatus

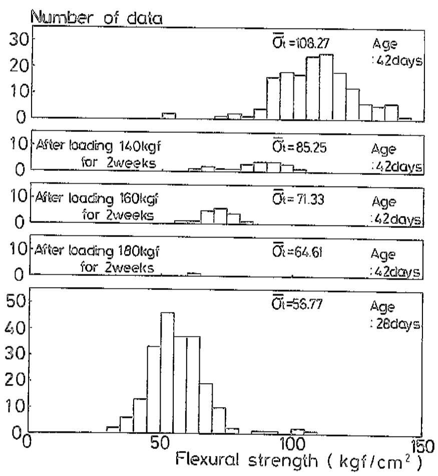

Fig.-10 Distributions of flexural strength

6 CONCLUSIONS

The degree of hydration reaction of concrete was formulated by the simple shaped differential equations to apply the analysis experimentally. A stress analysis method in which the problems on the heat of the hydration and the drying are dealt together was proposed. As a result of analyzing the stress occurred in a column, it is concluded that the relatively large stress which is almost equal to the tensile strength of concrete arises around the surface. There is every probability that cracks occur in concrete by the drying and the heat of hydration.

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