DETERMINATION OF THE DAMPING MATRIX IN DYNAMIC STRUCTURAL ANALYSIS OF A REACTOR CONTAINMENT

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ABSTRACT

In dynamic structural analysis of a reactor containment the mass matrix and the stiffness matrix can usually be determined from the physical properties of the undamped structural elements. But the damping matrix must be determined from appropriate assumptions on the damping characteristics of the system. This paper examines and compares various possible methods of determining the elements in the damping matrix. For each method considered the limitations and implications are illustrated by a numerical example. Typical numerical results show that as compared with an analysis using "nonproportional damping" matrix a design based on an analysis using "proportional damping" matrix can be much too conservative and a design based on an analysis disregarding the soil-structure interaction can be nonconservative.

1.0 INTRODUCTION

In analyzing the vibrational and shock response behavior of a reactor containment induced by internal and external conditions (such as thermal shock and earthquake motion) the containment building can be approximately idealized as an N-degree-of-freedom linear dynamic system with viscous damping (see, for example, Tsai et al. [1]). The equation of motion can be written as a single matrix equation.

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \tag{1}$$

where t is time, $\{F(t)\}$ is the given forcing function vector and $\{x\}$, $\{\dot{x}\}$ and $\{\ddot{x}\}$ are the unknown displacement vector, velocity vector and acceleration vector respectively. The NxN symmetric mass matrix $[M]$ and stiffness matrix $[K]$ can usually be determined from the physical properties of the undamped structural elements. But in each case the NxN symmetric damping matrix $[C]$ must be determined from appropriate assumptions on the damping characteristics of the system. It is the purpose of this paper to examine and compare various possible methods of determining the elements in the damping matrix and to illustrate the limitations and implications of these assumptions by numerical examples. The paper is intended to provide some helpful information in judicious choice of damping constants with or without experimental data.

2.0 A REPRESENTATIVE DYNAMIC MODEL

In order to illustrate the differences among various damping assumptions considered below let us choose a simple dynamic model which represents essential features of the horizontal motions of a typical reactor containment building Fig. 1(a). The model in Fig. 1(b) consists of 3 masses, 3 springs and 6 dashpots. Let m_1 be the mass of the foundation and m_2 and m_3 be the lumped mass representation of the total mass of the containment structure and internal building (not including the foundation). Let k_1 be the spring constant representing the horizontal soil-structure interaction and the spring constants k_2, k_3, and masses m_2, m_3 be so chosen that the three natural frequencies of the simple dynamic model coincide with the first three natural frequencies of the vibrational modes associated with the horizontal motions of the soil-structure system (analyzed by a more elaborate dynamic model or measured from a dynamic test). For such a model the mass and stiffness matrices in eq. (1) are respectively

$$[M] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, \quad [K] = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \tag{2}$$

It is the purpose of this paper to compare various possible assumptions on the elements in the damping matrix

$$[C] = \begin{bmatrix} c_{10}+c_{12}+c_{13} & -c_{12} & -c_{13} \\ -c_{12} & c_{12}+c_{23}+c_{20} & -c_{23} \\ -c_{13} & -c_{23} & c_{23}+c_{13}+c_{30} \end{bmatrix} \tag{3}$$

for the model.

As a numerical example representing a typical containment let us take the total mass

$$m = m_1+m_2+m_3 = 127.85\times10^6 \text{ lb}/g = 3.974 \times 10^6 \text{ lb sec}^2/\text{ft} \tag{4}$$

(where $g = 32.174$ ft/sec^2 is the gravitational acceleration) of which one third is the mass of the foundation

$$m_1 = m/3 = 1.325\times10^6 \text{ lb sec}^2/\text{ft} \tag{5}$$

If the foundation radius is

$$r_0 = 82 \text{ ft} \tag{6}$$

and the shear modulus and Poisson's ratio of the soil are

$$G = 20 \times 10^6 \text{ lb/ft}^2, \quad \nu = 0.44 \tag{7}$$

respectively, the horizontal soil-structure interaction spring constant can be computed as (See, for example, Richart et al. [2], Table 10-13)

$$k_1 = \frac{32(1-\nu)Gr_0}{7-8\nu} = 8.445\times10^9 \text{ lb/ft} \tag{8}$$

Suppose analyses of a more elaborate dynamic model give the first three circular natural frequencies of the horizontal modes of the soil-structure system as follows:

$$\omega_1 = 25.384 \text{ rad/sec}, \quad \omega_2 = 61.198 \text{ rad/sec}, \quad \omega_3 = 126.857 \text{ rad/sec} \tag{9}$$

These three conditions together with the condition (from eqs. (4) and (5))

$$m_2 + m_3 = 2m/3 = 2m_1 \tag{10}$$

uniquely determine the following four parameters of the model (such determination will be the subject of a forthcoming paper by the author)

$$m_2 = 0.86394m_1, \quad m_3 = 1.13606m_1, \quad k_2 = 0.86503k_1, \quad k_3 = 0.17006k_1 \tag{11}$$

Substituting these values into eqs. (2) we have

$$[M] = m_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.86394 & 0 \\ 0 & 0 & 1.13606 \end{bmatrix}, \quad [K] = k_1 \begin{bmatrix} 1.86503 & -0.86503 & 0 \\ -0.86503 & 1.03509 & -0.17006 \\ 0 & -0.17006 & 0.17006 \end{bmatrix} \tag{12}$$

where m_1 and k_1 are given by eqs. (5) and (8). When these matrices are used and [C] and {F(t)} are set to be null matrices eq. (1) defines an eigenvalue problem for the free vibration of the undamped dynamic model. The solution of this eigenvalue problem (see, for example, Hurty and Rubinstein [3], Chapter 3) gives the eigenvalues

$$\lambda_1 = \frac{k_1}{m_1\omega_1^2} = 9.89403,\ \lambda_2 = \frac{k_1}{m_1\omega_2^2} = 1.70223,\ \lambda_3 = \frac{k_1}{m_1\omega_3^2} = 0.39615 \qquad (13)$$

and the eigenvectors (or modal columns normalized to the greatest value of each mode)

$$\{\gamma^{(1)}\} = \begin{Bmatrix}0.15928\\0.32481\\1.00000\end{Bmatrix},\quad \{\gamma^{(2)}\} = \begin{Bmatrix}0.67710\\1.00000\\-0.34194\end{Bmatrix},\quad \{\gamma^{(3)}\} = \begin{Bmatrix}1.00000\\-0.76213\\0.04804\end{Bmatrix} \qquad (14)$$

The transformation matrix $[Y]$ formed from these modal columns is such that both $[M]$ and $[K]$ matrices are transformed into diagonal matrices as follows

$$[Y]^T[M][Y] = m_1\begin{bmatrix}1.25258 & 0 & 0\\0 & 1.45524 & 0\\0 & 0 & 1.50443\end{bmatrix} = \begin{bmatrix}{}^{\diagdown}M_{r\diagdown}\end{bmatrix} \qquad (15)$$

$$[Y]^T[K][Y] = k_1\begin{bmatrix}0.12660 & 0 & 0\\0 & 0.85490 & 0\\0 & 0 & 3.79763\end{bmatrix} = \begin{bmatrix}{}^{\diagdown}\omega_{r\diagdown}^2\end{bmatrix}\begin{bmatrix}{}^{\diagdown}M_{r\diagdown}\end{bmatrix} \qquad (16)$$

This means that the undamped dynamic model always possesses classical "normal modes" and its equations of motion can be uncoupled. In damped dynamic system, however, this property is generally violated except under special conditions to be discussed in the next section.

We note here that in practical design of reactor containments many more lumped masses than three should be taken and the vertical and rocking degrees of freedom should be considered simultaneously with the horizontal degrees of freedom. However the chosen simplified model for illustration purposes preserves all essential features of the analysis and the extension to cases with higher degrees of freedom is straightforward. Furthermore, energy dissipation in realistic structures can be due to mechanisms other than viscous damping. In this paper the total effects of damping due to a wide range of causes have been assumed to be replaced by equivalent viscous damping such that the same total energy is dissipated per cycle of vibration. (See, for example, Timoshenko [4].)

3.0 DETERMINATION OF THE DAMPING MATRIX SUCH THAT THE SYSTEM POSSESSES CLASSICAL NORMAL MODES ("PROPORTIONAL DAMPING")

The following assumptions of the $[C]$ matrix allow eq. (1) to be uncoupled and therefore allow the damped system to possess classical normal modes. It is shown in Caughey [5] that if a damped system possesses classical normal modes, these modes are identical with the normal modes for the corresponding undamped system. The response of each mode can be analyzed as a single degree of freedom system and the total response of the system obtained by the mode superposition method. This has been the usual approach in practical design of reactor containments.

3.1 Damping Matrix is Proportional to the Mass Matrix

In this case the $[C]$ matrix in eq. (1) is assumed to be

$$[C] = 2\xi[M] \qquad (17)$$

where the constant ξ can be chosen to satisfy the critical damping ratio D (either determined experimentally, given as a design criterion, or conjectured from engineering judgement) for any one of the N normal modes.

If the first mode with the lowest circular frequency ω_1 is used as the criterion we have

$$\beta = D_1\omega_1 \tag{18}$$

This will imply successively lower damping ratios for the higher modes in the following manner

$$D_r = \beta/\omega_r = D_1\omega_1/\omega_r \ , \ (r = 2, 3, \ldots N) \tag{19}$$

For example let us require that the critical damping ratio for the first mode of the representative dynamic model be 3.5%. We have $D_1 = 0.035$ and from eqs. (18), (17) and (12)

$$[C] = 2D_1\omega_1[M] = 0.07m_1\omega_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.86394 & 0 \\ 0 & 0 & 1.13606 \end{bmatrix} \tag{20}$$

Comparing this matrix with eq. (3) we identify the individual damping constants in the dynamic model as follows:

$$c_{12} = c_{13} = c_{23} = 0, \quad c_{10} = 0.07m_1\omega_1, \ c_{20} = 0.0605m_1\omega_1, \ c_{30} = 0.0795m_1\omega_1 \tag{21}$$

From eqs. (15) and (20) the following transformation results in a diagonal matrix

$$[\gamma]^T[C][\gamma] = 2D_1\omega_1\left[\diagdown M_r \diagdown\right] = 0.07\omega_1\left[\diagdown M_r \diagdown\right] \tag{22}$$

Substitution of eqs. (15), (16) and (22) and the transformation

$$\{x\} = [\gamma]\{\eta\} \tag{23}$$

into eq. (1) results in a set of three uncoupled equations of motion

$$\left[\diagdown M_r \diagdown\right]\{\ddot{\eta}\} + 2D_1\omega_1\left[\diagdown M_r \diagdown\right]\{\dot{\eta}\} + \left[\diagdown \omega_r^2 \diagdown\right]\left[\diagdown M_r \diagdown\right]\{\eta\} = [\gamma]^T\{F(t)\} \tag{24}$$

Hence the damped system possesses classical normal modes. If we compare the r-th uncoupled equation of this set with the equation of motion of a single degree of freedom system

$$M_r\ddot{\eta}_r + 2D_r\omega_r M_r\dot{\eta}_r + \omega_r^2 M_r\eta_r = \{\gamma^{(r)}\}^T\{F(t)\} \tag{25}$$

we can determine the effective critical damping ratio for the first mode

$$D_1 = \frac{D_1\omega_1}{\omega_1} = D_1 = 0.035 \tag{26}$$

as expected, and those for the second and third modes (using eqs. (9))

$$\left.\begin{aligned} D_2 &= \frac{D_1\omega_1}{\omega_2} = 0.4148D_1 = 0.01452 \\ D_3 &= \frac{D_1\omega_1}{\omega_3} = 0.2001D_1 = 0.00700 \end{aligned}\right\} \tag{27}$$

which are lower than D_1. Eqs. (27) are numerical examples of eq. (19).

In order to compare the responses of the dynamic model under various damping assumptions let us consider the base to be subjected to a horizontal harmonic acceleration of amplitude A and circular frequency ω such that the response displacements of m_1, m_2 and m_3 relative to the base can be obtained by solving eq. (1) with

$$\{x\} = \{X\}e^{i\omega t} \ , \ \{F(t)\} = -Ae^{i\omega t}\{M\} \tag{28}$$

where $\{M\}$ is a column matrix with elements m_1, m_2 and m_3. The solution for the complex amplitudes $\{X\}$ is given by (see, for example [3], Chapter 10)

$$\{X\} = -A[Z]^{-1}\{M\} \tag{29}$$

with the impedance matrix given by

$$[Z] = -\omega^2[M] + i\omega[C] + [K] \tag{30}$$

The corresponding complex amplitudes $\{X_0\}$ under static loading

$$\{F(t)\} = -A\{M\} \tag{31}$$

are obtained by taking $\omega = 0$ in eqs. (29) and (30)

$$\{X_0\} = -A\,[K]^{-1}\,\{M\} \tag{32}$$

Then the "dynamic magnification factor" for the j-th mass is given by the ratio of the absolute values of the complex amplitudes X_j and X_{0j}.

$$(MF)_j = \frac{|X_j|}{|X_{0j}|} \tag{33}$$

In the case of damping matrix [C] as given by eq. (20) and forcing frequency $\omega = \omega_1 =$ the fundamental circular natural frequency of the dynamic model we have from eqs. (12), (20) and (30)

$$[Z] = m_1\omega_1^2 \begin{bmatrix} 17.45266+0.071i & -8.55863 & 0 \\ -8.55863 & 9.37727+0.06048i & -1.68258 \\ 0 & -1.68258 & 0.54652+0.07952i \end{bmatrix} \tag{34}$$

Substituting eqs. (11) and (34) into eq. (29) we find the absolute value of the complex amplitude for the mass m_3

$$|X_3| = 17.97375\ (A/\omega_1^2) \tag{35}$$

Substituting eqs. (11) and (12) into eq. (32) we find the corresponding value under static loading

$$|X_{03}| = 11.99241\ (Am_1/k_1) \tag{36}$$

Finally substituting eqs.(35) and (36) into eq. (33) and using the value of λ_1 from eq. (13) we obtain the dynamic magnification factor of m_3 relative to the base

$$(MF)_3 = 1.49876\lambda_1 = 14.82878 \tag{37}$$

This same result can be obtained by superposition of the responses of m_3 due to the three normal modes (the "normal mode method") rather than the "frequency response method" used here.

3.2 Damping Matrix is Proportional to the Stiffness Matrix

In this case the [C] matrix in eq. (1) is assumed to be

$$[C] = \alpha[K] \tag{38}$$

where the constant α can be chosen to satisfy the critical damping ratio D for any one of the N normal modes.

If the first mode damping ratio D_1 is used as the criterion we have

$$\alpha = 2D_1/\omega_1 \tag{39}$$

This will imply successively higher damping ratios for the higher modes in the following manner

$$D_r = \alpha\omega_r/2 = D_1\omega_r/\omega_1\ ,\ (r = 2, 3, \ldots N) \tag{40}$$

For example let us require that the critical damping ratio for the first mode of the representative dynamic model be 3.5%. We have $D_1 = 0.035$ and from eqs. (39), (38) and (12)

$$[C] = \frac{2D_1}{\omega_1}[K] = \frac{0.07k_1}{\omega_1}\begin{bmatrix} 1.86503 & -0.86503 & 0 \\ -0.86503 & 1.03509 & -0.17006 \\ 0 & -0.17006 & 0.17006 \end{bmatrix} \tag{41}$$

Comparing this matrix with eq. (3) we identify the individual damping constants in the dynamic model as follows:

$$\left.\begin{aligned} & C_{13} = C_{20} = C_{30} = 0, \quad C_{10} = 0.07k_1/\omega_1, \\ & C_{12} = 0.06055k_1/\omega_1, \quad C_{23} = 0.01190k_1/\omega_1 \end{aligned}\right\} \tag{42}$$

From eqs. (16) and (41) the following transformation results in a diagonal matrix

$$[\gamma]^T[C][\gamma] = \frac{2D_1}{\omega_1}\left[\diagdown\omega_r^2\diagdown\right]\left[\diagdown M_r\diagdown\right] = \frac{0.07}{\omega_1}\left[\diagdown\omega_r^2\diagdown\right]\left[\diagdown M_r\diagdown\right] \tag{43}$$

Hence the damped system possesses classical normal modes. Comparison of the three elements in this diagonal matrix with the coefficient for $\dot{\eta}_r$ in eq. (25) for a single degree of freedom system gives $D_1 = 0.035$ as expected and $D_2 = 0.08438$, $D_3 = 0.17491$ which are higher than D_1, and consistent with eq. (40).

If this damped system is subjected to a horizontal harmonic acceleration of amplitude A and circular frequency ω_1 at the base the dynamic magnification factor of m_3 relative to the base is according to eqs. (29) through (33) found to be

$$(MF)_3 = 13.61243 \tag{44}$$

which is lower than the value given in eq. (37) for case 3.1. This is to be expected since in case 3.1 the effective damping ratios for the second and third normal modes are lower than the first mode damping whereas in the present case they are higher.

3.3 Damping Matrix is a Linear Combination of the Mass and Stiffness Matrices

This is the case first investigated by Lord Rayleigh [6]. Although Rayleigh did not use matrix algebra his results are equivalent to those stated below.

In this case the [C] matrix in eq. (1) is assumed to be

$$[C] = 2\beta[M] + \alpha[K] \tag{45}$$

where the constants α and β can be chosen to satisfy one of the following two conditions:

3.3.1 Critical damping ratios for any two of the N normal modes can be specified - If the first two modes are used as the criteria we have

$$\alpha = \frac{2(\omega_2D_2-\omega_1D_1)}{\omega_2^2-\omega_1^2}, \quad \beta = \frac{\omega_1\omega_2(\omega_2D_1-\omega_1D_2)}{\omega_2^2-\omega_1^2} \tag{46}$$

The damping ratios for the higher modes will then be given by

$$D_r = \beta/\omega_r + \alpha\omega_r/2, \quad (r = 3,4,\ldots N) \tag{47}$$

which can be either higher or lower than D_1 and D_2. For example let us require

$$D_1 = D_2 = 0.035 \tag{48}$$

for the representative dynamic model. Then eqs. (46) and (9) give

$$\left.\begin{aligned} \alpha &= \frac{2D_1}{\omega_1+\omega_2} = \frac{2D_1}{\omega_1}(0.29314) = 0.02052/\omega_1 \\ \beta &= \frac{D_1\omega_1\omega_2}{\omega_1+\omega_2} = D_1\omega_1(0.70686) = 0.02474\omega_1 \end{aligned}\right\} \qquad (49)$$

and eqs. (45) and (12) give

$$[C] = 0.04948m_1\omega_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.86394 & 0 \\ 0 & 0 & 1.13606 \end{bmatrix} + \frac{0.02052k_1}{\omega_1} \begin{bmatrix} 1.86503 & -0.86503 & 0 \\ -0.86503 & 1.03509 & -0.17006 \\ 0 & -0.17006 & 0.17006 \end{bmatrix} \qquad (50)$$

Comparing this matrix with eq. (3) we identify the individual damping constants in the dynamic model as follows:

$$\left.\begin{aligned} c_{13} &= 0, \quad c_{10} = 0.04948m_1\omega_1 + 0.02052k_1/\omega_1, \quad c_{20} = 0.04275m_1\omega_1 \\ c_{30} &= 0.05621m_1\omega_1, \quad c_{12} = 0.01775k_1/\omega_1, \quad c_{23} = 0.00349k_1/\omega_1 \end{aligned}\right\} \qquad (51)$$

From eqs. (15), (16) and (50) the following transformation results in a diagonal matrix

$$[\gamma]^T[C][\gamma] = 0.04948\omega_1\left[\diagdown M_r \diagdown\right] + \frac{0.02052}{\omega_1}\left[\diagdown \omega_r^2 \diagdown\right]\left[\diagdown M_r \diagdown\right] \qquad (52)$$

Hence the damped system possesses classical normal modes. Comparison of the three elements in this diagonal matrix with the coefficient for $\dot{\eta}_r$ in eq. (25) gives $D_1 = D_2 = 0.035$ as expected and $D_3 = 0.05623$ which is higher than D_1 and D_2 and consistent with eq. (47).

If this damped system is subjected to a horizontal harmonic acceleration of amplitude A and circular frequency ω_1 at the base the dynamic magnifcation factor of m_3 relative to the base is according to eqs. (29) through (33) found to be

$$(MF)_3 = 14.82741 \qquad (53)$$

which is only slightly less than the value obtained in eq. (37) for case 3.1 and is higher than the value obtained in eq. (44) for case 3.2.

3.3.2 <u>Critical damping ratio D_μ for the μth mode (which has the minimum damping ratio among all the normal modes) can be specified</u> - In terms of α and β the damping ratio for the μth mode can be written as (See eqs. (18) and (39))

$$D_\mu = \beta/\omega_\mu + \alpha\omega_\mu/2 \qquad (54)$$

In order for the μth mode to have the minimum damping ratio among all the normal modes we must have

$$\frac{dD_\mu}{d\omega_\mu} = 0 = -\beta/\omega_\mu^2 + \alpha/2 \qquad (55)$$

or

$$\omega_\mu^2 = 2\beta/\alpha \qquad (56)$$

Substituting eq. (56) into eq. (54) we determine

$$\alpha = D_\mu/\omega_\mu, \quad \beta = D_\mu\omega_\mu/2, \quad (1 \le \mu \le N) \qquad (57)$$

The damping ratios for other modes will then be given by

$$D_r = \frac{D_\mu}{2}\left(\frac{\omega_\mu}{\omega_r} + \frac{\omega_r}{\omega_\mu}\right), \quad (r = 1, 2, \ldots \mu-1, \mu+1, \ldots N) \qquad (58)$$

which are higher than D_μ.

For example let us require the first mode to have the minimum damping ratio $D_1 = 0.035$ for the representative dynamic model. Then we set $\mu = 1$ in eq. (57) which, when substituted in eq. (45), gives

$$[C] = D_1\omega_1 [M] + \frac{D_1}{\omega_1}[K]$$
$$= 0.035 m_1\omega_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.86394 & 0 \\ 0 & 0 & 1.13606 \end{bmatrix} + \frac{0.035 k_1}{\omega_1} \begin{bmatrix} 1.86503 & -0.86503 & 0 \\ -0.86503 & 1.03509 & -0.17006 \\ 0 & -0.17006 & 0.17006 \end{bmatrix} \quad (59)$$

Comparing this matrix with eq. (3) we identify the individual damping constants in the dynamic model as follows:

$$\left.\begin{aligned} c_{13} &= 0 \ , \ c_{10} = 0.035(m_1\omega_1 + k_1/\omega_1) \ , \ c_{20} = 0.03024 m_1\omega_1 \\ c_{30} &= 0.03976 m_1\omega_1 \ , \ c_{12} = 0.03028 k_1/\omega_1 \ , \ c_{23} = 0.00595 k_1/\omega_1 \end{aligned}\right\} \quad (60)$$

From eqs. (15), (16) and (59) the following transformation results in a diagonal matrix

$$[\gamma]^T[C][\gamma] = 0.035\omega_1 \left[\diagdown M_r \diagdown\right] + \frac{0.035}{\omega_1}\left[\diagdown \omega_r^2 \diagdown\right]\left[\diagdown M_r \diagdown\right] \quad (61)$$

Hence the damped system possesses classical normal modes. Comparison of the three elements in this diagonal matrix with the coefficient for $\dot{\eta}_r$ in eq. (25) gives $D_1 = 0.035$ as expected and $D_2 = 0.04946$, $D_3 = 0.09097$ which are higher than D_1 and consistent with eq. (58). For this particular case of $\mu = 1$ the general trend of higher damping ratios for higher modes is similar to case 3.2, but contrary to case 3.1.

If this damped system is subjected to a horizontal harmonic acceleration of amplitude A and circular frequency ω_1 at the base the dynamic magnification factor of m_3 relative to the base is according to eqs. (29) through (33) found to be

$$(MF)_3 = 14.82692 \quad (62)$$

which is slightly lower than the values obtained in eq. (53) for case 3.3.1 and eq. (37) for case 3.1 and is higher than the value obtained in eq. (44) for case 3.2.

3.4 <u>The Transformed Damping Matrix is a Power Series of the Transformed Stiffness Matrix</u>

This is the extension of Rayleigh's assumption eq. (45) by Caughey and O'Kelly [7].

It is always possible to find a real congruence transformation $[\phi]$ which will reduce the mass matrix in eq. (1) to an identity matrix

$$[\phi]^T[M][\phi] = [I] \quad (63)$$

Then according to [7] a necessary and sufficient condition for the damped dynamic system to possess classical normal modes is that the transformed damping matrix is expressible in terms of the transformed stiffness matrix in the following way

$$[\phi]^T[C][\phi] = \sum_{j=0}^{N-1} \alpha_j \left([\phi]^T[K][\phi]\right)^j \quad (64)$$

On the other hand, if classical normal modes exist they are identical with the normal modes for the corresponding undamped system [5]. A real orthogonal transformation $[\psi]$ consisting of the normal mode shapes can uncouple the equations of motion eq. (1) by forming the following three diagonal matrices:

$$[\psi]^T[\phi]^T[M][\phi][\psi] = [\psi]^T[\psi] = [I] \tag{65}$$

$$[\psi]^T[\phi]^T[K][\phi][\psi] = \left[\diagdown \omega_r^2 \diagdown\right] \tag{66}$$

$$[\psi]^T[\phi]^T[C][\phi][\psi] = \left[\diagdown 2D_r\omega_r \diagdown\right] \tag{67}$$

We may assume the series in eq. (64) to contain $s(1 \le s \le N)$ number of non-zero coefficients α_j. These coefficients may be solved to satisfy any s number of the critical damping ratios D_r from an sxs matrix equation partitioned from the NxN matrix equation eq. (67). Then the damping matrix can be determined from eq. (64) as follows:

$$[C] = ([\phi]^T)^{-1}\left(\sum_{j=0}^{N-1}\alpha_j([\phi]^T[K][\phi])^j\right)[\phi]^{-1} \tag{68}$$

Finally substituting the [C] matrix thus determined to eq. (67) we can find what effective damping ratios D_r are implied for those modes with unspecified damping ratios.

For example, for the representative dynamic model with [M] matrix given in eq. (12) the $[\phi]$ matrix which satisfies eq. (63) is found to be

$$[\phi] = m_1^{-\frac{1}{2}}\begin{bmatrix} 1 & 0 & 0 \\ 0 & (0.86394)^{-\frac{1}{2}} & 0 \\ 0 & 0 & (1.13606)^{-\frac{1}{2}} \end{bmatrix} = m_1^{-\frac{1}{2}}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.07587 & 0 \\ 0 & 0 & 0.93821 \end{bmatrix} \tag{69}$$

Then from eqs. (12) and (69) we have

$$[\phi]^T[K][\phi] = \frac{k_1}{m_1}\begin{bmatrix} 1.86503 & -0.93066 & 0 \\ -0.93066 & 1.19811 & -0.17166 \\ 0 & -0.17166 & 0.14969 \end{bmatrix} \tag{70}$$

The solution of the equation of motion for the free vibration of the undamped system

$$[\phi]^T[M][\phi]\{\ddot{p}\} + [\phi]^T[K][\phi]\{p\} = 0\ , \ (\{x\} = [\phi]\{p\}) \tag{71}$$

gives three natural frequencies identical to those in eqs. (9) and three modal columns (eigenvectors normalized to satisfy eq.(65)) as follows:

$$\{\psi^{(1)}\} = \begin{Bmatrix} 0.14232 \\ 0.26975 \\ 0.95235 \end{Bmatrix},\ \{\psi^{(2)}\} = \begin{Bmatrix} 0.56129 \\ 0.77051 \\ -0.30213 \end{Bmatrix},\ \{\psi^{(3)}\} = \begin{Bmatrix} 0.81529 \\ -0.57754 \\ 0.04175 \end{Bmatrix} \tag{72}$$

These will be also the normal mode shapes for the damped system. The square transformation matrix $[\psi]$ consisting of these three modal columns does indeed satisfy eqs. (65) and (66) when eqs. (12) and (69) are used.

Let us assume the series in eq. (64) to contain two non-zero coefficients α_0 and α_1. Then with N = 3 and $\alpha_2 = 0$ eq. (64) becomes

$$[\phi]^T[C][\phi] = \alpha_0[I] + \alpha_1[\phi]^T[K][\phi] \tag{73}$$

and eq.(67) becomes

$$\left[\diagdown 2D_r\omega_r \diagdown\right] = \alpha_0[I] + \alpha_1\left[\diagdown \omega_r^2 \diagdown\right] \tag{74}$$

where the relations eqs. (65) and (66) have been used. The coefficients α_0 and α_1 can be solved by specifying any two of the three damping ratios D_r. If we specify $D_1 = D_2 = 0.035$ the values of α_0 and α_1 can be solved from the following 2 x 2 matrix equation partitioned from eq. (74)

$$\begin{bmatrix} 2D_1\omega_1 & 0 \\ 0 & 2D_2\omega_2 \end{bmatrix} = \alpha_0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \alpha_1 \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix} \tag{75}$$

giving

$$\left.\begin{aligned} \alpha_0 &= \frac{2D_1\omega_1\omega_2}{\omega_2+\omega_1} = 2D_1\omega_1(0.70686) = 0.04948\omega_1 \\ \alpha_1 &= \frac{2D_1}{\omega_2+\omega_1} = \frac{2D_1}{\omega_1}(0.29314) = 0.02052/\omega_1 \end{aligned}\right\} \tag{76}$$

Then the desired damping matrix is determined from eqs. (68), (69) and (70)

$$\begin{aligned} [C] &= ([\phi]^T)^{-1}\left(0.04948\omega_1[I] + \frac{0.02052}{\omega_1}[\phi]^T[K][\phi]\right)[\phi]^{-1} \\ &= 0.04948m_1\omega_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.86394 & 0 \\ 0 & 0 & 1.13606 \end{bmatrix} + \frac{0.02052k_1}{\omega_1} \begin{bmatrix} 1.86503 & -0.86503 & 0 \\ -0.86503 & 1.03509 & -0.17006 \\ 0 & -0.17006 & 0.17006 \end{bmatrix} \end{aligned} \tag{77}$$

which is identical to eq. (50) for case 3.3.1.

Finally the third uncoupled equation in eq. (74) gives

$$D_3 = \frac{1}{2\omega_3}(\alpha_0 + \alpha_1\omega_3^2) = \frac{1}{2\omega_3}(0.04948\omega_1 + 0.02052\omega_3^2/\omega_1) = 0.05623 \tag{78}$$

which is identical to the value of D_3 for the numerical example in case 3.3.1. This example shows that if we take

$$\alpha_0 = 2\beta,\ \alpha_1 = \alpha,\ \alpha_2 = \alpha_3 = \ldots = \alpha_{N-1} = 0 \tag{79}$$

in eq. (64) then eq. (68) reduces to Rayleigh's assumption eq. (45).

As an alternative to eq. (64) Caughey [5] gave a sufficient (but not necessary) condition for the existence of classical normal modes as

$$[\phi]^T[C][\phi] = \sum_{N=1}^{\infty}\sum_{\ell=0}^{N-1} a_{n\ell}\,([\phi]^T[K][\phi])^{\ell/n} \tag{80}$$

where sufficient number of the non-zero coefficients $a_{n\ell}$ may be chosen to satisfy a desired number of critical damping ratios D_r in eq. (67). Eq. (80) provides an even wider class of possible expressions for [C] which uncouples eq. (1) than eqs. (45) and (64). It is shown in [7] that under certain conditions the series in eq. (80) can be expressed in the form of the series in eq. (64). A numerical example of eq. (80) is given in [5].

3.5 The Elements in the Transformed Damping Matrix are Proportional to the Modal Critical Damping Ratios

Caughey [5] showed that a necessary and sufficient condition that a damped dynamic system possessed classical normal modes was that the damping matrix be diagonalized by the same transformation which uncoupled the undamped system as indicated by eqs. (65), (66) and (67). Eq. (67) represents a case where the elements in the transformed damping matrix are proportional to the modal critical damping ratios. If we assign all N values of the critical damping ratios D_r in this equation the damping matrix can be uniquely determined from eq. (67) as follows:

$$[C] = ([\phi]^T)^{-1}([\psi]^T)^{-1} \begin{bmatrix} \diagdown \\ & 2D_r\omega_r \\ & & \diagdown \end{bmatrix} [\psi]^{-1}[\phi]^{-1} \tag{81}$$

This is the most satisfactory method of determining a proportional damping matrix without ambiguity on the effective damping ratios for various modes.

For example let us require $D_1 = D_2 = D_3 = 0.035$ for the representative dynamic model. Then substituting these values of D_r and eqs. (9), (69) and (72) into eq. (81) we obtain

$$[C] = m_1\omega_1 \begin{bmatrix} 0.28711 & -0.08277 & -0.00771 \\ -0.08277 & 0.19179 & -0.02942 \\ -0.00771 & -0.02942 & 0.09031 \end{bmatrix} \tag{82}$$

Comparing this matrix with eq. (3) we identify the individual damping constants in the dynamic model as follows:

$$\left.\begin{array}{lll} c_{10} = 0.19663m_1\omega_1, & c_{20} = 0.07960m_1\omega_1, & c_{30} = 0.05318m_1\omega_1 \\ c_{12} = 0.08277m_1\omega_1, & c_{23} = 0.02942m_1\omega_1, & c_{13} = 0.00771m_1\omega_1 \end{array}\right\} \tag{83}$$

If this damped system is subjected to a horizontal harmonic acceleration of amplitude A and circular frequency ω_1 at the base the dynamic magnification factor of m_3 relative to the base is according to eqs. (29) through (33) found to be

$$(MF)_3 = 14.82751 \tag{84}$$

which is higher than the corresponding value in eq. (53) for case 3.3.1 and lower than that in eq. (37) for case 3.1 as the modal damping ratios D_1, D_2 and D_3 for the present case lie between these two cases.

Nielsen [8] presented a method wherein by assuming the dynamic system to be "simply coupled", "close coupled" or "far coupled" the elements in the [K] and [C] matrices can be determined from experimental data of mode shapes, frequencies and damping ratios for a few normal modes in such a manner that a few columns of the matrix equations (66) and (67) are satisfied. This method is not suitable for determining [C] matrix when complete information on [M] and [K] matrices (and hence frequencies and mode shapes) is given since the damping matrix determined by satisfying a few columns of the matrix equation (67) does not, in general, satisfy the full matrix equation (67), which is required for uncoupling of all equations of motion and for existence of classical normal modes. Although the resulting damping matrix will not uncouple the equations of motion, Nielson's method does provide a means of estimating approximate damping values based on limited experimental data if the damping mechanism of the structure behaves essentially as "proportional damping."

4.0 DETERMINATION OF THE DAMPING MATRIX IF THE SYSTEM DOES NOT POSSESS CLASSICAL NORMAL MODES ("NONPROPORTIONAL DAMPING")

The methods examined in section 3.0 apply only if the type and amount of damping do not vary significantly throughout the structural system. Otherwise the damping matrix cannot be proportional to any combination of the mass and stiffness matrices. The equations of motion eq. (1) cannot be uncoupled and the damped system does not possess classical normal modes.

No critical damping ratio can be assigned to the entire system. Each element of the damping matrix must be determined individually resulting in a "nonproportional damping" matrix. (See, for example, Hurty and Rubinstein [3], Chapter 9). This is the general case of a damped linear dynamic system whose equations of motion eq. (1) cannot be solved by the mode superposition method.

This type of damping mechanism occurs, for example, in the dynamic model of a reactor containment for seismic analysis taking into account the soil-structure interaction. The damping in the structure is mainly of the internal structural type whereas the soil damping consists of the geometrical type (or "radiation damping") due to wave propogation as well as internal damping of hysteretic and plastic types. While a small critical damping ratio of 1 to 4 percent may usually be assigned to the internal structural damping the soil damping can amount to 50 percent or higher. In such case Hadjian [9] proposed that a proportional damping matrix be first determined by any of the methods in section 3.0 with appropriate internal structural damping ratios. Next those elements representing soil damping can be replaced by higher values computed, for example, by the method proposed by Richart et al. [2]. The resulting equations of motion with this "nonproportional damping" matrix can then be solved by the step-by-step integration method or the frequency response method. Alternatively the method of Foss [10] in terms of 2N complex eigenvalues and 2N complex eigenvectors can be used to solve eq. (1) with nonproportional damping.

4.1 A Numerical Example of Nonproportional Damping

Consider the numerical example in section 3.5 in which the critical damping ratios for all three normal modes are 3.5% which is a typical value of internal structural damping for a concrete containment. If we wish to consider the soil damping the damping constant $c_{10} = 0.19663 m_1 \omega_1$ in eqs. (83) should be replaced by a higher value. This value may be evaluated as follows: According to Table A-2 in Richart et al. [2] the geometrical damping ratio is approximately

$$D_g = 0.288 \left[\frac{7-8\nu}{32(1-\nu)} \frac{m}{\rho r_0^3} \right]^{-\frac{1}{2}} \tag{85}$$

Substituting the values of m, r_0 and ν from eqs. (4), (6) and (7) into eq. (85) and taking the soil density

$$\rho = 120 \text{ lb/ft}^3/g \tag{86}$$

we have

$$D_g = 0.47 \tag{87}$$

Adding to this a damping ratio of 0.05 to account for the soil internal damping due to hysteretic and viscous effects (as recommended in [2], p. 347) we should have the total horizontal soil damping ratio

$$D_x = 0.52 = \frac{c_{10}}{2\sqrt{k_1 m}} \tag{88}$$

When values of m, m_1, k_1 and ω_1 from eqs. (4), (5), (8) and (9) are used eq. (88) gives

$$c_{10} = 1.90523 \times 10^8 \text{ lb sec/ft} = 5.66463\, m_1 \omega_1 \tag{89}$$

Replacing c_{10} in eq. (83) by this value the damping matrix eq. (82) is modified to

$$[C] = m_1\omega_1 \begin{bmatrix} 5.75511 & -0.08277 & -0.00771 \\ -0.08277 & 0.19179 & -0.02942 \\ -0.00771 & -0.02942 & 0.09031 \end{bmatrix} \tag{90}$$

which is the nonproportional damping matrix for the problem. This matrix cannot be derived by any of the methods in section 3.0.

If this model of soil-structure system is subjected to a horizontal harmonic acceleration of amplitude A and circular frequency ω_1 at the base the impedance matrix is according to eqs. (30), (12) and (90)

$$[Z] = m_1\omega_1^2 \begin{bmatrix} 17.45266+5.75511i & -8.55863-0.08277i & -0.00771i \\ -8.55863-0.08277i & 9.37727+0.19179i & -1.68258-0.02942i \\ -0.00771i & -1.68258-0.02942i & 0.54652+0.09031i \end{bmatrix} \tag{91}$$

the absolute value of the complex amplitude for m_3 is according to eqs. (29), (11) and (91)

$$|X_3| = 7.56320\ (A/\omega_1^2) \tag{92}$$

and the dynamic magnification factor of m_3 relative to the base is according to eqs. (33), (36) and (92)

$$(MF)_3 = 6.23983 \tag{93}$$

This value is only 0.42 times the corresponding value of $(MF)_3$ in eq. (84) for case 3.5 in which proportional damping ratio of 0.035 is assumed for all three normal modes and the soil damping is not taken into account. This comparison, together with comparison of the values of $(MF)_3$ for various cases in section 3.0, shows that the system responses vary very little among various assumptions of proportional damping whereas the use of nonproportional damping matrix considering soil damping greatly reduces the responses.

4.2 Comparison of Case 4.1 with Proportional Damping $D_1 = D_2 = D_3 = 0.0833$

In order to estimate what proportional damping ratio will result in a response of m_3 given by eq. (93) for nonproportional damping let us compute the value of $(MF)_3$ for case 3.5 with

$$D_1 = D_2 = D_3 = \frac{0.035}{0.42} = 2.38 \times 0.035 = 0.0833 \tag{94}$$

Eq. (81) shows that for this case the damping matrix can be obtained by multiplying every element in eq. (82) by a factor of 2.38

$$[C] = m_1\omega_1 \begin{bmatrix} 0.68332 & -0.19699 & -0.01835 \\ -0.19699 & 0.45646 & -0.07002 \\ -0.01835 & -0.07002 & 0.21494 \end{bmatrix} \tag{95}$$

The dynamic magnification factor of m_3 relative to the base is according to eqs. (29) through (33) found to be

$$(MF)_3 = 6.22580 \tag{96}$$

This is only slightly lower than the value of $(MF)_3$ in eq. (93) for the case of nonproportional damping. Hence to account for structural damping as well as soil damping the equivalent proportional damping ratio of 8.3% should be used for all three normal modes.

4.3 Comparison of Case 4.1 with Dynamic Model Fig. 1(c) without Soil-Structure Interaction

If soil-structure interaction is assumed negligible and the foundation is assumed to be rigid and to have same motions as the ground the dynamic model for the containment Fig. 1(a) can be taken as the two degree of freedom system Fig. 1(c). According to eqs. (11) the mass and stiffness matrices for this model may be expressed as

$$[\bar{M}] = \begin{bmatrix} m_2 & 0 \\ 0 & m_3 \end{bmatrix} = m_1 \begin{bmatrix} 0.86394 & 0 \\ 0 & 1.13606 \end{bmatrix} \tag{97}$$

$$[\bar{K}] = \begin{bmatrix} k_2+k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} = k_1 \begin{bmatrix} 1.03509 & -0.17006 \\ -0.17006 & 0.17006 \end{bmatrix} \tag{98}$$

When these matrices are used and [C] and {F(t)} are set to be null matrices eq. (1) defines an eigenvalue problem whose solution gives the natural frequencies

$$\bar{\omega}_1 = 27.924 \text{ rad/sec}, \quad \bar{\omega}_2 = 88.384 \text{ rad/sec} \tag{99}$$

and modal columns (normalized to the greatest value of each mode)

$$\left\{\bar{\gamma}^{(1)}\right\} = \begin{Bmatrix} 1.00000 \\ 5.46506 \end{Bmatrix}, \quad \left\{\bar{\gamma}^{(2)}\right\} = \begin{Bmatrix} 1.00000 \\ -0.13915 \end{Bmatrix} \tag{100}$$

The transformation matrix $[\bar{Y}]$ formed from these modal columns is such that both $[\bar{M}]$ and $[\bar{K}]$ matrices are transformed into diagonal matrices as follows

$$[\bar{\gamma}]^T[\bar{M}][\bar{\gamma}] = m_1 \begin{bmatrix} 34.79451 & 0 \\ 0 & 0.88594 \end{bmatrix} = \left[\diagdown \bar{M}_r \diagdown\right] \tag{101}$$

$$[\bar{\gamma}]^T[\bar{K}][\bar{\gamma}] = k_1 \begin{bmatrix} 4.25548 & 0 \\ 0 & 1.08571 \end{bmatrix} = \left[\diagdown \bar{\omega}_r^2 \diagdown\right] \left[\diagdown \bar{M}_r \diagdown\right] \tag{102}$$

Let us choose method 3.3.1 to determine the damping matrix satisfying the condition that the critical damping ratios for both normal modes be 3.5% as expressed by eq. (48). Then substituting these damping ratios and eqs. (99) into eqs. (46) we have

$$\left.\begin{aligned} \alpha &= \frac{2D_1}{\bar{\omega}_1}\left(\frac{1}{1+\bar{\omega}_2/\bar{\omega}_1}\right) = \frac{2D_1}{\bar{\omega}_1}(0.24009) = 0.01681/\bar{\omega}_1 \\ \beta &= \frac{D_1\bar{\omega}_1}{1+\bar{\omega}_1/\bar{\omega}_2} = D_1\bar{\omega}_1\,(0.75991) = 0.02660\bar{\omega}_1 \end{aligned}\right\} \tag{103}$$

and eqs. (45), (97) and (98) give

$$[C] = 0.0532m_1\bar{\omega}_1 \begin{bmatrix} 0.86394 & 0 \\ 0 & 1.13606 \end{bmatrix} + \frac{0.01681k_1}{\bar{\omega}_1}\begin{bmatrix} 1.03509 & -0.17006 \\ -0.17006 & 0.17006 \end{bmatrix} \tag{104}$$

Comparing this matrix with the damping matrix for the model Fig. 1(c)

$$[C] = \begin{bmatrix} c_{12}+c_{23} & -c_{23} \\ -c_{23} & c_{23}+c_{13} \end{bmatrix} \tag{105}$$

we identify the individual damping constants for the two degree of freedom dynamic model as follows:

$$c_{13} = 0.06044m_1\bar{\omega}_1, \quad c_{23} = 0.00286k_1/\bar{\omega}_1, \quad c_{12} = 0.04596m_1\bar{\omega}_1 + 0.01454k_1/\bar{\omega}_1 \tag{106}$$

From eqs. (101), (102) and (103) the following transformation results in a diagonal matrix

$$[\bar{Y}]^T[C][\bar{Y}] = 0.0532\bar{\omega}_1 \left[\bar{M}_r\right] + \frac{0.01681}{\bar{\omega}_1}\left[\bar{\omega}_r^2\right]\left[\bar{M}_r\right] \tag{107}$$

Hence the damped system possesses classical normal modes.

If this damped system is subjected to a horizontal harmonic acceleration of amplitude A and circular frequency ω_1 at the base the impedance matrix is according to eqs. (30), (97), (98) and (104)

$$[Z] = m_1\omega_1^2 \begin{bmatrix} 9.37727 + 0.20704i & -1.68258 - 0.02571i \\ -1.68258 - 0.02571i & 0.54652 + 0.09219i \end{bmatrix} \tag{108}$$

The absolute value of the complex amplitude for m_3 is according to eqs. (29), (97) and (108)

$$|X_3| = 4.95551\ (A/\omega_1^2) \tag{109}$$

The corresponding value under static loading is according to eqs. (32), (97) and (98)

$$|X_{03}| = 8.99225(Am_1/k_1) \tag{110}$$

Substituting eqs. (109) and (110) into eq. (33) and using the value of λ_1 from eq. (13) we obtain the dynamic magnification factor of m_3 relative to the base

$$(MF)_3 = 5.45247 \tag{111}$$

which is lower than $(MF)_3 = 6.23983$ in eq. (93) for the three degree of freedom model with nonproportional damping and is much lower than $(MF)_3 = 14.82751$ in eq. (84) for the three degree of freedom model with proportional damping. This implies that a design based on the model Fig. 1(c) is less conservative than a design based on model Fig. 1(b) with soil damping. This fact is even more obvious if we compare the value of the displacement amplitude $|X_3|$ in eq. (109) with $|X_3| = 7.56320\ (A/\omega_1^2)$ in eq. (92) for case 4.1. On the other hand the results for the simpler model render better approximations (although on the nonconservative side) than those of case 3.5 where the soil-structure interaction is taken into account but the proper amount of soil damping has not been accounted for either by nonproportional damping as in case 4.1 or by higher equivalent proportional damping as in case 4.2.

5.0 CONCLUSIONS

If the type and amount of damping do not vary significantly throughout the structure the various methods in section 3.0 may be used to determine a "proportional damping" matrix. Otherwise the method in section 4.0 can be used to determine a "nonproportional damping" matrix. Numerical results for a simple representative dynamic model of containment-soil system Fig. 1(b) show that

(a) The dynamic responses eqs. (37), (44), (53), (62) and (84) by using various proportional damping matrices in section 3.0 differ very little.

(b) The dynamic response eq. (93) by using a nonproportional damping matrix (taking into account proper amount of soil damping) is only 0.42 times the corresponding response eq. (84) by assuming all normal modes to have internal structural damping ratio of 3.5%. This means that a design based on proportional damping with internal structural damping ratios can be much too conservative.

(c) In order to use proportional damping matrix to obtain the response eq. (93) resulting from use of the nonproportional damping matrix the modal damping ratios must be taken 8.3% for all modes as shown in section 4.2.

(d) The dynamic responses eqs. (109) and (111) for the model Fig. 1(c) disregarding the soil-structure interaction are lower than the responses eqs. (92) and (93) for the model Fig. 1(b) including the soil-structure interaction. This means that a design disregarding the soil-structure interaction can be nonconservative.

(e) If soil-structure interaction is taken into account in an analysis the proper amount of soil damping must be accounted for either by using nonproportional damping matrix as in section 4.1 or by using higher equivalent proportional damping ratios as in section 4.2.

The above conclusions are based upon dynamic responses at one point m_3 of the structure under a harmonic forcing function of one circular frequency ω_1. If other points of the structure and other forcing functions are considered and if more elaborate dynamic models are used different numerical values will result but the essential trend of the results will be similar. This paper provides the designer with a tool for evaluating such different numerical values whenever the need arises.

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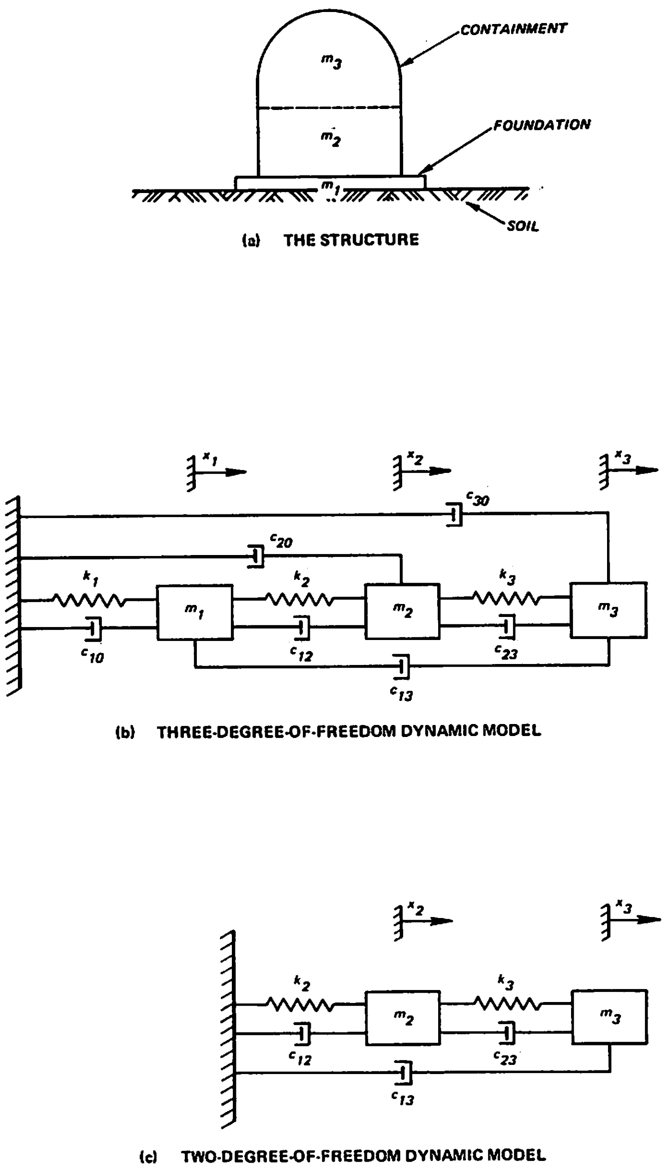

FIGURE 1 DYNAMIC MODELS OF A REACTOR CONTAINMENT