Mathematical Modellings and Computational Methods for Structural Analysis of LMFBRs

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Abstract

In this paper, two aspects of nuclear reactor problems are discussed, modelling techniques and computational methods for large scale linear and nonlinear analyses of LMFBRs. For nonlinear fluid-structure interaction problem with large deformation, arbitrary Lagrangian-Eulerian description is applicable. For certain linear fluid-structure interaction problem, the structural response spectrum can be found via "added mass" approach. In a sense, the fluid inertia is accounted by a mass matrix added to the structural mass. The fluid/structural modes of certain fluid-structure problem can be uncoupled to get the reduced added mass. The advantage of this approach is that it can account for the many repeated structures of nuclear reactor. In regard to nonlinear dynamic problem, the coupled nonlinear fluid-structure equations usually have to be solved by direct time integration. The computation can be very expensive and time consuming for nonlinear problems. Thus, it is desirable to optimize the accuracy and computation effort by using implicit-explicit mixed time integration method.

Many computational methods have been proposed to solve the fluid structure interaction problem in nuclear reactor analyses. For nonlinear fluid-structure interaction problem with large deformation, mixed Lagrangian-Eulerian formulation or Arbitrary Lagrangian-Eulerian (ALE) formulation is appropriate [9,22]. The Arbitrary Lagrangian-Eulerian description has been developed by Hirt et al. [1], Donea et al. [4], Belytschko and Kennedy [2], and Hughes et al. [3] among others. The virtue of ALE is that an arbitrary reference frame can be designed to describe a rather complicated problem such as free surface flow which would otherwise be very cumbersome to be treated in pure Eulerian Description or pure Lagrangian Description.

To this end, let us denote the spatial region by R_x and denote the material region by R_X. Then, we can construct an arbitrary reference region R_χ which can be considered as a "reference frame". The R_χ is related to R_x and R_χ by transformations which are defined specifically according to our convenience. If we choose R_x as the R_χ, we have an Eulerian description. On the other hand, if R_X is picked as the R_χ, we have a Lagrangian description. In general, the "reference frame" can be any well defined region with a well transformation related to R_x and R_χ. Interested readers may want to refer [1-3] and [7,9] for further information. Related shell structure mechanics can be found in [13,14]. The governing equations for fluid are

$$\rho \mathring{v}_i = \tau_{ij,j} + b_i \quad , \tag{1}$$

$$\mathring{\rho} = -\rho v_{i,i} \quad , \tag{2}$$

$$\tau_{ij} = -p\delta_{ij} + \mu(v_{i,j}+v_{j,i}) \tag{3}$$

where ρ is the density, $\underset{\sim}{b}$ is the body force, p is the pressure, $\underset{\sim}{v}$ is the velocity vector and μ is the dynamic viscosity. ",i" denotes spatial derivatives and "$\circ$" denotes the material time derivative. The variation equation or weak form corresponding to (1-3) is

$$\begin{aligned}
&\int_{R_x} \rho v_{i,t[\chi]} \bar{u}_i \, dR_x + \int_{R_x} \rho v_{i,j} w_j \, dR_x \\
&+\int_{R_x} - p\bar{u}_{i,i} \, dR_x + \int_{R_x} 2\mu v_{(i,j)} \bar{u}_{(i,j)} \, dR_x \\
&-\int_{R_x} b_i \bar{u}_i \, dR_x + \int_{\partial R_x} h_i \bar{u}_i \, d\Gamma_x = 0 \quad ,
\end{aligned} \tag{4}$$

where $\underset{\sim}{\bar{u}}$ is the test function and $\underset{\sim}{w}$ is the convective velocity. The corresponding semi-discrete equations are

$$\underset{\sim}{M}\,\underset{\sim}{a} + \underset{\sim}{N}(\underset{\sim}{v}) + \underset{\sim}{K}_\mu \underset{\sim}{v} - \underset{\sim}{G}\,\underset{\sim}{P} = \underset{\sim}{F} \quad , \tag{5}$$

where $\underset{\sim}{M}$ is the consistent ALE mass matrix, $\underset{\sim}{N}$ is the nonlinear convective inertia force, $\underset{\sim}{K}_\mu$ is the viscous matrix, $\underset{\sim}{G}$ is the gradient matrix and $\underset{\sim}{F}$ is the generalized force vector. The ALE can handle moving boundary such as sloshing phenomena naturally. Application to liquid storage tanks analyses can be found in [10,11,17,18]. However, the implementation to computer codes is rather involved.

For some fluid-structure interaction problem the fluid can be assumed to be inviscid and the system is subjected to small disturbance; in such case, the governing equations become linear. Then, the response spectrum for the structure with fluid influence can be studied via lumped mass or simple added mass [5].

The lumped mass approach can be done by tributary area method, in which liquid in the vicinity of a structural nodal points is lumped as additional mass at that nodal point. The added mass matrix can also be found by finite element method in a consistent approach [5].

The semi-discretized equation for fluid-structure system in pressure-structure formulation can be expressed as,

$$\underset{\sim}{M}_F\ddot{\underset{\sim}{P}} + \underset{\sim}{C}_F\dot{\underset{\sim}{P}} + \underset{\sim}{K}_F\underset{\sim}{P} - \underset{\sim}{R}\ddot{\underset{\sim}{u}} = \underset{\sim}{f}_F \quad , \tag{6}$$

and

$$\underset{\sim}{M}_s\ddot{\underset{\sim}{u}} + \underset{\sim}{C}_s\dot{\underset{\sim}{u}} + \underset{\sim}{K}_s\underset{\sim}{u} + \underset{\sim}{R}^T\underset{\sim}{P} = \underset{\sim}{f}_s \tag{7}$$

The subscript 's' refers to structure part while the subscript 'F' refers to fluid part. $\underset{\sim}{M}$, $\underset{\sim}{C}$ and $\underset{\sim}{K}$ are the mass matrix, damping matrix and stiffness matrix respectively. $\underset{\sim}{f}$ is the generalized force vector and $\underset{\sim}{R}$ is the coupling matrix between fluid and structure; $\underset{\sim}{P}$ is the pressure and $\underset{\sim}{u}$ is the displacement vector. If the fluid is incompressible, the above equations may be combined to yield,

$$(\underset{\sim}{M}_s + \underset{\sim}{R}^T\underset{\sim}{K}_F^{-1}\underset{\sim}{R})\ddot{\underset{\sim}{u}} + \underset{\sim}{C}_s\dot{\underset{\sim}{u}} + \underset{\sim}{K}_s\underset{\sim}{u} = \underset{\sim}{f}_s - \underset{\sim}{R}^T\underset{\sim}{K}_F^{-1}\underset{\sim}{f}_F \quad , \tag{8}$$

where the term $\underset{\sim}{R}^T\underset{\sim}{K}_F^{-1}\underset{\sim}{R}$ has the form of an "added mass" matrix. The problem has been reduced to one involving structure displacements only. The mode shapes and frequencies of the system can be determined, but the process of extracting eigenvalues is expensive especially for large system. The consistent "added mass" matrix is symmetric but it is a full matrix. One can diagonalize the consistent matrix in such way that the diagonal terms are in proportion to the diagonal terms of the consistent mass matrix or equal to the sum of each row in the matrix. An alternate approach proposed by Au-Yang et al. [12] is to compute the consistent matrix by series expansion method. The advantage of all these added mass approach as mentioned above is that once the fluid mass matrix is evaluated, the problem can be solved by the usual methods of structural dynamic analysis. As a word of caution, when tributary mass matrix or diagonalized consistent mass matrix is used, there is a tendency to underestimate the natural frequency of the system. The drawback is due to the inability of the mass matrix to account for the cross-coupling between nodal points [5].

To circumvent these difficulties, a simplified added mass approach is to uncouple fluid/structure modes to get a reduced added mass.

Let $$\overline{\underset{\sim}{f}}_s = \underset{\sim}{f}_s - \underset{\sim}{R}^T\underset{\sim}{K}_F^{-1}\underset{\sim}{f}_F \quad , \tag{9}$$

then eq. (8) can be written as,

$$\underset{\sim}{M}_s\ddot{\underset{\sim}{u}} + \underset{\sim}{C}_s\dot{\underset{\sim}{u}} + \underset{\sim}{K}_s\underset{\sim}{u} = \overline{\underset{\sim}{f}}_s - (\underset{\sim}{R}^T\underset{\sim}{K}_p^{-1}\underset{\sim}{P})\ddot{\underset{\sim}{u}} \quad . \tag{10}$$

Let $$\underset{\sim}{u} = \underset{\sim}{\phi}\,\underset{\sim}{d}$$

such that

$$\underset{\sim}{\phi}^T\underset{\sim}{M}_s\underset{\sim}{\phi} = \underset{\sim}{1} \tag{11}$$

$$\underset{\sim}{\phi}^T\underset{\sim}{K}_s\underset{\sim}{\phi} = \underset{\sim}{\Lambda}_s \tag{12}$$

$$\underset{\sim}{\phi}^T\underset{\sim}{C}_s\underset{\sim}{\phi} = \underset{\sim}{\xi}_s \quad . \tag{13}$$

Therefore,

$$\underset{\sim}{1}\ddot{\underset{\sim}{d}} + \underset{\sim}{\xi}_s\dot{\underset{\sim}{d}} + \underset{\sim}{\Lambda}_s\underset{\sim}{d} = \underset{\sim}{\phi}^T\overline{\underset{\sim}{f}}_s - \underset{\sim}{\phi}^T\underset{\sim}{R}^T\underset{\sim}{K}_f^{-1}\underset{\sim}{\phi}\,\ddot{\underset{\sim}{d}} \tag{14}$$

or

$$\underset{\sim}{M}\ddot{\underset{\sim}{d}} + \underset{\sim}{\xi}_s\dot{\underset{\sim}{d}} + \underset{\sim}{\Lambda}_s\underset{\sim}{d} = \underset{\sim}{\phi}^T\overline{\underset{\sim}{f}}_s \tag{15}$$

$$\underset{\sim}{M} = \underset{\sim}{1} + \underset{\sim}{\phi}^T \underset{\sim}{R}^T \underset{\sim}{K}_p^{-1} \underset{\sim}{R} \underset{\sim}{\phi} \quad . \tag{16}$$

The reduced system is easy to compute and is much smaller. Either step by step integration or modal analysis is applicable to solve the equation. If the fluid/structure modes can be uncoupled, the above technique is very useful especially for system with many identical structural components. Only one set of eigen modes is needed for each type of structural component.

For incompressible viscous fluid-structure interaction, the incompressibility constraint is one of the more difficult aspect of numerical treatment. One of the possible way to handle the incompressibility is by penalty Formulation [8]. The continuity equation for incompressible fluid is

$$u_{i,i} = 0 \tag{17}$$

which is replaced by

$$u_{i,i} = -\frac{1}{\lambda} p \tag{18}$$

where λ is the penalty parameter. It was shown by Teman in [23] that, under appropriate hypotheses, the solution to the penalty problem converges to the solution of full incompressible problem as $\lambda \to \infty$. An advantage of the penalty formulation is that pressure and the continuity equation are eliminated. Physically, the penalty parameter λ may be interpreted as a large dilatational viscosity.

The resulting semi-discrete equations for fluid-structure interaction problem may involve many thousand degrees of freedom. To get the solution by direct time integration can be very costly. One method to reduce the cost is to have algorithms which use different time integration methods in different parts of the mesh. There are, in general, two classes of integration scheme, implicit and explicit method. Advantages and disadvantages of the methods have been discussed in many literatures [6,15,16,19,20,21]. Although implicit method can be unconditionally stable, it is expensive to compute for each time step and is governed by the desired accuracy. On the other hand, explicit method is easy to compute, the time step is governed by stability limit. The idea of mixed-time implicit-explicit method is to use different methods with different time steps in different part of the solution domain to optimize the solution accuracy and costs. Examples can be found in references [9,10,24].

References

[1] HIRT, C. W., AMSDEN, A. A., and COOK, J. L., "An Arbitrary Lagrangian-Eulerian Computing Method for all Flow Speeds," Journal Computational Physics, Vol. 14, pp. 227-253, 1974.

[2] BELYTSCHKO, T., and KENNEDY, J. M., "Computer Models for Subassembly Simulation," Nuclear Engineering Design, Vol. 49, pp. 17-38, 1978.

[3] HUGHES, T. J. R., LIU, W. K., and ZIMMERMANN, T. K., "Lagrangian-Eulerian Finite Element Formulation for Incompresible Viscous Flow," Journal of Computer Methods in Applied Mechanics and Engineering, Vol. 29, No. 3, pp. 329-349, 1981.

[4] DONEA, J., FASOLI-STELLA, P., and GIULIANI, S., "Lagrangian and Eulerian Finite Element Techniques for Transient Fluid-Structure Interaction Problems," Trans. Fourth Int. Conf. on Structural Mechanics in Reactor Technology, San Francisco, August 1977.

[5] RUSH, R. H., and JACKSON, J. E., "Treatment of Hydrodynamic Effects for Toroidal Containment Vessels," Nuclear Engineering and Design, Vol. 53, pp. 217-222, 1979.

[6] HUGHES, T. J. R., and LIU, W. K., "Implicit-Explicit Finite Elements in Transient Analysis," Journal of Applied Mechanics, Vol. 45, pp. 371-378, 1978.

[7] LIU, W. K., "Development of Finite Element Procedures for Fluid-Structure Interaction," EERL 80-06, California Institute of Technology, Pasadena, California, August 1980.

[8] HUGHES, T. J. R., LIU, W. K., and BROOKS, A., "Finite Element Analysis of Incompressible Viscous Flow by the Penalty Function Formulation," Journal of Computational Physics, Vol. 30, pp. 1-60, 1979.

[9] LIU, W. K., "Finite Element Procedures for Fluid-Structure Interaction and Application to Liquid Storage Tanks," Nuclear Engineering and Design, Vol. 65, pp. 221-238, 1981.

[10] LIU, W. K., and MA, D. C., "Computer Implementation Aspects for Fluid-Structure Interaction Problems," Computer Methods in Applied Mechanics and Engineering, Vol. 31, pp. 129-148, 1982.

[11] MA, D. C., LIU, W. K., GVILDYS, J., and CHANG, Y. W., "Seismic Behavior of Liquid-Filled Shells," Nuclear Engineering and Design, Vol. 70, pp. 437-455, 1982.

[12] AU-YANG, M. K., and GALFORD, J. E., "A Structural Priority Approach to Fluid-Structure Interaction Problems," Journal of Pressure Vessel Technology, ASME, Vol. 103, pp. 142-150, May 1981.

[13] HUGHES, T. J. R., and LIU, W. K., "Nonlinear Finite Element Analysis of Shells: Part I - Three Dimensional Shells," Computer Methods in Applied Mechanics and Engineering, Vol. 26, pp. 331-362, 1981.

[14] HUGHES, T. J. R., and LIU, W. K., "Nonlinear Finite Element Analysis of Shells: Part II - Two Dimensional Shells," Computer Methods in Applied Mechanics and Engineering, Vol. 27, pp. 167-182, 1981.

[15] LIU, W. K., and BELYTSCHKO, T., "Mixed Time Implicit-Explicit Finite Elements for Transient Analysis," Computers and Structures, Vol. 15, pp. 445-450, 1982.

[16] BELYTSCHKO, T., and MULLEN, R., "Stability of Explicit-Implicit Mesh Partition in Time Integration," International Journal Numerical Method in Engineering, Vol. 12, pp. 1575-1586, 1978.

[17] LIU, W. K., MA, D. C., CHANG, Y. W., and LEE, H. S., "Seismic Analysis of Liquid Storage Tanks," Computer Analysis of Large-Scale Structures, ed. by K. C. Park and R. F. Jones, Jr., AMD-Vol. 49, ASME, pp. 69-80, 1981.

[18] LIU, W. K., and MA, D. C., "Coupling Effect Between Liquid Sloshing and Flexible Fluid-Filled System," Nuclear Engineering and Design, Vol. 72, pp. 345-357, 1982.

[19] LIU, W. K., "Development of Mixed Time Partition Procedures for Thermal Analysis of Structures," International Journal for Numerical Methods in Engineering, Vol. 19, pp. 125-140, 1983.

[20] LIU, W. K., and LIN, J., "Stability of Mixed Time Integration Schemes for Transient Thermal Analysis," Numerical Heat Transfer Journal, Vol. 5, pp. 211-222, 1982.

[21] LIU, W. K., and ZHANG, Y. F., "Unconditionally Stable Implicit-Explicit Algorithms for Coupled Thermal Stress Waves," to appear in Computers and Structures, 1983.

[22] BELYTSCHKO, T., "Fluid-Structure Interaction," Computers and Structures, Vol. 12, pp. 459-469, 1980.

[23] TEMAN, R., "Navier-Stokes Equation," North-Holland, Amsterdam, 1977.

[24] LIU, W. K., and ZHANG, Y. F., "Improvement of Mixed Time Implicit-Explicit Algorithm for Thermal Analysis of Structures," Computer Methods in Applied Mechanics and Engineering, 1983.