Nonlinear Response Analysis in Frequency Domain

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Abstract

This paper describes the analytical method of the nonlinear soil-structure intention due to base mat uplift. The superstructure is idealized as lumped masses and beams. The soil is represented as linear element in Boundary Element Method. The base mat is assumed to be rigid and on an elastic half space. The analysis is performed iteratively in the frequency domain and also in the time domain.

1. Introduction

Recently a dynamic analysis that includes soil-structure interaction effects has been frequently required in the design of nuclear power plant. The soil-structure interaction analysis is mostly executed in the frequency domain in order to reduce the computer time and its capacity. Because the dynamic analysis in the frequency domain is performed by applying suitable boundary conditions those are illustrated frequency dependent. It was difficult to apply the analysis in the frequency domain to the nonlinear problems. However, a new method for the solution of the nonlinear problems has been developed recently. This paper proporses the analytical method for the nonlinear soil-structure interaction caused by the base mat uplift in the frequency domain.

2. Method of Analysis

The equation of motion of dynamic excited systems in the linear range are written as:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=-[M]\{1\}\ddot{y} \tag{1}$$

where [M], [C] and [K] are the mass, viscous damping and stiffness matrices respectively. {u} is the displacement vector, and the dots indicate time derivatives. $\ddot{y}$ is excitation.

Due to harmonic excitation $\ddot{y}=\tilde{y}\, e^{i\omega t}$, equation given as follows:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=-[M]\{1\}\tilde{y}e^{i\omega t} \tag{2}$$

for hysteretic damped systems the equations are

$$[M]\{\ddot{u}\}+[K^*]\{u\}=-[M]\{1\}\tilde{y}e^{i\omega t} \tag{3}$$

where [K*], is complex stiffness matrix.

From eq.(3) the transformed differential equation can be written as:

$$(-\omega^2[M]+[K^*])\{\tilde{u}\}=-[M]\{1\}\tilde{y} \tag{4}$$

or in brief form $\qquad [K_S]\{\tilde{u}\}=-[M]\{1\}\tilde{y} \qquad (5)$

where ∿ indicate the value in the frequency domain.

2.1 Superstructure

The outline of the analysis model is shown in Fig.1. The super-structure is treated as the lumped masses and beams. Each mass has three degrees of freedoms (i.e. vertical, horizontal and rotational). Each beam element complex matrix $[K_e^*]$ is

$$[Ke^*]=(1+i\cdot 2h)[Ke] \tag{6}$$

where $[K_e]$ is each beam element stiffness matrix, i is the imaginary unit, and h is the critical damping ratio.

2.2 Soil

Modeling of the soil region is made as follows. The soil was assumed as the homogeneous isotoropic linear elastics. Under these assumptions the displacement equations of motion can be written as follows. [2]

$$(C_1^2-C_2^2)u_{i,ij}+C_2^2u_{j,ii}+b_i=\ddot{u}_i \tag{7}$$

where commas indicate space derivatives. The propagation speed of the dilatational and distortional waves are given as

$$C_1=\sqrt{\frac{\lambda+2\mu}{\rho}}, \quad C_2=\sqrt{\frac{\mu}{\rho}} \tag{8}$$

where λ and μ are the Lamé's constants, and ρ is the density of the soil. Stress and strains are given by

$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}) \tag{9}$$

$$\sigma_{ij}=\lambda\varepsilon_{mm}\delta_{ij}+2\mu\varepsilon_{ij} \tag{10}$$

where δ_{ij} is the Kronecker delta.

From eq.(7), the transformed equation of motion can written as follows:

$$(C_1^2-C_2^2)\tilde{u}_{j,ji}+C_2^2\tilde{u}_{i,jj}+\tilde{b}_i=-\omega^2\tilde{u}_i \tag{11}$$

The solution of the above equation under condition that a unit harmonic point load acts on the surface of the elastic half-space is obtained as the fundamental solution. The fundamental solution is calculated numerically. [4]

The displacement-force relationship of the soil is evaluated by the boundary element method. The displacement-force relationship is expressed by the integral equations as follows:

$$\tilde{u}_i+\int_\Gamma \tilde{u}\tilde{p}^*\,d\Gamma=\int_\Gamma \tilde{p}\,\tilde{u}^*\,d\Gamma \tag{12}$$

Where Γ is the boundary, u^* and P^* are the fundamental solution and the fundamental traction respectively.

Boundary elements are placed on the surface of the half-space soil as shown in Fig.2. With above assumptions the integral around singular point of fundamental solution is 0, and the second term of left hand side of the equation is 0. Thus the equation is rewritten as follows:

$$\tilde{u}_i=\int\tilde{p}\,\tilde{u}^*\,d\Gamma \tag{13}$$

When the linear boundary element shown in Fig. 3 is adopted, the discretized equation is obtained from eq. (13) as follows:

$$\tilde{u}_i=\sum_{j=1}^{n}\tilde{p}_i\,G_{ij} \tag{14}$$

or the matrix form is

$$\{\tilde{u}\}=[G]\{\tilde{p}\} \tag{15}$$

The k element component of the matrix [G] is

$$[G_K]=[\textstyle\int_{\Gamma K}[\phi]\tilde{u}_1^*\,d\Gamma,\ \cdots\cdots\cdots,\int_{\Gamma K}[\phi]\tilde{u}_i^*\,d\Gamma,\ \cdots\cdots\cdots] \tag{16}$$

where

$$[\phi]=[\{\phi_1\},\{\phi_2\},\{\phi_3\},\{\phi_4\}] \tag{17}$$

$$\{\phi_1\}^T=\frac{1}{4}\{(1-\xi)(1-\eta),\ (1+\xi)(1-\eta),\ (1+\xi)(1+\eta),\ (1-\xi)(1+\eta)\} \tag{18}$$

the impedance matrix (dynamic stiffness matrix) of soil is obtained as follows:

$$[Ks]=[G]^{-1} \tag{19}$$

and eq.(14) is rewritten as

$$[Ks]\{\tilde{u}\}=\{\tilde{p}\} \tag{20}$$

2.3 Interface of soil and super-structure

The base mat is assumed to be rigid. Relationship between the displacement of the base node and the displacement of a node on the surface of the soil are written as follows:

$$u_B=u_i, \quad v_i=\ell\ w_B+v_B \tag{21}$$

where u, v, w are horizontal, vertical and rotational displacement respectively, the subscript B and i indicate the base node of the superstructure and a node on the surface of the soil respectively, and ℓ is distance between i and B, as is shown in Fig.4. These relationships are written by Lagrange multipliers as follows:

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{Bmatrix} u_B \\ u_i \\ \lambda_u \end{Bmatrix} = 0 \qquad \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -\ell \\ 1 & -1 & -\ell & 0 \end{bmatrix} \begin{Bmatrix} v_i \\ v_B \\ w_B \\ \lambda_v \end{Bmatrix} = 0 \tag{22}$$

The Lagrange multipliers are equivalent to the reaction force acting between the base node and the node of the soil surface. Thus, the complete equations are constructed. But, because of the equations that include Lagrange multipliers are positive semidefinite, the equation must be solved by the algorizm considered these condition.

2.4 Uplift problem

When uplift is caused, the Lagrange multipliers become negative. The residual forces P_R are follows:

$$P_R=\lambda_P \tag{23}$$

The excitation function is shown as a finite sum of a Fourier series.

$$\ddot{y}=Re\sum_{s=0}^{N/2} \tilde{y}e^{i\omega_s t} \tag{24}$$

Here N is the number of digitized point of the input motion. The transformed differential non-linear equation can be written.

$$[Ks]\{\tilde{u}\}=[m]\{1\}\tilde{y}+\tilde{P}_R \tag{25}$$

Where P_R are evaluated in the time domain and $\tilde{P}_R$ are transformed from P_R.
The displacements in the time domain is obtained as a finite sum of a Fourier series.

$$\{u\}=Re\sum_{s=0}^{N/2}\{\tilde{u}\}e^{i\omega st} \tag{26}$$

2.5 Iterative calculation

Eq.(26) contains nonlinear parts which do not allow a solution closed in the frequency domain. Therefore, an iterative algorithm has to be used which should adapt the base mat reaction response on to the response of the calculated deformations.

The iterative process is shown in Fig.5. The convergence of the solution is checked as follows:

$$\| P_{+R}^{j}-P_R^{j-1} \| / \| P_R^{j-1} \| < \varepsilon \tag{27}$$

Where superscript indicate iteration cycles, and ε is a factor of accuracy.

3. Conclusion

The procedure presented here can be applied to the response analysis of the nonlinear problem caused by base mat uplift in the frequency domain. And this procedure can be extended for response analysis with elastic base mat uplift by adoption of the FEM solid element.

References

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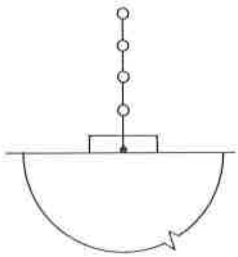

Fig. 1 Analysis model

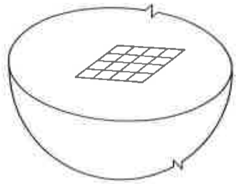

Fig. 2 Boundary element

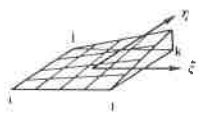

Fig. 3 Linear element

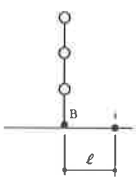

Fig. 4 Base mat stiffness

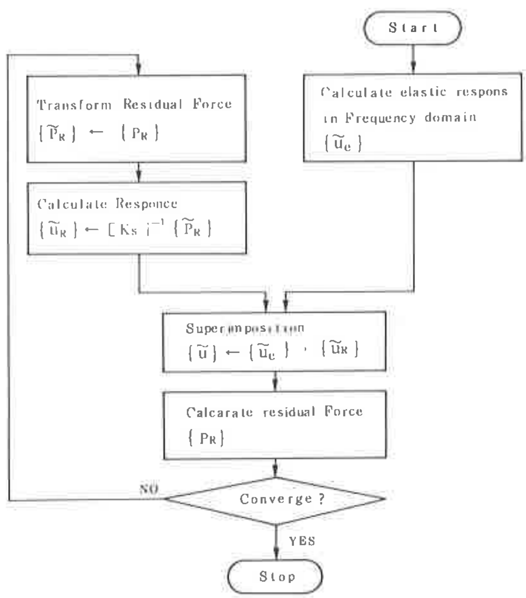

Fig. 5