# On the Dynamic Response of Ductile Piping Containing Stable Cracks

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The choice of ductile materials for nuclear reactor piping loops enables them to tolerate large flaws. Indeed, it is often considered a safety feature when cracks can grow to large proportions and still remain stable, and the leak-before-break criterion is frequently discussed in the context of longitudinal cracks several pipe diameters long or circumferential cracks that reduce the net cross section of the order of one-half the original load-carrying area. The demonstration of the stability of such cracks is often the end of a safety analysis of a cracked piping loop.

However, even though such large cracks may be found to be stable, their presence could dramatically affect the response of the piping loop to abnormal transient loads under seismic or accident conditions. The subsequent dynamic response of the cracked piping could in turn affect the stability of the crack.

The effects of large but stable cracks on the dynamic structural response of a piping loop will depend upon the size and location of the crack as well as upon the nature of the loading and supports. In order to develop an understanding of the generic effects of cracks on the structural behavior of piping loops, the more fundamental problem of the cracked perfectly-plastic beam has been studied under various end and loading conditions.

The presence of cracks can result not only in permanent transverse deformation of a different order of magnitude but also in entirely different modes of plastic response. The results are highly dependent upon the end conditions of a beam and the related bending moment distribution along its length. The simple mechanical models which have been employed to study the phenomenon involved enable the analyst to understand easily these dependencies while also providing numerical solutions to some elementary benchmark problems upon which more sophisticated techniques of analysis might be built.

As one might expect, results show that while the magnitude of beam response can be sensitive to relatively small (less than 10-20% reduction in load-carrying cross section) cracks located midway between supports, the effects of end conditions are small for such cracks. However, for large cracks located less symmetrically the beam response can be highly dependent upon the end conditions and the size and location of a crack. These configurations can result in deformed shapes less intuitive, and thus the experience gained from the simple models can be valuable in identifying worst-case crack locations in real piping loops.

## 1. Introduction

Large plastic deformations under dynamic loading conditions can present formidable problems for structural analysis, but the basic mechanical phenomena can also be captured with simple models and experiments. The rigid-perfectly plastic beam is an especially useful first approximation for studying the damage imparted by impact or blast loading conditions, and this simple model is capable of predicting not only the mode of deformation but also its magnitude.

The work by Parkes [1] on the permanent deformation of a cantilever struck transversely at its tip illustrates well the efficacy of the simple model. In this work the moment-curvature relationship was taken to be that shown in Figure 1(a), and the geometry of the problem is shown in Figure 1(b). The initially stationary beam of mass $m\ell$ is struck by the impact mass $M$, whereby a plastic hinge develops at the end and travels toward the root of the beam. Equations of rigid-body dynamics predict different behaviors for light and heavy strikers, defined according to whether the dimensionless parameter $\beta = m\ell/2M$ is small (heavy striker) or large (light striker).

Parkes performed experiments on mild-steel cantilevers, striking their tips with either a high speed bullet or a freely-falling heavy weight. A sample of his experimental results is shown in Figure 2. The final configuration of the beam is clearly affected by the extremes of the parameter $\beta$, and these shapes are predicted by the limiting cases of the simple rigid-plastic model.

Extensions of this work to an encastre beam struck at any point in its span was made easily by Parkes [2], and he generalized the work also to include frames [2]. Many others have applied equivalent models to a variety of loading conditions, and end conditions (e.g., Lee and Symonds [3], Symonds [4]), and they may be applied also to a piping loop subjected to impact or seismic excitation to ascertain the resulting permanent damage.

## 2. Effects of Cracks

When cracks are present, the results described above can be invalidated by the beam breaking prematurely at the crack location. However, this is unlikely to happen in structures made of flaw-tolerant ductile materials such as the stainless steels of liquid metal fast breeder reactor designs. These materials are so tough that axial through-wall cracks whose length is 10 times the wall thickness of piping or circumferential cracks whose length is 30 times the thickness are stable [5]. Thus the primary effect of such cracks will be to alter locally the moment curvature relation along the pipe length. Thus the beam results discussed above must be altered to take this into account.

A simple model for the cracked rigid-perfectly plastic beam has been developed by Petroski [6], and it is capable of predicting the permanent deformation of a cracked beam subjected to dynamic loading. Among the salient features of the results are the earlier initiation of yielding, the alteration of the modes of plastic deformation, and the greater degree of permanent damage.

These models are being applied to a variety of end conditions and crack locations and sizes, thereby providing the capability for determining analytical expressions to determine worst-case crack locations under various loading and end conditions. Worst-case configurations can subsequently be analyzed with more sophisticated models employing more accurate but also more expensive finite element and other computational techniques.

## 3. Some Simple Experiments

In order to demonstrate the effects of stable cracks on the plastic deformation of beams subjected to impact, a simple experimental set-up was employed. The apparatus is shown in Fig. 3 and consists of a nearly frictionless slider which moves freely on a rail mounted vertically on a wall. Sample beams were cantilevered from the slider assembly, which was then released from a predetermined height. A built-up rigid anvil was positioned so that it was struck by the descending beam at a position 1/4" from the beam's free end.

The beams consisted of 8" lengths cut from a 1/4" x 1/4" rod of scrap aluminum, which was employed because it demonstrated a good degree of bendability even in the presence of deep saw cuts. Cracks were simulated by saw cuts half-way through the beam, and they were made at positions one-quarter and one-half the distance from the beam's root to its free end. Table I shows the matrix of specimens employed.

The beams were dropped from two heights, 28.2" and 41.2" above the anvil, and outlines of the permanently deformed beams are shown in Figs. 4 and 5. In these figures the beams are oriented for easy comparison with Parkes's results, in which the weight was dropped on a stationary beam.

The deformed beams in Fig. 4 were dropped from 28.2", and there was no appreciable effect of the cracks on the maximum deformation at the tip. However, there is clearly a difference in shape of the deformed beams. The uncracked beam, Fig. 4(a), deformed with a single hinge at its root, just as did the beams Parkes struck with a heavy striker. When a crack is present, however, a second plastic hinge also develops at the crack location. Since the saw cuts had a width of approximately 1/16", the beam sections on either side of the "crack" could rotate through a small angle before closing the crack. Thus for these test conditions there was no noticeable difference in effect between cracks in tension and those in compression. (That this will not always be the case is demonstrated below.) A crack closer to the root of the beam, Fig. 4(d), resulted in a greater relative rotation across the crack plane, but the corresponding hinge angle at the root was accordingly reduced because of the greater energy absorbed at the crack location.

When the beams were dropped from the higher height of 41.2", several new phenomena became evident in the deformed specimens shown in Fig. 5. The maximum deformation at the tip of course increases under the greater impact, but there is also a discernible increase of tip deformation in the cracked beams over that of the uncracked counterpart dropped from the same height.

The effect of crack closure is clearly seen in Figs. 5(g) and 5(h). In the case where the crack closes there is a distinct plastic hinge at the beam's root, whereas in the case where the crack can open up without obstruction, the root remains undeformed because all the energy is absorbed at the crack location. The two deformed shapes are different in both the number of hinges and the magnitude of the total deformation.

## 4. Analytical Models

The above experimental results indicate that a rigid-perfectly plastic material model and the usual kinematic assumption of local plastic hinges should prove efficacious in producting a simple analytical model for the observed behavior when the yield moment at a crack location is taken as a function of the crack depth. Details of such a model are described in Reference [6]. The problem of crack closure requires a further modification of the model, but it can be treated by limiting the angle through which the two sides of the beam can rotate relative to each other.

Once the motion of the cracked beam is modelled, the stability of the crack may be checked since the deformation enables one to calculate crack-opening angle for use in a stability criterion. Such models are being developed and will serve for making parametric studies not only of simple beams but also of more complex systems representing cracked piping in various configurations. These studies can identify worst case scenarios and provide first-order solutions to benchmark problems, which should prove useful for verifying large computer piping codes.

Acknowledgements

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Table I. Schedule of Beams Tested

| Specimen Designation Fig. 4 or 5 | Distance from Root to Crack (in) | Crack in Tension or Compression | Height of Drop (in) | Permanent Tip Deflection (in) |
|---|---|---|---|---|
| (a) | no crack | -- | 28.2 | 1.67 |
| (b) | 2.5 | ten. | 28.2 | 1.83 |
| (c) | 2.5 | comp. | 28.2 | 1.83 |
| (d) | 1.0 | comp. | 28.2 | 1.75 |
| (e) | no crack | -- | 41.2 | 2.25 |
| (f) | 2.5 | ten. | 41.2 | 2.40 |
| (g) | 1.0 | comp. | 41.2 | 2.40 |
| (h) | 1.0 | ten. | 41.2 | 2.86 |

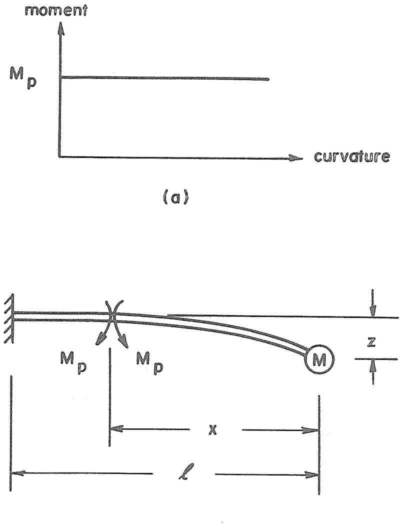


Figure 1. (b)

(a) Moment-curvature relation for rigid-perfectly plastic material; (b) Cantilever beam of that material deforming with moving plastic hinge a distance x from the tip (after Parkes [1]).

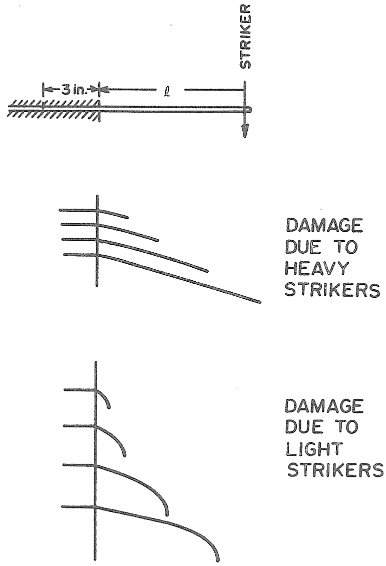


Figure 2. Permanent deformation of uncracked cantilever beams subjected to impact near their tips (after Parkes [1]).

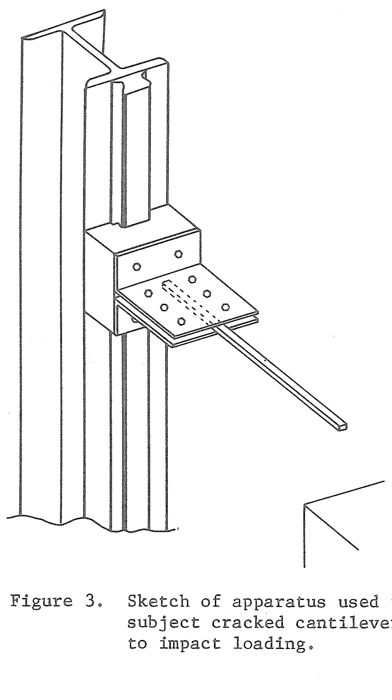

Figure 3. Sketch of apparatus used to subject cracked cantilever beams to impact loading.

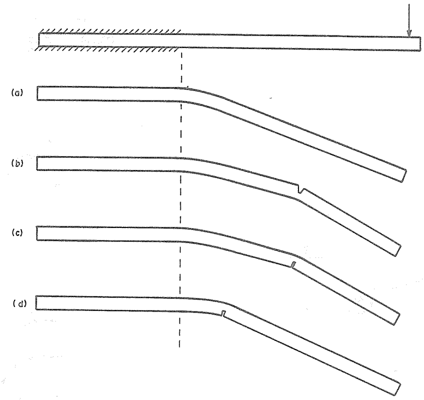


Figure 4. Permanent deformations of cracked cantilever beams subjected to lesser impact than in Fig. 5.

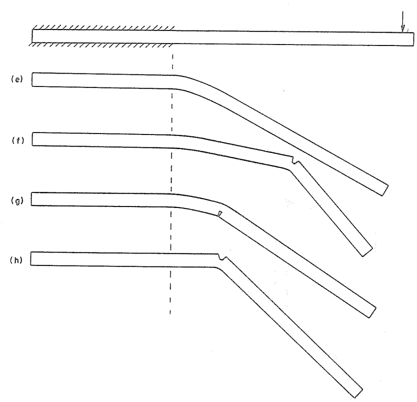


Figure 5. Permanent deformation of cracked cantilever beam subjected to greater impact than in Fig. 4.