



Transactions of the **13th International Conference on Structural Mechanics in Reactor Technology (SMiRT 13)**, Escola de Engenharia - Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil, August 13-18, 1995

## Finite element analysis of shells with internal ribs

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**ABSTRACT:** This paper presents the finite element method algorithm for shells with internal ribs. It is based on the contribution (Kutyłowski 1993), where the six parameters theory for such shells was considered. The internal ribs were modelled by the constraints put on the displacement field. The linear, static solution was considered.

Curved, quadrilateral element with 72 nodes parameters was taken into consideration (18 parameters in every node – six general displacements and their first derivatives in two orthogonal directions). The constitutive equations were completed by the shear stresses distribution coefficient. It was useful for modelling different cases of cooperation between shell facings and ribs. Products of Hermite polynomials of the third degree, as a basis, for the displacement field and the linear function for the constraints were used. Two orthogonal constraints which were connected together with meridional and parallel ribs were considered. Some numerical examples for testing the element and for solving the problem of the shells with internal ribs were presented.

### 1 FINITE ELEMENT METHOD ALGORITHM

The numerical solution for cylindrical shells is presented. The curved quadrilateral finite element in this case was used (Fig. 1). The displacement vector for the element was given as follow:

$$\mathbf{u} = \{\delta_1, \delta_2, \delta_3, u_1, u_2, u_3\}^T, \quad \bar{\mathbf{u}} = \{\bar{\delta}_1, \bar{\delta}_2, \bar{\delta}_3, \bar{u}_1, \bar{u}_2, \bar{u}_3\}^T \quad (1)$$

where the first vector is determined in global coordinates and the second one in local coordinates. Let's notice that:

$$\bar{\mathbf{u}} = \mathbf{T} \mathbf{u} \quad (2)$$

$\mathbf{T}$  is a transformation matrix. The strain vector is described as:

$$\boldsymbol{\varepsilon} = \{\gamma_1, \gamma_2, \gamma_3, \varepsilon_{13}, \varepsilon_{23}, \kappa_{11}, \kappa_{21}, \kappa_{12}, \kappa_{22}, \varepsilon_{11}, \varepsilon_{21}, \varepsilon_{12}, \varepsilon_{22}\}^T \quad (3)$$

The strain displacement relation:

$$\boldsymbol{\varepsilon} = \mathbf{L} \bar{\mathbf{u}} \quad (4)$$

where  $\mathbf{L}$  is an operator described by no more than the first derivatives (Appendix A). The stress vector is determined as:

$$\boldsymbol{\sigma} = \mathbf{D} \boldsymbol{\epsilon} \tag{5}$$

where  $\mathbf{D}$  is an elasticity matrix and

$$\boldsymbol{\sigma} = \{N^{13}, N^{23}, N^{33}, M^{13}, M^{23}, M^{11}, M^{12}, M^{21}, M^{22}, N^{11}, N^{12}, N^{21}, N^{22}\}^T \tag{6}$$

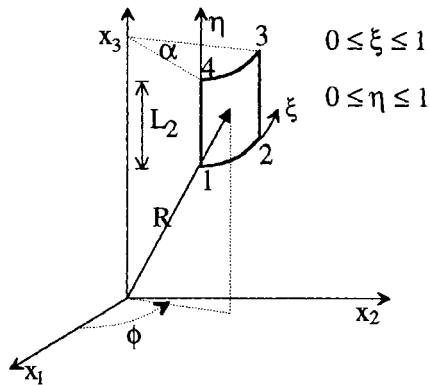


Fig. 1 Curved quadrilateral finite element

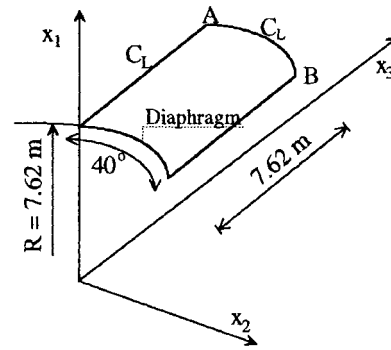


Fig. 2 Cylindrical panel

The constitutive equations were completed with  $\psi$  coefficient. It described shear stresses distribution. Coefficient  $\psi$  equals one for the constant distribution and 5/6 for parabolic distribution. The displacement vector is given by:

$$\mathbf{u} = \mathbf{N} \boldsymbol{\delta} \tag{7}$$

$\mathbf{N}$  – is a shape matrix. The components of this matrix there are the products of Hermite cubic polynomials in two orthogonal directions.

$\boldsymbol{\delta}$  – is a vector of nodal parameters (general 72 degrees of freedom, that means six general displacements and their first derivatives at every node).

$$\boldsymbol{\delta} = \{\boldsymbol{\delta}^1, \boldsymbol{\delta}^2, \boldsymbol{\delta}^3, \boldsymbol{\delta}^4\}^T \tag{8}$$

where

$$\boldsymbol{\delta}^i = \{\delta_1^i, \delta_{1,1}^i, \delta_{1,2}^i, \delta_2^i, \delta_{2,1}^i, \delta_{2,2}^i, \delta_3^i, \delta_{3,1}^i, \delta_{3,2}^i, u_1^i, u_{1,1}^i, u_{1,2}^i, u_2^i, u_{2,1}^i, u_{2,2}^i, u_3^i, u_{3,1}^i, u_{3,2}^i\}^T \tag{9}$$

The dimension of the shape matrix  $\mathbf{N}$  is 6 x 72. Let's write down only one line of the displacement vector  $\mathbf{u}$  :

$$\begin{aligned} u_3 = & u_3^1 N_1 N_2 + u_3^2 N_3 N_2 + u_3^3 N_3 N_4 + u_3^4 N_1 N_4 + \\ & + u_{3,1}^1 N_5 N_2 + u_{3,1}^2 N_7 N_2 + u_{3,1}^3 N_7 N_4 + u_{3,1}^4 N_5 N_4 + \\ & + u_{3,2}^1 N_1 N_6 + u_{3,2}^2 N_3 N_6 + u_{3,2}^3 N_3 N_8 + u_{3,2}^4 N_1 N_8. \end{aligned} \tag{10}$$

where

$$\begin{aligned}
 N_1 &= 1 - 3 \xi^2 + 2 \xi^3, & N_2 &= 1 - 3 \eta^2 + 2 \eta^3, & N_3 &= 3 \xi^2 - 2 \xi^3, \\
 N_4 &= 3 \eta^2 - 2 \eta^3, & N_5 &= (\xi - 2 \xi^2 + \xi^3) \alpha R, & & \\
 N_6 &= (\eta - 2 \eta^2 + \eta^3) L_2, & N_7 &= (\xi^3 - \xi^2) \alpha R, & N_8 &= (\eta^3 - \eta^2) L_2.
 \end{aligned} \tag{11}$$

The relation (4) can be write down as:

$$\boldsymbol{\epsilon} = \mathbf{L} \bar{\mathbf{u}} = \mathbf{L} \mathbf{T} \mathbf{N} \boldsymbol{\delta} = \mathbf{B} \boldsymbol{\delta} \tag{12}$$

$$\text{where } \mathbf{B} = \mathbf{L} \mathbf{T} \mathbf{N} \tag{13}$$

The stiffness matrix and the vector of external forces were obtained by means of the standard algorithm. The numerical integration in nine points (for three in one direction) for the Hermite polynomials of the third degree was used. There have been taken into account two orthogonal constraints, which were connected with meridional and parallel ribs. One can analyse the shell in traditional meaning (without constraints) for  $\psi = 5/6$  and  $2h = 0$  (Fig.4). The shell with constraints can be considered for  $\psi = 0$  and for parameter  $2h \neq 0$ . The shear stresses are transversed only by constraints in this case. The constraints using the tensor components in local coordinates are as follow:

$$\begin{aligned}
 -\frac{\bar{u}_1}{R} + \frac{\bar{u}_{3,\xi}}{\alpha} + \bar{\delta}_1 &= \bar{\gamma}_1 = 0 & \text{meridional constraints} \\
 \frac{\bar{u}_{3,\eta}}{L_2} + \bar{\delta}_2 &= \bar{\gamma}_2 = 0 & \text{parallel constraints}
 \end{aligned} \tag{14}$$

In physical components in global coordinates the above equations are written down as:

$$\begin{aligned}
 -(-u_1 \sin\phi + u_2 \cos\phi) / R - \delta_1 \sin\phi + \delta_2 \cos\phi + [(u_1 \cos\phi + u_2 \sin\phi)_{,\xi}] / (\alpha R) &= \gamma_1 = 0 \\
 [(u_1 \cos\phi + u_2 \sin\phi)_{,\eta}] / L_2 + \delta_3 &= \gamma_2 = 0
 \end{aligned} \tag{15}$$

The general equilibrium equation have been obtained from Lagrange functional:

$$\underline{L} = L_1 + L_2 \tag{16}$$

where

$$\begin{aligned}
 L_1 &= \int_S \frac{1}{2} \boldsymbol{\sigma}^T \boldsymbol{\epsilon} dS - \int_S \mathbf{f} \mathbf{u} dS \\
 L_2 &= \int_L \boldsymbol{\lambda} \boldsymbol{\gamma} dL
 \end{aligned} \tag{17}$$

The Lagrange multipliers have been approximated by:

$$\boldsymbol{\lambda} = \underline{\mathbf{N}} \boldsymbol{\omega} \tag{18}$$

Two orthogonal constraints have been considered. Then  $\boldsymbol{\omega}$  has been described as:

$$\omega = \{ \lambda_1, \lambda_2 \}^T \tag{19}$$

and

$$\underline{N} = \{ 1-\rho, \rho \}^T, \quad \rho = [ \xi \text{ or } \eta ] \tag{20}$$

$$\gamma = N' \delta \tag{21}$$

Hermite cubic polynomials were the base for the  $N'$  shape matrix. The Lagrange functional for the all construction may be written as (summation over the elements "e"):

$$\underline{L} = \frac{1}{2} \sum_e \delta_e^T \mathbf{K}^e \delta_e - \sum_e \delta_e^T \mathbf{f}^e + \sum_e \delta_e^T \mathbf{G}^e \omega_e \tag{22}$$

The above relation can be presented in the following form:

$$\begin{aligned} \mathbf{K} \delta - \mathbf{G} \omega &= \mathbf{F} \\ \mathbf{G}^T \delta &= \mathbf{0} \end{aligned} \tag{23}$$

where

$\mathbf{K}$  – is the stiffness matrix,

$\mathbf{G} = \underline{N} N'$

$\omega$  – is the vector of nodal parameters of the Lagrange multipliers.

## 2 NUMERICAL EXAMPLE

The FEM algorithm was applied for the cylindrical panel. In this case  $q = 4.48 \text{ KN/m}^2$  (panel weight), the shell thickness  $2H = 0.0762 \text{ m}$ ,  $\nu = 0$  and  $E = 21500 \text{ MPa}$  (Sohrabuddin 1977). A quarter of the shell with marked the symmetry axis is shown on the Fig. 2. It has also been assumed that the displacements on the diaphragm, within its surface equal zero. The accuracy of the solution comparing it with (Sohrabuddin 1977) was satisfied. The shell with constraints (marked by dot lines) for various divisions (Fig. 3) next has been analysed. The cross section of the shell is shown on the Fig. 4.

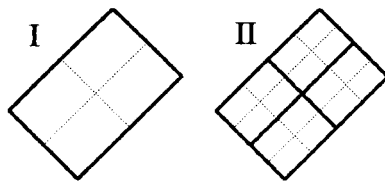


Fig. 3 Mesh and constraints

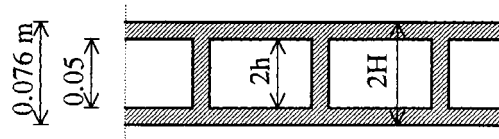


Fig. 4 Cross section

Table 1. illustrates the  $u_1$  displacement for points A and B for the shell with various ribs height ( $2h$  changes from 0.00635 m to 0.0635 m). In this case the shear stresses coefficient  $\psi = 5/6$ . The  $u_1$  displacement is measured in normal direction to the middle surface. It was analysed for the mesh shown on the Fig. 3 (I – one element and II – four elements, constraints were marked by dot lines). In the Table 2. displacements of A and B points for  $\psi = 0$  are placed (the shear stresses are transferred only through the ribs). In this case the number of constraints increases from one to nine (I division) and from one to two (II division). It is worth to notice that the similar displacement is

obtained in case I with five constraints, as well as, II division with two constraints. In the above two cases on the same part of the shell there are two ribs but in the second one case the displacement  $u_1$  is a little bit smaller, because of the II division. The displacement  $u_1$  for I and II division and for  $\psi = 1$  is presented on the Fig. 5 (the dot line – I division and the second line – II division). Table 3 contains  $u_1$  displacements for the shell with or without constraints for various  $\psi$ . For example for point B (division I)  $u_1$  displacement for  $\psi = 0$  is 10 times greater than for  $\psi = 5/6$ . For increasing number of ribs and for II division  $u_1$  displacement is greater about two times.

Table 1.

2 h [m]	$u_1$ for points	
	A	B
	$u_1 * 10^{-3}$ m	$u_1 * 10^{-2}$ m
0.00635	1.9171	- 4.2316
0.0127	1.6054	- 4.5469
0.0254	0.7055	- 5.3957
0.0508	- 3.6264	- 9.6129
0.0635	- 11.8041	- 18.1021

Table 2.

const – raints number	division I for points	
	A	B
	$u_1 * 10^{-2}$ m	$u_1 * 10^{-1}$ m
1	206.81	- 8.4389
2	8.2751	- 3.9371
3	5.7897	- 2.9619
4	3.6811	- 2.2351
5	2.9924	- 1.9576
7	- 0.5188	- 0.9535
9	- 0.5081	- 0.9093
const – raints number	division II for points	
	A	B
	$u_1 * 10^{-2}$ m	$u_1 * 10^{-1}$ m
1	3.8418	- 2.3237
2	2.4988	- 1.9833

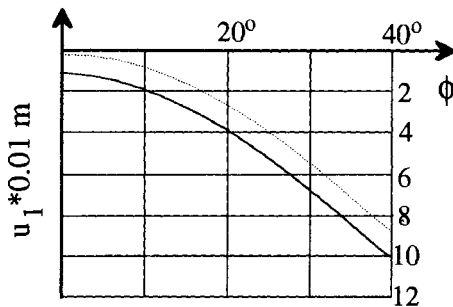


Fig. 5 Displacement  $u_1$  for  $\psi = 1$

Table 3.

division	without constraints $\psi = 5/6$	with constraints $\psi = 5/6$	with constraints $\psi = 0$
	point B	point B	point B
	$u_1 * 10^{-2}$ m	$u_1 * 10^{-2}$ m	$u_1 * 10^{-2}$ m
I	-9.9314	-9.6129	-84.389
II	-10.567	-10.578	-23.237

### 3 CONCLUSION

The FEM algorithm was presented with the numerical analysis. The following problems were discussed:

- various number of constraints in the element,
- various height of the ribs.

It was stated that an increasing number of constraints in the element causes the construction more stiffer. For the shear stresses distribution coefficient  $\psi = 0$  the shell with ribs was modelled (the shear stresses were transferred only by ribs). The presented FEM algorithm let us to analyse the shell with internal orthogonal placed ribs.

### REFERENCES

- Kutyłowski R.: 1993. The new concept of the director nonlinear theory of internal ribbed shells. *Proc. 12th SMiRT*: 129 – 134. Amsterdam: Elsevier.
- Sohrabuddin A., Irons B. M., Zienkiewicz O. C. 2 1977. Thick and thin shells structures. *Int. J. Num. Meth. Engng.*

### APPENDIX A:

1	0	0	$-\frac{1}{R}$	0	$\frac{1}{\alpha R} \frac{\partial}{\partial \xi}$
0	1	0	0	0	$\frac{1}{L_2} \frac{\partial}{\partial \eta}$
0	0	1	0	0	0
0	0	$\frac{1}{\alpha R} \frac{\partial}{\partial \xi}$	0	0	0
0	0	$\frac{1}{L_2} \frac{\partial}{\partial \eta}$	0	0	0
$\frac{1}{\alpha R} \frac{\partial}{\partial \xi}$	0	$\frac{1}{R}$	0	0	0
0	$\frac{1}{\alpha R} \frac{\partial}{\partial \xi}$	0	0	0	0
$\frac{1}{L_2} \frac{\partial}{\partial \eta}$	0	0	0	0	0
0	$\frac{1}{L_2} \frac{\partial}{\partial \eta}$	0	0	0	0
0	0	0	$\frac{1}{\alpha R} \frac{\partial}{\partial \xi}$	0	$\frac{1}{R}$
0	0	0	0	$\frac{1}{\alpha R} \frac{\partial}{\partial \xi}$	0
0	0	0	$\frac{1}{L_2} \frac{\partial}{\partial \eta}$	0	0
0	0	0	0	$\frac{1}{L_2} \frac{\partial}{\partial \eta}$	0