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Equivalent Soil Resonators for Layered Half Spaces

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Abstract

Impedance functions have been calculated for various types of soils and a rigid foundation :

- homogeneous soils
- layered soils
- homogeneous soil with embedded foundation with and without material soil damping.

The modelisation was axisymetric and finite elements were used.

The method takes into account the generally neglected coupled terms between horizontal and rocking movement.

Comparisons were made with some published results by other methods and satisfying agreements were found.

1. Introduction

Many soil-structure interaction codes use frequency dependent springs and dampers to simulate the behaviour of the soil. These impedance approaches assume the soil to be an elastic, homogeneous and isotropic medium [1] or layered [3] and foundation as circular.

A method for computing impedances, or inverse functions called compliances, was described in Reference 2 ; it is based on the Fourier Interpretation of a transient calculation.

In the present paper, we present calculations of soil impedances, using the Ref [2] method, for the circular foundations, superficial or embedded, resting on an homogeneous or two-layers soil. Some comparisons are made with published results obtained following other methods [1], [3].

2. Description of the method

The method presented by reference 2 uses transient calculations performed on a finite element model submitted to two different impulsions : the Fourier Transforms of the response, compared with the Fourier components of the impulsions, allows to derive impedances for translation and rocking. The transient calculations were performed for axisymetric geometries using the INCA code [4]. The calculations were stopped at the physical time corresponding to the first return to initial position. This proves sufficient for obtained stabilized impedance values.

If $F(t)$ is the excitation, $u(t)$ the foundation time history displacement, $F(\omega)$ and $u(\omega)$ their respective Fourier Transforms, for each frequency the complex frequency dependent impedance $k(\omega)$ is given by the following ratio :

$$k(\omega) = \frac{F(\omega)}{u(\omega)}$$

We applied this method taking in account the terms coupling the horizontal and rocking vibrations.

Given $F(t)$ and $M(t)$, respectively horizontal impulsion in the x direction and rocking moment impulsion around a horizontal axis y perpendicular to the x axix, we performed two successive calculations mentioned by the two superscrips for which $F(t)$ or $M(t)$ was applied.

Hence, four equations were obtained, in which k_{11}, k_{12}, k_{21}, k_{22} are the frequency dependent unknown impedances, u and ψ the horizontal displacement and the rotation around a horizontal axis :

$$
\begin{aligned}
F(t) &= k_{11}\, u^{(1)}(t) + k_{12}\psi^{(1)}(t) \\
0 &= k_{21}\, u^{(1)}(t) + k_{22}\psi^{(1)}(t) \\
M(t) &= k_{21}\, u^{(2)}(t) + k_{22}\psi^{(2)}(t) \\
0 &= k_{11}\, u^{(2)}(t) + k_{22}\psi^{(2)}(t)
\end{aligned}
\qquad (1)
$$

After obtaining the Fourier Transform of system (1), the k_{ij} values may be obtained :

$$
k_{11} = \frac{F(\omega)\psi^{(2)}(\omega)}{u^{(1)}(\omega)\psi^{(2)}(\omega) - u^{(2)}(\omega)\psi^{(1)}(\omega)} \qquad k_{21} = \frac{-M(\omega)\psi^{(1)}(\omega)}{u^{(1)}(\omega)\;{}^{(2)}(\omega) - u^{(2)}(\omega)\psi^{(1)}(\omega)}
$$

$$
k_{22} = \frac{M(\omega)\, u^{(1)}(\omega)}{u^{(1)}(\omega)\psi^{(2)}(\omega) - u^{(2)}(\omega)\psi^{(1)}(\omega)} \qquad k_{12} = \frac{-F(\omega)\, u^{(2)}(\omega)}{u^{(1)}(\omega)\psi^{(2)}(\omega) - u^{(2)}(\omega)\psi^{(1)}(\omega)} \qquad (2)
$$

where $M(\omega)$, $u(\omega)$, $F(\omega)$, $\psi(\omega)$ are respectively the Fourier Transforms of $M(t)$, $u(t)$, $F(t)$ and $\psi(t)$.

It has been verified that $k_{12} = k_{21}$.

Another way to express the above equations is to consider at the same time the results of the horizontal impulsion and impulsion rocking moment to obtain after taking the Fourier Transform as previously :

$$
\begin{aligned}
F(\omega) &= k_{11}(\omega)\, u(\omega) + k_{12}(\omega)\,\psi(\omega) \\
M(\omega) &= k_{12}(\omega)\, u(\omega) + k_{22}(\omega)\,\psi(\omega)
\end{aligned}
\qquad (3)
$$

This latter system makes it possible to derive the following values :

$$
\begin{aligned}
u(\omega) &= \frac{k_{22}(\omega)\, F(\omega) - k_{12}(\omega)\, M(\omega)}{k_{11}\, k_{22} - k_{12}^2} = A_1(\omega)\, F(\omega) - A_3(\omega)\, M(\omega) \\
\psi(\omega) &= \frac{k_{11}(\omega)\, M(\omega) - k_{12}(\omega)\, F(\omega)}{k_{11}\, k_{22} - k_{12}^2} = A_2(\omega)\, F(\omega) - A_3(\omega)\, F(\omega)
\end{aligned}
\qquad (4)
$$

This is to be compared with Deleuze formulation (1) :

$$
\begin{aligned}
u(\omega) &= \frac{F(\omega)}{GR_o}(f_{1H} + if_{2H}) + \frac{M(\omega)}{GR_o^2}(f_{1D} + if_{2D}) \\
\psi(\omega) &= \frac{F(\omega)}{GR_o^2}(f_{1D} + if_{2D}) + \frac{M(\omega)}{GR_o^3}(f_{1R} + if_{2R})
\end{aligned}
\qquad (5)
$$

R_o is the foundation radius, G the dynamic modulus of the soil.

The various compliances f_{1H}, f_{2H}, f_{1D}, f_{2D}, f_{1R}, f_{2R}, for a rigid circular foundation were computed and compared with those of Deleuze [1] concerning the flexible foundation.

We also compared our results with those found by Luco [3] from Fredholm integrals for a two layered medium.

H being the horizontal force and M the rocking moment, the formulation in Luco [3] is the following :

$$M = \frac{8G_1 a^3}{3(1-\nu_1)} \left[k_{MM} (a_o) + i a_o C_{MM} (a_o) \right] \alpha e^{i\omega t}$$

$$H = \frac{8G_1 a}{2 - \nu_1} \left[k_{HH} (a_o) + i a_o C_{HH}(a_o) \right] \Delta_H e^{i\omega t}$$

a_o is the dimensionless frequency, G_1 and ν_1 the dynamic modulus and Poisson coefficient of the upper medium of the two-layered soil.

3. Results and comments

Figures 1 and 2 correspond to compliances f_{1R} and f_{1H} for the circular, superficial foundation. For the flexible foundation (vertical stress proportional to rotation axis distance) the Deleuze [1] values were used. For the rigid foundation our method is used ; smaller compliances were conveniently found.

Figure 3 correspond to the variation versus frequency of moduli of rocking compliance for the rigid, superficial foundation, versus the Poisson coefficient.

Figures 4 to 7 correspond to impedance coefficients found for the superficial foundation resting on a two-layers soil.

Values found by LUCO [3] were obtained neglecting the cross terms coupling horizontal and rocking vibrations : they show a strong frequency dependance. Our values obtained without neglecting cross terms show very comparable shapes.

As dynamic analyses are generally made neglecting the random phase of frequency components, more useful comparisons may be performed on the amplitude terms of these components.

Figure 8 to 9 give variations versus frequency of translation and rocking compliance amplitudes. It may be seen that amplitude curves give smoother variations than impedance curves.

Figures 10 and 11 give a comparison of the same amplitude functions for a superficial foundation and an embedded foundation.

In figure 12, are compared two homogeneous soils of $\nu = 0.35$, one with an internal damping of 5%, the other one without internal damping, for the case of superficial foundations.

The observed difference well corresponds to the difference of damping.

4. Conclusion

Frequency dependent springs and dampers have been calculated for various soils and a rigid circular foundation.

The method takes into account the generally neglected compled terms between the horizontal and rocking vibrations.

Comparisons with other methods confirm the validity of our results.

The interest of this method is that, relatively to other methods, it can be applied to more complicated cases without too long a calculation time.

Furthermore, it has been shown that the calculation in amplitude terms is useful for the representation of the various results.

References

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[2] M. Farvacque, F. Gantenbein.
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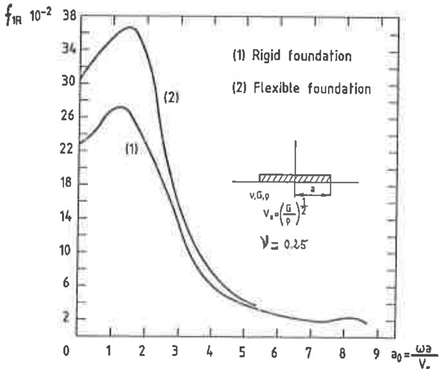

Fig. 1 - Comparison between Deleuze's approach and present rigid foundation's approach.

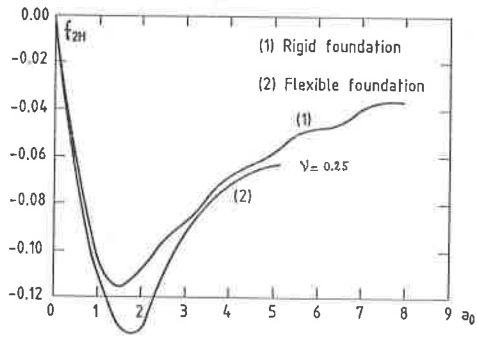

Fig. 2 - Comparison between Deleuze's approach and present rigid foundation's approach.

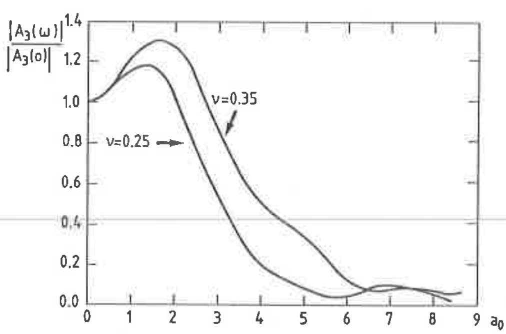

Fig. 3 - Comparison between two homogeneous soils

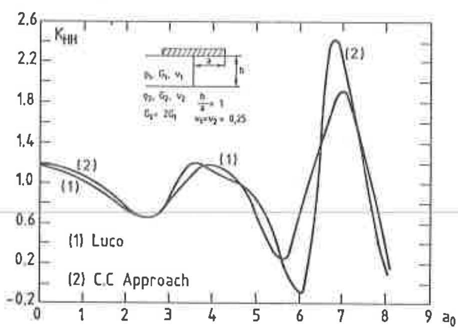

Fig. 4 - Comparison between Luco's approach and present study.

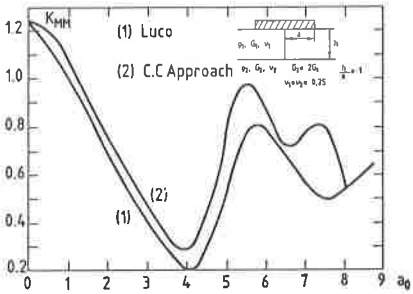

Fig. 5 - Comparison between Luco's approach and present study.

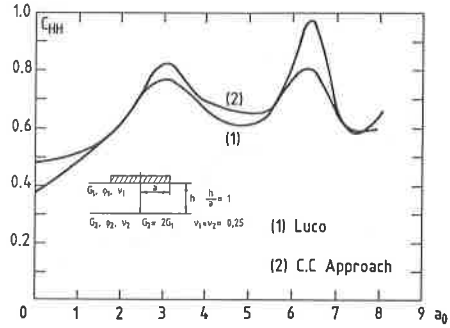

Fig. 6 - Comparison between Luco's approach and present study.

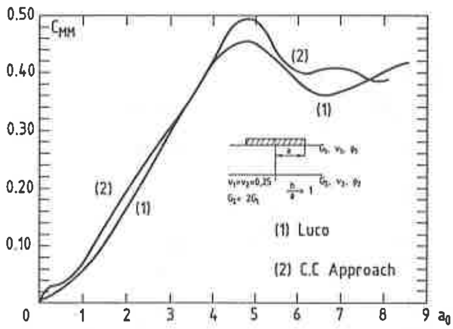

Fig. 7 - Comparison between Luco's approach and present study.

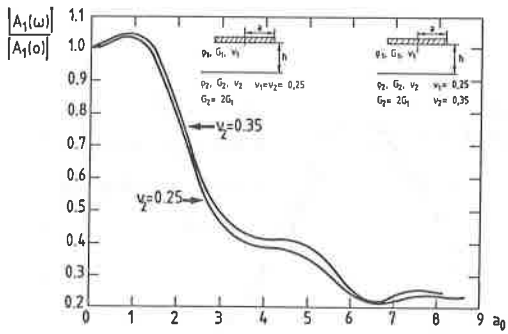

Fig. 8 - Comparison between two-layered soils with $\frac{h}{a} = 1$.

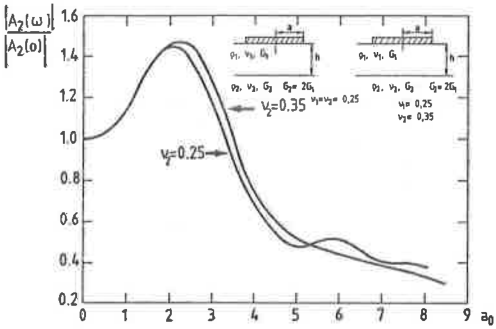

Fig. 9 - Comparison between two-layered soils with $\frac{h}{a} = 1$.

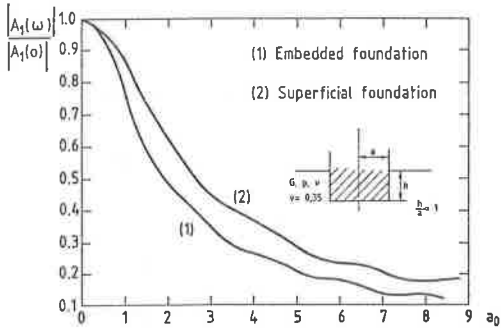

Fig. 10 - Comparison between embedded and superficial foundation.

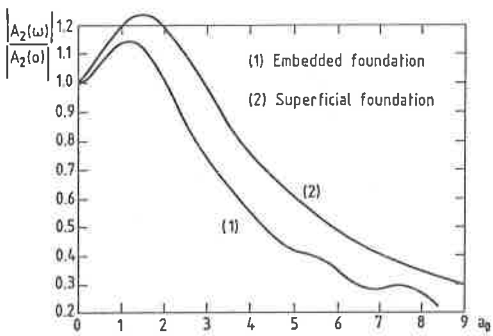

Fig. 11 - Comparison between embedded and superficial foundation.

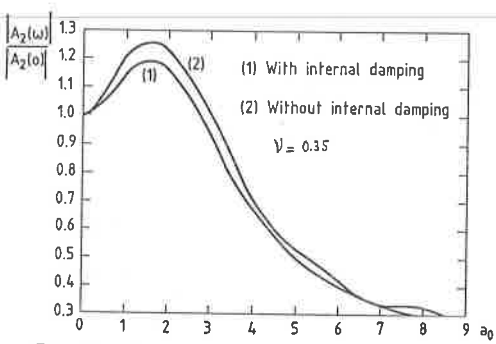

Fig. 12 - Comparison between two homogeneous soils with and without damping of 5%.