Probabilistic Seismic Response of Nuclear Power Plants

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Most current methods of seismic analysis of nuclear power plants assume the dynamic characteristics of the soil surrounding the structure as deterministic. However, the results of laboratory and field evaluations of moduli and dampings of cohesive and granular materials generally show a significant scatter. Since the properties of the soil surrounding a structure have a prime role in its seismic behavior it is clear that the random character of the soil should be accounted for in the analyses.

Accordingly, an analytical method which takes into account the uncertainties in the soil properties was developed and is presented in this paper. The procedure is based on the complex response method and by means of theory of perturbations the stiffness and damping characteristics of the soil are considered as random parameters. A finite element computer code, DARE, was written to evaluate the dynamic response of massive structures founded on soils having nondeterministic properties. The results are given in terms of mean values of maximum accelerations, response spectra, shear stresses, etc. Upper and lower bounds of the same quantities are also provided. These limits are computed according to the confidence band determined for the dynamic properties of the foundation soils.

1. Introduction

Nuclear power plants are heavy, massive structures usually embedded at a considerable depth in a soil deposit. An important aspect in the seismic design of these structures is the evaluation of the dynamic interaction between the soil and the structure. Since such interaction effects may significantly affect the response of the structure and of the sensitive equipment, it is of paramount importance that in the seismic analyses the relevant characteristics of the soil-structure interaction problem be considered realistically.

A mathematical interaction model which has proved to yield sufficiently accurate results for engineering purposes, was proposed at the University of California,Berkeley [1,2].The method uses the finite element techniques for the performance of complete interaction of a plane strain model, and by means of an iterative procedure the strain dependent nature of the nonlinear soil characteristics is considered. The analytical method includes viscous boundaries to simulate the three dimensional effect and transmitting boundaries which absorb any wave effects emanating from the structure, thus simulating the effects of an extensive soil deposit. The input seismic motion can be specified either as a deterministic acceleration time history [2] or as a stationary random process [3-5]. The dynamic soil properties are assumed deterministic.

There exists a large body of evidence showing that the commonly used deterministic assumption for dynamic soil properties, i.e., shear modulus and damping ratio, is hardly tenable. A much better hypothesis would be to treat these two soil parameters as random variables. To include such uncertainties in seismic analyses of nuclear power plants, it is a common practice to use what may be called the three-analysis approach, i.e., one analysis is carried out with the mean value of the dynamic properties determined from field and laboratory tests, and other two using the properties corresponding to the upper and lower envelopes. This is a rough approach to account for the uncertainties in soil properties.

Accordingly, to properly take into account the randomness of the shear modulus and damping ratio in the dynamic soil-structure interaction analyses a probabilistic method based on the theory of perturbations was developed. This approach was included in an interaction model similar to that mentioned above, yielding a computer program called DARE [6]. Results obtained by the new method are in excellent agreement with the results of corresponding simulation analyses. Furthermore, the probabilistic results (given in terms of mean values, and confidence limits) can be obtained at a fraction of the cost of both the simulation (Monte Carlo) and the three-analysis approach results.

2. Probabilistic Method of Analysis

The objective of probabilistic analyses is to obtain probability distributions of the response (i.e., stresses, strains, response spectra, etc.) in different components of a system. A probabilistic determination of the response requires a knowledge of the nature and magnitude of the loads, material characteristics affecting the response, and underlying assumptions of the method of analysis adopted. A proper account of the uncertainties at every stage of the response determination procedure is extremely important for reliable results.

From comparisons with the actual behavior of the Humboldt Bay Power Plant in the Ferndale earthquake of June 7, 1975, Valera et al. [7] concluded that the method of analysis proposed by Seed at al [1,2] was adequate from the engineering point of view. (A similar method of analysis was adopted in the research reported herein).

Recent investigations have shown that the random nature of the seismic excitation can be incorporated in the dynamic response analyses of soil-structure systems by means of random vibration theory. In these studies the input motion was assumed to be a Gaussian, stationary and zero-mean random process characterized either by an equivalent power spectral density [3,4] or by a mean maximum acceleration together with probabilistic normalized power spectral density, defined in terms of its mean spectrum and covariances [5].

On the other hand, the influence from the soil material uncertainties on the seismic response of soil-structure systems has been evaluated mostly through parametric studies using deterministic models (i.e., using dynamic compliance functions). Similarly, finite element computations have been carried out using: (a) The mean value of the curve shear modulus-shear strain obtained from laboratory and field tests, and (b) the upper and lower curves enveloping the testing results [7]. Although these approaches allow to gain some insight with regard to the significance of the various material parameters, they do not account for their statistical nature.

The handling of the material uncertainties is rather difficult. One posible way of solving the problem would be to use Monte Carlo methods. For example, if the shear modulus, G, of the foundation soil is known to have a wide scatter and if its probability distribution is known, the analysis problem may be solved by generating a sample of G values consistent with its probability law, and the analysis performed for each value of G generated. The results can then be analyzed to produce the desired statistical response quantities. This method involves a great number of numerical computations and hence its practical use is limited. In this paper an alternate procedure based on perturbation techniques [8] coupled with numerical simulations is proposed, and used for computing the dynamic response of a hypothetical nuclear power plant. This method retains the formalism of the Monte Carlo method but eliminates the burden of solving the full problem for each set of soil parameter values.

2.1 Theoretical Formulation

The analytical model used to represent the soil-structure system is shown schematically in Fig. 1. The equation of motion for the finite element representation of the soil-structure system can be written

$$[M]\{\ddot{u}\} + [K]\ u = -\{m\}\ \ddot{y}\ (t) \tag{1}$$

where $[M]$ and $[K]$ are the usual plane strain mass and stiffness matrices, respectively; $\{u\}$ are the displacements of the nodal points relative to the rigid base; and $\{m\}$ is a vector related to $[M]$ and the direction of the rigid base acceleration, $\ddot{y}(t)$. Material damping may be included by forming $[K]$ from complex moduli: $G^* = G \exp(2i\beta)$, where β is the fraction of critical damping and $i = \sqrt{-1}$.

Using the complex response method [2] eq. (1) can be written in the frequency domain as

$$([K] - \omega_r^2 [M])\{U\}_r = -\{m\}\ \ddot{Y}_r \tag{2}$$

From this set of linear equations the displacement amplitudes $\{U\}_r$ may be obtained for each frequency ω_r, $r = 0,1,2, \ldots,$ of the input motion by Gaussian elimination.

The material properties involved in the equation of motion are the shear modulus, damping ratio, and unit weight. Statistical evaluations [9] of laboratory and field determinations of shear moduli and damping ratios have shown that both parameters should be considered as random variables. On the other hand, since values of unit weight fall within a narrow band, it may be assumed to be a deterministic variable. Consequently, $[K]$ and $[M]$ will be probabilistic and deterministic matrices, respectively. Therefore, using perturbation techniques, $[K]$ may be expressed as follows

$$[K] = [K_0] + [Q_1] + [Q_2] + \dots + [Q_i] \tag{3}$$

where $[K_0]$ is the unperturbed stiffness matrix (i.e. deterministic); and $[Q_i]$ are the stiffness matrices of first -, second -, ... , ith - order perturbation.

In this study perturbations of order higher than one were neglected because second-order effects are accounted for by the simulation procedure involved in the probabilistic model (see paragraph 2.2). Assuming that the perturbation is small, the solution of eq.(2) is given by

$$\{U\}_r = \{U_0\}_r + \{U_1\}_r \tag{4}$$

where $\{U_0\}_r$ is the unperturbed solution (i.e. deterministic); and $\{U_1\}_r$ is the perturbed solution (i.e. probabilistic). Substituting eqs. (3 and 4) in eq.(2), and grouping the terms of equal order, the following equations are obtained

<u>Zero order</u>

$$([K_0] - \omega_r^2 [M])\{U_0\}_r = -\{m\}\ddot{Y}_r \tag{5}$$

<u>First order</u>

$$([K_0] - \omega_r^2 [M])\{U_1\}_r = -[Q_1]\{U_0\}_r \tag{6}$$

It is most interesting to point out that these two equations are identical except for the loading vector. Thus the probabilistic solution of the problem can be obtained in a straightforward way by solving eq. (5) for unitary loading and then multiplying eqs. (5 and 6) by their corresponding loading vectors, the values of $\{U_0\}_r$ and $\{U_1\}_r$ are computed. In matrix form, the total solution is given by

$$\{U\}_r = \{U_0\}_r + \{U_1\}_r = (-[I] + [L]_r^{-1}[Q_1])[L]_r^{-1}\{m\})\ddot{Y}_r \tag{7}$$

where $[I]$ is the unitary matrix; and $[L]_r^{-1} = ([K_0] - \omega_r^2 [M])^{-1}$

2.2 <u>Probabilistic Model</u>

In order to obtain the probabilistic solution $\{U\}_r$ given by eq. (7), it is necessary first to generate a sample of matrices $[Q_1]$ consistent with its probability distribution, and then to carry out the analysis for each value of $[Q_1]$ generated. This procedure is equivalent to the use of a Monte Carlo approach; however it does not require solving the equation of motion for each value of $[Q_1]$ since eq. (7) involves only products between the unitary deterministic solution, $[L_r]^{-1}$, and the probabilistic stiffness matrix, $[Q_1]$. Henceforth, the numerical simulation can be performed advantageously.

The probabilistic complex matrix $[Q_1]$ (assembled similarly to $[K_0]$) is made up of random complex moduli, ΔG^*. Using the statistical concept of coefficient of variation, CV, to represent the soil modulus and damping scattering, it can be shown [9] that ΔG^* can be evaluated from

$$\Delta G^* = G_0^* (R_G \cdot CV_G + 2i\, \beta_0\, R_\beta \cdot CV_\beta) \tag{8}$$

where G_0^* is the average of the G^* values determined from a testing program; R_G and R_β are random numbers having normal distribution, N(0,1); CV_G and CV_β are the coefficients of variation for G and β, respectively; and β_0 is the average damping ratio resulting from a testing program.

The values of CV_G and CV_β are calculated from statistical analyses of field and laboratory determinations of G and β. Since various testing techniques are used to cover the full range of strains of interest in earthquake engineering problems, and they necessarily involve different uncertainty levels, it is expected that in general CV_G and CV_β be strain dependent. As an example, in Fig. 2a the mean values and uncertainty bands of G and β for granular soils are presented. The corresponding coefficients of variation are shown in Fig. 2b. As should be expected, the uncertainty in β is larger than in G. It is also evident that β determinations at low strain levels are less reliable. Similarly, it seems that the larger scatter in G values occurs at a strain level around 3×10^{-2} % , which corresponds roughly to the strain where resonant column and cyclic triaxial (direct shear) results overlap. Hence, this scatter is believed to be caused mainly by the fundamental differences between testing techniques.Accordingly, this scatter could be partially eliminated if a reliable testing procedure covering the full strain range of interest were available.

Another source of uncertainty in the ordinates of the modulus-strain curves is the process involved in rendering compatible the maximum shear moduli (i.e. low-strain level, 10^{-4} %) determined from laboratory and field tests. This type of uncertainty may be taken into account in the probabilistic method by simply assigning to CV_G a value other than zero at the low-strain level (10^{-4}%).

Upper and lower bounds can be obtained from the statistical analysis of the dynamic responses computed through the simulation procedure outlined above.

Upper bound

$$\{U\}_{ub} = \{U_o\} + \alpha S$$

Lower bound

$$\{U\}_{\ell b} = \{U_o\} - \alpha S \qquad (9)$$

where $\{U_o\}$ is the deterministic or equivalently the mean solution; S is the standard deviation; and α is a parameter which controls the width of the response uncertainty band (i.e. α = 1 corresponds to 68% probability interval).

3. Validation of the Probabilistic Method of Analysis

The theory presented previously was incorporated into a new computer code, DARE [6] , which is a plane-strain finite element program; the strain-dependent nature of the soil characteristics is considered through an interative procedure.

In order to validate the proposed probabilistic method of analysis, parallel computations were performed by the probabilistic approach and the Monte Carlo method, for the layered soil deposit shown in Fig. 3. The soil characteristics included in the figure correspond to the iterated, strain compatible properties for the excitation shown in Fig. 4.

To generate the sample of seismic responses, 20 sets of soil properties (G and β) were produced assuming that the scatter around the mean values of G and β shown in Fig. 2a conforms with a normal probability distribution having coefficients of variation as indicated in Fig 2b. Then 20 deterministic analyses (Monte Carlo) were carried out using program DARE. The results were then evaluated statistically to produce mean values and confidence limits. Similarly, using the theory proposed above, a set of 20 seismic responses was generated and the corresponding mean values and confidence limits evaluated. The comparison between maximum acceleration amplification ratios obtained with both procedures is shown in Fig. 5. It can be seen that the probabilistic method yields similar results to those given by the Monte Carlo method. The small differences observed may be attributed to the relatively limited sample (20 simulations) considered in the analyses. In fact, it has been shown [9] that the agreement between both approaches improves with the number of simulations.

4. Application to a Soil-Structure System

With the intent of studying the implications of including nondeterministic soil properties in the seismic response of a soil-structure system, a hypothetical nuclear power plant was considered. The model used is similar to that shown in Fig. 1. The modulus and damping-strain curves for the foundation material are depicted in Fig. 2a, and the low strain modulus in Fig. 3; the deterministic analysis was carried out using the mean curve of Fig. 2a. For the probabilistic calculations the values of CV_G and CV_β were assumed to be strain independent and equal to 10%. The characteristics of the structure were: unit weight = 21 kN/m³; shear modulus = 288 MPa; damping ratio = 3%; and Poisson's ratio = 0.3. Three iterations were required to obtain strain-compatible soil properties. The input motion, considered to be acting at the model base-rock, is presented in Fig. 4.

One of the objectives of dynamic soil-structure interaction analyses is to determine response floor spectra for design of sensitive equipment. Uncertainty in the generation of floor spectra arises from the definition of the input motion and material characteristics. The use of probabilistic representations of the excitation in floor spectra determinations has been presented elsewhere [3-5]. The method proposed in this paper allows computation of floor spectra taking into account the uncertainties in the foundation soils parameters (shear modulus and damping ratio).

Calculated deterministic and upper bound level (α = 1) floor spectra are shown in Fig. 6. It may be seen that in the low frequency range the ordinates of the upper bound spectrum are significantly higher than those of the deterministic case. This is an important aspect for the design of low-frequency sensitive equipment. Alternatively, in the high frequency range the spectral shapes are similar although the upper bound spectral ordinates are not amplified by a constant factor. For this particular case the effect of the properties uncertainties is more pronounced in the low frequency range; however, this may not be so for cases where, for example, soil and structure stiffnesses are not significantly different. Accordingly, it should be agreed that the effect of the randomness of the soil properties on the seismic response of nuclear power plants could be considerable, and consequently it should be agreed that the effect of the randomness of the soil properties on the seismic response of nuclear power plants could be considerable, and consequently it should be evaluated on a routine basis in all designs.

5. Conclusions

It has been long recognized that considerable judgement is required to determine representative soil characteristics and that it is usually necessary to consider ranges of properties for analyses purposes. A new procedure, which considers such uncertainties, has been developed for evaluating the seismic response of soil-structure systems by combining elements from the theory or perturbations, the finite element method, and the complex response method.

Results obtained by the new probabilistic method are in excellent agreement with corresponding results obtained by the Monte Carlo method and have the added advantage that the computer costs are a small fraction of those generated by the Monte Carlo. (The computer time required by a probabilistic analysis is about 1.7 times an equivalent deterministic analysis). The new method of analysis provides confidence limits on all results; hence, they are potentially more useful for the design of critical components.

References

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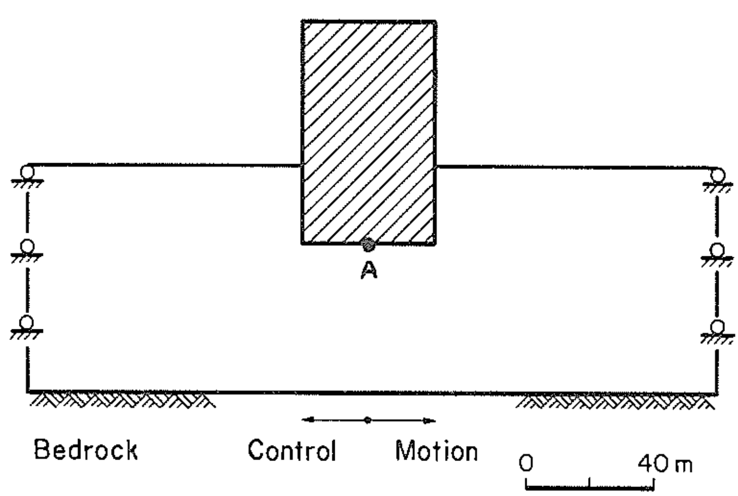


Fig 1. Soil – Structure Model

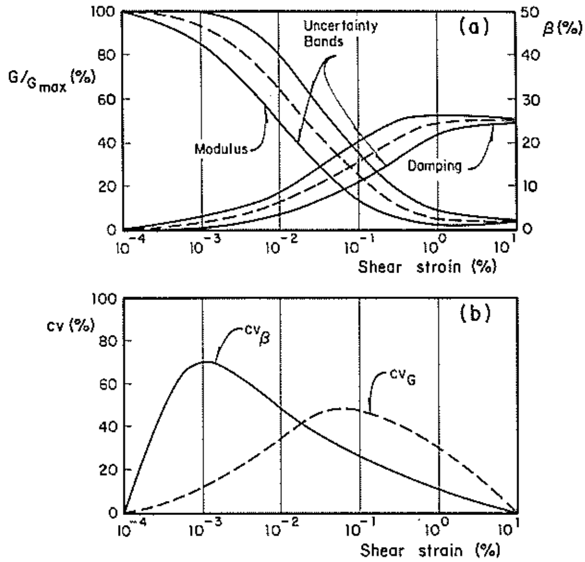


Fig 2. Shear Modulus, Damping and Coefficients of Variation for Granular Soils

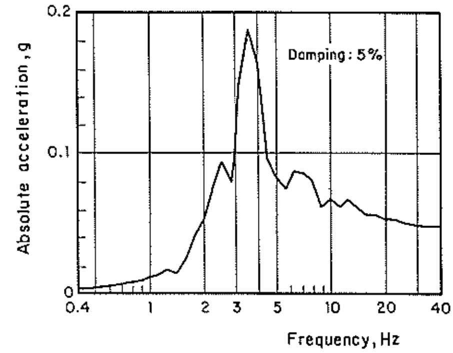


Fig 4 Response Spectrum for Control Motion

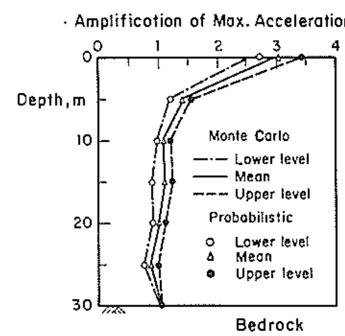
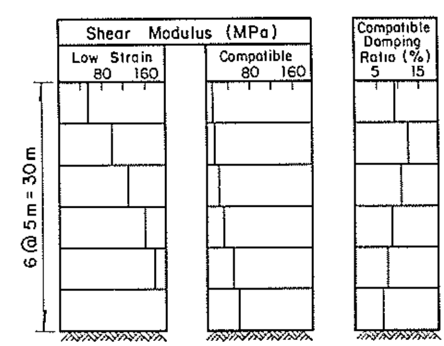


Fig 5 Validation of the Probabilistic Method



Dry Unit Weight = 17 kN/m^3
Poisson's Ratio = 0.33

Fig 3. Dynamic Properties of Layered Sand Deposit

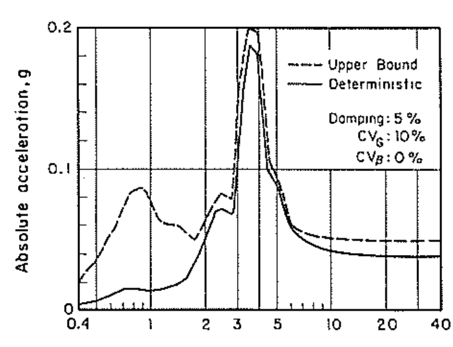


Fig 6 Response Spectra at Point A