DETAILED ANALYSES OF THREE ELBOW-PIPE ASSEMBLIES FOR TYPICAL LOAD CASES

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--- SUMMARY ---

In order to understand the mechnical properties of some kinds of elbow-pipe assemblies, which will be used for the piping systems of fast breeder reactors, the detailed elastic analyses are performed for typical three loading conditions.

Many analytical works concerning the structural behavior of elbows have been made on the assumption that any planar section taken around the arc of the elbow deforms identically. However, for general elbow-pipe assemblies, this assumption is not valid at sections where the elbow is attached to other pipes or elbows, and then more accurate but expensive finite element techniques are required to include end-effects. It appears, therefore, that there have been not so many analytical studies including end-effects on the elbow-pipe assemblies such as U-bends.

In this paper, we perform the detailed elastic analyses for a series of elbow-pipe assemblies under the basic loading conditions including end-effects by use of the recently developed special elbow elements of the FINAS (FBR Inelastic Structural Analysis System) finite element analysis system, and then discuss the design margins of the existing simplified methods.

The elbow-pipe models investigated in this paper are as follows,

(1) L-shaped model; Pipe(5D length)-90°Elbow-Pipe(5D length)
(2) U-shaped model-1; Pipe(5D length)-180°Elbow-Pipe(5D length)
(3) U-shaped model-2; Pipe(5D length)-90°Elbow-Pipe(1D length)-90°Elbow -Pipe(5D length)

where D is the diameter of the pipe.

Three load cases are considered for each model; at one end of this assembly, (1) in-plane lateral load, (2) axial load (3) out-of-plane lateral load, and the other end is fixed.

The circumferential and longitudinal stress distributions are obtained in the normalized form for nine cases of all elbow-pipe models and loading conditions. Analysis results show clearly the inclination of stresses decreasing along the longitudinal direction of the pipe near the elbow-pipe intersections. The observation of these results suggests that, in the case of U-shaped elbow-pipe assembly, connecting adjacent two 90°elbows with a short straight pipe would lead to the significant reduction of the stresses at the center section of the U-shaped assembly.

The calculated stresses are compared with those obtained by the existing simplified methods, such as C_2 index of ASME Code Section III and the detailed stress index from the minimum potential energy method, and thereby the conservatism of these simplified methods are evaluated quantitatively.

Further, the flexibility characteristics of the elbow-pipe assemblies are evaluated in all cases and compared with the corresponding quantities obtained by the flexibility analysis method of ASME Code Section III and the beam theory, respectively.

1. Introduction

The coolant piping systems used in large nuclear power plants such as LMFBR (Liquid Metal Fast Breeder Reactor) are of major importance with regard to considerations of high safety and integrity. The curved pipes or elbows, which have a high degree of flexibility in comparison with that of straight pipes especially for thin-walled large diameter system , are useful to reduce the thermal expansion forces induced on the pressure vessels and supporting structures that connect to the piping systems. Since this flexibility, however, may result in severe strain concentrations of the elbow, the adequate stress and strain distributions should be considered in the piping design. Thus, special finite elements have been developed to analyze the structural behavior of elbows [1-3], and applied to a number of elastic and inelastic problems. Among then, the pipe bend elements proposed by Marcal[1] have already been used in the inelastic analyses of several piping systems[5,6].

Except a few attempts[2-4,7], many analytical works on the elbow behavior have been made on the assumption that any planner section taken across the arc of the elbow deforms identically. However for general elbow-pipe assemblies, this assumption is not valid especially at sections where the elbow is attached to other pipes or elbows, so that expensive but more accurate finite element methods are required to include the elbow-pipe interactions or end effects. For this reason, even in the elastic resion, it appears that there have been not so many analytical studies including end-effects on the elbow-pipe assemblies such as U-bends.

In this paper, we perform the detailed elastic analyses for three typical elbow-pipe assemblies, as can be seen in FBR plants, under basic loading conditions including end-effects.

The recently developed special elbow elements of the FINAS (FBR Inelastic Structural Analysis System) systems[8] which is under development at PNC (Power Reactor and Nuclear Fuel Development Corporation) was used in these analyses. The effects including the interaction of 90° elbows and straight pipes are shown with respect to circumferential and longitudinal stress distributions of the elbow-pipe assemblies. Furthermore, the calculated stress indices and flexibility factors are compared with those obtained by the simplified methods of analysis in ASME Code Section III.

2. Elbow-pipe assembly models

The elbow-pipe assembly models investigated here are shown in Fig. 1. Three typical combinations of 90° elbows (include 180° elbows) and straight elbows are selected as follows:

(1) L-shaped model: Pipe(5D length)-90°elbow-pipe(5D length)
(2) U-shaped model 1: Pipe (5D length)-180°elbow-pipe(5D length)
(3) U-shaped model 2: Pipe (5D length)-90°elbow-pipe(1D length)-90°elbow-pipe-(5D length)

where D is the outer diameter of the pipe. All elbows have outer diameter of D=812.8 mm, bend radius of R=1219.2mm, wall thickness of t=11.1mm, and consequently pipe factor of λ=0.0842. With respect to the loading conditions, the following three directional load cases are considered at one end of each model as shown in Fig. 1:

(A) in-plane lateral load: P_1
(B) axial load: P_2
(C) out-of-plane lateral load: P_3

The other end of the assembly is fixed throughout this study, and the temperature is maintained 530°C. The material of the assembly is 316 stainless steel which has Young's modulus of $E=1.59\times10^4$ kg/mm^2 and Poisson's ratio of 0.3044.

3. The method of detailed elastic analysis

Since specific attention was devoted to the effects of including the elbow-pipe interactions, the special elbow element ELBOW 6 of the FINAS system[3] was used in the present investigations. ELBOW 6 is a doubly curved quadrilateral thin shell element developed by the authors especially for the detailed analysis of elbow-pipe assemblies. ELBOW 6 is summarized in Fig. 2, leaving details of theoretical basis in [3]. The elbow-pipe assembly is divided into finite length curved or straight rings, and each ring is modeled by ELBOW 6 elements around the circumferential direction as shown in Fig. 3 for U-shaped models.

4. Analysis Results

The stress distributions in the circumference of the pipe were obtained at several cross sections for all elbow-pipe assemblies mentioned above. Among them, Fig. 4 was cited as a typical example to show the complex stress distributions at the elbow-pipe intersection of the U-shaped model 2 subjected to out-of-plane lateral load. Fig. 5 shows how the circumferential stress distribution σ_ϕ changes at the cross sections D, E, and F (see Fig. 1) for the same model.

The maximum circumferential stresses at every cross section of the pipe were plotted along the longitudinal direction in Figs.6 and 7 for U-shaped model 1 and model 2, respectively. It is noteworthy that, in the case of U-shaped model 2, the significant reduction of the maximum stress can be recognized in the 1D length-pipe connecting two 90°elbows of the assembly.

Now concider the maximum stress intensity distribution along the pipe for three elbow-pipe assemblies subjected to in-plane and out-of-plane lateral loads, respectively. Figs. 8-13 show the maximum normalized stress intensity distributions calculated by FINAS, comparing with those obtained by C_2 index of ASME Code Section III and detailed stress index. In these figures, the maximum resultant moment at either end of the elbows was adjusted to the unity as shown in Table 1, and the stress intensities at every cross section were determined in the following manner.

(1) C_2 index of ASME Code Section III [9]

NB-3685 of ASME Code Section III gives C_2 index for the resultant moment; i.e. C_2=1.95/ $\lambda^{\frac{2}{3}}$ for elbows and C_2=1 for straight pipes, where λ is a pipe factor, and C_2 = 10.15 for the elbows investigated herein. C_2 index yields constant stress intensity along the pipe length as shown in Figs. 8-13.

(2) Maximum stress intensity by the detailed stress indices

On the other hand, the stress intensities using the detailed stress indices were determined by stress components calculated as follows;

$$\begin{Bmatrix} \sigma_\phi \\ \sigma_\theta \\ \tau_{\phi\theta} \end{Bmatrix} = Z \begin{bmatrix} (\sigma_\phi)^*_{in} & (\sigma_\phi)^*_{out} & 0 \\ (\sigma_\theta)^*_{in} & (\sigma_\theta)^*_{out} & 0 \\ 0 & 0 & (\tau_{\phi\theta})^*_{tor} \end{bmatrix} \begin{Bmatrix} M_{in} \\ M_{out} \\ M_{tor}/2 \end{Bmatrix}$$

where $(\sigma_\phi)^*_{in}$, $(\sigma_\theta)^*_{out}$,-----, $(\tau_{\phi\theta})^*_{tor}$ are the detailed stress indices obtained by the minimum potential energy method[10], for in-plane bending moment M_{in} , out-of-plane bending moment M_{out} ,and tortional moment M_{tor} , respectively, and Z is the section modulus of cross section. At every cross section, the maximum stress intensity was chosen among the stress intensities evaluated at 75 points in the circumference of the pipe.

(3) Maximum stress intensities by FINAS

Furthermore, the stress intensities by FINAS were determined at 48 points in the circumference of the pipe, and the maximum one was determined at every cross section. From Figs.8-13, it can be seen that the ratio of maximum stress intensity obtained by FINAS to C_2 index is about 80-90% and 60% for in-plane and out-of-plane bending, respectively, as pointed out by Ohtsubo[4]. And then, it should be noticed that the detailed stress indeces might sometimes underestimate the stress intensity even in the elbow parts, especially where the torsional moment is dominant, for example, at point A in Fig. 9, point C in Fig. 11, points B and E in Fig. 13. This facts is due to that the stress index for tortional moment is taken as $(\tau_{\phi\theta})^*_{tor}$ =1.

The comparison of Figs. 10 and 12 suggests that, for in-plane bending, the insertion of a short(1D length) straight pipe in the U-shaped elbow-pipe assembly would lead to the significant reduction(50%) of the maximum stress around the center section of the assembly. On the other hand, as can be seen in Fig. 11 and Table 1 for the U-shaped models subjected to out-of-plane lateral load, the stress intensity takes relatively lower value even at the center section where the resultant moment is maximum, because the tortional moment is dominant compared with the out-of-plane bending moment.

Table 3 shows the flexibility characteristics calculated by FINAS for a number of segments of the elbow-pipe assemblies, compared with those obtained by the simplified flexibility analysis method of ASME Code Section III. In this paper, the flexibility characteristic was defined on the component basis as the ratio of the relative rotation of the both ends of the segment calculated by the above two methods to those obtained by beam theory. The flexibilities evaluated by the detailed analysis(FINAS) were lower than those obtained by the simplified analysis method in almost all segments of the cases treated here.

5. Conclusion

(1) The detailed elastic analyses of a number of elbow-pipe assemblies were performed for typical three loading conditions in order to understand the mechanical properties of them. Analysis results showed clearly the inclination of stresses decreasing along the longitudinal direction of the pipe near the elbow-pipe intersections.
(2) The observation of the results obtained here suggests that, in the case of U-shaped elbow-pipe assembly, connecting adjacent two 90°elbows with a short straight pipe would lead to the significant reduction of the maximum stress at the center section of the assembly, particularly for in-plane bending.
(3) The calculated maximum stress intensities by the detailed analysis were about 10-20 % and 50% lower than C_2 index of ASME Code Section III for in-plane bending and out-of-plane bending, respectively. This fact implies the conservatism of the simplified stress factor method using C_2 index.
(4) The flexibilities obtained by the detailed analysis for the elbow-pipe assmblies were generally lower than those obtained by the flexibility analysis of ASME Code Section III.

A new economical refined elbow element using the reduced integration technique will be developed elsewhere[11].

6. Acknowledgement

The authors are grateful to advices apposite to the evaluation of the analysis results by K. Okabayashi and T. Yuhara of FBR Project, Power Reactor and Nuclear Fuel Development Corporation.

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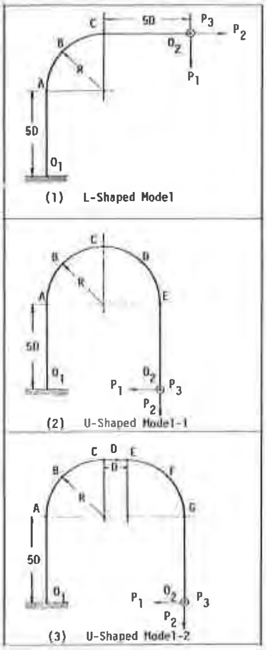
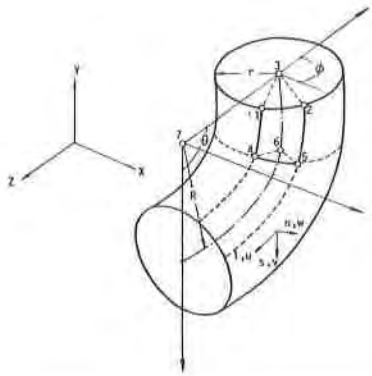
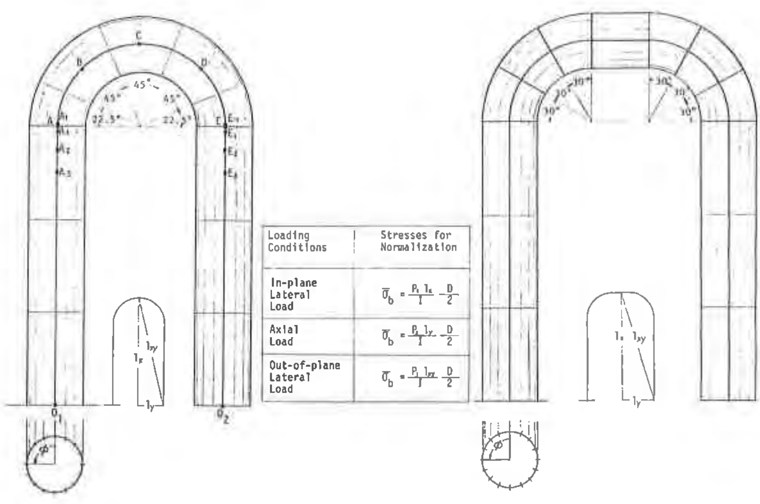


Fig. 1 Elbow-Pipe Assembly Models



Configuration	doubly curved quadrilateral thin shell element
Displacement Function	Longitudinal Displacement u: Fourier Series Circumferential Displacement v: Normal Displacement w: 2-dimensional 3rd-order Hermite function
Number of Nodes	6
Co-ordinates	Rectangular,3-dimension: X,Y, and Z
Degree of Freedom	Elbow Node : 26 $u_1, u_2, -----, u_m; \partial u_1/\partial l, \partial u_2/\partial l, ----- \partial u_m/\partial l$ Surface Node : 8 $v, \partial v/\partial l, \partial v/\partial s, \partial^2 v/\partial l \partial s$, $w, \partial w/\partial l, \partial w/\partial s, \partial^2 w/\partial l \partial s$. Total: 84
Stress Components	$\sigma_\phi, \sigma_\theta, \tau_{\phi\theta}$
Strain Components	$\varepsilon_\phi, \varepsilon_\theta, \gamma_{\phi\theta}$

Fig. 2 Summary of ELBOW 6 Element



Loading Conditions	Stresses for Normalization
In-plane Lateral Load	$\bar{\sigma}_b = \frac{P_1 l_x}{I} \cdot \frac{D}{2}$
Axial Load	$\bar{\sigma}_b = \frac{P_2 l_y}{I} \cdot \frac{D}{2}$
Out-of-plane Lateral Load	$\bar{\sigma}_b = \frac{P_3 l_{xy}}{I} \cdot \frac{D}{2}$

Fig. 3 Finite Element Idealization for U-shaped Models

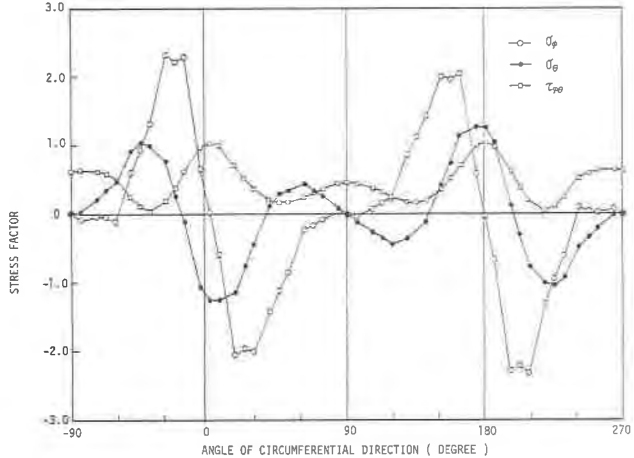


Fig. 4
Stress Distributions at the ELBOW-Pipe Intersection (E) of U-shaped Model 2 Subjected to Out-of-plane Bending

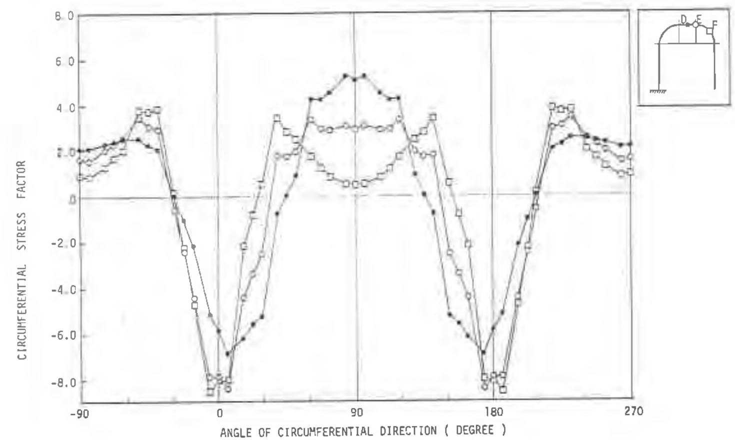


Fig. 5
Circumferential Stress Distributions at D, E, and F Sections of U-shaped Model 2 Subjected to Out-of-plane Bending

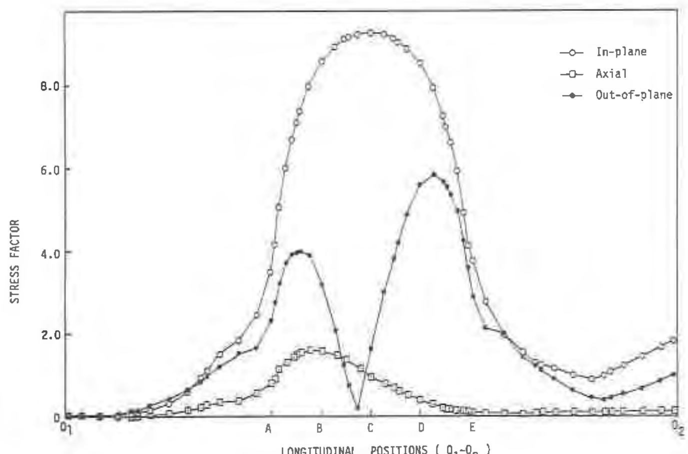


Fig. 6
Maximum Circumferential Stress Distributions along the Pipe for U-shaped Model 1

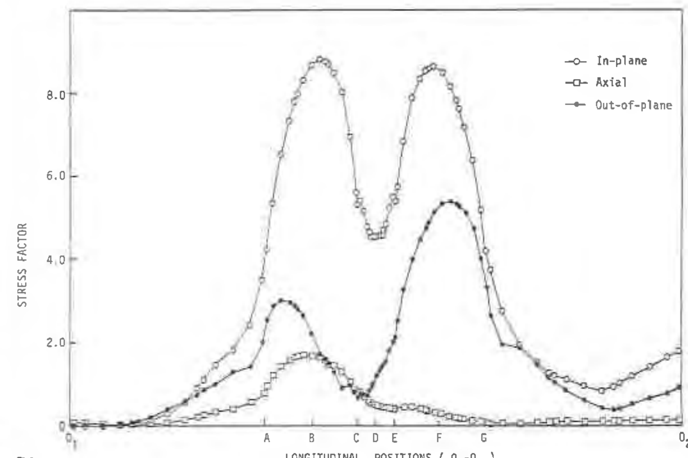


Fig. 7
Maximum Circumferential Stress Distributions along the Pipe for U-shaped Model 2

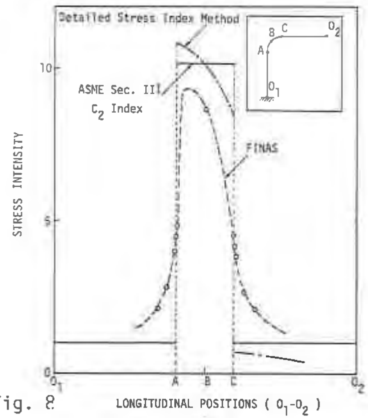


Fig. 8

Maximum Stress Intensity along the Pipe of L-shaped Model for the In-plane Lateral Load

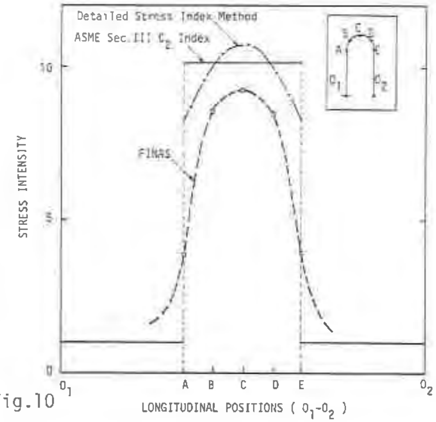


Fig.10

Maximum Stress Intensity along the Pipe of U-shaped Model 1 for the In-plane Lateral Load

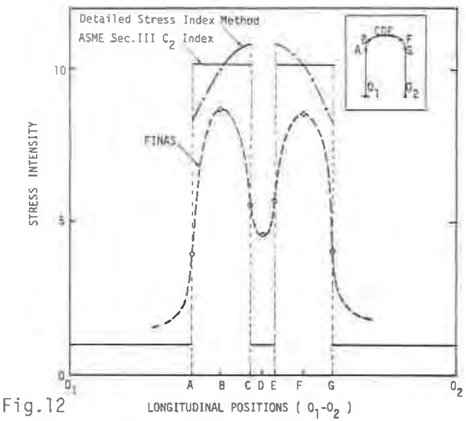


Fig.12

Maximum Stress Intensity along the Pipe of U-shaped Model 2 for the In-plane Lateral Load

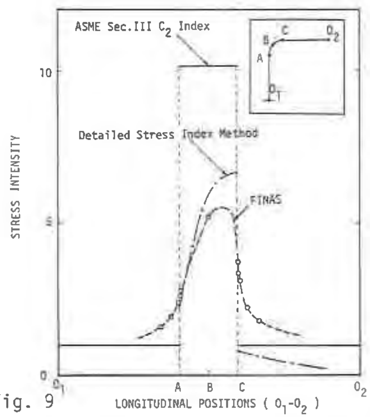


Fig. 9

Maximum Stress Intensity along the Pipe of L-shaped Model for the Out-of-plane Lateral Load

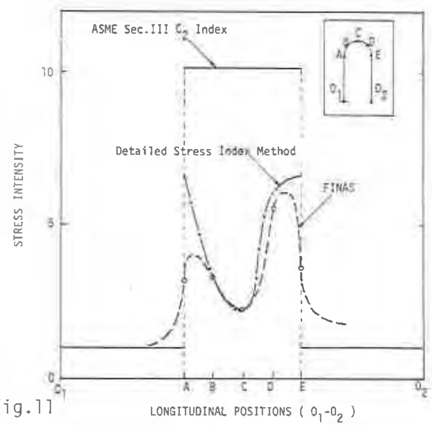


Fig.11

Maximum Stress Intensity along the Pipe of U-shaped Model 1 for the Out-of-plane Lateral Load

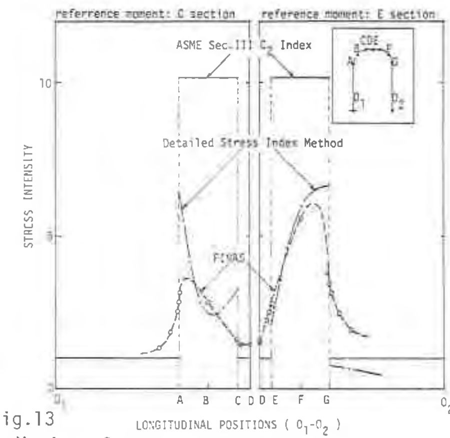


Fig.13

Maximum Stress Intensity along the Pipe of U-shaped Model 2 for the Out-of-plane Lateral Load

Table 1 Maximum Stress Intensity along the Pipe of U-shaped Model 2 for the Out-of-plane Lateral Load

Evaluation Point (see Fig.3a)	Moment In-plane	Moment Axial	Moment Out-of-plane	Moment Resultant	Section III C_2 Index	FINAS	Detailed Stress Index Method
A_{-3}	——	0.62	0.45	0.77	1.00	1.52	0.77
A_{-2}	——	0.69	0.45	0.83	1.00	2.10	0.83
A_{-1}	——	0.74	0.45	0.86	1.00	2.92	0.86
A	——	0.75	0.45	0.87	10.15	3.22	6.64
A_{+1}	——	0.75	0.46	0.88	10.15	3.56	——
B	——	0.37	0.91	0.99	10.15	3.33	3.43
C	——	0.22	0.97	1.00	10.15	2.25	2.22
C	——	0.22	0.97	1.00	10.15	2.25	2.22
D	——	0.69	0.60	0.91	10.15	5.60	6.13
E_{-1}	——	0.75	0.01	0.75	10.15	3.93	——
E	——	0.75	0.00	0.75	10.15	3.63	6.66
[illegible]	——	0.74	0.00	0.74	1.00	3.32	0.74
E_2	——	0.69	0.00	0.69	1.00	2.42	0.69
E_1	——	0.62	0.00	0.62	1.00	1.98	0.62

Table 2 Comparison of Flexibility Coefficients

Model	Section	In-plane K_F	In-plane K_S	Axial K_F	Axial K_S	Out-of-plane (Bending) K_F	Out-of-plane (Bending) K_S	Out-of-plane (Torsion) K_F	Out-of-plane (Torsion) K_S
L-Shaped	O_1 - A	0.98	1.00	0.98	1.00	0.98	1.00	0.96	1.00
	A^1 - B	16.94	19.59	13.26	19.58	2.96	3.10	2.34	2.09
	A - C	17.65	19.59	17.72	19.55	3.88	4.38	2.52	2.64
	B - C	18.46	19.59	15.42	19.42	7.65	9.61	2.57	2.91
	C - O_2	0.99	1.00	----	----	1.01	1.00	----	----
	O_1 - O_2	4.95	5.42	2.00	2.12	2.56	2.84	5.85	6.13
U-Shaped 1	O_1 - A	1.01	1.00	0.99	1.00	0.99	1.00	0.95	1.00
	A^1 - B	18.17	19.59	16.08	19.59	2.96	3.83	1.29	4.05
	A - C	18.36	19.59	17.04	19.59	0.41	0.08	7.50	10.05
	B - C	18.52	19.59	18.37	19.58	2.15	2.71	17.61	27.26
	E - O_2	0.99	0.99	----	----	1.00	1.00	----	----
	O_1 - O_2	10.21	10.83	6.54	6.95	4.31	4.78	3.17	3.37
U-Shaped 2	O_1 - A	0.99	1.00	0.99	1.00	1.00	1.00	0.95	1.00
	A^1 - B	17.91	19.59	16.65	19.60	1.19	2.21	3.03	4.08
	A - C	18.20	19.59	17.91	19.59	0.46	0.70	18.07	22.38
	B - C	18.45	19.59	19.51	19.58	1.72	1.65	4.09	4.59
	C - E	0.97	1.00	1.03	1.00	0.99	1.00	0.99	1.00
	G - O_2	0.98	0.99	----	----	1.00	1.00	----	----
	O_1 - O_2	9.11	9.75	6.13	6.58	3.62	4.07	3.01	3.25

Note: Flexibility coefficients for A-B section of the pipe are defined as

$$K_F = (\theta^F)_{AB} / (\theta^*)_{AB} , \quad K_s = (\theta^S)_{AB} / (\theta^*)_{AB}$$

where $(\theta^F)_{AB}$, $(\theta^S)_{AB}$, $(\theta^*)_{AB}$ are relative rotation of A-B section obtained by the detailed analysis method (FINAS), the flexibility analysis method (Sec.III, ASME), and the beam theory, respectively.