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AIRPLANE CRASH MODELLING: ASSESSMENT OF THE RIERA MODEL

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ABSTRACT

The Riera approach is the most common method of defining a loading curve of a missile impacting a structure. After a brief reminder of the Riera assumptions (soft crash for crushed parts, instantaneous deceleration of uncrushed part due to local crushing strength of crushing part), some physical insights are developed. According to this model, the loading curve F(x, t), where the section S(x) at distance x (x=x (t)) from missile nose is under crushing, is given by the sum of a term Re(x), corresponding to the mechanical crushing strength of the section S(x), and of a term $\mu(x)*v(t)^2$, corresponding to the inertial force exerted at section S(x) by mass elements of density $\mu(x)$, whose velocity decreases from v to zero. Some analytical considerations are proposed regarding the missile motion when impacting a rigid target and its energy dissipation. Results from models of missile and target structures performed with the fast dynamic Finite Element code RADIOSS are used to assess the robustness, accuracy and limits of Riera model. Significant discrepancies between these approaches are found for commercial aircraft. Models by fast dynamic explicit code display loading curves characterised by higher and sharper peaks, whereas the velocity decrease of the uncrushed part is delayed. Riera model assumptions are therefore questioned and analysed. Those discrepancies are mainly due to higher distribution peaks of crushing strength and mass density and due to some hard inclusions in aircraft structure, like the engines and the junction of wing to fuselage. Soft impact is no longer relevant for such parts, whose action is then characterised by high peaks of short duration. When rounding the loading curve and spreading the impact area, the shear rupture is underestimated. Regarding concrete damaging, the Riera approach appears too optimistic. In order to keep the simplicity of the Riera approach and to improve its accuracy, the authors propose to evaluate the loading curve by the Riera approach during fuselage and wing crushing and by SDOF elastoplastic models for engines during their impact. To evaluate damage to the structure correctly, the loading force is to be distributed either locally (turbine shaft of engine) or along lines (periphery of fuselage, line between engines).

Keywords: airplane crash, Riera model, loading curve, RADIOSS, hard impact, concrete structure

1. REMINDER OF THE RIERA MODEL AND PHYSICAL INSIGHTS

1.1 Basic assumptions of the Riera model

In his fundamental paper [ref. 1], Prof. Riera evaluates the impact of an aircraft that frontally impinges a rigid target by using the following hypotheses:

- The aircraft is modelled by a stick with mass distribution $\mu(x)$ and crushing strength distribution Re(x), x being the distance along the fuselage from aircraft nose up to the section Σ that undergoes the crushing.
- The crushing strength Re induces instantaneous and homogeneous deceleration dv/dt in the remaining uncrushed part of the missile M-m, M being the whole mass and m being the mass of the crushed part : $(M-m).dv/dt = -Re$ (1)
- There is no rebound of crushed part (soft impact).

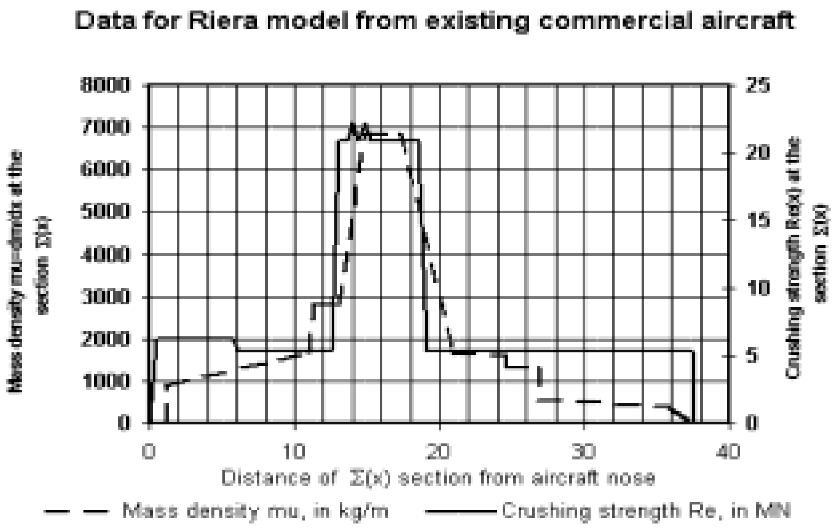

Figure 1 – Typical diagrams for crushing strength Re distribution and mass density μ distribution along aircraft fuselage

1.2 Fundamental Riera formula

Applying the second Newton law to the system (M-m) leads to $d[(M-m).v]/dt = -F$ (2)

where F is the impinging force, v=dx/dt is the velocity of the uncrushed part of the aircraft and d[ ]/dt is the derivation with respect to the time variable t.

Deriving equation (2) induces: $v.d(M-m)/dt + (M-m).dv/dt = -F \Rightarrow -v.dm/dt + (M-m).dv/dt = -F$ (3)

Considering $dm/dt=(dm/dx).(dx/dt) = \mu(x).v$ and substituting the relation (1) in (3), it appears:

$$F = Re + \mu.v^2 \quad (4)$$

This remarkable formula, valid even for deformable targets (v is then the missile relative velocity with respect to the target), means that the impinging force exerted by the missile on the target is the sum of two terms, a structural term and an inertial term. The first term Re is the mechanical resistance brought by the crushing aircraft structure and the term $\mu.v^2$ is the inertial force brought by the mass under crushing, whose velocity drops from v to zero.

The curve $F(t) = Re + \mu.v^2$ is the loading curve corresponding to the action of the aircraft during its impact on the target.

The distributions of the crushing strength Re and mass density μ along fuselage, see for instance Fig. 1, show a peak corresponding to the fuselage/wings liaison. At this junction are both higher fuselage resistance, due to the junction itself, and higher mass density, due to the kerosene tank inside the wings and the fuselage. The diagram of the loading function F(t) therefore displays a peak corresponding to the impact of the fuselage/wing junction. The peak is roughly equal to $(F_{peak}) = Re)_{peak} + (\mu)_{peak}. V_w^2$ and the peak duration is roughly equal to the length of the fuselage/wing junction divided by the velocity V_w.[1]

[1] V_w is the velocity of the uncrushed part of the aircraft at the beginning of wings impact

In addition, according to the second Newton law and the hypothesis of soft impact (no rebound), the whole area under the loading function F(t) is rigorously equal to the momentum $M.V_0$ of the aircraft prior to impact.

It is quite difficult to acquire such distributions from aircraft manufacturer, as this is proprietary information.

1.3 Some mechanical considerations on the Riera model

When the target is deformable, the computation has to take the motion of the structure when impacted into account. The Riera model can easily be programmed on a worksheet, when the structure is assimilated to an elastoplastic spring mass system, even if there is no simple analytical solution. When the target is rigid, some results may be expressed with analytical formulations.

Considering: $$dv/dt = (dv/dx).(dx/dt) = (dv/dx).v \quad (5)$$

And substituting (5) in (1), we obtain: $$(M-m).(dv/dx)v = -Re \Rightarrow v.dv = -[Re/(M-m)].dx \quad (6)$$

Assuming a rigid target, the differential equation (6) may be integrated as follows:

$$V_0^2 - v^2(x) = \int_0^x [2.Re(\xi)/(M-m(\xi))].d\xi \quad (7)$$

where V_0 is the velocity of the aircraft prior to its impact and $m(\xi) = \int_0^\xi \mu(s).ds$ (8)

in particular $m(L) = M$, L being the total length of the aircraft.

The value of the velocity v during the crushing depends only on the initial velocity V_0 and the distribution of crushing strength Re(x) and of mass density $\mu(x)$ along the fuselage.

Let us consider V(x) defined as follows:

$$V^2(x) = \int_0^x [2.Re(\xi)/(M-m(\xi))].d\xi \quad (9)$$

The function $V^2(x)$ is monotonously growing and reaches its maximum value $V^2(L)$.

If $V_0 < V(L)$, there is a distance X_c, such as $V_0 = V(X_c)$, and for this distance X_c, the velocity v is equal to 0: such a distance X_c corresponds to the crushed length at the end of impact. Graphically, the intersection of the curve $y = V^2(x)$ by the line $y = V_0^2$ provides the crushed length (intersection abscissa Xc), and the distribution of the square of the velocity v^2 along the fuselage during the crushing (the difference between $y = V_0^2$ and $y = V^2(x)$).

If $V_0 > V(L)$, there is no intersection; there is, however, the end of crushing by the whole “consumption” of the aircraft.

The next figure shows the variation of $V_0^2 - v^2(x)$ along the fuselage for an aircraft with distribution of crushing strength Re and mass density as per Figure 1. As the initial velocity just before impact is V_0 = 100m/s, the crushed length is about 20m.

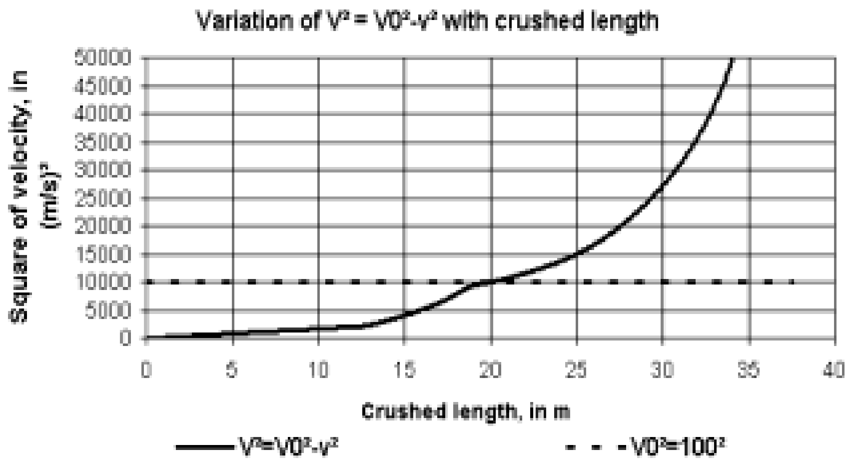

Figure 2 –Diagram $V^2 = V_0^2 - v^2$ vs. crushed length for an aircraft with the characteristics of Figure 1. Geometrical determination of crushed length Xc through intersection of V^2 curve by V_0^2 curve.

From equations (7) and (9):

$$V_0^2 - v^2(x) = V^2(x) \quad (10)$$

The equation of aircraft motion during impact is then given by:

$$t = \int_0^x \{1/ [V_0^2 - V^2(\xi)]^{1/2}\}.d\xi \quad (11)$$

The variation of the force F with time, F(t), is given by the set of equations (4) and (11).

The next figure presents the loading curve F(t) of the aircraft characterized by distributions from Figure 1 impacting a rigid target at 100m/s.

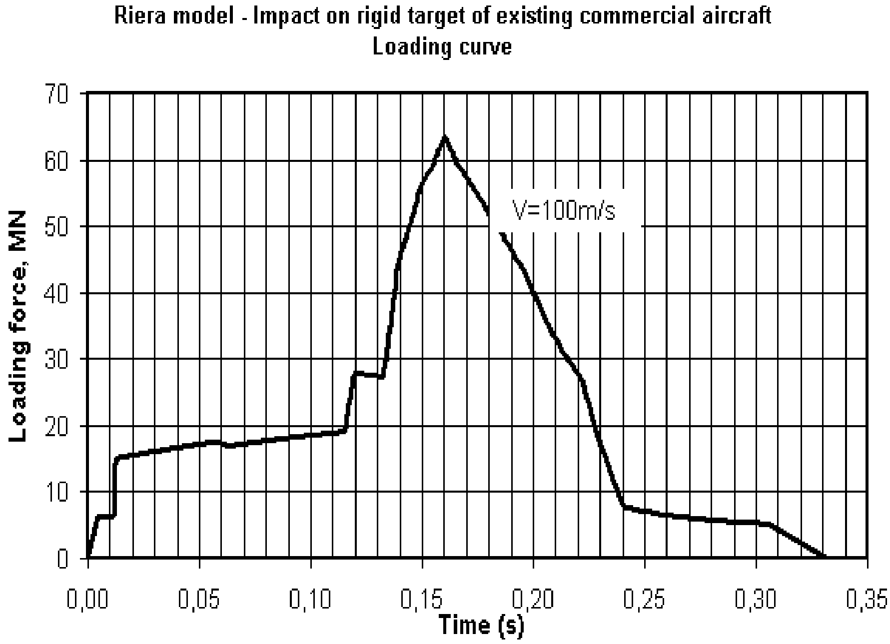

Fig.3 – Loading curve F(t) during the impact with hypotheses from Fig. 1 and initial velocity of 100m/s

It can be demonstrated that the variation in kinetic energy during the impact is given by:

$$½.M.V_0^2 – ½.(M-m).v^2(x) = \int_0^x Re(\xi).d\xi + ½.\int_0^x \mu.v^2(\xi)d\xi \qquad (12)$$

The energy consumption during the impact is shared between the work of the crushing strength Re on the crushed length and the kinetic energy of the successive mass elements $dm = \mu * d\xi$, whose velocity decreases from v to zero during their soft impact.

If $V_0 < V(L)$, the stopping length of aircraft X_c is such as v = 0 (or $V_0 = V(X_c)$), then:

$$½.M.V_0^2 = \int_0^{Xc} Re(\xi).d\xi + ½.\int_0^{Xc} \mu.v^2.d\xi \qquad (13)$$

In terms of energy at the end of impact, it can be considered that the initial kinetic energy $E_{c,0} = ½.M.V_0^2$ is shared between an energy of missile deformation $E_{d,p} = \int_0^{Xc} Re(\xi).d\xi$, and a kinetic energy transmitted to the target $E_{c,d}=½\int_0^{Xc} \mu.v^2.d\xi$.

1.4 Application of the Riera model to a cylindrical barrel

Let us consider a cylindrical barrel, characterized by cross section S, length L, mass per volume unit ρ and elastic stress/deformation relationship (Young modulus E) up to the constant plastic limit σ_p. This barrel is assumed to be frontally impacting a rigid target at initial velocity V_0. In this case the distributions Re(x) and $\mu(x)$ are uniform and defined by: $Re= \sigma_p.S$ and $\mu = \rho.S$, then, by (7) and (8):

$$m(x) = \int_0 ^x \mu.ds=\mu.x \quad \text{and} \quad V_0^2 – v^2 = \int_0^x [2.\ \sigma_p.S/(\mu.L-\mu.\xi)].d\xi = -2.\ (\sigma_p.S/\mu).Log(1-x/L)$$

The crushed length X_c at the end of the motion corresponding to v=0, thus $X_c=L\{1-exp[-\mu.V_0^2 / (2\sigma_p.S)]\}$.

The velocity v is given by $v^2 = V_0^2 + 2.\ (\sigma_p.S/\mu).Log(1-x/L)$ for $x \le X_c$.

The time equation of the motion x(t) is given by:

$$dx/dt = v, \text{ thus } t = \int_0^x [V_0^2 + 2.\ (\sigma_p.S/\mu).Log(1-\xi/L)]^{-1/2}.d\xi$$

with some algebraic manipulations, it can be transformed into:

$$V_0.t = \int_0^x \{1 - [\mathrm{Log}(1-\xi/L) / \mathrm{Log}(1- x_c/L)]\}^{-1/2}.d\xi$$

The evaluation of energy repartition by (13) is interesting:

$$E_{d,p} = \int_0^{Xc} \mathrm{Re}(\xi).d\xi = \sigma_p.S.\ X_c = \sigma_p.S.\ L\{1\text{-}\exp[-\mu.V_0^2 / (2\sigma_p.S)]\}$$

Let us introduce $\beta = (2.\sigma_p.S/\mu.V_0^2)$ and $\alpha = E_{d,p} / E_{c,0}$:

$$\alpha = \left(\int_0^{Xc} \mathrm{Re}(\xi).d\xi\right) / \left(½.M.V_0^2\right) = \beta.[1\text{-}\exp(-1/\beta)] \qquad (14)$$

The coefficient β characterizes the ratio of the crushing strength Re on the inertial force $\mu.V_0^2$, whereas the coefficient α characterises the sharing of initial kinetic energy $E_{c,0}$ in the missile deformation energy $E_{d,p} = \alpha.\ E_{c,0}$ and in the kinetic energy $E_{c,d} = (1-\alpha).\ E_{c,0}$ laid on the rigid target.

Considering the Riera classification mentioned in reference [2], a missile with $\beta<0.1$ induces a soft impact, the inertial term is largely greater than the crushing strength one, the energy corresponding to the work of crushing strength is far smaller than the kinetic energy brought on the target by mass deposit. Symmetrically, a missile with $\beta>1$ induces a hard impact: the crushing strength is greater than the inertial force and the initial kinetic energy is dissipated by missile deformation rather than by kinetic energy deposit.

This definition of coefficient β produces the following comments:

- The coefficient β that characterizes the impact is not an intrinsic coefficient, it incorporates the initial velocity prior to impact.
- An intermediate class of impacts should be considered, corresponding to coefficients β between 0.1 (soft impact) and 1 (hard impact).
- The Riera hypothesis is consistent with soft impacts, but not necessarily with hard impacts.

The following figure represents the diagram $\alpha(\beta)$ and the relevant impacts classification.

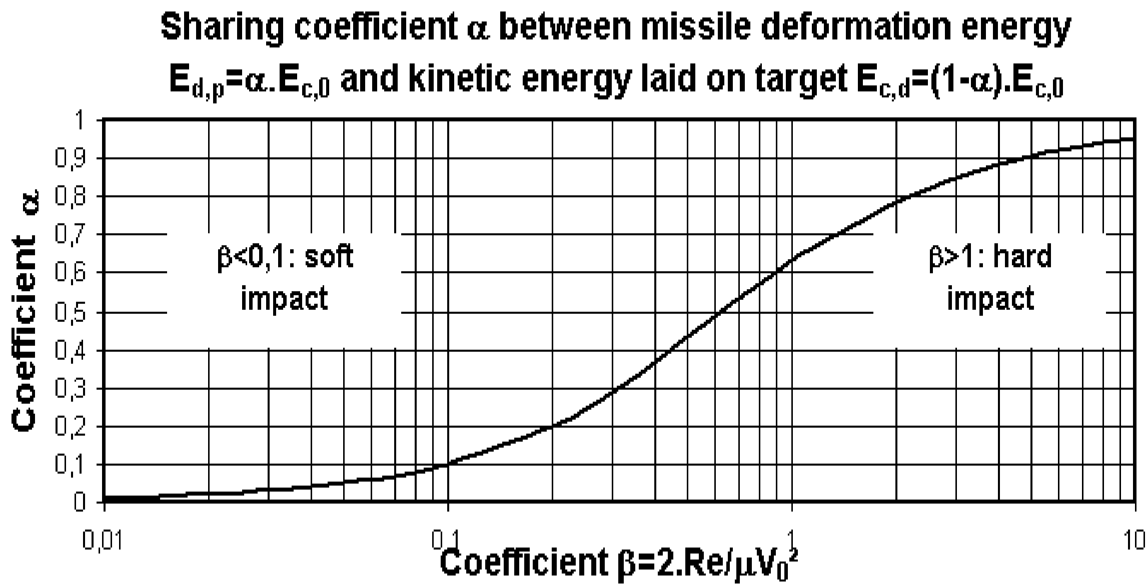

Figure 4 – Sharing of energy dissipation vs. type of impact.

The coefficient β of the above barrel may be generalized to a missile with non-uniform distribution Re(x) and μ(x). In this case, the coefficient β depends on the position x along the fuselage and is defined by:

$$\beta(x) = 2.\mathrm{Re}(x)/(\mu(x).V_0^2) \qquad (15)$$

The equation (7) becomes:

$$V_0^2 - v^2 = \int_0^x [2.\mathrm{Re}(\xi)/(M-m(\xi))].d\xi = \int_0^x [\beta(\xi).\mu(x).V_0^2/(M-m(\xi))].d\xi$$

$$= V_0^2 . \int_0^x [\beta(\xi)/(M-m(\xi))].\mu(x).d\xi = V_0^2 . \int_0^{m(x)} [\beta(\xi)/(M-m)].dm$$

with $m(\xi) = \int_0^{\xi} \mu(s).ds$ and $dm = \mu(\xi).d\xi$

An aircraft whose coefficient β remains constant along the fuselage gives the velocity variation v through:

$$V_0^2 - v^2 = V_0^2 . \int_0^{m(x)} [\beta/(M-m)].dm = - \beta.V_0^2 .Log (1 - m(x)/M)$$

The necessary mass to be crushed to stop the aircraft (v=0) is then given by:

$m(x=Xc) = m_c = M.[1-\exp(-1/\beta)]$

If β=0.1, m_c = M.[1-exp(-1/β)]=0,999955M: the aircraft is almost completely crushed.

If β=1, m_c = M.[1-exp(-1/β)]≈0,63M: 1/3 of whole mass is still uncrushed.

For intermediate values of β, the crushed mass varies between M and 2/3M.

If β=10, m_c = M.[1-exp(-1/β)]≈0,095M: only 9.5% of the aircraft mass is crushed.

When the coefficient β is not constant but has small variations with m, the above considerations are still valid. Nevertheless, it seems advisable to verify that values of β remain less than 1, to remain in line with the soft impact hypothesis (no rebound).

1.5 For a simple model for hard impact

When the soft impact hypothesis is not relevant, another simplified model should be considered; this is the single-degree-of-freedom elastoplastic model characterized by mass M uniformly distributed along length L, rigidity K and crushing strength Re. Rigidity K allows a linear elastic behaviour for the missile if the initial velocity V_0 is less than the rebound velocity after crushing $V_r=Re/(M.K)^{1/2}$; for a higher initial velocity, plastic deformation does appear, $\varepsilon_p=(1/\beta).(1-V_r^2/V_0^2)$, with $\beta=2.Re/(\mu.V_0^2)$ and μ=M/L, (β and μ having the same definition as proposed by Prof. Riera). The peak value of the loading function is Re when $V_0 \geq V_r$, and $Re.(V_0/V_r)$ when $V_0 \leq V_r$, the peak duration Δt is equal to $\pi.M.V_r/Re$ when $V_0 \leq V_r$, and roughly equal to $M.V_0/Re = (1/\beta).(2.L/V_0)$ when $V_0 > V_r$.

In such a model, the rebound velocity V_r after crushing is an intrinsic parameter that is not dependent upon the initial velocity V_0; it can be used to characterize the elastic behaviour. Such a model is convenient for engine impact and can be easily programmed on a worksheet.

2. ASSESSMENT OF THE RIERA MODEL

2.1 Modelling aircraft structure impact on concrete structure by fast dynamic FE code

In order to assess the robustness of Nuclear Power Plant structures with respect to commercial aircraft impact, IRSN has produced several models of impact by existing commercial aircrafts using the fast dynamic finite element (FE) code RADIOSS. The aircrafts are modelled with several thousand elements (trusses characterized by a linear mass density, a section and as behaviour an elastic-perfectly plastic law) and the targets are modelled, when deformable, with several hundred of thousand volumic elements for reinforced or pre-stressed concrete, whose behaviour is characterized by the Ottosen-type law with smeared cracks. The resulting force at contact, the loading force, is compared with the force obtained by the Riera approach, whose Re and μ distributions are obtained from FE modelling of aircrafts.

2.2 Loading curve: Riera rounded curve vs. FE sharp curve

The loading curve F(t) obtained with the FE code, see Figure 5, differs from the loading curve from the Riera approach; in particular, F(t) from Riera presents a rounded peak whereas the peak of FE F(t) is sharp. After some investigations it appears that the sharper peak of F(t) curve by FE code is due to the following reasons:

- the kerosene, housed in the first part of the wings, acts as a concentrated mass,
- the fuselage/wing junction is on a short length and the angle formed by the wings is very open,
- the engine impacts and wing junction impacts are simultaneous and additive,
- the engine impact is a hard impact, with rebound, marked by a high peak with short duration.

The impact duration is shortened with respect to the Riera model as a consequence of the sharpest peak of F(t) curve by RADIOSS.

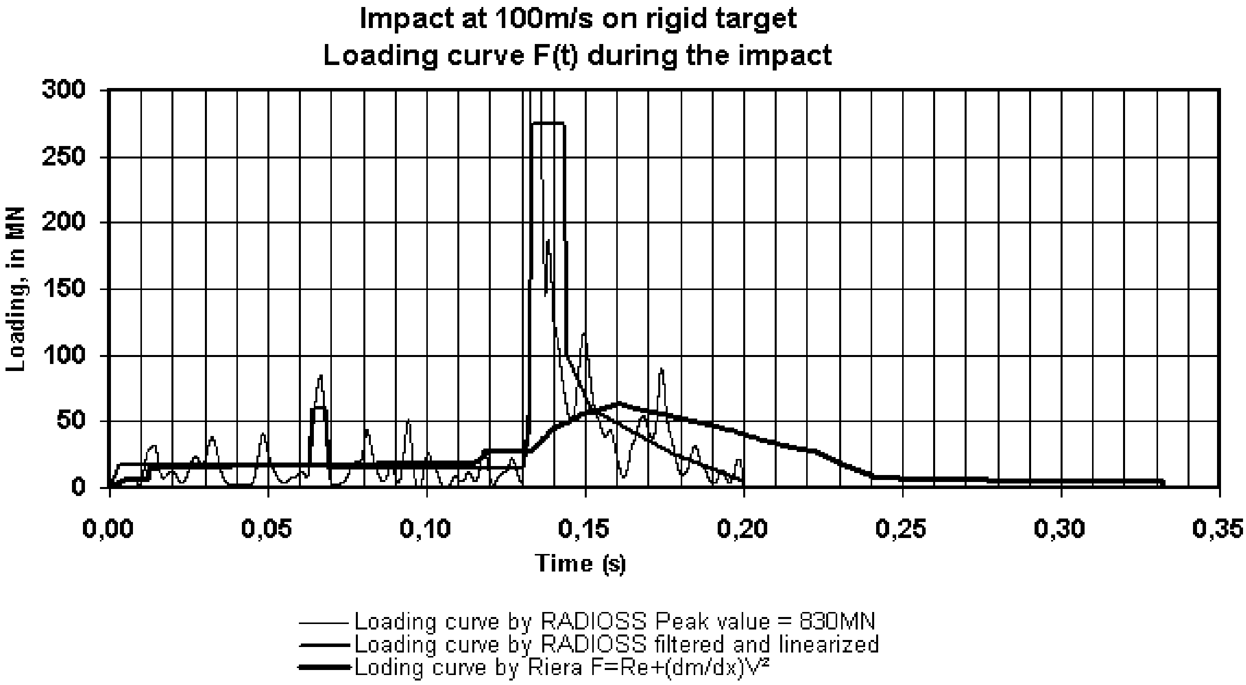

Figure 5 – Comparison of loading curves by FE model, by FE model after filtering and by the Riera approach

The huge value of F(t) peak is due to the simultaneous impacts of the wing junction and the 2 engines; the peak is accentuated by the rebound. As the peak duration is very short and corresponds to a high frequency solicitation, the loading curve has been filtered beyond 50hz (peak duration becomes 10ms), with conservation of the area under the curve (i.e. the momentum). The FE model peak, even after filtering the effects beyond 50hz, may reach about four times the Riera model peak.

2.3 Impacted area: simplified areas vs. front surface of aircraft

The shape and dimensions of the impacted area on target are sensitive topics: the greater the area, the lower the pressure exerted by the impact loading. The area dimensions vary over time. The area impacted is limited to the front surface of the aircraft: increasing the impact area minimizes the shear force and the bending moment in the impacted structure. The results of studies performed by the authors show that aircraft action during the impact varies over time (the resultant F(t) is not constant) and space (the action is not uniform on the whole area). In particular, the action is concentrated along the most rigid parts of aircraft which are located on the periphery of the fuselage, in front of the engines, of course, and along the section of the wings between their junction with the fuselage up to the engine(s) liaison. Due to the dimensions of such rigid parts with respect to the wall dimensions of the impacted structure, the loading may be considered as linearly distributed (periphery of fuselage and wings) and locally exerted (engine).

2.4 Velocity distribution: instantaneous deceleration vs. shock wave transmission

The Riera model considers the deceleration of the uncrushed part of aircraft to be instantaneous and uniform. It is an approximation: the impact causes shock waves that carry the impact information to the rest of the aircraft. Such information travels inside the aircraft at a celerity roughly equal to $(E/\rho)^{1/2}$ for compression waves, where E is the longitudinal deformation modulus and ρ the mass density. The celerity of such waves may be strongly reduced, when the modulus E is weakened by the crushing during impact: it is likely that the wave celerity (i.e. sound celerity) is lower than the aircraft velocity. In this case, the uncrushed part of aircraft is not informed of the crushing event and is not subjected to any deceleration. Such a phenomenon tends to accentuate the peak due to the inertial force $\mu.v^2$ with a higher value of v. The results from the FE models clearly show that deceleration is delayed and that the celerity distribution is not homogeneous.

The hypothesis of soft impact in the Riera approach induces a perfectly plastic behavior for the aircraft during impact: the result is instantaneity of impact transmission within the aircraft. Any model taking such shock waves into consideration will introduce elastic behavior before plastic deformation.

2.5 Damaged concrete structure: shear rupture is underestimated

When comparing the damaging of the same reinforced concrete structure by an FE aircraft model impact and by a set of forces replacing aircraft action with the same loading curve, it appears that the latter underestimates the deformation of a concrete structure when the resulting loading force is uniformly distributed over a too large area. When analyzing the resistance of a concrete structure subjected to an impact from a commercial aircraft, this approximation tends to underestimate heavily the damaging of the structure by shearing force. This is due to the distribution of mass, strength and rigidity within the structure of such an aircraft.

2.6 Conclusions: over-optimistic results from the Riera model

The Riera model permits an evaluation of the impact force exerted by an aircraft on structures which is simple and interestingly realistic, but the necessary hypotheses for such simplicity underestimate the damaging effect of impact on the structure. In particular, the hypothesis of soft impact is convenient for fuselage crushing, but less relevant for hard inclusions like the engines and the wing section between the engines. The distribution of the loading over time is, therefore, more concentrated (with sharpest peak) and the distribution inside the impacted area is not uniform and subsequently also presents concentrations, namely in front of the hardest parts (periphery of fuselage, engine shaft and wing caissons). By underestimating the damage to the impacted structure by shear force, the Riera approach leads to an optimistic assessment of the resistance of concrete structures to commercial aircraft impacts.

3 SOME IDEAS FOR RECONCILING THE SIMPLICITY OF THE RIERA APPROACH AND THE COMPLEXITY OF AIRCRAFT STRUCTURE IN RESISTANCE ASSESSMENT OF BUILDINGS

From a mechanical point of view, a commercial aircraft is a very rigid and heavy V-shaped beam (wings) supporting a softer and less dense tube (fuselage). Wing rigidity is necessitated by engine and fuselage supporting and wing weight comes mainly from the incorporated kerosene tanks.

For a simple and accurate evaluation of the loading force of an aircraft impacting a structure, the idea is to keep the Riera approach for fuselage and wing crushing and to consider the hard impact of the engines separately. For the Riera model, crushing strength and mass density distributions are to be thoroughly established, especially at fuselage/wing junction. For the engines a classical elastoplastic spring mass system is sufficiently accurate to predict the impact force, which should be added to the loading force peak by Riera.

Concerning the damage to the impacted structure, effect of aircraft structure is to be considered: attention should be paid to the dimensions of the impacted areas, which are slenderer than generally assumed, and to the non uniform distribution of the loading force within those areas. For a simple and accurate evaluation of damage to impacted structure, the loading force should be distributed:

- before the peak and during the impact of the fuselage front section, around the fuselage periphery section,
- during the peak, while the wing/fuselage junctions hit the structure, along the line between the engines,
- during the peak, in addition, the impact forces of engines should be locally exerted.

REFERENCES

[1] Riera J.D., On the stress analysis of structures subjected to aircraft impact forces, *Nucl. Eng. & Des*. 8, 1968, 415 – 426.

[2] Riera J.D., Basic concepts and load characteristics in impact problems, *Introductory Report, RILEM-CEB-IABSE-IASS Interassociation Symposium on Concrete Structures under Impact and Impulsive Loading*, Berlin, June 1982, 7-29

